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Nonrelativistic quantum effects of the Lorentz symmetry violation on the Morse potential

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Abstract

We search for Lorentz symmetry violation effects at low-energy regime by exploring the Dirac equation in $(1+1)$-dimensions and the possibility of dealing with quantum systems with spherical symmetry. We bring a discussion about the influence of the Lorentz symmetry violation effects on the spectrum of molecular vibrations caused by the coupling between a fixed vector field and the derivative of the fermionic field. Further, we discuss the influence of this Lorentz symmetry violation background on the revival time.

Keywords: Lorentz symmetry violation, Morse potential, quantum revivals, derivative of the fermionic field

1. Introduction

The Weinberg–Salam–Glashow Standard Model (SM) [1] has achieved enormous success in explaining the origin of quantum particles and their properties. The mass of these particles also has a mechanism that explains its origin: the Anderson–Higgs Mechanism. However, despite containing the description of the fundamental forces (the weak interaction, the electromagnetic interaction and the strong interaction), the gravitational interaction is outside the model. Another problem is that SM does not explain the unbalance matter-antimatter. Then, through these gaps, the search for a more fundamental theory is fully justified.

The idea of extending the Higgs Mechanism opens up the possibility that new background fields that violate the Lorentz symmetry [2, 3] could be detected. In this way, effective theories that consider renormalizability have begun to be proposed [4]. These theories are collected in a proposal known as the Standard Model Extension (SME) [5]. In short, if we relax the normalization condition as an effective theory, we achieve the non-minimal formulation beyond SM [6, 7]. This possibility allows us to explore scenarios with spontaneous Lorentz symmetry breaking in which background fields can appear [8, 9]. These fields must appear tenuously in low-energy physics. The emergence of these fields from the spontaneous symmetry breaking can modify the transport properties of particles [10]. Thereby, we can establish new investigations which are different from the usual ones (Large Hadrons Collider) to investigate a physics beyond SM. Indeed, SME has inspired many works, such as, works that cover several different aspects of fermion systems [11, 12], CPT-probing experiments [13], the electromagnetic CPT- and Lorentz-odd term [14] and the nineteen electromagnetic CPT-even and Lorentz-odd coefficients [15]. Recently, we have studied Lorentz symmetry breaking effects on a fermion in the context of nonrelativistic quantum mechanics [16–20].

In this work, we follow the seminal work of Kostelecký and Lane [3], where they proposed several models of studying the fermionic sector of SME at low-energy regime [11, 12]. Our focus is on the Lorentz symmetry violation effects caused by the coupling between a fixed vector field f^{μ} and the derivative of the fermionic field. In another perspective, it is worth citing the coupling between the derivative of the scalar field and a fixed vector field in which has been studied within

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the Casimir effect [21, 22]. In the context of relativistic quantum mechanics, the influence of this coupling between the derivative of the scalar field and a fixed vector field on a set of central potentials has been studied in [23–25]. Hence, inspired by [21–24], we raise a discussion about possible changes in the spectrum of molecular vibrations caused by the coupling between a fixed vector field f^{μ} and the derivative of the fermionic field. This gives us a perspective of searching for Lorentz symmetry violation effects at low-energy regime as proposed by Kostelecký and Lane [3]. Further, we discuss the influence of the coupling between a fixed vector field and the derivative of the fermionic field on the quantum revivals [26–29] related to the spectrum of molecular vibrations.

This paper is organized as follows: in section II, we introduce the Dirac equation (1+1)-dimensions with the coupling between a fixed vector field and the derivative of the fermionic field, and thus, obtain its nonrelativistic limit; in section III, we analyse the nonrelativistic effects of the Lorentz symmetry violation on the Morse potential [30–32]; in section 4, we extend our discussion to the revival time [26–29]; in section 5, we present our conclusions.

2. Nonrelativistic wave equation in a background of the Lorentz symmetry violation

In this section, our aim is to study the low-energy phenomena which can be influenced by the spontaneous Lorentz symmetry violation. It may occur on an energy scale in which SM is no longer valid. Our focus is on the fermionic sector of SME at low-energy regime [3, 11, 12]. Therefore, let us start by introducing the Dirac equation proposed in [3], where the description of the violation of the Lorentz symmetry is made by a fixed vector field in the form (we shall work with the units $\hbar = 1$ and $c = 1$):

$$\mathrm{i}\gamma^{\mu}\partial_{\mu}\Psi + if^{\mu}\,\gamma^{5}\,\mathrm{i}\,\partial_{\mu}\,\Psi = m\,\Psi. \tag{1}$$

The parameter f^{μ} determines the extent of the Lorentz symmetry violation [3]. Thereby, the term $B^{\mu} = f^{\mu}\gamma^{5}$ corresponds to a fixed vector field which yields a privileged direction in the spacetime. Therefore, the effects of the Lorentz symmetry violation are caused by the coupling between the fixed vector field $f^{\mu}\gamma^{5}$ and the derivative of the fermionic field. Henceforth, we work with the Dirac equation in (1 + 1)-dimensions ($\mathrm{d}s^2 = -\mathrm{d}t^2 + \mathrm{d}x^2$), where the Dirac matrices are defined in the form [33]:

$$\hat{\beta} = \sigma^{3};\;\; \hat{\alpha}^{1} = \sigma^{1};\;\; \hat{\beta}\,\gamma^{5} = \sigma^{2}. \tag{2}$$

The matrices $\sigma^{i} = (\sigma^{1}, \sigma^{2}, \sigma^{3})$ corresponds to the Pauli matrices

$$\sigma^{1} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix};\;\; \sigma^{2} = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix};\;\; \sigma^{3} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{3}$$

From now on, we assume that f^{μ} is given by

$$f^{\mu} = (0,\, \varsigma). \tag{4}$$

In this way, the Dirac equation (1) becomes

$$\begin{aligned} \mathrm{i}\,\frac{\partial\Psi}{\partial t} &= m\,\hat{\beta}\Psi - \mathrm{i}\,\hat{\alpha}^{1}\,\frac{\partial\Psi}{\partial x} + \varsigma\,\hat{\beta}\,\gamma^{5}\,\frac{\partial\Psi}{\partial x} \\ &= m\,\sigma^{3}\Psi - \mathrm{i}\sigma^{1}\,\frac{\partial\Psi}{\partial x} + \varsigma\,\sigma^{2}\,\frac{\partial\Psi}{\partial x}. \end{aligned} \tag{5}$$

With the purpose of searching for effects of the Lorentz symmetry violation in the nonrelativistic limit, let us write

$$\Psi(t,\, x) = \mathrm{e}^{-\mathrm{i}mt}\begin{pmatrix} \psi \\ \phi \end{pmatrix}, \tag{6}$$

where ψ is the large components and ϕ is the small components of $\Psi(t, x)$ [34]. Thereby, after some calculations, we obtain the Schrödinger equation (with the units $\hbar = 1$ and $c = 1$):

$$\mathrm{i}\,\frac{\partial\psi}{\partial t} = -\frac{1}{2m}(1 - \varsigma^{2})\frac{\partial^{2}\psi}{\partial x^{2}}. \tag{7}$$

An interesting aspect of the Dirac equation in (1 + 1)-dimensions is the possibility of dealing with quantum systems with spherical symmetry [35–37]. In (1 + 1)-dimensions, the Dirac equation (5) or its nonrelativistic limit given by equation (7) allow us to analyse the s-waves in a quantum system with spherical symmetry. Therefore, in the following section, we bring a discussion about the influence of the Lorentz symmetry violation effects on the Morse potential [30–32].

3. Morse potential

In this section, we focus on the influence of the background of the Lorentz symmetry violation determined by the fixed vector field (4) on the Morse potential [30–32]:

$$V(x) = V_{0}[\mathrm{e}^{-2\,a\,x} - 2\,\mathrm{e}^{-a\,x}], \tag{8}$$

where $V_0 > 0$ and $a > 0$ are constants. With the potential energy (8), the Schrödinger equation (7) becomes

$$\begin{aligned} \mathrm{i}\,\frac{\partial\psi}{\partial t} &= -\frac{1}{2m}(1 - \varsigma^{2})\frac{\partial^{2}\psi}{\partial x^{2}} \\ &\quad + V_{0}[\mathrm{e}^{-2\,a\,x} - 2\,\mathrm{e}^{-a\,x}]\psi. \end{aligned} \tag{9}$$

The solution to equation (9) is given by $\psi(t,\, x) = \mathrm{e}^{-\mathrm{i}\,E\,t}\,u(x)$, thus, after substituting this solution into equation (9), we obtain

$$\frac{\mathrm{d}^{2}u}{\mathrm{d}x^{2}} - \frac{2m\,V_{0}}{(1 - \varsigma^{2})}[\mathrm{e}^{-2\,a\,x} - 2\,\mathrm{e}^{-a\,x}]\,u + \frac{2mE}{(1 - \varsigma^{2})} = 0. \tag{10}$$

Let us define the dimensionless parameter:

$$y = \frac{2}{a}\sqrt{\frac{2m\,V_0}{(1-\varsigma^2)}}\;\mathrm{e}^{-a\,x}. \tag{11}$$

Thereby, equation (10) can be rewritten in the following form:

$$\frac{\mathrm{d}^2u}{\mathrm{d}y^2} + \frac{1}{y}\frac{\mathrm{d}u}{\mathrm{d}y} + \frac{2mE}{a^2(1-\varsigma^2)y^2}\,u + \frac{1}{a\,y}\sqrt{\frac{2m\,V_0}{(1-\varsigma^2)}}\,u - \frac{1}{4}\,u = 0. \tag{12}$$

With the purpose of obtaining bound state solutions, we assume that $E < 0$. Then, we define the parameters:

$$\gamma = \frac{1}{a}\sqrt{\frac{-2mE}{(1-\varsigma^2)}}; \qquad \delta = \frac{1}{a}\sqrt{\frac{2m\,V_0}{(1-\varsigma^2)}}. \tag{13}$$

Hence, equation (12) becomes

$$\frac{\mathrm{d}^2u}{\mathrm{d}y^2} + \frac{1}{y}\frac{\mathrm{d}u}{\mathrm{d}y} - \frac{\gamma^2}{y^2}\,u + \frac{\delta}{y}\,u - \frac{1}{4}\,u = 0, \tag{14}$$

whose solution is given by

$$u(y) = \mathrm{e}^{-y/2}\,y^{\gamma}\;{}_1\mathrm{F}_1\!\left(\gamma + \frac{1}{2} - \delta,\, \gamma + 1;\, y\right). \tag{15}$$

Note that $\gamma > 0$ (see equation (13)) and ${}_1F_1\left(\gamma + \frac{1}{2} - \delta,\, \gamma + 1;\, y\right)$ is the confluent hypergeometric function [38, 39]. We should observe that the asymptotic behaviour of this confluent hypergeometric function for large values of its argument is given by [39]

$${}_1F_1(A, B;\, y) \approx \frac{\Gamma(B)}{\Gamma(A)}\,\mathrm{e}^{y}\,y^{A-B}[1 + \mathcal{O}(|y|^{-1})]. \tag{16}$$

Since it diverges when $y \to \infty$, thus, the bound states solutions can be achieved by imposing that $A = -n$ (where $n = 0, 1, 2, 3, \ldots$), i.e. by imposing that

$$\gamma + \frac{1}{2} - \delta = -n. \tag{17}$$

It is worth observing that this condition guarantees that the confluent hypergeometric function becomes well-behaved when $y \to \infty$. Therefore, by using the parameters defined in equation (13), we obtain from equation (17):

$$E_n = -V_0\left[a\,\sqrt{\frac{(1-\varsigma^2)}{2m\,V_0}}\left(n + \frac{1}{2}\right) - 1\right]^2. \tag{18}$$

Therefore, equation (18) corresponds to the energy levels of the Morse potential (8) under the influence of the Lorentz symmetry violation background determined by the coupling between the fixed vector field $f^{\mu}\gamma^{5}$ and the derivative of the fermionic field. We can observe that the background of the Lorentz symmetry violation modifies the spectrum of energy of the Morse potential. The effects associated with the presence of the Lorentz symmetry violation background can be viewed through the presence of the parameter ς in the energy levels (18). Furthermore, from the definition of the parameter γ given in equation (13), we have that $\gamma > 0$. Thus, from equation (17), we have $\delta - \left(n + \frac{1}{2}\right) > 0$. Then, we obtain

$$n_{\max} < \frac{1}{a}\sqrt{\frac{2m\,V_0}{(1-\varsigma^2)}} - \frac{1}{2}, \tag{19}$$

otherwise, no bound states exist. Therefore, equation (19) shows that there is an upper limit to the quantum number n, which is influenced by the background of the Lorentz symmetry violation. The quantum number n takes values from zero to the upper limit ($n_{\max}$) given in equation (19). According to [31], the existence of this upper limit means that the number of energy levels is limited.

Finally, by taking $\varsigma \to 0$ in equations (18) and (19), we recover the energy levels of the Morse potential in the absence of the violation of the Lorentz symmetry [30–32].

4. Quantum revivals

In recent years, the appearance of quantum revivals has been discussed in the infinite square well [28, 29, 40–42], quantum pendulum [43], position-dependent mass systems [44], Rydberg atoms [45–47], graphene [48, 49] and under the influence of a spiral dislocation [50]. According to [26–29], quantum revivals occurs when the wave function recovers its initial shape at a time called the revival time. By considering a quantum system that possesses one quantum number n, the revival time is obtaining from the energy eigenvalues when we expand them about the central value n_1 of the quantum number n. In this way, the energy eigenvalues can be expanded in Taylor series as [26, 27]:

$$E_n \approx E_{n_1} + \left(\frac{\mathrm{d}E}{\mathrm{d}n}\right)_{n=n_1}(n - n_1) + \frac{1}{2}\left(\frac{\mathrm{d}^2E}{\mathrm{d}n^2}\right)_{n=n_1}(n - n_1)^2 + \cdots \tag{20}$$

Therefore, there are distinct time scales. The classical period is given by

$$T_{\mathrm{cl}} = \frac{2\pi\hbar}{\left|\left(\frac{\mathrm{d}E}{\mathrm{d}n}\right)_{n=n_1}\right|}, \tag{21}$$

while the revival time is defined by [26, 27]

$$\tau = \frac{4\pi\hbar}{\left|\left(\frac{\mathrm{d}^2E}{\mathrm{d}n^2}\right)_{n=n_1}\right|}. \tag{22}$$

Our interest in the revival time is focused on the influence of the Lorentz symmetry violation determined by the fixed vector field (4) on it. Thereby, with respect to the revival time (22), we obtain from the energy levels (18) (with $\hbar = 1$):

$$\tau = \frac{4\pi\,m}{a^2(1-\varsigma^2)}. \tag{23}$$

Hence, equation (23) shows that the revival time is influenced by background of the Lorentz symmetry violation determined by the coupling between the fixed vector field $f^{\mu}\gamma^{5}$ and the derivative of the fermionic field. By taking $\varsigma \to 0$ in equation (23), we thus obtain the revival time of a quantum particle subject to the Morse potential in the absence of the violation of the Lorentz symmetry.

5. Conclusions

We have studied effects of the Lorentz symmetry violation at low-energy regime, where the background of the Lorentz symmetry violation is caused by the coupling between a fixed vector field $f^{\mu}\gamma^{5}$ and the derivative of the fermionic field. By considering $(1+1)$-dimensions, we have studied the influence of the coupling between a fixed vector field $f^{\mu}\gamma^{5}$ and the derivative of the fermionic field on the Morse potential. We have seen that the spectrum of energy and the number of energy levels are influenced by the background of the Lorentz symmetry violation. In addition, we have seen that the revival time is influenced by the coupling between the fixed vector field and the derivative of the fermionic field.

The possibility of working with the Dirac equation in $(1+1)$-dimensions and dealing with quantum systems with spherical symmetry [35–37] also opens discussions about the search for Lorentz symmetry breaking effects in nanosystems. For instance, the Aharonov–Bohm effect [51, 52] is investigated in a nanosphere in [53]. The authors of [53] also introduce a model for a quantum ring in the spherical space. Clearly, the search for Lorentz symmetry breaking effects in nanosystems is not limited to the spherical symmetry. Quantum dots and quantum rings allow us to extend our discussion to the quantum systems with cylindrical symmetry [54–59]. In view of these kind of nanostructures, they can be a good hint about searching for effects of the coupling between a fixed vector field $f^{\mu}\gamma^{5}$ and the derivative of the fermionic field at low-energy regime.

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