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Complexity of charged anisotropic spherically symmetric fluids in $f(\mathcal{G})$ gravity

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Abstract

The previous ideology of complexity factor for the dynamical spheres [Herrera *et al* 2018, *Phys. Rev. D*, **98**, 104059] is extended for the influence of charge. A dynamical spherically symmetric non-dissipative and dissipative self-gravitating structure is examined in the presence of Maxwell $f(\mathcal{G})$ gravity to examine the complexity factor. The pattern of evolution is studied with the minimal complexity constraint. The complexity factor remains the same for the structure of fluid distribution, while we examine homologous constraints for the most basic evolution pattern. We calculate the structure scalars which play an important role in order to understand the fundamental properties of the system. The fluid is geodesic as well as shearing for the dissipative case and there is a large number of solutions. In the non-dissipative fluid distribution, a shear-free, homogeneous and isotropic, geodesic fluid correlates with the evolving homologous and vanishing complexity condition. The implication of the condition of vanishing complexity factor and stability are discussed at the end.

Keywords: gravitation, anisotropy, complexity, hydrodynamics

1. Introduction

Various revolutionary characteristics of the current cosmos are analyzed by cosmological consideration and it is concluded that the cosmos is expanding at an accelerated rate [1]. This has attained a remarkable interest in cosmology. This rate of expanding universe is a result of a puzzling force called dark energy which is assumed to have a large negative pressure. Modified theories of gravity (MTG) play an important role to reveal the enigmatic nature of this energy. These interesting theories have gained comprehensive significance to study the features of cosmic expansion. These theories are developed by the alternation of scalar invariants as well as by replacing the corresponding generic functions of these scalars in the action function. These modified theories are named as Brans–Dicke theory, $f(R)$ theory, $f(\mathcal{G})$ theory which is also known as the modified Gauss–Bonnet (GB) theory, etc.

Modified GB gravity [2] is obtained by the modification in the Einstein action with $f(\mathcal{G})$, where $\mathcal{G}$ is GB scalar and is a topological invariant in 4 dimensions. We may get several fascinating cosmic results with GB invariant in the brane-world higher-dimensional approach [3]. In string theory, the GB theory emerges naturally for the effective action in low energy limit for analyzing the late-time cosmology in the context of GB terms [4]. For a reasonable choice of function f, the GB theory allows us to discuss transition phases (from retardation to acceleration), solar system testing, inflationary epochs, and crossing the phantom division line [5, 6]. In comparison to $f(R)$ with modified GB gravity, this theory depicts more fascinating alternative results for dark energy [7]. De Felice and Tsujikawa [8] proposed cosmologically viable $f(G)$ models with the conditions $f_{\mathcal{G}\mathcal{G}} > 0$ and they also analyzed many $f(\mathcal{G})$ patterns that satisfy these criteria. They also discussed the correspondence of these cosmologically feasible $f(\mathcal{G})$ models with Solar system constraints [9]. One may read the articles [10–18] for additional information on the modified gravity theories and dark energy.

The term 'system' is based on the Latin word 'systēma.' A system is defined by its distinct structure, which organizes all of its components in a certain order. The effect of external and internal forces can change the nature of a system. Complex systems are defined as systems with a complicated structure due to the irregular arrangement of their components

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or various types of interactions. The concept of complexity in any system corresponds to something that may be the source of complications in that system. Several efforts have been made in the last few years to build a precise definition of complexity [19–24]. There is still no accepted precise definition of complexity that can be used across disciplines.

Two elementary structures, a perfect crystal, and an isolated ideal gas form the basis for the concept of complexity in mathematical physics. Because both of these structures are demonstrations of primitive substitution, they have a low level of complexity. Let us summarize their basic features, which are order and information. The structure of a perfect crystal is made up of symmetrically ordered particles, so the amount of information required to define it, is minimum, because it may be thoroughly described by analyzing a single element. However, a large number of arbitrarily moving particles of an ideal gas predict a disordering in the system. The properties of any one molecule of an isolated ideal gas cannot be abandoned in such a scenario for a complete knowledge of its structure. As a result, complete maximum data is required to determine the model of an isolated ideal gas.

This analysis revealed that, despite their differences in behavior, both systems have a low level of complexity. This indicates that structure and order exhibit the least amount of complexity; as a result, we must redefine the concept of complexity and add additional parameters to describe it. For ideal gases and perfect crystals, Lopez-Ruiz and his collaborators [25, 26] introduced the concept of disequilibrium, which is maximal and zero, respectively. To define the complexity factor, it may be necessary to ensure that it is zero for both perfect crystals and ideal gases. In order to understand complex systems, the definition of complexity has also been defined by energy density. The definition of complexity has also been expressed through energy density in order to study complex systems. Lloyd and Pagels [27] examined self-gravitating structures in the visible case and also formulated useful complexity factors for all physical structures. Crutchfield and Young [28] introduced a novel technique with the help of dimension and entropy to formulate the complexity of the system, as well as for non-linear dynamical systems, and also a method to formulate the basic equations.

Herrera and his collaborators [29–34] proposed an alternative approach for evaluating the complexity by considering the notions of disequilibrium and information in terms of energy density, as well as basic features of the fluid distribution (mass-radius ratio, pressure, heat dissipation, and luminosity) in the perspective of the theory of general relativity (GR). Herrera and his collaborators investigated the complexity of self-gravitational complex systems and proposed a new understanding of complexity based on a term scalar function which is generated by the decomposition of the Riemann tensor orthogonally. This approach is the simplest system, where energy density is homogeneous and pressure is isotropic, and vanishes. It is worth highlighting that the complexity factor can disappear not only in circumstances where the fluid configuration is homogeneous for energy density and locally isotropic pressure, but also when two components, anisotropic pressure, and inhomogeneous energy density, cancel out. Structure scalars are a form of mathematical tool used to represent self-gravitational complex systems and key properties for active gravitational mass, energy density inhomogeneity, pressure anisotropy, dissipative flux, and so on, all of which are vital when discussing the dynamics of self-gravitating complex objects [35–38].

Here is a brief summary of how the paper is put together. In the next portion, we calculated the field equations and the important variable for our article. Section 3 presents the concept of the complexity factor. In part 4, we analyzed the homologous and homogeneous expansion conditions (pattern of evolution). The vital effect of homogeneous evolution and vanishing the complexity factor conditions are explored in portions 5 and 6. The stability is discussed in part 7. In the second last section, the rest of the issues and results are examined. Finally, in the end, references and some significant equations are discussed in the appendix.

2. Equations of motion for studying non-static stellar systems in Maxwell-$f(\mathcal{G})$ gravity

In this section, we formulate the field equations for the modified GB theory and introduce other essential variables. In the presence of a charge, the $f(\mathcal{G})$ theory action is as follows

$$S = \int\left(l_m + \frac{f(\mathcal{G}) + R}{c}\right)\sqrt{-g}\,\mathrm{d}^4x + S_{\mathrm{e}}(g^{\gamma\nu}, \psi). \tag{2.1}$$

In above equation l_m represents the Lagrangian density of matter distribution, f is an arbitrary function of the GB invariant, $\mathcal{G} = R^2 - 4R_{\gamma\nu}R^{\gamma\nu} + R_{\gamma\xi\nu\eta}R^{\gamma\xi\nu\eta}$, R is Ricci scalar, c is the coupling constant which is taken as 8π, g is the determinant of the metric tensor and $S_{\mathrm{e}}(g^{\gamma\nu}, \psi)$ shows the charged action. Variation of this line element with respect to metric tensor gives us the field equation for $f(\mathcal{G})$ gravity as

$$R_{\gamma\nu} + \frac{1}{2}g_{\gamma\nu}R = 8\pi T_{\gamma\nu}, \quad \gamma, \nu = 0, 1, 2, 3, \tag{2.2}$$

where $T_{\gamma\nu}$ is the expression for the energy–momentum tensor and its value is given as

$$T_{\gamma\nu} = T^{(m)}_{\gamma\nu} + T^{(\mathcal{G})}_{\gamma\nu} + \mathcal{S}_{\gamma\nu}. \tag{2.3}$$

The matter part, modified part due to $f(\mathcal{G})$ gravity and electromagnetic part of the energy–momentum tensor are represented by $T^{(m)}_{\gamma\nu}$, $T^{(\mathcal{G})}_{\gamma\nu}$, and $\mathcal{S}_{\gamma\nu}$, respectively. The expression for the matter part of the energy–momentum tensor is given as

$$T^{(m)}_{\gamma\nu} = (\rho + P_\perp)V_\gamma V_\nu + P_\perp g_{\gamma\nu} + (P_r - P_\perp)\chi_\gamma\chi_\nu + q_\gamma V_\nu + V_\gamma q_\nu, \tag{2.4}$$

where q^γ, χ^γ, P_r, V^γ, $P_\perp$, and ρ show heat flux, four vectors, radial pressure, four velocity, tangential pressure, and energy density, respectively. For non-static case, these all quantities satisfy the following properties

$$\chi_\gamma\chi^\gamma = 1, \quad \chi_\gamma V^\gamma = 0, \quad V_\gamma V^\gamma = -1, \quad V_\gamma q^\gamma = 0. \tag{2.5}$$

We can write energy–momentum tensor in another form as follows

$$T^{(m)}_{\gamma\nu} = \rho V_\gamma V_\nu + \Pi_{\gamma\nu} + Ph_{\gamma\nu} + q(V_\gamma \chi_\nu + \chi_\gamma V_\nu), \quad (2.6)$$

where

$$P = \frac{P_r + 2P_\perp}{3}, \quad h_{\gamma\nu} = g_{\gamma\nu} + V_\gamma V_\nu,$$
$$\Pi_{\gamma\nu} = \Pi\left(\chi_\gamma \chi_\nu - \frac{1}{3}h_{\gamma\nu}\right), \quad \Pi = -(P_\perp - P_r). \quad (2.7)$$

We suppose the spherically symmetric line element, coupled with locally anisotropic matter that can radiate heat or energy. The fluid that we consider can be expressed by energy–momentum tensor, here it is taken with two unequal principal stresses (P_r, $P_\perp$) which depend on scalar function (Π). The energy–momentum tensor for the modified $f(\mathcal{G})$ theory can be expressed as

$$\begin{aligned} T^{(\mathcal{G})}_{\gamma\nu} = &\frac{1}{k}[(4R_{\gamma\xi}R^{\xi}_{\nu} - 2RR_{\gamma\nu} - 2R_{\gamma\xi\delta\eta}R^{\xi\delta\eta}_{\nu} + 4R_{\gamma\xi\nu\eta}R^{\xi\eta})f_{\mathcal{G}} \\ &+ \frac{1}{2}g_{\gamma\nu}f(\mathcal{G}) - 2Rg_{\gamma\nu}\nabla^2 f_{\mathcal{G}} \\ &+ 2R\nabla_\gamma\nabla_\nu f_{\mathcal{G}} - 4R^{\xi}_{\gamma}\nabla_\nu\nabla_\xi f_{\mathcal{G}} - 4R^{\xi}_{\nu}\nabla_\gamma\nabla_\xi f_{\mathcal{G}} \\ &+ 4R_{\gamma\nu}\nabla^2 f_{\mathcal{G}} + 4g_{\gamma\nu}R^{\xi\eta}\nabla_\xi\nabla_\eta f_{\mathcal{G}} - 4R_{\gamma\xi\nu\eta}\nabla^\xi\nabla^\eta f_{\mathcal{G}}]. \end{aligned} \quad (2.8)$$

The value of d' Alembert operator is given by $\nabla^2 = \nabla^\gamma \nabla_\gamma$ and $f_{\mathcal{G}} = \frac{\mathrm{d}f(\mathcal{G})}{\mathrm{d}\mathcal{G}}$. Furthermore, the expression for the electromagnetic field tensor is given as

$$\mathcal{S}_{\gamma\nu} = \frac{1}{4\pi}\left(-F^{\beta}_{\gamma}F_{\beta\nu} + \frac{1}{4}g_{\gamma\nu}F^{\beta\delta}F_{\beta\delta}\right), \quad (2.9)$$

The value $F_{\gamma\nu} = \phi_{\gamma,\nu} - \phi_{\nu,\gamma}$ describes the Maxwell field tensor and $\phi_\nu = \phi(t, r)\delta^0_\nu$ describes the four potential. The expression for the space-time of the spherically symmetric dissipative matter configuration is shown as

$$\mathrm{d}s^2 = -A^2\mathrm{d}t^2 + B^2\mathrm{d}r^2 + C^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2), \quad (2.10)$$

where metric coefficients are the functions of t and r. The non-zero components of energy–momentum tensor given in equation (2.6) are shown as

$$T^{(m)}_{00} = \rho A^2, \quad T^{(m)}_{01} = -qAB, \quad T^{(m)}_{11} = P_r B^2,$$
$$T^{(m)}_{22} = T^{(m)}_{33} = P_\perp C^2. \quad (2.11)$$

The expression for the 4-velocity and 4-vector is given as

$$V_\gamma = (-A, 0, 0, 0), \quad \chi_\gamma = (0, B, 0, 0). \quad (2.12)$$

The above-mentioned components meet the conditions given below

$$V^\gamma = A^{-1}\delta^\gamma_0, \quad q^\gamma = qB^{-1}\delta^\gamma_1, \quad \chi^\gamma = B^{-1}\delta^\gamma_1. \quad (2.13)$$

For the aforementioned metric, the Maxwell field equations are as follows:

$$4\pi\sigma AB^2 = \phi'' - \left(\frac{A'}{A} + \frac{B'}{B} - 2\frac{C'}{C}\right)\phi',$$

by integrating the above expression, we obtain the following results

$$\phi' = \frac{SAB}{C^2}.$$

We attain the non-zero components of equation (2.9) as follows

$$\mathcal{S}_{00} = \frac{1}{8\pi}\frac{\phi'^2}{B^2} = \frac{S^2A^2}{8\pi C^4},$$
$$\mathcal{S}_{11} = \frac{1}{8\pi}\frac{\phi'^2}{A^2} = \frac{S^2B^2}{8\pi C^4},$$
$$\mathcal{S}_{22} = \frac{\phi'^2C^2}{8\pi A^2B^2} = \frac{S^2}{8\pi C^2}.$$

By utilizing the previous equations, the field equations are formulated as

$$\begin{aligned} 8\pi\left(\rho A^2 - T^{(\mathcal{G})}_{00} + \frac{S^2A^2}{8\pi C^4}\right) = &\frac{\dot{C}}{C}\left(\frac{\dot{C}}{C} + 2\frac{\dot{B}}{B}\right) - \left(\frac{A}{B}\right)^2 \\ &\times\left[\left(\frac{C'}{C}\right)^2 + 2\frac{C''}{C} - \left(\frac{B}{C}\right)^2 \right. \\ &\left. - 2\frac{B'C'}{BC}\right], \end{aligned} \quad (2.14)$$

$$8\pi(T^{(\mathcal{G})}_{01} + qAB) = -2\left(\frac{C'\dot{B}}{RB} + \frac{A'\dot{C}}{AC} - \frac{\dot{C}'}{C}\right), \quad (2.15)$$

$$\begin{aligned} &8\pi\left(P_rB^2 - T^{(\mathcal{G})}_{11} + \frac{S^2B^2}{8\pi C^4}\right) \\ &= \left(\frac{B}{A}\right)^2\left[\left(2\frac{\dot{A}}{A} - \frac{\dot{C}}{C}\right)\frac{\dot{C}}{C}\right. \\ &\left.-2\frac{\ddot{C}}{C} - \left(\frac{A}{C}\right)^2\right] \\ &+\left(2\frac{A'}{A} + \frac{C'}{C}\right)\frac{C'}{C}, \end{aligned} \quad (2.16)$$

$$\begin{aligned} &8\pi\left(P_\perp C^2 - T^{(\mathcal{G})}_{22} + \frac{S^2}{8\pi C^2}\right) \\ &= \left(\frac{C}{B}\right)^2\left[\frac{A''}{A} + \frac{C''}{C}\right. \\ &\left.+\frac{C'}{C}\left(\frac{A'}{A} - \frac{B'}{B}\right) - \frac{A'B'}{AB}\right] \\ &-\left(\frac{C}{A}\right)^2\left[\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C}\right. \\ &\left.-\frac{\dot{A}}{A}\left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + \frac{\dot{B}\dot{C}}{BC}\right], \end{aligned} \quad (2.17)$$

The derivative with respect to time and radius are denoted by dot and prime, respectively. It is interesting to analyze the role of the electromagnetic field on the existence as well as the evolution of compact objects in the existence of an electromagnetic field. If the gravitational collapse of the objects occurs in a charged medium, the Reissner–Nordstrom will be its resultant product. Yousaf and his collaborator [39–41] inferred that the charged terms are trying to reduce the collapse rate. It is necessary to state that dissipation and shear viscosity is not added to the system because there is a chance of absorption in the components of collapsing fluid. Shear viscosity is absorbed in the $P_\perp$, and P_r and dissipated in the P_r, q, and ρ. Now, we define the acceleration a_γ, and expansion Θ in four dimensions, whose values are given as

$$a_\gamma = V_{\gamma;\nu}V^\nu, \quad \Theta = V^\gamma_{;\gamma}, \tag{2.18}$$

by using the value of expansion and acceleration, we define shear $\sigma_{\gamma\nu}$ whose expression is written as

$$\sigma_{\gamma\nu} = V_{(\gamma;\nu)} + a_{(\gamma}V_{\nu)} - \frac{1}{3}\Theta h_{\gamma\nu}. \tag{2.19}$$

The acceleration and expansion gain their new form with the help of equation (2.9) and appears as

$$a_1 = \frac{A'}{A}, \quad a = \sqrt{a^\gamma a_\gamma} = \frac{A'}{AB}, \tag{2.20}$$

$$\Theta = \frac{1}{A}\left(\frac{\dot{B}}{B} + 2\frac{\dot{C}}{C}\right). \tag{2.21}$$

Again by using equation (2.9) the values of the scalar of shear is written as

$$\sigma^{\gamma\nu}\sigma_{\gamma\nu} = \frac{2}{3}\sigma^2, \tag{2.22}$$

and its nonzero components are

$$\sigma_{11} = \frac{2}{3}B^2\sigma, \quad \sigma_{22} = \frac{\sigma_{33}}{\sin^2\theta} = -\frac{1}{3}C^2\sigma, \tag{2.23}$$

where

$$\sigma = \frac{1}{A}\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right). \tag{2.24}$$

By utilizing the value of expansion and σ in the equation (2.15), we get the new form of this equation

$$4\pi[Bq - (W_4\dot{f}_{\mathcal{G}} + W_5 f'_{\mathcal{G}} + W_6\dot{f}'_{\mathcal{G}})] = \frac{1}{3}(\Theta - \sigma)' - \sigma\frac{C'}{C}. \tag{2.25}$$

The terms W_4, W_5, and W_6 in the above mathematical expression are given in the appendix. The mass function introduced by Misner and Sharp [42] can be shown as

$$m = \frac{C}{2}\left[1 + \left(\frac{\dot{C}}{A}\right)^2 - \left(\frac{C'}{B}\right)^2\right] + \frac{S^2}{2C}. \tag{2.26}$$

The proper time derivative can be defined as

$$\mathrm{D}_T = \frac{1}{A}\frac{\partial}{\partial t}, \quad U = \mathrm{D}_T C < 0, \tag{2.27}$$

here, the velocity of a collapsing fluid is represented by U, it is defined as the derivative of the areal radius with respect to proper time. With help of mass function and velocity, we formulated an expression given as

$$E \equiv \frac{C'}{B} = \sqrt{\left(1 - \frac{2m}{C} + U^2 + \frac{S^2}{4}\right)}. \tag{2.28}$$

By inserting the above value in the equation (2.25), we get the expression as follows

$$4\pi\left[q - \frac{1}{B}(W_4\dot{f}_{\mathcal{G}} + W_5 f'_{\mathcal{G}} + W_6\dot{f}'_{\mathcal{G}})\right] = E\left[\frac{1}{3}\mathrm{D}_C(\Theta - \sigma) - \frac{\sigma}{C}\right], \tag{2.29}$$

the value of radial proper derivative D_C is given as

$$\mathrm{D}_C = \frac{1}{C'}\frac{\partial}{\partial r}. \tag{2.30}$$

The time derivative and radial derivative of the mass function and utilizing the field equations, the mass function appears in the following equations, respectively

$$\mathrm{D}_T m = -4\pi\left(P_r U + qE - \frac{UT^{(\mathcal{G})}_{11}}{B^2} + \frac{ET^{(\mathcal{G})}_{01}}{AB}\right)C^2, \tag{2.31}$$

$$\mathrm{D}_C m = 4\pi\left(\rho + \frac{qU}{E} - \frac{T^{(\mathcal{G})}_{00}}{A^2} + \frac{UT^{(\mathcal{G})}_{01}}{ABE}\right)C^2 + \frac{SS'}{CC'}, \tag{2.32}$$

and we attain the expression for the mass function

$$m = 4\pi\int_0^r\left(\rho + \frac{qU}{E} - \frac{T^{(\mathcal{G})}_{00}}{A^2} + \frac{UT^{(\mathcal{G})}_{01}}{ABE}\right)C^2C'\mathrm{d}r + \int_0^r\frac{SS'}{C}\mathrm{d}r \tag{2.33}$$

After some simplification, the final equation for the mass function along with modified terms and electromagnetic part appears as

$$\frac{3m}{C^3} = 4\pi\rho - \frac{4\pi}{C^3}\int_0^r C^3\left[\mathrm{D}_C\rho - \frac{3}{C}\left(\frac{qU}{E} - \frac{T^{(\mathcal{G})}_{00}}{A^2} + \frac{UT^{(\mathcal{G})}_{01}}{ABE}\right)\right]C'\mathrm{d}r + \frac{3}{2C^3}\left(\frac{S^2}{C} + \int_0^r\frac{S^2}{C^2}\mathrm{d}r\right). \tag{2.34}$$

2.1. Matching conditions and structure scalars: a way to describe complex astrophysical systems

Darmois conditions are the matching conditions for the bounded fluid distribution. Vaidya space-time is considered as an outside boundary, it can be described as

$$\mathrm{d}s^2 = -\left[1 - \frac{2M(\upsilon)}{r} + \frac{Q^2}{r^2}\right]d\upsilon^2 - 2\mathrm{d}r\mathrm{d}\upsilon + r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2), \tag{2.35}$$

$M(\upsilon)$ represents total mass and υ is the value of retarded time. By connecting the Vaidya space-time and the complete nonadiabatic sphere for the surface $r_\Sigma = C =$ constant, we require

$$m(t, r) \approx M(v) \Longleftrightarrow Q \approx s, \tag{2.36}$$

$$P_r = -\frac{T_{01}^{(\mathcal{G})}}{AB} - \frac{T_{11}^{(\mathcal{G})}}{B^2}, \tag{2.37}$$

$$\frac{M}{C^2} - 4\pi C\left[\frac{T_{01}^{(\mathcal{G})}}{AB} + \left(P_r + \frac{T_{11}^{(\mathcal{G})}}{B^2} + 2\pi S^2\right)\right] = 0. \tag{2.38}$$

Equation (2.36) gives the relation between the mass function of exterior and interior line elements and if both these are equal then their corresponding charge terms are also equal. The relationship between dark source terms and radial pressure is represented in equation (2.37) while equation (2.38) describes the interaction of the mass function of the exterior with an electromagnetic field as well as dark source terms.

In this section, we use an approach from the previous articles [43, 44] to determine the structure scalars, these scalars are essential components, as we analyze the complexity of the system by using these scalars. We obtained these structure scalars by using the technique of splitting of Riemann tensor orthogonally. The only electric part of the Weyl tensor is studied because the magnetic part vanishes for the spherically symmetric case. The expression for the electric part is given as follows

$$\mathbb{E}_{\gamma\nu} = \mathcal{C}_{\gamma\xi\nu\lambda}V^{\xi}V^{\lambda}, \tag{2.39}$$

here $\mathcal{C}_{\gamma\xi\nu\lambda}$ represents the Weyl tensor and its expression is

$$\mathcal{C}_{\gamma\nu\lambda\xi} = (g_{\gamma\nu\mu\sigma}g_{\lambda\xi\tau\rho} - \eta_{\gamma\nu\mu\sigma}\eta_{\lambda\xi\tau\rho})V^{\mu}V^{\tau}\mathbb{E}^{\sigma\rho},$$
$$\rho, \tau, \sigma, \mu = 0, 1, 2, 3. \tag{2.40}$$

The Levi–Civita tensor is denoted with $\eta_{\gamma\nu\mu\sigma}$ and $g_{\gamma\nu\mu\sigma} = g_{\gamma\mu}g_{\nu\sigma} - g_{\gamma\sigma}g_{\nu\mu}$. The nonzero values of the above-mentioned tensors are expressed as

$$\mathbb{E}_{11} = \frac{2}{3}A^2\mathcal{E}, \quad \mathbb{E}_{22} = -\frac{1}{3}B^2\mathcal{E}, \quad \mathbb{E}_{33} = \mathbb{E}_{22}\sin^2\theta, \tag{2.41}$$

whereas $\mathcal{E}$ represents the Weyl scalar. We can write $\mathbb{E}^{\gamma\nu}$ in another form as

$$\mathbb{E}^{\gamma\nu} = \mathcal{E}\left(\chi^{\gamma}\chi^{\nu} + \frac{1}{3}h^{\gamma\nu}\right), \tag{2.42}$$

where

$$\mathcal{E} = -\frac{1}{2C^2} + \frac{1}{2B^2}\left[\left(\frac{C'}{C} + \frac{B'}{B}\right)\left(\frac{C'}{C} - \frac{A'}{A}\right) - \frac{C''}{C} + \frac{A''}{A}\right] + \frac{1}{2A^2}\left[\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right)\left(\frac{\dot{C}}{C} + \frac{\dot{A}}{A}\right) - \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C}\right]. \tag{2.43}$$

For evaluating the scalar variables, we defined the tensors, with the help of Riemann tensor, which are given as,

$$Y_{\gamma\nu} = R_{\gamma\lambda\nu\xi}u^{\lambda}u^{\xi}, \tag{2.44}$$

$$X_{\gamma\nu} =^{*} R^{*}_{\gamma\lambda\nu\xi}u^{\lambda}u^{\xi} = \frac{1}{2}\eta^{\alpha\nu}_{\gamma\nu}R^{*}_{\alpha\nu\lambda\xi}u^{\lambda}u^{\xi}, \tag{2.45}$$

where $R^{*}_{\gamma\nu\lambda\xi} = \frac{1}{2}\eta_{\alpha\nu\lambda\xi}R^{\alpha\nu}_{\gamma\nu}$ and values of the above tensors are given as,

$$X_{\gamma\nu} = \frac{1}{3}X_T h_{\gamma\nu} + X_{TF}\left(\chi_{\gamma}\chi_{\nu} - \frac{1}{3}h_{\gamma\nu}\right), \tag{2.46}$$

$$Y_{\gamma\nu} = \frac{1}{3}Y_T h_{\gamma\nu} + Y_{TF}\left(\chi_{\gamma}\chi_{\nu} - \frac{1}{3}h_{\gamma\nu}\right), \tag{2.47}$$

The values of the trace-free and trace components of above-mentioned scalars inclusive with electromagnetic part are formulated as

$$X_T = 8\pi\rho^{(\text{eff})} + \frac{S^2}{C^4}, \tag{2.48}$$

$$X_{TF} = -4\pi\Pi^{(\text{eff})} - \mathcal{E} + \frac{S^2}{C^4}, \tag{2.49}$$

$$Y_T = 4\pi(\rho^{(\text{eff})} + 3P_r^{(\text{eff})} - 2\Pi^{(\text{eff})}) + \frac{S^2}{C^4}, \tag{2.50}$$

and

$$Y_{TF} = \mathcal{E} - 4\pi\Pi^{(\text{eff})} + \frac{S^2}{C^4}, \tag{2.51}$$

the values of the $\rho^{(\text{eff})}$, $P_r{}^{(\text{eff})}$, and $P_\perp{}^{(\text{eff})}$ are given in appendix A. Now, by utilizing field equations, equations (2.26), and (2.43), the expression for the mass function appears

$$\frac{3m}{C^3} = 4\pi\left(\rho - \Pi + T_{AB}^{(\mathcal{G})} + \frac{3S^2}{8\pi C^4}\right) - \mathcal{E}. \tag{2.52}$$

At this instance $T_{AB}^{(\mathcal{G})} = -\frac{T_{00}^{(\mathcal{G})}}{A^2} + \frac{T_{11}^{(\mathcal{G})}}{B^2} - \frac{T_{22}^{(\mathcal{G})}}{C^2}$. By using equations (2.34), (2.51), and (2.52), we obtain the following expression for the structure scalar Y_{TF} as

$$Y_{TF} = \frac{4\pi}{C^3}\int_0^r C^3\left[D_C\rho - \frac{3}{C}\left(\frac{qU}{E} + T_{BD}^{(\mathcal{G})}\right)\right]C'\mathrm{d}r - 8\pi\Pi - \frac{3}{2C^3}\int_0^r \frac{S^2}{C^2}\mathrm{d}r + 4\pi T_{AB}^{(\mathcal{G})} + \frac{S^2}{C^4}, \tag{2.53}$$

also $T_{BD}^{(\mathcal{G})} = -\frac{T_{00}^{(\mathcal{G})}}{A^2} + \frac{UT_{01}^{(\mathcal{G})}}{EAB}$, and value of scalar function X_{TF} appears as

$$X_{TF} = \frac{4\pi}{C^3}\int_0^r\left[\frac{3S^2}{8\pi C^2} - C^3(D_C\rho - \frac{3}{C}\left(\frac{qU}{E} + T_{BD}^{(\mathcal{G})}\right))C'\right]\mathrm{d}r - 4\pi T_{AB}^{(\mathcal{G})} + \frac{S^2}{C^4}. \tag{2.54}$$

A differential equation for the scalar function and inhomogeneity energy density could be stated as [45]

$$\left(4\pi\rho + \frac{X_{TF}}{4\pi} + T_{AB}^{(\mathcal{G})} + \frac{S^2}{8\pi C^4}\right)' = 4\pi qB(\Theta - \sigma) + \frac{3SS'}{C^4} - \frac{3C'(X_{TF} + \frac{S^2}{2C^4})}{C} + T_{CD}^{(\mathcal{G})}, \tag{2.55}$$

now, $T_{CD}^{(\mathcal{G})} = -\frac{12\pi T_{01}^{(\mathcal{G})}U}{AC} - \frac{3C'}{C}\left(\frac{T_{11}^{(\mathcal{G})}}{B^2} - \frac{T_{22}^{(\mathcal{G})}}{C^2}\right)$. In the non-dissipative case, last equation appears as

$$X_{TF} = 0 \Leftrightarrow \rho' = 0 \Leftrightarrow \frac{3SS'}{C^4} - \frac{3S^2C'}{2\pi C^5} - \left(T_{AB}^{(\mathcal{G})} + \frac{S^2}{8\pi C^4}\right)' + T_{CD}^{(\mathcal{G})} = 0. \tag{2.56}$$

Equation (2.57) has another form for the general dissipative state, which is given as

$$X_{TF} = 0 \Leftrightarrow \rho' = 4\pi qB(\Theta - \sigma) + \frac{3SS'}{C^4} - \frac{3S^2C'}{2\pi C^5} - \left(T_{AB}^{(\mathcal{G})} + \frac{S^2}{8\pi C^4}\right)' + T_{CD}^{(\mathcal{G})}. \tag{2.57}$$

As correspondence to the previous result, we can see here that in the presence of energy density and inhomogeneity, electromagnetic components play an important role in describing the scalar function X_{TF}.

3. The complexity factor

A major problem in the study of viable compact structures is the need to employ a wide variety of physical restrictions that satisfy the Einstein field equations. Note that even if the degree of anisotropy is small, the resulting changes in compact entities might be rather large [46]. As a result, the premise of isotropic pressure seems to be a very restricted condition, particularly when the compact structure is depicted as a high-density structure (such as neutron stars, for example). Isotropic pressure is also unstable due to the presence of physical aspects including shear, dissipation, inhomogeneity, and energy density [47]. The study of these physical features has attracted the attention of researchers during recent decades, which are significant in the field of astrophysics and cosmology. Therefore, from an astrophysical viewpoint, it is more feasible to relax the strong isotropic criteria and allow for local anisotropy among stellar matter. Since the number of physical parameters defining self-gravitating fluids exceeds the number of equations provided by the theory, additional conditions must be adopted to conclude the results of Einstein's field equations. In the same way, complexity is a significant and well-known idea in physics, when applied to the framework of GR as well as modified theories, this may be perfectly regarded to provide the additional required constraints. It is associated with one of the structure scalars labeled as a complexity factor, the idea which is proposed by Herrera *et al* [43]. To comprehend the composition and evolution of self-gravitating fluids, the complexity factor has great significance. The vanishing of this complexity factor makes our system less complex and more stable. In the analysis of self-gravitating fluids, the concept of stability has vital importance. The stellar remnants of collapsing phenomena are in a state of equilibrium. The state of being in equilibrium is the final fate of any phenomena occurring in our universe. So, in this regard, to understand the characterization and evolution of a system, the idea of the vanishing complexity factor, which we used in our manuscript, deals with it in an appropriate way.

Now that scientists have observed compact phenomena like neutron stars, black holes, and pulsars, they are shifting their focus from finding mathematical explorations to constructing viable physical models based on very exact observational evidence [48]. A lot of work is done by many scientists for the compact objects in the framework of complexity [49–54], so we can say that this work is related to astrophysics and cosmology.

The concept of complexity for the dynamical case is discussed by the two extra components. Fluid characteristics like anisotropic pressure, energy density, and viscosity are involved in the static scenario, while, in the dynamical scenario, two additional factors are discussed, named as the complexity of the pattern of evolution and structure of the system. Here, we further discussed the homologous condition and homogenous expansion in the context of kinematical consideration and dynamical consideration involving dissipative parameters. All these terms are shown by a single scalar function Y_{TF} which is named as the complexity factor [30].

3.1. Evolution and expansion criterion

The complexity factor Y_{TF} of the system has been taken (mentioned in the previous section) in this part. Here, we have to examine the pattern of evolution as well as check out whether these patterns can simplify the system or not. Homogeneous expansion ($\Theta' = 0$) and homologous evolution are the two simplest evolutionary processes because they are focused on fully basic ideas. In the non-dissipative situation, we will investigate both evolution pattern strategies, and find that (in the next portion) both these patterns support each other. In the generalized, dissipative scenario, the rationale provided in the next section allows us to accept homologous evolution as the simplest model.

3.1.1. Homologous evolution. equation (2.25) with homologous condition becomes

$$D_C\left(\frac{U}{C}\right) = \frac{4\pi}{E}\left[q - \frac{1}{B}(W_4\dot{f}_G + W_5 f_G' + W_6\dot{f}_G')\right] + \frac{\sigma}{C}. \tag{3.1}$$

We can get the following result by integrating the previous equation

$$U = a(t)C + C\int_0^r \left[\frac{4\pi q}{E} - \frac{4\pi}{EB}(W_4\dot{f}_G + W_5 f_G' + W_6\dot{f}_G') + \frac{\sigma}{C}\right]C'\mathrm{d}r, \tag{3.2}$$

Here a is integration function when its value is added in the previous equation, it appears

$$U = \frac{U_\Sigma}{C_\Sigma}C - C\int_r^{r_\Sigma}\left[\frac{4\pi q}{E} - \frac{4\pi}{EB}(W_4\dot{f}_G + W_5 f_G' + W_6\dot{f}_G') + \frac{\sigma}{C}\right]C'\mathrm{d}r. \tag{3.3}$$

From the preceding equations (3.2) and (3.3), we get $U \sim C$, it is a common characteristic of homologous evolution [55]. We can get these results if the fluid is shear-free and non-dissipative, or if two integral parts cancel one another. We have concentric shells for two areal radii C_{I} and C_{II} as

$$\frac{C_{\mathrm{I}}}{C_{\mathrm{II}}} = \text{constant}. \tag{3.4}$$

The equation above clearly shows that the evolution pattern corresponding to the homologous criteria is the simplest throughout the evolution of the fluid distribution. As a result, for a homologous evaluation

$$U = a(t)C, \quad a(t) \equiv \frac{U_\Sigma}{C_\Sigma}. \tag{3.5}$$

It is deduced that C is a separable function, therefore

$$C = C_1(t)C_2(r). \tag{3.6}$$

In equation (3.6), we take the separable form of $C(t, r)$ as $C(t, r) = C_1(t)C_2(r)$. Here C_1 and C_2 are the functions that depend upon the temporal and radial coordinates of the spherically symmetric spacetime. If we take $C_1(t) = t^0 = 1$, then our results will describe the static case of relativistic spheres. Thus, the involvement of $C_1(t)$ in the analysis makes our spherically symmetric system behave dynamically during the homologous development of the body. Finally, the homologous condition is achieved by combining equations (3.4) and (3.6) in equation (2.28) as

$$\frac{4\pi B}{C'}\left[q - \frac{1}{B}(W_4\dot{f}_{\mathcal{G}} + W_5 f'_{\mathcal{G}} + W_6\dot{f}'_{\mathcal{G}})\right] + \frac{\sigma}{C} = 0. \tag{3.7}$$

3.1.2. Homogeneous expansion. Another evolution pattern that would be defined as 'simple' is homogeneous expansion. Equation (2.28) gives the expression as

$$4\pi\left[q - \frac{1}{B}(W_4\dot{f}_{\mathcal{G}} + W_5 f'_{\mathcal{G}} + W_6\dot{f}'\mathcal{G})\right] = -\frac{C'}{B}\left[\frac{1}{3}D_C(\sigma) + \frac{\sigma}{C}\right]. \tag{3.8}$$

The homologous condition is imposed to the homogeneous expansion, yielding $D_C(\sigma) = 0$, meaning that $\sigma = 0$, which shows that dissipation is zero. According to the above equation, shear-free fluid, homogeneous expansion scalar corresponds to the non-dissipative case. In this case, the fluid is also homogenous, as demonstrated by equation (3.8).

4. Some kinematical considerations for radiating and non-radiating stars

Equation (3.7) presents a homogeneous condition that can be stated as

$$4\pi Bq - 4\pi(W_4\dot{f}_{\mathcal{G}} + W_5 f'_{\mathcal{G}} + W_6\dot{f}'_{\mathcal{G}}) = -\frac{C'\sigma}{C}. \tag{4.1}$$

By inserting the value of homogeneous condition into equation (2.29), we attain

$$(\Theta - \sigma)' = 0. \tag{4.2}$$

After putting the values of Θ and σ, above equation becomes

$$(\Theta - \sigma)' = \left(\frac{3\dot{C}}{AC}\right)' = 0. \tag{4.3}$$

equation (3.6) gives the following result

$$A' = 0. \tag{4.4}$$

The fluid is believed to be geodesic as a result of this. By reparametrizing the coordinate r, we can likewise put $A = 1$ without affecting generality. The converse is also true, therefore geodesic constraints demand homogeneous fluid which is stated as

$$\Theta - \sigma = \frac{3\dot{C}}{C}. \tag{4.5}$$

By solving the preceding equation for the center, we obtain $(\Theta - \sigma)' = 0$ for $C \sim r$. Taking successive derivatives with respect to r, we obtain the above equation close to the center given as

$$\frac{\partial^n(\Theta - \sigma)}{\partial r^n} = 0, \qquad n > 0. \tag{4.6}$$

Hence, in both conditions, the geodesic and homologous are involved with one another. This result is fully compatible with [29].

In the non-dissipative scenario, the homologous constraint demands that the fluid be both shear-free and geodesic. In this non-dissipative condition, the shear-free condition necessitates the homologous constraint. The homogeneous expansion with equation (3.8) in the non-dissipative case yields

$$\sigma' C + 3\sigma C' = 12\pi(W_4\dot{f}_{\mathcal{G}} + W_5 f'_{\mathcal{G}} + W_6\dot{f}'_{\mathcal{G}}), \tag{4.7}$$

It becomes as follows after simplification

$$\sigma = \frac{12\pi}{C^3}\int_0^r C^3(W_4\dot{f}_{\mathcal{G}} + W_5 f'_{\mathcal{G}} + W_6\dot{f}'_{\mathcal{G}})\mathrm{d}r + \frac{h(t)}{C^3}, \tag{4.8}$$

where arbitrary integration function is denoted by $h(t)$. As B becomes zero at the center when the radius is zero, it means $h(t) = 0$, and hence $\sigma = 0$. $\Theta' = 0$ is also obtained from (2.29) if we put $\sigma = 0$. Finally, in the case of the non-dissipative condition,

$$\sigma = 0 \Leftrightarrow U \sim B \Leftrightarrow \Theta' = 0. \tag{4.9}$$

Again the homologous condition is implied with the homogeneous expansion condition in the non-dissipative case and the result is compatible with [29].

It is really important to note that the scalar function $Y_{TF} = 0$ if a shear-free geodesic fluid remains geodesic and shear-free throughout its evolution, as proved by [44]. A system persists shear-free, if it starts its evolution from the initial point, and if $Y_{TF} = 0$ the fluid is geodesic. This is just another reason why $Y_{TF} = 0$ should be used as the complexity factor. Equation (2.29) with the value of $\Theta' = 0$ becomes as

$$\sigma' + \frac{3\sigma C'}{C} + 12\pi[qB - (W_4\dot{f}_{\mathcal{G}} + W_5 f'_{\mathcal{G}} + W_6 \dot{f}'_{\mathcal{G}})] = 0. \tag{4.10}$$

The preceding equation is solved as

$$\sigma = -\frac{12\pi}{C^3}\int_0^r C^3[qB - (W_4\dot{f}_{\mathcal{G}} + W_5 f'_{\mathcal{G}} + W_6 \dot{f}'_{\mathcal{G}})]\mathrm{d}r. \tag{4.11}$$

Until we assume modified terms become zero and $q = 0 = \sigma$, the homologous condition violates the previous statement. It explains the difficulty of simultaneously using both constraints in the presence of dissipation, modified as well as charge terms.

5. Dynamical observations

In the general dissipative case, the anisotropic fluid corresponding to the homologous condition is geodesic. Equation (8.5) becomes as follows when this condition is met

$$\mathrm{D}_T U = -\frac{m}{C^2} - 4\pi P_r C + \frac{S^2}{2C^3} + \frac{4\pi C T_{11}^{(\mathcal{G})}}{B^2} - D_{11}. \tag{5.1}$$

With the help of scalar Y_{TF} in the preceding equation, we attain

$$3\frac{\mathrm{D}_T U}{C} = -4\pi(\rho + 3P_r - 2\Pi) + Y_{TF} - 4\pi T_{AB}^{(\mathcal{G})} + \frac{12\pi T_{11}^{(\mathcal{G})}}{B^2} - \frac{3D_{11}}{C} + \frac{S^2}{C^4}. \tag{5.2}$$

Field equations and equation (2.43) gave the following result by using mass function

$$4\pi\left(\rho + 3P_r - 2\Pi - T_{CD}^{(\mathcal{G})} + \frac{S^2}{8\pi C^4}\right) = -\frac{2\ddot{C}}{C} - \frac{\ddot{B}}{B}, \tag{5.3}$$

the value $T_{CD}^{(\mathcal{G})} = T_{00}^{(\mathcal{G})} + \frac{T_{11}^{(\mathcal{G})}}{B^2} + \frac{T_{22}^{(\mathcal{G})}}{C^2}$. By utilizing equation (2.27), the preceding equation becomes

$$3\frac{\mathrm{D}_T U}{C} = \frac{3\ddot{C}}{C}. \tag{5.4}$$

By using the above value in equation (5.2), we obtain the following result for the scalar Y_{TF}

$$Y_{TF} = \frac{\ddot{C}}{C} - \frac{\ddot{B}}{B} - 4\pi T_{DE}^{(\mathcal{G})} - \frac{S^2}{2C^4} + \frac{3D_{11}}{C}, \tag{5.5}$$

here $T_{DE}^{(\mathcal{G})} = T_{00}^{(\mathcal{G})} + \frac{T_{00}^{(\mathcal{G})}}{A^2} + \frac{3T_{11}^{(\mathcal{G})}}{B^2} + \frac{2T_{22}^{(\mathcal{G})}}{C^2}$. By submitting the value of equation (3.5) in equation (5.2), we attain

$$3\left(\dot{a}(t) + a(t)\frac{\dot{C}}{C}\right) = Y_{TF} + \frac{12\pi T_{11}^{(\mathcal{G})}}{B^2} - \frac{3D_{11}}{C} - 4\pi(\rho + 3P_r - 2\Pi + T_{AB}^{(\mathcal{G})}) + \frac{S^2}{2C^4}. \tag{5.6}$$

We formulate the value of B from equation (5.5) by integrating it and considering a system that has zero complexity, we obtain

$$B = C_1(t)\left[b_1(r)\int\left(\frac{1}{C_1^2(t)}\mathrm{e}^{\int(\frac{S^2}{2C^4} + \frac{3D_{11}}{C} - 4\pi T_{DE}^{(\mathcal{G})})\frac{C_1(t)}{\dot{C}_1(t)}\mathrm{d}t}\right)\mathrm{d}t + b_2(r)\right], \tag{5.7}$$

Integration functions $b_1(r) = f(r)g(r)$ and $b_2(r) = f(r)h(r)$ are used here. The following form can be simply obtained from the above equation

$$B = C_1(t)C_2^{'}(r)\left[\tilde{b}_1(r)\int\left(\frac{1}{C_1^2(t)}\mathrm{e}^{\int(\frac{S^2}{2C^4} + \frac{3D_{11}}{C} - 4\pi T_{DE}^{(\mathcal{G})})\frac{C_1(t)}{\dot{C}_1(t)}\mathrm{d}t}\right)\mathrm{d}t + \tilde{b}_2(r)\right], \tag{5.8}$$

in the above equation $C_2'(r)\tilde{b}_1(r)$ and $C_2'(r)\tilde{b}_2(r)$ are the values of $b_1(r)$ and $b_2(r)$, respectively. We introduce new parameters to redefine the value of B

$$Z = \tilde{b}_1(r)\int\left(\frac{1}{C_1^2(t)}\mathrm{e}^{\int(\frac{S^2}{2C^4} + \frac{3D_{11}}{C} - 4\pi T_{DE}^{(\mathcal{G})})\frac{C_1(t)}{\dot{C}_1(t)}\mathrm{d}t}\right)\mathrm{d}t + \tilde{b}_2(r), \tag{5.9}$$

hence the value of B becomes as

$$B = ZC', \tag{5.10}$$

where $C' = C_1(t)C_2'(r)$.

5.1. The non-dissipative case

For the non-dissipative fluid, we can rewrite equation (5.5) as

$$\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} = 0 \Rightarrow Y_{TF} = \frac{3D_{11}}{C} - \frac{S^2}{2C^4} - 4\pi T_{DE}^{(\mathcal{G})}, \tag{5.11}$$

because the fluid is also shear-free, we can derive the conditions using equations (2.24) and (5.7), which are given as

$$b_1(r) = 0 \Rightarrow B = C_1(t)b_2(r) = C_1(t)C_2'(r)\tilde{b}_2(r). \tag{5.12}$$

We get $Z = 1$ by applying the constraints of the previous equation and reparametrizing r, which concludes that $B = C'$ [56]. The field equations can be expressed as

$$4\pi q = -\frac{\dot{Z}}{Z^2 C} - \frac{4\pi T_{01}^{(\mathcal{G})}}{ZC'}, \tag{5.13}$$

$$8\pi(P_r - P_\perp) = \frac{\dot{Z}\dot{C}}{ZC} + \frac{1}{Z^2C^2}\left(\frac{Z'C}{ZC'} + 1 - Z^2\right) + 8\pi\left(\frac{T_{11}^{(\mathcal{G})}}{(ZC')^2} - \frac{T_{22}^{(\mathcal{G})}}{C^2}\right) + \frac{2S^2}{C^4}. \tag{5.14}$$

For the value of $Z=1$, the above expression yields $P_r - P_\perp = \Pi = 0$; we also get $\rho' = 0$ by adopting the vanishing complexity constraint. We already know that with zero Weyl tensor and homogeneous energy density, dust with isotropic pressure remains shear-free and non-dissipative fluid [43, 57]. Of course, this system offers a simple setup and progresses in a homogeneous and vanishing complex manner. It is necessary to note that in the non-dissipative scenario, homogeneous and homologous expansion necessitate each other. Recently, we have studied the importance of curvature terms in the stability of non-dissipative systems [58–60].

5.2. The dissipative case

By utilizing the equations (2.24) and (5.11) in the dissipative constraints, we obtain the following result

$$\dot{\sigma} = -Y_{TF} + \left(\frac{\dot{C}}{C}\right)^2 - \left(\frac{\dot{B}}{B}\right)^2 - \frac{S^2}{2C^4} + \frac{3D_{11}}{C} - 4\pi T_{DE}^{(\mathcal{G})}. \tag{5.15}$$

By using the time derivative of equation (4.1), the previous equation yields

$$Y_{TF}\frac{C'}{C} = 4\pi Bq\left(\frac{\dot{q}}{q} + 2\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) - \left(\frac{S^2}{2C^4} + 4\pi T_{DE}^{(\mathcal{G})} - D_{12}\right)\frac{C'}{C}, \tag{5.16}$$

here $D_{12} = \frac{3D_{11}}{C} - 4\pi\frac{C}{C'}(W_4\dot{f}_\mathcal{G} + W_5 f'_\mathcal{G} + W_6\dot{f}'_\mathcal{G})$. By using the vanishing complexity constraint, we achieve

$$q = \frac{f_{(r)}}{B^2C} + \frac{1}{4\pi B^2C}\int BC'\left(\frac{S^2}{2C^4} + 4\pi T_{DE}^{(\mathcal{G})} - D_{12}\right), \tag{5.17}$$

also

$$\dot{q} = -q(\Theta + \sigma) + \frac{C'}{4\pi BC}\left(\frac{S^2}{2C^4} + 4\pi T_{DE}^{(\mathcal{G})} - D_{12}\right). \tag{5.18}$$

This type of solution can be found by applying the general approaches used in [61, 62]. The stationary state, in other words, the lack of transitory occurrences, can be considered the simplest dissipative state in a dissipative system [63]. If we assume a stationary state, we get

$$q^{(\text{eff})} = -\frac{\kappa T'}{A}, \tag{5.19}$$

the previous equation with the help of equation (5.19) appears as

$$T' = -\frac{f(r)}{\kappa AB}. \tag{5.20}$$

This result indicates that we cannot assist furthermore the consideration regarding the vanishing of relaxation time as a measure of zero complexity corresponding to the dissipative region.

6. Stability of non-complex systems

We can investigate the evolution of the scalar X_{TF}, which is defined in equation (8.6) and moreover characterized as

$$-\frac{3\dot{C}}{C}Y_{TF} - \dot{Y}_{TF} - 8\pi\dot{\Pi} - 4\pi(\rho + P_r)\sigma - 16\pi\Pi\frac{\dot{C}}{C} - 4\pi\left[Z - \mathcal{D} + \frac{1}{B}\left(q' - \frac{qC'}{C}\right)\right] + \mathcal{F} = 0, \tag{6.1}$$

the term $\mathcal{F}$ contains itself the correction and charge components

$$\mathcal{F} = 4\pi\left[\frac{\partial}{\partial t}\left(T_{AB}^{(\mathcal{G})} + \frac{3S^2}{8\pi C^4}\right) + \frac{3\dot{C}}{C}\left(T_{AB}^{(\mathcal{G})} + \frac{3S^2}{8\pi C^4}\right) + \frac{3}{C}\left(\frac{ET_{01}^{(\mathcal{G})}}{AB} - \frac{UT_{01}^{(\mathcal{G})}}{B^2}\right)\right]$$

The previous equation for the non-dissipative condition becomes

$$-\dot{Y}_{TF} - 8\pi\dot{\Pi} - 4\pi(Z - \mathcal{D}) + \mathcal{F} = 0.$$

We get $(\dot{\rho})' = 0$ by the time derivative of equation (2.55) and evaluate it at $t=0$. The only suitable solution is obtained by calculating the derivative with respect to the time of equation (6.1) and putting $Y_{TF}=0$

$$-\ddot{Y}_{TF} - 8\pi\ddot{\Pi} + 8\pi\dot{\Pi}\frac{\dot{C}}{C} + \dot{\mathcal{F}} - 4\pi\frac{\partial}{\partial t}(Z - \mathcal{D}) + \frac{3\dot{C}}{C}[4\pi(Z - \mathcal{D}) - \mathcal{F}] = 0. \tag{6.2}$$

By continuing this approach and using the t derivative of order n (for any $n > 0$), we can observe that the system could only deviate from the condition of the vanishing complexity factor if it deviates from the conditions of homogeneous energy density and isotropic pressure. At the initial stage for the dissipative system generally, we get

$$-\dot{Y}_{TF} - 8\pi\dot{\Pi} - 4\pi(\rho + P_r)\sigma - 16\pi\Pi\frac{\dot{C}}{C} + \mathcal{F} - 4\pi\left[Z - \mathcal{D} + \frac{1}{B}\left(q' - \frac{qC'}{C}\right)\right] = 0. \tag{6.3}$$

We would discuss the above equation in all respects. We would like to say that one can study the stability of the vanishing complexity constraint through the charge, heat flux, and correction terms of $f(\mathcal{G})$ gravity.

7. $f(\mathcal{G})$ theory and GR for the description of compact stellar objects

An expanding universe is predicted from observations of cosmic occurrences (such as Type Ia supernovas and cosmic microwave background radiation) [64–66]. Dark energy (DE) is the enigmatic cause of the rapid expansion. If anyone trying to figure out DE using GR, he is going to run across issues like cosmic coincidence and fine tuning. For these reasons, the action of GR is modulated, leading to MTG. One option is to extend GR into higher dimensions, which is defined as Lovelock gravity, and for $n = 4$ [67, 68], it corresponds to GR. The Ricci scalar R and GB invariant are two Lovelock scalars included in this theory. The second Lovelock scalar, which is a four-dimensional topological invariant with the formula $G = R^2 + R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu} - 4R_{\alpha\beta}R^{\alpha\beta}$ where $R_{\alpha\beta}$ and $R_{\alpha\beta\mu\nu}$ are the Ricci and the Riemann tensors, respectively, provides the five-dimensional Einstein GB gravity [69, 70].

Contrary to popular belief, various improvements to the innovative GR have been proposed by scientists throughout the last century. The Ricci scalar is used to substitute in these modifications with some arbitrary function like $f(R)$ where R is a Ricci scalar, $f(\mathcal{G})$ in which $\mathcal{G}$ is GB invariant, as well as many others, are explained in [71–79]. The accelerated expansion of the cosmos cannot be described by GR within its standard form unless an additional term is included in the exotic matter or gravitational Lagrangian [80, 81]. The outstanding phenomenon which is the expansion of the cosmos is studied by these modified theories [82].

Kanti [83] and his collaborators studied that the Ricci scalar can be neglected (means GR) when we examined the early time of the Universe, while higher-order curvature components would presumably be the most relevant in the high-energy phase of the early universe. According to their findings [83, 84], it is possible to get de Sitter solutions with a linearly expanding Milne phase by using a scalar-GB theory. In the astrophysical framework, the GB theory, in particular, has resulted in significant astrophysical differences. The fundamental prediction in these theories is that black holes are scalarized (see, for example, [85, 86]), implying the existence of a fifth force in order to the violation of the strong equivalence principle. In GR, there are no traversable wormholes, unless an exotic matter is added. This situation alters significantly if string theory corrections [87, 88] are applied. Without the use of exotic matter, they studied wormhole solutions in Dilatonic Einstein–Gauss–Bonnet theory (DEGB) [89].

GB gravity is a well-known modified theory that has been investigated several times in recent years [90, 91]. $f(\mathcal{G})$ gravity is the simplest form of modified gravity, which is extensively used and capable of reproducing any cosmological solution. It might, for example, contribute to the study of acceleration regimes and their transition to decelerated regimes, crossing the phantom dividing line and the inflationary era [92, 93]. The GB gravity is less restricted than $f(R)$ gravity [7]. Furthermore, as an alternative to dark energy, $f(\mathcal{G})$ gravity provides an effective framework for analyzing a variety of cosmic challenges [94]. Moreover, $f(\mathcal{G})$ gravity offers itself very well to study late time eras in an accelerating universe as well as finite time future singularities [7, 94]. Likewise, the cosmic accelerating nature is investigated in the context of several realistic models in $f(\mathcal{G})$ gravity [95, 96].

Although the ΛCDM model provides a simple explanation for DE in GR, it has flaws such as fine-tuning and cosmic coincidence. This leads researchers toward the MTG-like $f(\mathcal{G})$. This theory successfully explains the accelerated expansion of the cosmos as well as the transition of the cosmos from decelerated to accelerated [90]. Experiments based on the solar system are vital restraints on modified gravity because they evaluate how these theories differ from GR. For a gravity model to be considered feasible, it must be able to meet these restrictions, and this theory is successful in passing all solar system tests [9]. Numerous feasible $f(\mathcal{G})$ gravity approaches have been suggested in order to pass certain solar system limitations [95, 96], and further constraints on $f(\mathcal{G})$ models may emerge from the behavior of energy conditions and lead us to cover the shortcoming of GR [94, 97, 98].

De Felice and Tsujikawa [8] constructed the explicit forms of $f(\mathcal{G})$ models as well as developed conditions that must be met for these models to have cosmologically viable. This theory analyzes the dynamics of gravitational collapse and explains the thermodynamics of black holes [99]. Nojiri and Odintsov [100] examined numerous relationships between modified theories and came to the conclusion that such theories might explain inflation in the dark energy era. In the context of $f(\mathcal{G})$ gravity, Oikonomou [101] suggested bounce cosmology with a Type IV singularity at the bouncing point. He investigated if the unique bounce cosmology can be explained by mimetic vacuum $f(\mathcal{G})$ gravity. Nojiri *et al* [102] explored late-time acceleration, inflation, and bouncing cosmology, as well as some of the challenges and advancements in modified gravity. It is worth noting that, in this theory, anisotropic interior solutions describe more dense structures than GR.

8. Concluding remarks

In this era, the concept of complexity has attained remarkable importance as it provides help to scientists to understand, identify, and examine the components which are accountable for the appearance of complexity in the system. By pursuing this idea, we have examined the complexity emerging from the anisotropic spherical stellar objects in the context of modified GB gravity with the influence of charge. For this dynamical fluid, we have assumed two different epochs for estimating the complexity. In the first epoch, we analyzed the complexity of the structure and in the second epoch, we supposed the complexity of the pattern of evolution influenced with modified as well as charge components. In this article, we formulate field equations as well as mass functions with the help of the Misner-Sharp epoch.

We have calculated structure scalars (first epoch) for the modified GB theory and one of them, named Y_{TF}, is considered the complexity factor. The reason behind this is that it contains all the physical parameters like anisotropy pressure, inhomogeneous energy density, modified terms, dissipative as well as charge variables. For the static case, we regain the analogous result for the complexity factor by removing the dynamical electromagnetic and modified GB theory limitations. Also, Y_{TF} encloses the dissipative parameters as well as the pattern of evolution vanishing complexity factor progenitor from the homologous condition.

In the second epoch, we studied the complexity of the pattern of evolution by using two cases named the homogenous expansion and the homologous condition. In this pattern of evaluation, we analyzed geodesic fluid in the dissipative case, moreover, it is obvious that this type of flow displays the simplest pattern of evolution. Similarly, we observed the simplest pattern of evolution in the non-dissipative case such that the complexity factor Y_{TF} becomes zero by neglecting the effect of the electromagnetic part and modified components due to $f(\mathcal{G})$ gravity. Here, it is necessary to mention that in the non-dissipative case, complexity increases in the presence of charge as compared to previous work by Yousaf and his collaborators [103]. We also observe that the shear-free condition corresponds to the result $q=0$ (zero dissipation) if at the same time, both these cases of homogenous expansion, as well as the homologous condition, exist.

At the last, the stability of $Y_{TF}=0$ (complexity vanishes) is examined. For the non-dissipative scenario, it appears that stability will spread in time as long as the pressure is isotropic. The problem is substantially more complicated in the dissipative situation, because the system may diverge from the constraint $Y_{TF}=0$ due to dissipative values. For both cases, dissipative and non-dissipative, stability is influenced by charge components. All of our research work contains general relativity outcomes [29].

Data accessibility declaration

This published article contains all of the data examined or studied during this research.

Appendix A.

The additional curvature components related to GB gravity can be found as follows

$$\rho^{(\mathrm{eff})}=\rho A^2-T_{00}^{(\mathcal{G})}\quad P_r^{(\mathrm{eff})}=P_rB^2-T_{11}^{(\mathcal{G})}$$
$$P_\perp{}^{(\mathrm{eff})}=P_\perp C^2-T_{22}^{(\mathcal{G})},$$

where

$$T_{00}^{(\mathcal{G})}=\frac{A^2}{2}(\mathcal{G}f_{\mathcal{G}}-f)-W_1f_{\mathcal{G}}''-W_2f_{\mathcal{G}}'-W_3\dot{f}_{\mathcal{G}},$$
$$T_{01}^{(\mathcal{G})}=-W_4\dot{f}_{\mathcal{G}}-W_5f_{\mathcal{G}}'-W_6\dot{f}_{\mathcal{G}}',$$
$$T_{11}^{(\mathcal{G})}=-\frac{B^2}{2}(\mathcal{G}f_{\mathcal{G}}-f)-W_7\dot{f}_{\mathcal{G}}-W_8f_{\mathcal{G}}'-W_9\ddot{f}_{\mathcal{G}},$$
$$T_{22}^{(\mathcal{G})}=-\frac{C^2}{2}(\mathcal{G}f_{\mathcal{G}}-f)-W_{10}\ddot{f}_{\mathcal{G}}-W_{11}f_{\mathcal{G}}''-W_{12}\dot{f}_{\mathcal{G}}-W_{13}f_{\mathcal{G}}'-W_{14}\dot{f}_{\mathcal{G}}',$$

also

$$W_1=\frac{4}{B^4C^2}(C'^2A^2-A^2B^2-\dot{C}^2B^2),$$
$$W_2=\frac{4}{AB^5C^2}(B'B^2A\dot{C}^2+2C'C''BA^3-3C'^2B'A^3+B'B^2A^3-2C'\dot{C}B^2A\dot{B}),$$
$$W_3=\frac{4}{A^2B^3C^2}(3\dot{B}B^2\dot{C}^2-C'^2\dot{B}A^2+B^2\dot{B}A^2-2C''A^2B\dot{C}+2\dot{C}C'B'A^2),$$
$$W_4=\frac{4}{B^2A^3C^2}(2\dot{C}C'AB\dot{B}-A'A^2C'^2-2B^2\dot{C}A\dot{C}'+3A'B^2\dot{C}^2+A'B^2A^2),$$
$$W_5=\frac{4}{B^3A^2C^2}(\dot{C}^2B^2\dot{B}-2A'ABC'\dot{C}-3A^2\dot{B}C'^2+2C'A^2\dot{C}'B+\dot{B}B^2A^2),$$
$$W_6=\frac{4}{B^2A^2C^2}(A^2C'^2-B^2\dot{C}^2-B^2A^2),$$
$$W_7=\frac{4}{B^2A^5C^2}(2\dot{C}C'A'A^2B^2-A^2\dot{A}B^2C'^2-2AB^4\dot{C}\ddot{C}+3\dot{A}B^4\dot{C}^2+\dot{A}A^2B^2C^2),$$
$$W_8=\frac{4}{B^2A^3C^2}(\dot{C}^2A'B^2-2\dot{A}B^2C'\dot{C}-3A'A^2C'^2+2AB^2C'\ddot{C}+A'A^2B^2),$$
$$W_9=\frac{4}{B^2A^4C^2}(C'^2A^2B^2-\dot{C}^2B^4-A^2B^4),$$
$$W_{10}=\frac{4}{A^4B^3}(C''A^2BC-\dot{B}\dot{C}B^2C-CC'B'A^2),$$
$$W_{11}=\frac{4}{A^3B^4}(C\ddot{C}AB^2-A'A^2C'C-C\dot{C}\dot{A}B^2),$$
$$W_{12}=\frac{4}{B^3A^5}(\dot{A}A^2C'CB'-\dot{B}A^2C'CA'-AB^2\dot{B}C\ddot{C}-\dot{A}A^2BC''C+2A'A^2BC\dot{C}'-AB^2\ddot{B}C\dot{C}+A''A^2BC\dot{C}-2AA'^2BC\dot{C}+3\dot{A}\dot{B}B^2C\dot{C}-A'A^2B'C\dot{C}),$$
$$W_{13}=\frac{4}{A^3B^5}(\dot{A}B^2\dot{C}CB'-\dot{B}B^2\dot{C}CA'-AB^2B'C\ddot{C}-A'A^2BC''C+2AB^2\dot{B}C\dot{C}'+AB^2\ddot{B}BC'-A''A^2BCC'-2A\dot{B}^2BCC'+3A'B'A^2CC'-\dot{A}\dot{B}B^2C'C),$$
$$W_{14}=\frac{2}{A^3B^2}(C\dot{B}AC'+C\dot{C}A'B-C\dot{C}'AB).$$

Appendix B.

By utilizing equations (2.14)–(2.17), the non-vanishing components of the contracted Bianchi identities are given as

$$Z=T^{\gamma\nu}_{;\nu}V_\gamma=-\frac{1}{A}\left[\dot{\rho}+(\rho+P_r)\frac{\dot{B}}{B}+2(\rho+P_\perp)\frac{\dot{C}}{C}\right]-\frac{1}{B}\left[q'+2q\frac{(AC)'}{AC}\right]+\mathcal{D}=0,\tag{8.1}$$

here

$$\mathcal{D}=\frac{1}{2A}(\dot{\mathcal{G}}f_{\mathcal{G}}+\mathcal{G}\dot{f}_{\mathcal{G}}-\dot{f})-\frac{1}{A^3}\left[\frac{\partial}{\partial t}(W_1f''_{\mathcal{G}}+W_2f'_{\mathcal{G}}+W_3\dot{f}_{\mathcal{G}})\right]+\frac{1}{A}\left[\left(\frac{2\dot{A}}{A^3}-\frac{\dot{B}}{BA^2}-\frac{2\dot{C}}{CA^2}\right)(W_1f''_{\mathcal{G}}+W_2f'_{\mathcal{G}}+W_3\dot{f}_{\mathcal{G}})+\frac{1}{B^2}\frac{\partial}{\partial r}(W_4\dot{f}_{\mathcal{G}}+W_5f'_{\mathcal{G}}+W_6\dot{f}'_{\mathcal{G}})+\left(\frac{A'}{AB^2}-\frac{B'}{B^3}+\frac{2C'}{CB^2}\right)(W_4\dot{f}_{\mathcal{G}}+W_5f'_{\mathcal{G}}+W_6\dot{f}'_{\mathcal{G}})-\frac{\dot{B}}{B^3}(W_7\dot{f}_{\mathcal{G}}+W_8f'_{\mathcal{G}}+W_9\ddot{f}'_{\mathcal{G}})-\frac{2\dot{C}}{C^3}(W_{10}\ddot{f}_{\mathcal{G}}+W_{11}f''_{\mathcal{G}}+W_{12}\dot{f}_{\mathcal{G}}+W_{13}f'_{\mathcal{G}}+W_{14}\dot{f}'_{\mathcal{G}})\right],$$

and

$$T^{\gamma\nu}_{;\nu}\chi_\gamma=\frac{1}{B}\left[P_r'+(\rho+P_r)\frac{A'}{A}+2(P_r-P_\perp)\frac{C'}{C}-\frac{SS'}{4\pi R^4}\right]+\frac{1}{A}\left[\dot{q}+2q\left(\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)\right]+\mathbb{D}=0, \tag{8.2}$$

here

$$\mathbb{D}=-\frac{1}{2B}(\mathcal{G}'f_{\mathcal{G}}+\mathcal{G}f'_{\mathcal{G}}-f')-\frac{1}{B^3}\left[\frac{\partial}{\partial r}(W_7\dot{f}_{\mathcal{G}}+W_8f'_{\mathcal{G}}+W_9\ddot{f}_{\mathcal{G}})\right]+\frac{1}{B}\left[\left(\frac{\dot{B}}{BA^2}-\frac{\dot{A}}{A^3}+\frac{2\dot{C}}{CA^2}\right)(W_4\dot{f}_{\mathcal{G}}+W_5f'_{\mathcal{G}}+W_6\dot{f}'_{\mathcal{G}})+\frac{1}{A^2}(W_4\dot{f}_{\mathcal{G}}+W_5f'_{\mathcal{G}}+W_6\dot{f}'_{\mathcal{G}})+\left(\frac{2B'}{B^3}-\frac{A'}{AB^2}-\frac{2C'}{CB^2}\right)(W_7\dot{f}_{\mathcal{G}}+W_8f'_{\mathcal{G}}+W_9\ddot{f}_{\mathcal{G}})-\frac{A'}{A^3}(W_1f''_{\mathcal{G}}+W_2f'_{\mathcal{G}}+W_3\dot{f}_{\mathcal{G}})-\frac{2C'}{C^3}(W_{10}\ddot{f}_{\mathcal{G}}+W_{11}f''_{\mathcal{G}}+W_{12}\dot{f}_{\mathcal{G}}+W_{13}f'_{\mathcal{G}}+W_{14}\dot{f}'_{\mathcal{G}})\right].$$

The Bianchi identities can be rewrite by utilizing equation (2.20), (2.21), (2.28), the radial and proper time derivatives as

$$\mathrm{D}_T\rho+\frac{1}{3}(3\rho+P_r+2P_\perp)\Theta+\frac{2}{3}(P_r-P_\perp)\sigma+qE\mathrm{D}_C+2q\left(a+\frac{E}{C}\right)-\mathcal{D}=0, \tag{8.3}$$

$$\mathrm{D}_Tq+\frac{2}{3}q(2\Theta+\sigma)+E\mathrm{D}_CP_r+(\rho+P_r)a+(P_r-P_\perp)\frac{E}{C}-\frac{SS'}{4\pi BC^4}+\mathbb{D}=0. \tag{8.4}$$

Using the mass function, equations (2.16) and (2.28), the last equation could be reduced even further and appears as

$$\mathrm{D}_TU=-\frac{m}{C^2}-4\pi P_rC+Ea+\frac{S^2}{2C^3}+\frac{4\pi CT_{11}^{(\mathcal{G})}}{B^2}-D_{11}, \tag{8.5}$$

here

$$D_{11}=\frac{C}{2B^2}\left[\frac{B^2}{2}(\mathcal{G}f_{\mathcal{G}}-f)+W_7\dot{f}_{\mathcal{G}}+W_8f'_{\mathcal{G}}+W_9\ddot{f}_{\mathcal{G}}\right].$$

By utilizing equation (8.3) and [43, 45], the scalar function X_{TF} can be found as

$$(4\pi\rho^{(\mathrm{eff})}+\dot{X}_{TF})+\frac{1}{3}(2X_{TF}-Y_{TF}+X_T+Y_T)\times(\Theta-\sigma)C+12\pi q^{(\mathrm{eff})}\frac{CB'}{AB}=0. \tag{8.6}$$

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References

[1] Padmanabhan T 2002 Accelerated expansion of the universe driven by tachyonic matter *Phys. Rev.* D **66** 021301
[2] Nojiri S and Odintsov S D 2005 Modified *G*auss-*B*onnet theory as gravitational alternative for dark energy *Phys. Lett.* **631** 1 B
[3] Nojiri S, Odintsov S D and Ogushi S 2001 Cosmological and black hole brane-world universes in higher derivative gravity *Phys. Rev.* D **65** 023521
[4] Tsujikawa S and Sami M 2007 String-inspired cosmology: a late time transition from a scaling matter era to a dark energy universe caused by a *G* auss-*B* onnet coupling *J. Cosmol. Astropart. Phys.* **2007** 006
[5] Cognola G *et al* 2007 String-inspired *G* auss-*B* onnet gravity reconstructed from the universe expansion history and yielding the transition from matter dominance to dark energy *Phys. Rev.* D **75** 086002
[6] Amendola L, Charmousis C and Davis S C 2007 Solar system constraints on *G* auss-*B* onnet mediated dark energy *J. Cosmol. Astropart. Phys.* **2007** 004
[7] Santos J, Alcaniz J and Rebouças M 2006 Energy conditions and supernovae observations *Phys. Rev.* D **74** 067301
[8] De Felice A and Tsujikawa S 2009 Construction of cosmologically viable *f*(G) gravity models *Phys. Lett.* B **675** 1
[9] De Felice A and Tsujikawa S 2009 Solar system constraints on *f*(G) gravity models *Phys. Rev.* D **80** 063516
[10] Nashed G and Nojiri S 2022 Slow-rotating black holes with potential in dynamical chern-simons modified gravitational theory arXiv:2208.11498
[11] Sharif M and Yousaf Z 2014 Instability of a dissipative restricted non-static axial collapse with shear viscosity in *f*(*R*) gravity *J. Cosmol. Astropart. Phys.* **06** 019
[12] Loo T-H, De A, Arora S and Sahoo P 2022 Impact of curvature based geometric constraints on *f*(*R*) theory *Eur. Phys. J.* C **82** 705
[13] Bajardi F and D'Agostino R 2022 Late-time constraints on modified *G* auss-*B* onnet cosmology arXiv:2208.02677

[14] Yousaf Z, Bhatti M Z and Farwa U 2022 Dissipative collapse of cosmic structures in modified gravity *Chin. J. Phys.* **78** 363
[15] Yousaf Z, Bhatti M Z and Nasir M M M 2022 On the study of complexity for charged self-gravitating systems *Chin. J. Phys.* **77** 2078
[16] Yousaf Z, Bhatti M Z and Asad H 2022 Consequences of electric charge on anisotropic hyperbolically symmetric static spacetime *Phys. Scr.* **97** 055304
[17] Bhatti M Z, Yousaf Z, Yousaf M and Bamba K 2022 Dynamical analysis of charged fluid under nonminimally coupled gravity theory *Int. J. Mod. Phys.* D **31** 2240002
[18] Yousaf Z, Bhatti M Z and Asad H 2020 Gravastars in $f(R, T, R_{\mu\nu}T^{\mu\nu})$ gravity *Phys. Dark Univ.* **28** 100527
[19] Yousaf Z 2020 Definition of complexity factor for self-gravitating systems in palatini $f(R)$ gravity *Phys. Scr.* **95** 075307
[20] Cano P A, Hennigar R A and Marrochio H 2018 Complexity growth rate in lovelock gravity *Phys. Rev. Lett.* **121** 121602
[21] Ghodrati M 2017 Complexity growth in massive gravity theories, the effects of chirality, and more *Phys. Rev.* D **96** 106020
[22] Heras C L and Leon P 2022 Complexity factor of spherically anisotropic polytropes from gravitational decoupling *Gen. Relativ. Gravit.* **54** 138
[23] Maurya S *et al* 2022 Anisotropic strange star model beyond standard maximum mass limit by gravitational decoupling in $f(Q)$ gravity *Progr. Phys.* **70** 2200061
[24] Oertel M *et al* 2017 Equations of state for supernovae and compact stars *Rev. Mod. Phys.* **89** 015007
[25] Lopez-Ruiz R, Mancini H L and Calbet X 1995 A statistical measure of complexity *Phys. Lett.* A **209** 321
[26] Catalán R G, Garay J and López Ruiz R 2002 Features of the extension of a statistical measure of complexity to continuous systems *Phys. Rev.* E **66** 011102
[27] Lloyd S and Pagels H 1988 Complexity as thermodynamic depth *Ann. Phys.* **188** 186
[28] Crutchfield J P and Young K 1989 Inferring statistical complexity *Phys. Rev. Lett.* **63** 105
[29] Herrera L, Di Prisco A and Ospino J 2018 Definition of complexity for dynamical spherically symmetric dissipative self-gravitating fluid distributions *Phys. Rev.* D **98** 104059
[30] Herrera L 2018 New definition of complexity for self-gravitating fluid distributions: the spherically symmetric, static case *Phys. Rev.* D **97** 044010
[31] Herrera L, Di Prisco A and Ospino J 2022 Non-static fluid spheres admitting a conformal killing vector: exact solutions *Universe* **8** 296
[32] Herrera L, Di Prisco A and Carot J 2019 Complexity of the bondi metric *Phys. Rev.* D **99** 124028
[33] Herrera L, Prisco A D and Ospino J 2020 Quasi-homologous evolution of self-gravitating systems with vanishing complexity factor *Eur. Phys. J.* C **80** 1
[34] Herrera L, Di Prisco A and Ospino J 2021 Hyperbolically symmetric versions of lemaitre-tolman-bondi spacetimes *Entropy* **23** 1219
[35] Sharif M and Yousaf Z 2015 Instability of meridional axial system in $f(R)$ gravity *Eur. Phys. J.* C **75** 194
[36] Sharif M and Yousaf Z 2015 Radiating cylindrical gravitational collapse with structure scalars in $f(R)$ gravity *Astrophys. Space Sci.* **357** 49
[37] Yousaf Z 2019 Hydrodynamic properties of dissipative fluids associated with tilted observers *Mod. Phys. Lett.* A **34** 1950333
[38] Bhatti M Z, Yousaf Z and Ilyas M 2018 Existence of wormhole solutions and energy conditions in $f(R, T)$ gravity *J. Astrophys. Astron.* **39** 69
[39] Yousaf Z 2020 Construction of charged cylindrical gravastar-like structures *Phys. Dark Univ.* **28** 100509
[40] Yousaf Z *et al* 2019 Charged gravastars in modified gravity *Phys. Rev.* D **100** 024062
[41] Bhatti M Z and Yousaf Z 2016 Influence of electric charge and modified gravity on density irregularities *Eur. Phys. J.* C **76** 219
[42] Misner C W and Sharp D H 1964 Relativistic equations for adiabatic, spherically symmetric gravitational collapse *Phys. Rev.* **136** B571
[43] Herrera L *et al* 2009 Structure and evolution of self-gravitating objects and the orthogonal splitting of the riemann tensor, *Phys. Rev.* D **79** 064025
[44] Herrera L, Di Prisco A and Ospino J 2010 On the stability of the shear-free condition *Gen. Relativ. Gravit.* **42** 1585
[45] Herrera L 2011 Physical causes of energy density inhomogenization and stability of energy density homogeneity in relativistic self-gravitating fluids *Int. J. Mod. Phys.* D **20** 1689
[46] Di Prisco A, Herrera L and Varela V 1997 Cracking of homogeneous self-gravitating compact objects induced by fluctuations of local anisotropy *Gen. Relativ. Gravit.* **29** 1239
[47] Herrera L 2020 Stability of the isotropic pressure condition *Phys. Rev.* D **101** 104024
[48] Hehl F W, Kiefer C and Metzler R J 1998 *Black Holes: Theory and Observation: Proc. of the 179th WE Heraeus Seminar Held at Bad Honnef (18-22 August 1997(Springer: Germany)*
[49] Chatzisavvas K C *et al* 2009 Complexity and neutron star structure *Phys. Lett.* A **373** 3901
[50] Bogadi R S and Govender M 2022 Dynamical complexity and the gravitational collapse of compact stellar objects *Eur. Phys. J.* C **82** 475
[51] De Avellar M G B and Horvath J E 2012 Entropy, complexity and disequilibrium in compact stars *Phys. Lett.* A **376** 1085
[52] Bhatti M Z, Yousaf Z and Tariq Z 2021 Role of structure scalars on the evolution of compact objects in palatini $f(R)$ gravity *Chin. J. Phys.* **72** 18
[53] de Avellar M G B and Horvath J E 2013 Entropy, disequilibrium and complexity in compact stars: an information theory approach to understand their composition arXiv:1308.1033
[54] Adhitya H and Sulaksono A 2020 Complexity and neutron stars with crust and hyperon core *J. Phys. Conf. Ser.* **1572** 012012
[55] Hansen C J, Kawaler S D and Trimble V 2012 *Stellar Interiors: Physical Principles, Structure, and Evolution* (New York: Springer Science & Business Media)
[56] Herrera L and Santos N 2010 Collapsing spheres satisfying an euclidean condition *Gen. Relativ. Gravit.* **42** 2383
[57] Herrera L, Di Prisco A, Martin J, Ospino J, Santos N O and Troconis O 2004 Spherically symmetric dissipative anisotropic fluids: a general study *Phys. Rev.* D **69** 084026
[58] Bhatti M Z, Yousaf Z and Yousaf M 2022 Stability analysis of restricted non-static axial geometry in $f(R, T)$ gravity *Chin. J. Phys.* **77** 2617–38
[59] Bhatti M Z, Yousaf Z and Yousaf M 2022 Dynamical analysis for cylindrical geometry in non-minimally coupled $f(R, T)$ gravity *Int. J. Geom. Meth. Mod. Phys.* **19** 2250018
[60] Bhatti M Z, Yousaf M and Yousaf Z 2022 Junction conditions in perfect fluid $f(G, T)$ gravitational theory arXiv:2207.05965
[61] Thirukkanesh S and Maharaj S D 2009 Radiating relativistic matter in geodesic motion *J. Math. Phys.* **50** 022502
[62] Ivanov B V 2016 A different approach to anisotropic spherical collapse with shear and heat radiation *Int. J. Mod. Phys.* D **25** 1650049
[63] Israel W and Stewart J M 1976 Thermodynamics of nonstationary and transient effects in a relativistic gas *Phys. Lett.* A **58** 213

[64] Perlmutter S *et al* 1997 Measurements of the cosmological parameters Ω and Λ from the first seven supernovae at $z = 0.35$ *Astrophys. J.* **483** 565
[65] Riess A G *et al* 1998 Observational evidence from supernovae for an accelerating universe and a cosmological constant *Astron. J.* **116** 1009
[66] Tegmark M *et al* 2004 Cosmological parameters from sdss and wmap *Phys. Rev.* D **69** 103501
[67] Deruelle N 1989 On the approach to the cosmological singularity in quadratic theories of gravity: the kasner regimes *Nucl. Phys.* B **327** 253
[68] Deruelle N and Farina-Busto L 1990 Lovelock gravitational field equations in cosmology *Phys. Rev.* D **41** 3696
[69] Bhawal B and Kar S 1992 Lorentzian wormholes in *E* instein-*G* auss-*B* onnet theory *Phys. Rev.* D **46** 2464
[70] Deruelle N and Doležel T 2000 Brane versus shell cosmologies in *E* instein and *E* instein-*G* auss-*B* onnet theories *Phys. Rev.* D **62** 103502
[71] Koyama K 2016 Cosmological tests of modified gravity *Rep. Prog. Phys.* **79** 046902
[72] Bamba K and Odintsov S D 2015 Inflationary cosmology in modified gravity theories *Symmetry* **7** 220
[73] Cruz-Dombriz La *et al* 2012 Black holes, cosmological solutions, future singularities, and their thermodynamical properties in modified gravity theories *Entropy* **14** 1717
[74] Bamba K, Nojiri S and Odintsov S D 2013 Modified gravity: walk through accelerating cosmology arXiv:1302.4831
[75] Bamba K and Odintsov S D 2014 Universe acceleration in modified gravities: $f(R)$ and $f(T)$ cases arXiv:1402.7114
[76] Yousaf Z *et al* 2016 Influence of modification of gravity on the dynamics of radiating spherical fluids *Phys. Rev.* D **93** 064059
[77] Yousaf Z, Bamba K and Bhatti M Z 2016 Causes of irregular energy density in $f(R, T)$ gravity *Phys. Rev.* D **93** 124048
[78] Bhatti M Z, Yousaf Z and Yousaf M 2022 Effects of non-minimally coupled $f(R, T)$ gravity on the stability of a self-gravitating spherically symmetric fluid *Int. J. Geom. Meth. Mod. Phys.* **19** 2250120
[79] Nojiri S and Odintsov S D 2007 Introduction to modified gravity and gravitational alternative for dark energy *Int. J. Geom. Methods Mod. Phys.* **4** 115
[80] Faraoni V and Capozziello S 2011 *Beyond Einstein Gravity: A Survey of Gravitational Theories for Cosmology and Astrophysics* (Berlin: Springer)
[81] Dvali G, Gabadadze G and Porrati M 2000 4d gravity on a brane in 5d minkowski space *Phys. Lett.* B **485** 208
[82] Bamba K *et al* 2012 Dark energy cosmology: the equivalent description via different theoretical models and cosmography tests, *Astrophys. Space Sci.* **342** 155
[83] Kanti P, Gannouji R and Dadhich N 2015 Gauss-*B* onnet inflation *Phys. Rev.* D **92** 041302
[84] Kanti P, Gannouji R and Dadhich N 2015 Early-time cosmological solutions in *E* instein-scalar-*G* auss -*B* onnet theory *Phys. Rev.* D **92** 083524
[85] Nair R *et al* 2019 Fundamental physics implications for higher-curvature theories from binary black hole signals in the ligo-virgo catalog gwtc-1 *Phys. Rev. Lett.* **123** 191101
[86] Carson Z and Yagi K 2020 Probing string-inspired gravity with the inspiral-merger-ringdown consistency tests of gravitational waves, *Class. Quant. Grav.* **37** 215007
[87] Gross D J and Sloan J H 1987 The quartic effective action for the heterotic string *Nucl. Phys.* **291** 41
[88] Metsaev R and Tseytlin A A 1987 Order α' (two-loop) equivalence of the string equations of motion and the s-model weyl invariance conditions: Dependence on the dilaton and the antisymmetric tensor *Nucl. Phys.* B **293** 385
[89] Kanti P, Kleihaus B and Kunz J 2011 Wormholes in dilatonic Einstein–Gauss–Bonnet theory *Phys. Rev. Lett.* **107** 271101
[90] Cognola G *et al* 2006 Dark energy in modified *G* auss-*B* onnet gravity: late-time acceleration and the hierarchy problem, *Phys. Rev.* D **73** 084007
[91] Elizalde E *et al* 2010 Λ *CDM* epoch reconstruction from $f(R, G)$ and modified gauss-bonnet gravities *Class. Quant. Grav.* **27** 095007
[92] Visser M 1997 Energy conditions in the epoch of galaxy formation *Science* **276** 88
[93] Visser M 1997 General relativistic energy conditions: the hubble expansion in the epoch of galaxy formation *Phys. Rev.* D **56** 7578
[94] Santos J *et al* 2007 Energy conditions and cosmic acceleration *Phys. Rev.* D **75** 083523
[95] Sen A and Scherrer R J 2008 The weak energy condition and the expansion history of the universe, *Phys. Lett.* B **659** 457
[96] Santos J *et al* 2007 Lookback time bounds from energy conditions *Phys. Rev.* D **76** 043519
[97] Gong Y and Wang A 2007 Energy conditions and current acceleration of the universe *Phys. Lett.* B **652** 63
[98] Astashenok A V, Capozziello S and Odintsov S D 2015 Extreme neutron stars from extended theories of gravity *J. Cosmol. Astropart. Phys.* **2015** 001
[99] Sadjadi H M 2011 On the second law of thermodynamics in modified gauss-bonnet gravity *Phys. Scr.* **83** 055006
[100] Nojiri S and Odintsov S D 2011 Unified cosmic history in modified gravity: from $f(R)$ theory to lorentz non-invariant models *Phys. Rep.* **505** 59
[101] Oikonomou V 2015 Singular bouncing cosmology from gauss-bonnet modified gravity *Phys. Rev.* D **92** 124027
[102] Nojiri S, Odintsov S D and Oikonomou V K 2017 Modified gravity theories on a nutshell: inflation, bounce and late-time evolution *Phys. Rept.* **692** 1
[103] Yousaf Z, Bhatti M Z and Nasir M M M 2022 Role of $f(G)$ gravity in the study of non-static complex systems *Can. J Phys.* **100** 185