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An optical analogue for rotating BTZ black holes

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Abstract

We demonstrate an optical realization for the rotating BTZ black hole using the recent popular photon fluid model in an optical vortex but with a new proposed expression for the optical phase. We also give the numerical realization for the optical vortex to ensure that it can be generated experimentally. Different from the earlier suggestions for the analogue rotating black holes, our proposal includes an inner horizon in the analogue black hole structure. Such structure can keep for a long distance for the convenience of observing analogue Hawking or Penrose radiations.

Keywords: BTZ black holes, optical vortex, inner horizon, analogue gravity

(Some figures may appear in colour only in the online journal)

1. Introduction

The analogue gravity [1] had been well-known to provide a prospective avenue to study the properties of black holes. This stemmed from the thought put forward initially by Unruh [2], where it was that suggested there was a kinematical analogue between the motion of the sound wave in a convergent fluid flow and the motion of a scalar field in the background of Schwarzschild spacetime. For the analogue, the inhomogeneous flowing fluid was considered as the spacetime background and the sound waves as a massless scalar field could move under this spacetime background, but the sound waves cannot propagate through a surface which is determined by the relation that the fluid velocity is equal to the local sound velocity and behaves like the event horizon of a gravitational black hole. Thus, the analogue fluid can present many interesting phenomena about black holes, such as the event horizon [3], Hawking radiations [4], and so on. Due to this, many other physical systems, such as Bose–Einstein condensates (BEC) [5], superfluid He [6], slow light [7], electromagnetic waveguide [8], light in a nonlinear liquid [9], laser pulse filaments [10], trapped-ion systems [11], i.e. see the review [1], have been suggested as the physical systems of the analogue gravity. However, most of these analogue systems were made for the (1 + 1) dimensional spacetime, and the analogue phenomena of Schwarzschild black holes were usually presented.

Actually, there are also many proposals to simulate other types of black holes in the perspective of analogue gravity, such as rotating black holes [12–16] which could be relevant to the astrophysics, AdS and dS black holes [17, 18] for understanding the AdS/CFT or gravity/fluid duality. But, it is not easy to find the corresponding physical systems to finish the analogue with the feasible velocity distribution. A recent experiment [19] used the photon fluid [20] to realize the analogue for the rotating black hole. The photon fluid can carry the angular momentum and is the ideal candidate for this kind of analogue. In their experiment, the rotating analogue black hole is presented by an optical vortex propagating in a self-defocusing medium at room temperature [19, 21, 22], in which an ergosphere is formed to enclose the internal region in which rotating energy could be extracted [23]. In this paper, we will suggest a new type of optical vortex to simulate a new structure about BTZ (Banãdo, Teitelboim, Zanelli) [24] black hole spacetime, since the earlier models cannot include the inner horizon and present only the equatorial slice of the Kerr geometry.

As is well-known, the BTZ black hole is a solution to Einstein's field equation in a (2 + 1) dimensional theory of gravity with a negative cosmological constant. The (2 + 1) dimensional theory of gravity is usually considered a useful model to explore the foundations of classical and quantum gravity [25–28], and its solution of BTZ black holes is considered an effective avenue to realize this to some extent [29]. The BTZ black hole has the common features of a general black hole, i.e. it has an event horizon and an inner horizon in a rotating case, it appears as the final state of collapsing matter, and it has thermodynamic properties much like those of a (3 + 1)-dimensional black hole. Meanwhile, the BTZ black hole has some important different properties from the Schwarzschild or Kerr black hole, i.e. it is asymptotically anti-de Sitter rather than asymptotically flat and has no curvature singularity at the origin, which provides a simple platform for understanding the AdS/CFT correspondence or the properties of black holes relevant to string theory [30]. Therefore, it is significant to find the analogue systems to present the structure of BTZ black holes in the laboratory. It is noted that the nonrelativistic acoustic metric can match the rotating BTZ black hole metric in form [14], but the matching form for the fluid velocity is hard to be realized in the actual experimental systems. In this paper, we will give a method to simulate the metric approximately but realize nearly all the structures required by the BTZ black hole spacetime.

The paper is organized as follows. In the second section, the optical analog of BTZ black holes is reviewed. The corresponding quantities between the BTZ black hole and the acoustic metric are given, and the formation conditions for the horizon are depicted with a figure. In the third section, we discuss a concrete model using the optical vortex to realize the spacetime structure of BTZ black holes. In particular, we use the method of numerical simulation to present the feasibility of our suggested optical vortex. Finally, we give the conclusion and discussion for the future prospect in the fourth section.

2. Optical analogue of BTZ black holes

The crucial point for the analogue of the black hole is the formation of the horizons, which is usually dependent on the consideration of the black hole spacetime as a moving fluid, i.e. the fluid as the background is flowing along a direction to the region beyond the Newtonian escape velocity (i.e. the local velocity of sound), and the point where the velocity of background fluid equals the sound velocity represents the horizon of the black hole. For our purpose, the optical vortices are used since they can carry the angular momentum. In order to simulate a rotating acoustic BTZ black hole, there should be an effective method to form the inner horizon beside that realized in the earlier simulation for the rotating black holes where the event horizon and the ergosphere had been formed. In this section, we consider a monochromatic laser beam propagating through a bulk medium, as in [19, 21], and discuss how to relate it to a BTZ black hole metric.

We start with the nonlinear Schrödinger equation (NLSE) in the paraxial approximation, which governs the evolution of the electric field $E(x, y, z)$ of the vortex beam

$$\partial_z E = \frac{\mathrm{i}}{2k}\nabla_\perp^2 E + \mathrm{i}\frac{kn_2}{n_0}E\,|E|^2, \tag{1}$$

where z is the propagation direction, n_0 is the linear refractive index and n_2 is the third-order nonlinear coefficient, $k = \frac{2\pi n_0}{\lambda}$ is the longitudinal wave number, $|E|^2$ is the optical intensity, and $\nabla_\perp^2 E$ is defined with respect to transverse coordinates (x, y), which can accounts for diffraction while the nonlinear term describes the self-defocusing effect.

To present the relation with the fluid dynamics, the electric field can be written with the form, $E = \rho^{1/2}\mathrm{e}^{\mathrm{i}\phi}$. Thus, NLSE equation (1) becomes the hydrodynamic continuity and Euler equations [1, 21]

$$\partial_z\rho + \nabla\cdot(\rho\mathbf{v}) = 0, \tag{2}$$

$$\partial_z\psi + \frac{1}{2}v^2 + \frac{c^2 n_2}{n_0^3}\rho - \frac{c^2}{2k^2 n_0^2}\frac{\nabla^2\rho^{1/2}}{\rho^{1/2}} = 0, \tag{3}$$

where c is the speed of light, the optical intensity ρ corresponds to the fluid density, $\mathbf{v} = \frac{c}{kn_0}\nabla\phi \equiv \nabla\psi$ is the fluid velocity, and the third term is the quantum pressure which is usually ignored in the linearized process for the derivation of the analogue metric (see equation (4) below), but it actually plays an important role close to the horizons [31], determining the width of the horizon and regularizing the otherwise singular solution.

We linearize the density and the velocity potential with $\rho = \rho_0 + \rho_1$, $\psi = \psi_0 + \psi_1$, where ρ_0 and ψ_0 are related to the background fluid determined by the equations of motion, and ρ_1 and ψ_1 are small perturbations. Then, we put the new velocity potential and mass density after perturbations into equations (2) and (3). Finally, the two equations are combined into one about the fluctuation ψ_1. It is found that the fluctuation ψ_1 satisfies a wave equation, $\nabla^2\psi_1 = (1/\sqrt{-g})\partial_\mu(\sqrt{-g}\,g^{\mu\nu}\partial_\nu\psi_1) = 0$, with the metric given as [2, 21]

$$\mathrm{d}s^2 = \left(\frac{\rho_0}{c_s}\right)^2[-(c_s^2 - v_0^2)\mathrm{d}t^2 - 2v_r\mathrm{d}r\mathrm{d}t - 2v_\theta r\mathrm{d}\theta\mathrm{d}t + \mathrm{d}r^2 + (r\mathrm{d}\theta)^2], \tag{4}$$

where c_s is the local speed of sound and is related to the bulk pressure which is derived from the nonlinear interaction. The terms v_r and v_θ represent the radial and tangential velocity components, with their definitions as $v_r = \partial_r\psi_0$ and $v_\theta = \frac{1}{r}\partial_\theta\psi_0$, from which the total velocity is $v_t^2 = v_r^2 + v_\theta^2$. For the metric (4), it can model a spacetime about a black hole if the region $v_r > c_s$ exists where anything cannot escape, and it can model a spacetime about the ergosurface of a rotating black hole if the region $v_t > c_s > v_r$ exists where it is impossible for an observer to remain stationary relative to a distant observer. So, it is appropriate to simulate the properties of a rotating black hole, as made in [19], but the inner horizon cannot be found in their work. Here, we first see if the metric (4) can match the BTZ black hole metric in form.

The rotating BTZ black hole metric is given as [24]

$$\mathrm{d}s^2 = -\left(-M + \frac{r^2}{l^2} + \frac{J^2}{4r^2}\right)\mathrm{d}t'^2 + \frac{\mathrm{d}r^2}{-M + \frac{r^2}{l^2} + \frac{J^2}{4r^2}} + r^2\left(\mathrm{d}\theta' - \frac{J}{2r^2}\mathrm{d}t'\right)^2, \tag{5}$$

where $\Lambda = -\frac{1}{l^2}$ is the negative cosmological constant and the units with $c = G = 1$ are taken. M and J are the mass and angular momentum of the black hole, and are given as

$$M = \frac{r_+^2 + r_-^2}{l^2}, \qquad J = \frac{2r_+ r_-}{l}, \tag{6}$$

where r_+, r_- represent the positions of the outer and inner horizon of the black hole, respectively. They are determined by $-M + \frac{r^2}{l^2} + \frac{J^2}{4r^2} = 0$, which leads to the results, $r_\pm^2 = \frac{Ml^2}{2}\left(1 \pm \sqrt{1 - \left(\frac{J}{Ml}\right)^2}\right)$. We may assume without loss of generality that $J \geqslant 0$ and assume that $Ml \geqslant J$ to ensure the existence of an event horizon at $r = r_+$. In particular, the ergosurface is specified at the position $r_E = \sqrt{M}\, l = \sqrt{r_+^2 + r_-^2}$.

To compare conveniently with the metric (4), we make the transformation for the BTZ black hole metric from the coordinates t', θ' to t, θ according to the following forms

$$\mathrm{d}t = \mathrm{d}t' - \frac{\sqrt{1 - (-M + \frac{r^2}{l^2} + \frac{J^2}{4r^2})}}{-M + \frac{r^2}{l^2} + \frac{J^2}{4r^2}},$$

$$\mathrm{d}\theta = \mathrm{d}\theta' - \frac{J}{2r^2}\frac{\sqrt{1 - (-M + \frac{r^2}{l^2} + \frac{J^2}{4r^2})}}{-M + \frac{r^2}{l^2} + \frac{J^2}{4r^2}}. \tag{7}$$

This changes the metric (5) into the form

$$\begin{aligned} \mathrm{d}s^2 = &-\left(-M + \frac{r^2}{l^2}\right)c_s^2\mathrm{d}t^2 \\ &- 2c_s\sqrt{1 - \left(-M + \frac{r^2}{l^2} + \frac{J^2}{4r^2}\right)}\mathrm{d}t\mathrm{d}r \\ &+ Jc_s\mathrm{d}t\mathrm{d}\theta + \mathrm{d}r^2 + r^2\mathrm{d}\theta^2, \end{aligned} \tag{8}$$

where the speed of light is introduced again but with the local expression c_s.

Thus, we can link the nonrelativistic metric (4) with the BTZ black hole metric (8) up to the overall factor $\left(\frac{\rho_0}{c_s}\right)^2$ by letting

$$v_r = c_s\sqrt{1 - \left(-M + \frac{r^2}{l^2} + \frac{J^2}{4r^2}\right)}, \tag{9}$$

$$v_\theta = c_s\frac{J}{2r}. \tag{10}$$

It is seen that the metric (4) is valid only in the range where the radial velocity $v_r = c_s\sqrt{1 - \left(-M + \frac{r^2}{l^2} + \frac{J^2}{4r^2}\right)}$ is well defined, and this holds for $R_- < r_- < r_+ < R_+$ with

$$R_\pm^2 = \frac{(1+M)l^2}{2}\left[1 \pm \sqrt{1 - \left(\frac{J}{(1+M)l}\right)^2}\right], \tag{11}$$

where $R_\pm$ represents the boundary that the analogue can be made. It is easy to check for the consistency of the horizons, which is determined in the fluid aspect by the relation $v_r(r) = c_s(r)$ just at $r_\pm$ that is determined in the BTZ black hole aspect. Similarly, it has the same result with $v_t(r) = c_s(r)$ for the determination of the ergosurface. Moreover, to ensure that the ergosphere can be observed in the analogue system, it requires $r_E < R_+$, which leads

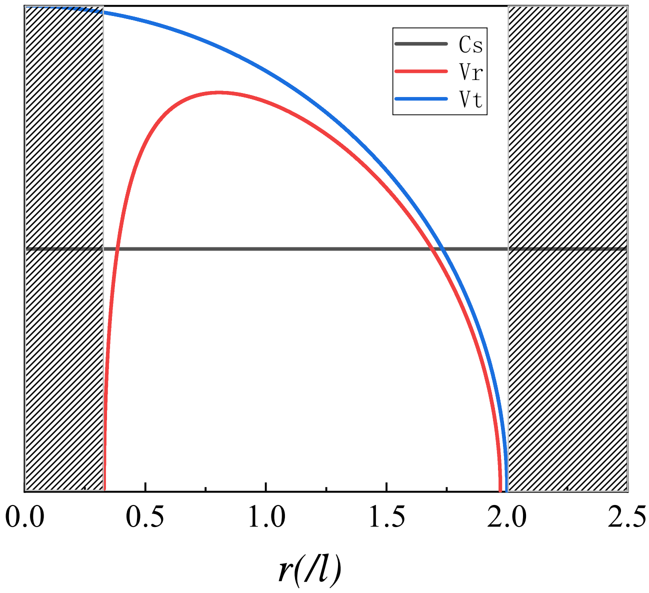

Figure 1. The velocities of the fluid as a function of the radius for the representation of the rotating BTZ black hole analogue with $M = 3$ and $J = 1.3l$. The red line represents the radial velocity which leads to the horizons at the intersections with the black line of the sound speed. The blue line represents the total velocity which gives the position for the ergosurface when it matched with the sound speed. The shaded gray regions represent the invalid range in which the acoustic analogue breaks down because the radial velocity is ill defined.

to a new constrained condition $Ml^2 > J^2$. Therefore, under all these constrained conditions, it is obtained that the acoustic metric realized by the optical vortex can be regarded as the analogue of the BTZ black hole with the nearly perfect match in form up to an overall factor.

In figure 1, we present the actual conditions required for the fluid as the realization of the analogue BTZ black hole, with the parameters $M = 3$, $J = 1.3l$ $(< \sqrt{M}\, l)$, which gives $r_+ \approx 1.69l$, $r_- \approx 0.38l$, $r_E \approx 1.73l$, $R_+ \approx 1.98l$ and $R_- \approx 0.33l$ by the intersections of the line of the local speed of sound with the lines of $v_r(r)$ and $v_t(r)$. Note that here c_s is assumed to be a constant without loss of generality, but in the actual physical systems such as BEC [5] or the optical vortex [19, 21], the velocity c_s is not constant. In fact, the critical point for the analogue is the realization of the relations, i.e. $v_r(r) = c_s(r)$, $v_t(r) = c_s(r)$ as stated above. R_+ and R_- give the boundary beyond which the simulation of an acoustic rotating BTZ black hole would be invalid. Obviously, they satisfy the constraint: $R_- < r_- < r_+ < r_E < R_+$. Besides the above-mentioned constraints, we still have to consider the continuity of the fluid flow in the practical simulation. In the case here, it is required to fine-tune the density as $\rho \propto \frac{1}{r\sqrt{1 - \left(-M + \frac{r^2}{l^2} + \frac{J^2}{4r^2}\right)}}$ to satisfy the continuity equation $\nabla \cdot (\rho \mathbf{v}) = 0$.

3. Optical realization

In the last section, we have presented how to simulate the BTZ black hole using the fluid model. However, the required form for the radial and tangential velocities is hard to be found in the real physical system. In the earlier experiment [19], a monochromatic laser beam, with the electric field $E_0 = \sqrt{\rho_0(r)}\exp(im\theta)$, where m is the topological charge integer of the optical vortex, is used to simulate the background spacetime as a photon fluid, in which the radial velocity $v_r(r) = \frac{c}{kn_0}\frac{-\pi}{\sqrt{r_0 r}}$ and tangential velocity $v_\theta(r) = \frac{c}{kn_0}\frac{m}{r}$ is obtained respectively. This can present the phenomena of rotating black holes like the ergosphere, but it does not include the inner horizon which is also a vital component for the structure of rotating black holes. In this section, we consider a special Gauss optical vortex with the electric field

$$E_0 = \sqrt{\rho_0(r)}\exp\left(im\theta - 2i\pi(Ar - B\ln r - Cr^2)\right), \quad (12)$$

where $\rho_0(r) = \rho_0\left(\frac{r}{\sigma}\right)^{1/2}\exp\left(-\frac{2r^2}{\sigma^2}\right)$ takes the Gaussian-like form, and ρ_0, A, B, C are the constants. The corresponding phase is given as $\varphi(r) = -2\pi(Ar - B\ln r - Cr^2)$, which leads to the new expressions for the radial and tangential velocities,

$$v_r(r) = \frac{-2\pi c}{kn_0}\left(13 - \frac{3}{r} - 5r\right), \quad (13)$$

$$v_\theta(r) = \frac{c}{kn_0}\frac{m}{r}, \quad (14)$$

when the optical vortex is injected into a self-defocusing medium (nonlinear refractive index $n_2 < 0$) without cavity. Here, the radial velocity is obtained by taking the radial derivative of the phase φ and the constants are taken as $A = 13$, $B = 3$, $C = 2.5$ for matching the geometric structure of the BTZ black holes in the last section, i.e. see figure 2.

In figure 2, we present the numerical results with the evolution in equation (1) for the change of the radial velocity $v_r(r)$, the total velocity $v_t(r) = \sqrt{v_\theta^2 + v_r^2}$, and the speed of sound $c_s(r) = \sqrt{c^2|\gamma|\rho_0(r)/n_0^3}$ (where γ is the nonlinear coefficient of the medium) with the radius r. It is seen that the corresponding structure is determined by the relations, i.e. $v_r(r) = c_s(r)$ for the horizons and $v_t(r) = c_s(r)$ for the ergosurface, which are seen by the intersections of velocity curves, i.e. the left (right) intersection of velocity curves $v_r(r)$ and $c_s(r)$ determines the position of the inner (outer) horizon, and the intersection of velocity curves $v_t(r)$ and $c_s(r)$ determines the position of the ergosurface. This is nearly the same as the requirement of simulating BTZ black holes as in figure 1, but the boundary of the valid analogue determined by $v_r(r) = 0$, or $R_+ \approx 2.34$ and $R_- \approx 0.25$, is larger than that for the analogue BTZ black hole in figure 1.

The upper panel of figure 2 presents the analogue structure at the distance of 0.5 millimeters for the propagation of the optical beam in the nonlinear medium, which shows that the nonlinear interaction makes the analogue black hole structure form quickly. When the beam propagates in the medium for a distance of 14 centimeters, it is seen in the lower panel of figure 2 that the analogue BTZ black hole structure remains well. This shows the stability of the analogue black hole structure for the whole process of the optical vortex passing through the nonlinear medium, and it is advantageous for further studies about the analogous properties of black holes, i.e. Hawking radiations. It is noted that Hawking radiation had been discussed before using the optical vortex and a very interesting phenomenon about resonance enhancement of Hawking radiation was found [15] in which the quantum potential was considered when the analysis is near the horizon. Although they gave an attractive model including a white-hole horizon and the corresponding ergosurface, and a black-hole horizon and the corresponding ergosurface, the inner horizon required for the rotating BTZ black holes is still not included. It is significant to see if this phenomenon of Hawking radiation enhancement exists for our suggested model which included a complete BTZ black

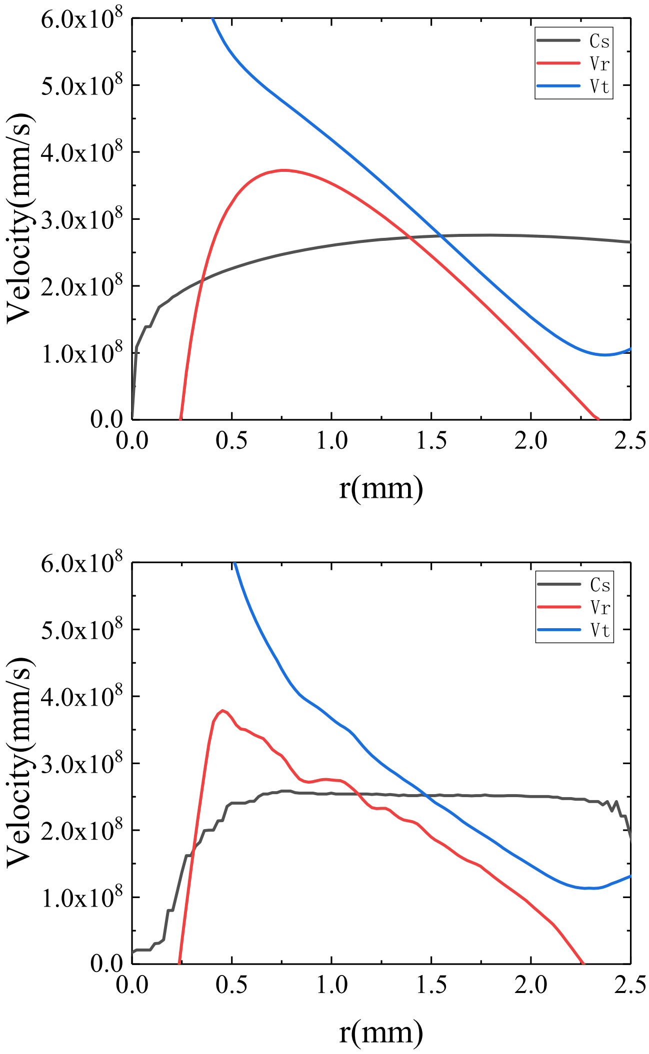

Figure 2. The velocities of the fluid as a function of the radius obtained by numerically calculating the evolution equation (1) with the initial velocities as in equations (13) and (14). The parameters are taken according to the actual experimental requirements, i.e. the linear refractive index $n_0 = 1.5$, the nonlinear coefficient $|\gamma| = 4.4 \times 10^{-7}\ \mathrm{cm}^2\ \mathrm{W}^{-1}$, the power of the laser $p = 140$ mW, the beam waist of the laser $\sigma = 5$ mm, and the topological charge integer $m = 20$. The upper (lower) panel presents the results as the propagation distance of the optical vortex into the medium is 0.5 mm (14 cm). The lines have the same meaning as in figure 1.

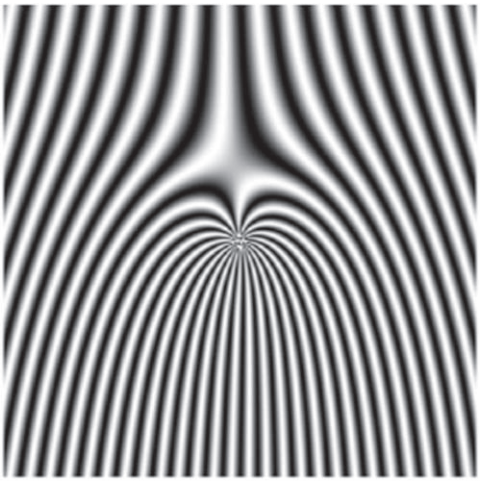

Figure 3. Hologram for the optical vortex satisfying the required phase given in equation (12). The bifurcated part represents the topological charge integer $m = 20$.

hole spacetime structure. This would be investigated in our future work.

Moreover, if the topological charge takes a smaller number, i.e. $m = 2$, the analogue BTZ black hole structure still exists, but the distance between the outer horizon and the ergosurface is a little small and cannot be displayed in the plot at the scale of millimeters. Notwithstanding, we give an approximate fluid model to present all the elements required by the BTZ black holes. The next crucial question is: can the form in equation (12) about the optical vortex be realized in the present experiment? In the following, we will interpret this using the numerical simulation.

Due to the widespread technological applications in manipulating small macroscopic particles [32], the cooling and trapping of neutral atoms [33], BEC [34], driving micro fabricated gears for creating micrometre-scale motors [35] and so on, there has been increasing interest in the generation of optical vortex which has helical wavefront and undefined phase at an isolated zero amplitude point in recent years. Several methods for generating optical vortex have been reported, including the use of computer-generated holograms [36], geometric mode converters [37, 38], spatial light modulators (SLM) [39–41], and spiral phase plates [42–44]. Here we suggest a method using liquid crystal spatial SLM to generate the optical vortex with the requirement given in equation (12). The crystal spatial SLM can regulate the amplitude and phase of the optical waves by the electro-optical effect of the liquid crystal. In order to generate such an optical vortex, one has to obtain the hologram of the phase $\varphi(r)$ at first, as given in figure 3 using the numerical simulation, and then crave an optical grating according to the hologram and load the grating into the pure-phase SLM of liquid crystal. When a planar light is guided through the SLM with the loaded holographic optical grating, the required optical vortices would be generated by the exiting light. The phase and the near-field intensity are simulated numerically, as given in figures 4 and 5. It is noted that the center of the optical vortex is dark in figure 5. However, when the vortex beam is injected into the nonlinear self-defocusing medium, the central dark core will shrink, and the bright ring will expand and become flattened for the light intensity of the transverse profile, as seen in figure 2. In particular, it will form optical solitons under certain experimental conditions.

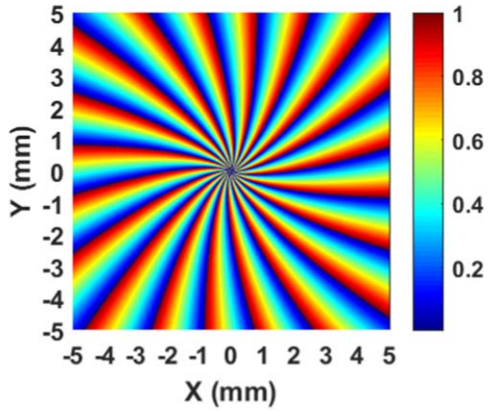

Figure 4. The simulated phase of the optical vortex. The phase increases according to the color from blue to red. The repeated time of the color is the topological charge integer $m = 20$.

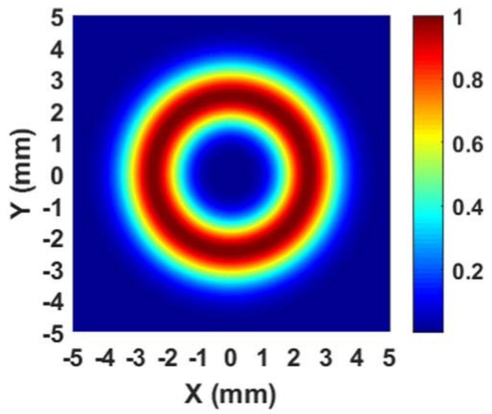

Figure 5. The simulated intensity of the optical vortex. The intensity increases according to the color from blue to red.

4. Conclusion and discussion

In this paper, we review the earlier suggestion using the photon fluid in an optical vortex to simulate the rotating black holes. It is noted that their analogue did not include the inner horizon that is characteristic of the rotating black hole. We use the same physical systems to give a kind of analogue black hole including the inner horizon. At first, we compare the acoustic metric obtained from the photon fluid model with the BTZ black hole metric and give the expressions for the flowing velocity of the fluid in the radial and tangential directions. With these, we have described the spacetime structure of the BTZ black hole in the analogue fluid model and determined the conditions for the formation of the inner and outer horizon and the ergosurface. However, the radial velocity in equation (9) is not integrable along the radius and so it cannot find an exact expression for the phase of the optical vortex. Fortunately, an approximate form for the phase has been found, and all the structures required for the BTZ black hole can be presented. We have also given a suggestion about how to generate the optical vortex and obtained the properties of its wavefront and intensity using the method of numerical simulation.

With regard to the interesting properties related to AdS/CFT or the string theory that could be presented using the BTZ black hole, it is significant to realize the analogue BTZ black hole in the lab which would provide a physical platform for the corresponding exploration. As far as we know, there was a suggestion to simulate the BTZ black hole in the real physical system, so our work might make up a helpful suggestion for future experiments about analogue gravity.

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