Commun. Theor. Phys. **71** (2019) 421–426 Vol. 71, No. 4, April 1, 2019

Holographic Dark Energy Model is Consistent with Pantheon SN Ia Data*

Xin Li (李昕),† Su-Ping Zhao (赵素平),‡ Li Tang (唐丽),§ and Hai-Nan Lin (林海南)¶

Department of Physics, Chongqing University, Chongqing 401331, China

(Received October 10, 2018; revised manuscript received November 26, 2018)

Abstract *We constrain three cosmological models, i.e.,* Λ*CDM model, holographic dark energy (HDE) model and* $R_h = ct$ *model by using the recent Pantheon compilation of type Ia supernovae (SN Ia), the direction measurements of Hubble parameter* $H(z)$*, and the baryon acoustic oscillations (BAO). The spatial curvature is considered in the* Λ*CDM model and the HDE model. We show that the HDE model in a spatially flat and HDE dominate universe has the same behavior as* $R_h = ct$ *model if the characteristic parameter of the HDE model* C_0 *approaches to infinity. Numerical results show that the* Λ*CDM model is the best favoured one among the three models. The HDE model is consistent with observational data, the best fitting value of* C_0 *is around* 0.8*, which implies that the* $R_h = ct$ *model should be modified to be compatible with the present cosmological observational data. Combing all the datasets, we give strict constraint on the Hubble constant, where* $h_0 = 0.694 \pm 0.020$ *for the* Λ*CDM model and* $h_0 = 0.689 \pm 0.019$ *for the HDE model. Our results imply that the tension of Hubble constant between Planck collaborations and Riess et al. has been partially relaxed. The constraint on the spatial curvature is also given, where* $\Omega_{k0} = -0.066 \pm 0.165$ *for the* Λ*CDM model and* $\Omega_{k0} = 0.029 \pm 0.067$ *for the HDE model.*

DOI: 10.1088/0253-6102/71/4/421

Key words: holographic dark energy, cosmological observations, supernova, general

1 Introduction

The standard cosmological model, i.e. the ΛCDM model is well consistent with various astronomical observations, such as type-Ia supernovae[1–2] (SN Ia), baryon acoustic oscillations (BAO), cosmic microwave background (CMB) radiation from WMAP[3] and Planck satellites.[4] The ΛCDM model states that our universe is mainly constituted of three components, namely, baryonic matter, cold dark matter, and cosmological constant. The cosmological constant is the most popular model of dark energy, which could explain the accelerated expansion of the universe. However, the cosmological constant will face fine tuning problem,[5] if it originates from vacuum energy density. To understand the nature of dark energy and to solve the fine tuning problem, various models have been proposed to discuss the dynamics of the dark energy.[6] Instead of considering dark energy as a cosmological constant, it is interesting to search alternate models of dark energy and test them using the current astronomical observations.

Melia[7] has proposed an alternate model of dark energy, namely, $R_h = ct$ model. Melia's model suggests that the Hubble horizon of the universe is always equal to the light traveling distance throughout the whole history of universe. It was shown that various astrophysical data are well consistent with the $R_h = ct$ model.[8–12] In fact, the $R_h = ct$ model has connections with holographic dark energy (HDE) model.[13] The HDE model is inspired by the holographic principle,[14] and it assumes that the infrared cut-off of vacuum energy density, which is relevant to dark energy is the size of the future event horizon of our universe. In an early work, Zhang and Wu[15] constrained the HDE model using the gold sample of SN Ia combined with CMB and the large-scale structure (LSS), and the results can be fitted well with the observational data. In Refs. [16–17], various dark energy models have been discussed by using the observational data, and the best fitting results of the HDE model can also be fitted well with the observational data. Moreover, in Ref. [18] for recent review, the HDE model is in good agreement with several cosmological observations.

The Planck collaborations have combined the CMB data with various astrophysical data to give tight constraint on models of dark energy and modified gravity.[19] However, Riess *et al.*[20] have measured the Hubble constant by using four geometric distance calibrations of Cepheid, and the measured value of Hubble constant is more than 3σ tension with the global measurement from CMB.[4] In some other works,[21–23] an evidently smaller value of C_0 can be obtained when the Planck CMB data

---

*Supported by the National Natural Science Foundation of China under Grant Nos. 11603005 and 11775038.

†E-mail: lixin1981@cqu.edu.cn

‡E-mail: zhaosp@cqu.edu.cn

§E-mail: tang@cqu.edu.cn

¶E-mail: linhn@cqu.edu.cn

are used for the HDE model. Recently, a large sample of Supernova cosmology data[24] called Pantheon has been released. The redshift of the data is ranging from 0.01 to 2.3. The wider range of redshift could help to find or constrain the behavior of the dynamics of dark energy. It is interesting to test the alternate models of dark energy and to constrain the Hubble constant with the Pantheon SN Ia data.

In this paper, we will test three cosmological models (ΛCDM, HDE, and $R_h = ct$) by using various astronomical probes, which include the Pantheon SN Ia data, $H(z)$ and BAO. The rest of the paper is arranged as follows: In Sec. 2, we briefly review the ΛCDM model, the HDE model and the $R_h = ct$ model, and present the relation between the HDE model and the $R_h = ct$ model. In Sec. 3, we introduce the observational datasets, which are utilized to constrain the cosmological parameters, and present the best-fitting results of the model parameters. Finally, discussion and conclusion are presented in Sec. 4.

2 Theoretical Models

According to the cosmology principle, the universe is homogeneous and isotropic on large-scale, which can be described by the Friedmann-Robertson-Walker metric as

$$\mathrm{d}s^2 = c^2\mathrm{d}t^2 - a^2(t)\Big(\frac{\mathrm{d}r^2}{1-kr^2} + r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2)\Big), \quad (1)$$

where $a(t)$ is the cosmic scale factor, c is the speed of light, $k = \pm 1, 0$ denotes the spatial curvature of universe and corresponds to a closed, open and flat universe, respectively.

Given the equation of state of each component of the universe, one can find the solutions of Einstein equations in FRW spacetime. In the simplest ΛCDM model, we only consider our universe being composed of matter (including baryonic matter and cold dark matter) and dark energy. The dark energy in ΛCDM model is represented by the cosmological constant. Taking the spatial curvature into consideration, we can find the equation for the Hubble parameter $H \equiv \dot{a}/a$

$$H(z) = H_0\sqrt{\Omega_{m0}(1+z)^3 + \Omega_{k0}(1+z)^2 + \Omega_{\Lambda 0}}\,, \quad (2)$$

where we have used the relation $a(1+z) = 1$ to derive the above equation, H_0 is the Hubble constant, $\Omega_{i0} \equiv \rho_{i0}/\rho_{\rm cr}$ $(i = m, \Lambda)$ is the normalized energy density today, $\rho_{\rm cr} = 3c^2H_0^2/8\pi G$ is the critical density of the universe, and $\Omega_{k0} = -kc^2/H_0^2$ is the spatial curvature. The total content of the universe is normalized to unity, i.e. $\Omega_{m0} + \Omega_{k0} + \Omega_{\Lambda 0} = 1$.

In the FRW spacetime, the particle horizon is given as

$$R_p = ac\int_0^t \frac{\mathrm{d}t}{a}\,, \quad (3)$$

where c is the speed of light in vacuum. By making use of Eq. (2), one can obtain

$$(1+z)R_p = \frac{c}{H_0}\int_0^z \frac{\mathrm{d}z}{E(z)}\,, \quad (4)$$

where $E(z) \equiv H(z)/H_0$ is the dimensionless Hubble parameter. In the ΛCDM model, the relation between the luminosity distance and redshift is given as[25]

$$d_L(z) = (1+z)\frac{c}{H_0}|\Omega_{k0}|^{-1/2}\mathrm{sinn}\Big(|\Omega_{k0}|^{1/2}\int_0^z \frac{\mathrm{d}z'}{E(z')}\Big)\,, \quad (5)$$

where

$$\mathrm{sinn}(x) = \begin{cases} \sin(x), & (\Omega_{k0} < 0), \\ x, & (\Omega_{k0} = 0), \\ \sinh(x), & (\Omega_{k0} > 0). \end{cases}$$

The dimensionless distance modulus is defined by

$$\mu_{\rm th}(z) = 5\log_{10}\Big[\frac{d_L(z)}{\rm Mpc}\Big] + 25\,. \quad (6)$$

The $R_h = ct$ model requires that the Hubble horizon of universe is always equal to the light traveling distance throughout the whole history of universe,[10,26–27] where $R_h = c/H$ denotes the Hubble horizon. Then, the relation between the luminosity distance and redshift can be directly derived from the Einstein field equation and the relation $R_h = ct$. It is given as

$$d_L(z) = (1+z)\frac{c}{H_0}\ln(1+z)\,. \quad (7)$$

Accordingly, the dimensionless Hubble parameter in the $R_h = ct$ Universe is given by

$$E(z) = 1 + z\,. \quad (8)$$

Comparing the ΛCDM model with the $R_h = ct$ model, one can find that the $R_h = ct$ model can be equivalently considered as a flat ΛCDM model with equation of state $\omega = p/\rho = -1/3$ for the total contents of universe.

The HDE model suggests that the energy density of dark energy is relevant to the future event horizon of our universe[13,18]

$$R_f = ac\int_t^\infty \frac{\mathrm{d}t}{a}\,. \quad (9)$$

The expression of the energy density of dark energy being relevant to holographic principle is given by

$$\rho_{\rm de} = 3C_0^2M_P^2R_f^{-2}\,, \quad (10)$$

where C_0 is a constant parameter, and M_P is the Planck mass. From Eq. (10), the equation of state of dark energy satisfies

$$\omega_{\rm de} = -\frac{1}{3} - \frac{2\sqrt{\Omega_{\rm de}}}{3C_0}\,, \quad (11)$$

where $\Omega_{\rm de} \equiv \rho_{\rm de}/\rho_{\rm cr} = C_0^2/H^2R_f^2$ is the normalized density of holographic dark energy. While the universe is dominated by holographic dark energy, one can find from Eq. (10) and Friedmann equation that $H^{-1} \propto a^{1-1/C_0}$. If C_0 approaches to infinity, then R_f approaches to C_0a or C_0t and $\omega_{\rm de}$ approaches to $-1/3$. In this limitation, we find that R_f in HDE model has the same behavior as R_h in the $R_h = ct$ model.

The relation between the luminosity distance and redshift in the HDE model takes the same form to Eq. (5), since the background geometry, i.e., the FRW metric is unchanged in the HDE model. However, the $E(z)$ in Eq. (5) is different from the one in ΛCDM model Eq. (2). In a spatially flat universe, namely, $\Omega_k = 0$, $E(z)$ in the HDE model satisfies the following relations[18]

$$E(z) = \left(\frac{\Omega_{m0}(1+z)^3}{1-\Omega_{\rm de}(z)}\right)^{1/2}, \tag{12}$$

$$\frac{{\rm d}\Omega_{\rm de}}{{\rm d}z} = -\frac{2\Omega_{\rm de}(1-\Omega_{\rm de})}{1+z}\left(\frac{\sqrt{\Omega_{\rm de}}}{C_0}+\frac{1}{2}\right). \tag{13}$$

In a spatially non-flat universe, $E(z)$ in the HDE model satisfies the following relations[28]

$$\frac{1}{E(z)}\frac{{\rm d}E(z)}{{\rm d}z} = -\frac{\Omega_{\rm de}}{1+z}\left(\frac{\Omega_k-3}{2\Omega_{\rm de}}+\frac{1}{2}+\sqrt{\frac{\Omega_{\rm de}}{C_0^2}+\Omega_k}\right), \tag{14}$$

$$\frac{{\rm d}\Omega_{\rm de}}{{\rm d}z} = -\frac{2\Omega_{\rm de}(1-\Omega_{\rm de})}{1+z}\left(\sqrt{\frac{\Omega_{\rm de}}{C_0^2}+\Omega_k}+\frac{1}{2}-\frac{\Omega_k}{2(1-\Omega_{\rm de})}\right), \tag{15}$$

where

$$\Omega_k(z) = \frac{\Omega_{k0}(1+z)^2}{E(z)^2}, \tag{16}$$

and the initial condition for solving Eqs. (14), (15) are $E(0)=1$, $\Omega_{\rm de0}=1-\Omega_{m0}-\Omega_{k0}$.

3 Observational Constraints

In this section, we use the available astrophysical data to constrain the above cosmological models: the ΛCDM model, HDE model, and $R_h = ct$ model. For the former two models we consider spatially flat and non-flat cases separately. The non-flat ΛCDM model and HDE model are called *o*CDM model and KHDE model, respectively.

3.1 *Observational Data*

The observational data used here comprise the type Ia supernovae (SN Ia), the direct measurement of Hubble parameter ($H(z)$), and the baryon acoustic oscillation (BAO).

For the SN Ia data, we used the most up-to-date and the largest compilation called "Pantheon" recently released by Ref. [24]. The Pantheon compilation consists of 1048 SN Ia in the shift range $0.01 < z < 2.3$. Compared with the previous compilation such as Union2.1[29] and JLA,[30] the number of SN Ia in the Pantheon is significantly enlarged and the redshift range is much wider.

The observed distance modulus can be obtained using the linear relation

$$\mu_{\rm obs} = m - M_b + \alpha\times X_1 - \beta\times\mathcal{C}, \tag{17}$$

where m is the apparent magnitude, M_b is the absolute magnitude of SN Ia, X_1 and $\mathcal{C}$ are the stretch and color parameters respectively, α and β are the coefficients of stretch and color corrections respectively. Usually the nuisance parameters α and β are fitted together with cosmological parameters. In the Pantheon compilation, the authors proposed a new method called BEAMS with Bias Corrections (BBC) to calibrated the SNe. According to the BBC method, the nuisance parameters α and β are determined by fitting to a randomly chosen reference cosmology. Once α and β are determined, we do not need to fit them any more in other cosmology fits. For the Pantheon dataset, the authors reported the apparent magnitude after correcting for stretch and color, so we do not need to do the stretch and color correction any more. Thus there is only one free parameter remain, namely the absolute magnitude M_b.

We constrain M_b simultaneously with the cosmological parameters. The best-fitting parameters are the ones, which can minimize the χ^2,

$$\chi^2_{\rm SN} = \Delta\boldsymbol{\mu}^{\rm T}\boldsymbol{C}^{-1}\Delta\boldsymbol{\mu}, \tag{18}$$

where $\Delta\boldsymbol{\mu} = \boldsymbol{\mu}_{\rm obs} - \boldsymbol{\mu}_{\rm th}$ is the vector of distance residuals and $\boldsymbol{C}$ is the total covariance matrix between each SN Ia. The total covariance matrix can be written as the sum of statistical covariance and systematic covariance,

$$\boldsymbol{C} = \boldsymbol{D}_{\rm stat} + \boldsymbol{C}_{\rm sys}, \tag{19}$$

where the statistical covariance $\boldsymbol{D}_{\rm stat}$ is a diagonal matrix. Both $\boldsymbol{D}_{\rm stat}$ and $\boldsymbol{C}_{\rm sys}$ can be found in Ref. [24].

Next, we consider the constraints from $H(z)$ data, which provides the direct measurement for the Hubble parameter at different redshift. In general, there are two methods to directly measure the Hubble parameter, i.e. the differential age of galaxies (DAG) method and the BAO method. However, only the former method is independent of cosmological model. In this paper, we use the 30 $H(z)$ data measured from the DAG method compiled in Ref. [31]. The χ^2 for $H(z)$ data is

$$\chi^2_{\rm Hz} = \sum_{i=1}^{N}\frac{[H(z_i)-\hat{H}_i]^2}{\sigma^2_{\hat{H}_i}}, \tag{20}$$

where $H(z_i)$, $\hat{H}_i$ and $\sigma_{\hat{H}_i}$ are the theoretical Hubble parameter at redshift z_i, the observed Hubble parameter, and the uncertainty of $\hat{H}_i$, respectively.

Finally, the third data we use in this paper is the BAO data, which provides a "standard ruler" for length scale in cosmology. This standard ruler is characterized by the comoving sound horizon r_d

$$r_d = \int_{z_d}^{\infty}\frac{c_s(z){\rm d}z}{H(z)}, \tag{21}$$

where $c_s(z)$ is the sound speed at redshift z, z_d is the redshift of drag epoch.[32] Following Ref. [33], r_d is recorded as a free parameter.

The effective distance measurement $D_V(z)$ can be depicted as

$$D_V(z) = \left[\frac{d_L^2(z)}{(1+z)^2}\frac{cz}{H(z)}\right]^{1/3}. \tag{22}$$

Moreover, BAO data can estimate the ratio of the effective distance to the comoving sound horizon

$$R(z) = \frac{D_V(z)}{r_d}. \tag{23}$$

In this paper, we use the BAO data points compiled in Table 1 of Ref. [34], but we exclude the point of $z = 2.34$. The χ^2 of BAO data is given by

$$\chi^2_{\rm BAO} = \sum_{i=1}^{N}\frac{[R_{\rm th}(z_i) - R_{\rm obs}(z_i)]^2}{\sigma^2_{R_i}}. \tag{24}$$

Eventually, all the data sets are combined to constrain the cosmological model. The total χ^2 of the all data sets is the sum of each χ^2,

$$\chi^2_{\rm total} = \chi^2_{\rm SN} + \chi^2_{\rm Hz} + \chi^2_{\rm BAO}. \tag{25}$$

The values of the best-fit model parameters generate the corresponding minimal χ^2 (or χ^2/dof, where dof=$N-k$ is the degree of freedom, and N is the total number of data, k is the number of free parameters).

The Akaike information criterion (AIC)[35] and the Bayesian information criterion (BIC)[36] are used to pick up the model which are best consistent with the observational data. They are defined by

$$\mathrm{AIC} = \chi^2_{\min} + 2k, \tag{26}$$

$$\mathrm{BIC} = \chi^2_{\min} + k\ln N, \tag{27}$$

where $\chi^2_{\min}$ is the minimum of χ^2, k is the number of free parameters, and N is the number of data points. The model which has the smallest AIC or BIC is the best one. Usually, one can choose a reference model and calculate the difference of AIC or BIC of other models with respect to the reference model. Here we choose the ΛCDM as the reference model, and define

$$\Delta\mathrm{AIC}_{\rm model} = \mathrm{AIC}_{\rm model} - \mathrm{AIC}_{\Lambda\rm CDM}, \tag{28}$$

$$\Delta\mathrm{BIC}_{\rm model} = \mathrm{BIC}_{\rm model} - \mathrm{BIC}_{\Lambda\rm CDM}. \tag{29}$$

For AIC, the model is strongly supported when $0 < \Delta\mathrm{AIC} < 2$, is less supported when $4 < \Delta\mathrm{AIC} < 7$, and is essentially unsupported when $\Delta\mathrm{AIC} > 10$.[17] For BIC, there is positive, strong and extremely strong evidence against the model with $2 < \Delta\mathrm{BIC} < 6$, $6 < \Delta\mathrm{BIC} < 10$, and $\Delta\mathrm{BIC} > 10$, respectively.[37]

3.2 *Results*

In our paper, we analyse the characteristic of holographic dark energy comparing with the standard cosmological model. Firstly, we use the SN Ia data alone to constrain the parameters. Due to the degeneracy of the Hubble constant h_0 ($h_0 = H_0/100\ \mathrm{km\cdot s^{-1}\cdot Mpc^{-1}}$) with the absolute magnitude M_b, the two parameters can not be constrained simultaneously. Here, we fix $h_0 = 0.7$ and free M_b. The best-fitting parameters are list in Table 1, where in the last two rows we also list the ΔAIC and ΔBIC values for each model. According to the information criteria, the flat ΛCDM is the best model, while the $R_h = ct$ model is decisively disfavored. According to AIC, the *o*CDM, HDE, and KHDE models could not be excluded, although they are not as good as ΛCDM. According to BIC, however, *o*CDM and HDE models are strongly disfavoured while KHDE model is decisively excluded. In any case, the universe prefers being spatially flat.

Secondly, the $H(z)$ data are added to the SN Ia data to constrain the cosmological parameters. The inclusion of the $H(z)$ data breaks the degeneracy between h_0 and M_b, thus the two parameters can be limited contemporaneously. The best-fitting parameters are reported in Table 2. Compared with Table 1, we can see that the parameters from SN Ia+$H(z)$ are consistent with that from SN Ia alone. The Hubble parameter in $R_h = ct$ model is considerably smaller than the local measurement and Planck result, while in the rest models, the Hubble parameters are between the local H_0 and Planck H_0. Similar to the SN Ia only case, the $R_h = ct$ model is decisively excluded, and the ΛCDM is the best model.

Table 1 The best-fitting results for different cosmological models from SN Ia alone.

	ΛCDM	*o*CDM	HDE	KHDE	$R_h = ct$
Ω_{m0}	0.298 ± 0.022	0.324 ± 0.068	0.274 ± 0.062	0.261 ± 0.047	–
C_0	–	–	0.855 ± 0.263	0.904 ± 0.286	–
Ω_{k0}	–	-0.068 ± 0.171	–	0.004 ± 0.066	–
M_b	-19.351 ± 0.011	-19.354 ± 0.014	-19.354 ± 0.015	-19.352 ± 0.011	-19.238 ± 0.004
χ^2	1026.862	1026.715	1026.908	1026.953	1114.914
χ^2/dof	0.982	0.983	0.983	0.984	1.065
ΔAIC	0	1.853	2.046	3.911	86.052
ΔBIC	0	6.808	7.000	13.821	81.097

Table 2 The best-fitting results for different cosmological models from SN Ia+$H(z)$.

	ΛCDM	oCDM	HDE	KHDE	$R_h = ct$
Ω_{m0}	0.301 ± 0.020	0.330 ± 0.064	0.286 ± 0.043	0.289 ± 0.050	–
C_0	–	–	0.804 ± 0.181	0.845 ± 0.251	–
Ω_{k0}	–	-0.079 ± 0.162	–	-0.039 ± 0.204	–
M_b	-19.381 ± 0.054	-19.374 ± 0.056	-19.387 ± 0.056	-19.381 ± 0.064	-19.489 ± 0.050
h_0	0.690 ± 0.018	0.694 ± 0.020	0.690 ± 0.019	0.692 ± 0.021	0.624 ± 0.014
χ^2	1041.498	1041.277	1041.376	1041.356	1131.545
χ^2/dof	0.969	0.970	0.970	0.971	1.052
ΔAIC	0	1.779	1.878	3.858	88.047
ΔBIC	0	6.762	6.861	13.823	83.064

Table 3 The best-fitting results for different cosmological models from SN Ia+BAO.

	ΛCDM	oCDM	HDE	KHDE	$R_h = ct$
Ω_{m0}	0.296 ± 0.021	0.316 ± 0.067	0.257 ± 0.016	0.263 ± 0.070	–
C_0	–	–	0.924 ± 0.013	0.867 ± 0.359	–
Ω_{k0}	–	-0.052 ± 0.168	–	0.022 ± 0.072	–
M_b	-19.352 ± 0.010	-19.355 ± 0.013	-19.353 ± 0.011	-19.353 ± 0.015	-19.238 ± 0.004
r_d	148.476 ± 1.898	148.885 ± 2.333	148.471 ± 1.921	150.147 ± 5.873	134.156 ± 1.084
χ^2	1031.562	1031.475	1031.649	1031.571	1124.484
χ^2/dof	0.982	0.982	0.983	0.983	1.069
ΔAIC	0	1.913	2.087	4.009	90.922
ΔBIC	0	6.873	7.047	13.930	85.962

Table 4 The best-fitting results for different cosmological models from SN Ia+$H(z)$+BAO.

	ΛCDM	oCDM	HDE	KHDE	$R_h = ct$
Ω_{m0}	0.299 ± 0.020	0.323 ± 0.064	0.277 ± 0.043	0.278 ± 0.043	–
C_0	–	–	0.837 ± 0.195	0.795 ± 0.218	–
Ω_{k0}	–	-0.066 ± 0.165	–	0.029 ± 0.067	–
M_b	-19.378 ± 0.054	-19.372 ± 0.056	-19.382 ± 0.056	-19.390 ± 0.058	-19.238 ± 0.004
h_0	0.691 ± 0.018	0.694 ± 0.020	0.692 ± 0.018	0.689 ± 0.019	0.623 ± 0.014
r_d	150.169 ± 3.699	150.080 ± 3.719	150.330 ± 3.814	153.261 ± 8.045	134.156 ± 1.084
χ^2	1046.212	1046.055	1046.240	1046.063	1141.101
χ^2/dof	0.969	0.970	0.970	0.970	1.056
ΔAIC	0	1.843	2.028	3.851	92.890
ΔBIC	0	6.831	7.016	13.828	87.901

Thirdly, the BAO data are combined with the SN Ia data to make an analysis, where the sound horizon r_d is an additional parameter. The best-fitting results are given in Table 3. The best-fitting parameters are consistent with Table 1 and Table 2. In the $R_h = ct$ model, the r_d is considerably smaller than the rest models. The $R_h = ct$ model is decisively disfavoured and the ΛCDM is still the best model. Compared with Table 1, we see that adding the BAO data to SN Ia does not change the main conclusion.

Finally, the combination of SN Ia + $H(z)$ + BAO data are utilized to constrain the cosmological models. The best-fitting results are presented in Table 4. From Table 4, we can see that very similar conclusion can be arrived compared with the previous three cases.

In summary, based on the above analysis, the observational astrophysical data, either the SN Ia alone or the combination of SN Ia with other data, strongly exclude $R_h = ct$ model and favor ΛCDM model, but the HDE model could not be ruled out yet. In addition, the data prefers a spatially flat universe.

4 Discussion and Conclusion

In this paper, we combined the latest observation of Pantheon SN Ia dataset with 30 $H(z)$ and 6 BAO measurements to give strict constraint on three cosmological models, i.e., the ΛCDM model, the HDE model and the $R_h = ct$ model. For ΛCDM and HDE models, we consider two cases, i.e., the universe is spatially flat or non-flat. Numerical results show that the ΛCDM model is the best favoured one among the three models. And the HDE model is consistent with observational data. However, the $R_h = ct$ model is decisively disfavored by the observational data. In the $R_h = ct$ model, the best fitting Hubble constant is remarkably lower than the one given by Planck collaborations[4] and Riess *et al.*[20] Thus, the present cosmological observations do not support the $R_h = ct$ model. We also showed that the HDE model in a spatial flat universe has similar behaviour to the $R_h = ct$ model if C_0 approaches to infinity. However, the best fitting value of C_0 is around 0.8, which implies that the $R_h = ct$ model should be modified to be compatible with the present cosmological observations. Applying the $H(z)$ data and BAO data, we have given the constraint on Hubble constant where $h_0 = 0.694 \pm 0.020$ for the ΛCDM model and $h_0 = 0.689 \pm 0.019$ for the HDE model. Our results imply that the tension of Hubble constant between Planck collaborations and Riess *et al.* has been partially relaxed. The constraint on the spatial curvature is $\Omega_{k0} = -0.066 \pm 0.165$ for the ΛCDM model and $\Omega_{k0} = -0.029 \pm 0.067$ for the HDE model, implying that the astrophysical data are consistent with a spatially flat universe. In summary, the $H(z)$, BAO and Pantheon SN Ia data show that the $R_h = ct$ model is unfavored while the HDE can not be excluded.

In a recent work,[23] the HDE model can be excluded by the Planck 2015 observation combined with the BAO data, the JLA SN Ia data and the $H(z)$ data, which is caused by the H_0 tension. The CMB data carries the cosmological information since the end of the inflation. At the early stage of universe, the dark energy does not dominate the evolution of the universe. One property of the HDE model shows that the smaller C_0 makes the universe accelerate faster.[13] It is expected that the combined observations of CMB data and other astronomical data will prefer a smaller C_0 than the one fitted by SN Ia data. There is tension of H_0 between the observation of Planck collaborations[4] and Riess *et al.*[20] Thus, to avoid possible conflict between Planck data and SN Ia data, we do not use the Planck data to constrain the HDE model.

References

[1] A. G. Riess, *et al.*, Astron. J. **116** (1998) 1009.
[2] S. Perlmutter, *et al.*, Astrophys. J. **517** (1999) 565.
[3] C. L. Bennett, *et al.*, [WMAP Collaboration], Astrophys. J. **208** (2013) S20.
[4] P. A. R. Ade, *et al.*, [Planck Collaboration], Astron. Astrophys. **594** (2016) A13.
[5] I. Zlatev, L. M. Wang, and P. J. Steinhardt, Phys. Rev. Lett. **82** (1999) 896.
[6] E. J. Copeland, M. Sami, and S. Tsujikawa, Int. J. Mod. Phys. D **15** (2006) 1753.
[7] F. Melia, Mon. Not. Roy. Astron. Soc. **382** (2007) 1917.
[8] F. Melia, Astrophys. J. **764** (2013) 72.
[9] F. Melia and T. M. McClintock, Astron. J. **150** (2015) 119.
[10] F. Melia, Mon. Not. Roy. Astron. Soc. **464** (2017) 1966.
[11] J. J. Wei, X. F. Wu, and F. Melia, Astron. J. **149** (2015) 165.
[12] J. J. Wei, X. F. Wu, and F. Melia, Mon. Not. Roy. Astron. Soc. **463** (2016) 1144.
[13] M. Li, Phys. Lett. B **603** (2004) 1.
[14] G. 't Hooft, arXiv:gr-qc/9310026 (1993).
[15] X. Zhang and F. Q. Wu, Phys. Rev. D **72** (2005) 043524.
[16] M. Li, X. D. Li, and X. Zhang, Sci. China Phys. Mech. Astron. **53** (2010) 1631.
[17] Y. Y. Xu and X. Zhang, Eur. Phys. J. C **76** (2016) 588.
[18] S. Wang, Y. Wang, and M. Li, Phys. Rep. **696** (2017) 1.
[19] P. A. R. Ade, *et al.*, [Planck Collaboration], Astron. Astrophys. **594** (2016) A14.
[20] A. G. Riess, *et al.*, Astrophys. J. **826** (2016) 56.
[21] M. Li, X. D. Li, Y. Z. Ma, X. Zhang, and Z. Zhang, JCAP **09** (2013) 021.
[22] J. F. Zhang, M. M. Zhao, Y. H. Li, and X. Zhang, JCAP **04** (2015) 038.
[23] R. Y. Guo, J. F. Zhang, and X. Zhang, arXiv:1809.02340 [astro-ph.CO].
[24] D. M. Scolnic, *et al.*, Astrophys. J. **859** (2018) 101.
[25] D. W. Hogg, arXiv:astro-ph/9905116.
[26] F. Melia, Mon. Not. Roy. Astron. Soc. **382** (2007) 1917.
[27] F. Melia, A. Shevchuk, Mon. Not. Roy. Astron. Soc. **419** (2012) 2579.
[28] M. Li, *et al.*, JCAP **12** (2009) 014.
[29] N. Suzuki, D. Rubin, C. Lidman, *et al.*, Astrophys. J. **746** (2012) 85.
[30] M. Betoule, *et al.*, [SDSS Collaboration], Astron. Astrophys. **568** (2014) A22.
[31] M. Moresco, *et al.*, JCAP **05** (2016) 014.
[32] D. J. Eisenstein and W. Hu, Astrophys. J. **496** (1998) 605.
[33] I. Tutusaus, *et al.*, Astron. Astrophys. **602** (2017) A73.
[34] C. Cheng, *et al.*, Sci. China Phys. Mech. Astron. **58** (2015) 599801.
[35] H. Akaike, IEEE Trans. Automatic Control **19** (1974) 716.
[36] G. Schwarz, Ann. Statist. **6** (1978) 461.
[37] R. E. Kass, and A. E. Raftery, J. Amer. Stat. Assoc. **90** (1995) (430).