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Rotating effects on the thermophysical properties of a two-dimensional GaAs quantum ring

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Abstract

In this work, we have investigated the rotating effect on the thermodynamic properties of a 2D quantum ring. Accordingly, we have considered the radial potential of a 2D quantum ring and solved the Schrödinger equation in the presence of the Aharonov–Bohm effect and a uniform magnetic field for the considered potential. According to the solution of the equation, we calculated the eigenvalues and eigenfunctions of the considered system. Using the calculated energy spectrum, we obtained the partition function and thermodynamic properties of the system, such as the mean energy, specific heat, entropy and free energy. Our results show that the rotating effect has a significant influence on the thermophysical properties of a 2D quantum ring. We also study other effects of the rotating term: (1) the effect of different values of rotating parameters, and (2) the effect of negative rotation on the thermodynamic properties of the system. Our results are discussed in detail.

Keywords: Schrödinger equation, quantum ring, rotation effect, thermophysical properties

(Some figures may appear in colour only in the online journal)

1. Introduction

In recent years, the treatment of 2D quantum rings at the nanoscale has been studied under various conditions, such as specific types of confinement potential, the Aharonov–Bohm effect and external fields. These conditions cause different physical phenomena that affect the thermodynamic properties of the considered system. Notably, the influence of a radial potential modifies the Landau levels, resulting in an enhancement in magnetization and the persistent current [1, 2]. Furthermore, quantum rings exhibit thermodynamic properties, as demonstrated through determination of the partition function and other properties, such as the mean energy, entropy and specific heat. These properties can be affected by many factors, such as the confinement potential and external fields [3–10]. Quantum rings have been significantly considered by many scientists due to their physical background and industrial applications. These subjects contain the study of quantum rings. Tamborenea *et al* [11] studied the dynamics of the angular momentum in narrow quantum rings under the Rashba and Dresselhaus spin–orbit interaction. They indicated that the different angular momentum states are coupled. Gonzalez *et al* [12] considered magnetic field and topological defects, and described spin–orbit and Zeeman effects on the electronic properties of quantum rings. They showed that the absorption intensity for the considered transitions reduces compared to when the interactions are absent. Hashemi *et al* [13] investigated the effect of polarons on harmonic generation of a tuned quantum ring. They revealed that the electron–phonon interaction has a great effect on the second and third harmonic generation of quantum rings. Also, quantum rings have wide utilization in nano-flash memories [14], photonic devices [15], qubits for spintronic quantum computing [16, 17], magnetic random access memory, recording mediums and other spintronic devices [18]. For more information, the readers can refer to [19–25].

The study of rotational effects in quantum mechanics has a long history and many researchers have shown an interest in this topic [26–28]. In this regard, the Coriolis force in a rotating frame is analogous to the Lorentz force inserted on a charged particle due to a magnetic field [29]. Tsai and Neilson [30] demonstrated that the quantum interference effect is analogous to the explicit Aharonov–Bohm effect [31]. Deep understanding of how rotation effects quantum systems offers substantial insights into the fundamental treatment of particles. However, the study of rotational effects in quantum rings is an interesting recent subject. In this regard, the rotational effects in a quantum ring system have been analytically studied [32, 33]. Furthermore, recent numerical calculations have demonstrated important rotational effects on electronic states, the persistent current and magnetization in quantum rings [17, 34]. Altogether, the influence of rotation in the description of quantum rings has significant consequences on the energy levels and quantum behavior of the system. Therefore, the study of the spatial distribution of particles and the thermodynamic properties of quantum rings under rotational effects is still an open context.

From this viewpoint, this study illustrated the influence of rotation on the thermodynamic properties of quantum rings by explicitly considering the Aharonov–Bohm effect and a uniform magnetic field. In this work, we obtained the radial equation to illustrate the system. For this goal, we use the Schrödinger equation with minimal coupling, where rotation effects are incorporated by inserting the momentum factor with an effective four-parameter. In the considered equation, we insert a radial potential term that is related to the radius of a ring. It is worth noting that we limited the considered system into 2D motion and neglected the degree of freedom in the z-direction. Also, the probability distribution is illustrated for different values of the rotational quantities. Furthermore, using analytical analysis, we investigated the rotational effects on the energy levels, partition function and thermodynamic properties, such as the mean energy, entropy, specific heat and free energy. In this regard, this paper is organized as follows: in section 2, we present an in-depth description of the equation of motion and provide its mathematical computations and illustrate the concepts of the quantum ring. Also, this section shows the effects of rotation on the energy levels and thermodynamic properties of the system. In section 3, we discuss our results in detail. Finally, in section 4, we present our conclusions according to our findings and highlight the importance of the studied rotation effects in quantum mechanics.

2. Mathematical framework

In this section, we describe the quantum motion of a charged particle in a rotating state in the presence of the Aharonov–Bohm effect and a uniform magnetic field. Also, we neglect the particle spin effect. In this regard, we calculate the equation of motion and, correspondingly, the radial equation of motion. Accordingly, we obtain the particle wavefunctions and eigenenergies. Following the model indicated in chapter 17 of [35], we consider the Schrödinger equation with minimal coupling. We substitute the rotational and electromagnetic effects in the Schrödinger equation as

$$\rho^{\nu} \to \rho^{\nu} - \mu A^{\nu}_{\text{eff}}\ (\nu = 0, 1, 2, 3), \tag{1}$$

where the effective four-potential A^{ν}_{eff} consists of the electromagnetic four-potential A^{ν}_{ele} and the gauge field for the rotating state $\boldsymbol{A}^{\nu}_{\text{rot}} = (A^{0}_{\text{rot}}, A_{\text{rot}})$. Here, $\boldsymbol{A}^{\nu}_{\text{ele}} = (A^{0}_{\text{ele}}, \boldsymbol{A}_{\text{ele}} = \boldsymbol{A}_1 + \boldsymbol{A}_2)$ represents the electromagnetic four-potential. As mentioned below, the effective vector potential A_{ele} stands for the Aharonov–Bohm potential. Also, it shows the potential related to a uniform magnetic field in the z-direction, with $\boldsymbol{\nabla} \bullet \boldsymbol{A}_{\text{ele}} = 0$ and $A^{0}_{\text{ele}} = 0$. We introduce the magnetic flux tube for the Aharonov–Bohm field as

$$e\boldsymbol{A}_1 = \left(0, -\frac{\varphi}{\rho}, 0\right), e\boldsymbol{B}_1 = \left(0, 0, -\varphi\frac{\delta(\rho)}{\rho}\right). \tag{2}$$

The parameter $\varphi = \frac{e\Phi}{2\pi\hbar} = \frac{\Phi}{\Phi_0}$ is due to the Aharonov–Bohm flux, in which Φ indicates the magnetic flux and $\Phi_0 = \frac{h}{e}$ denotes the quantum of magnetic flux. The potential vector causes the uniform magnetic field in the z-direction and is defined as

$$\boldsymbol{A}_2 = \left(0, \frac{1}{2}B\rho, 0\right), \boldsymbol{B}_2 = (0, 0, B). \tag{3}$$

Moreover, the third configuration that specifies the gauge field for the rotating frame is

$$A^{\nu}_{\text{rot}} = \left(-\frac{1}{2}(\Omega \times \boldsymbol{r})^2, \Omega \times \boldsymbol{r}\right), \tag{4}$$

where Ω is the angular velocity. We suppose that Ω has only the z-direction; thus, the coordinate vector is $\boldsymbol{r} = \rho\hat{\rho}$ in cylindrical coordinates and consequently we indicate $\Omega \times \boldsymbol{r} = \Omega\rho\hat{\varphi}$. Furthermore, it is clear that the Aharonov–Bohm flux has translational invariance in the z-direction; therefore, we can eliminate this direction by considering $p_z = z = 0$ [36–38]. Therefore, we constrain the motion of electrons only in the plane. Thus, we must consider the combined effects, including the rotation Ω, the potential $\boldsymbol{A}_1$ and $\boldsymbol{A}_2$, and the field $\boldsymbol{B}_2$. Ultimately, the equation of motion is defined as

$$\frac{1}{2\mu}(\boldsymbol{p} - e\boldsymbol{A}_{\text{ele}} - \mu\Omega \times \boldsymbol{r})^2\Psi - \frac{1}{2}\mu(\Omega \times \boldsymbol{r})^2\Psi + V(\boldsymbol{r})\Psi = E\Psi, \tag{5}$$

where $V(\boldsymbol{r})$ shows the radial potential of a 2D ring represented by

$$V(\rho) = \frac{\mu\omega_0^2}{8}\left(\rho - \frac{\rho_0^2}{\rho}\right)^2, \tag{6}$$

where ρ_0 indicates the average radius of the ring and also shows the minimum point of the potential (figure 1 represents the potential $V(\rho)$). Also, the parameter ω_0 defines the strength of the transverse confinement. It is worth mentioning that the radial

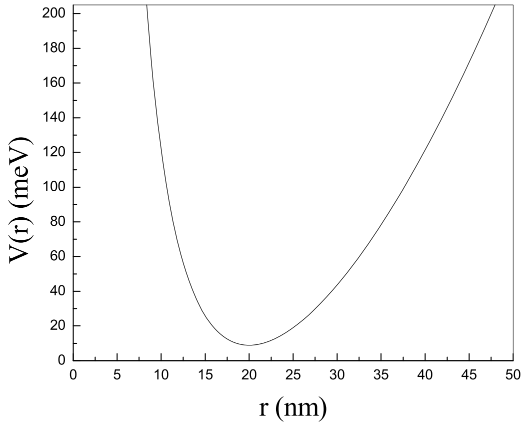

Figure 1. The radial potential illustrating a ring of average radius $\rho_0 = 20$ nm and $\hbar\omega_0 = 40$ meV.

potential, equation (6), illustrates a localized ring of finite width and provides a theoretical subject to describe electronic states, as well as their relation to the magnetic field in a 2D ring [1, 2]. By substituting the field and potential forms in equation (5), we deduce

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\Psi}{\partial\rho}\right) - \frac{1}{\rho^2}\left[\left(\frac{\partial}{\mathrm{i}\partial\varphi} - \varphi\right)^2 + \frac{\mu^2\omega_0^2\rho_0^4}{4\hbar^2}\right]\Psi$$
$$- \rho^2\left(\frac{\mu^2\omega_c^2}{4\hbar^2} + \frac{\mu^2\Omega\omega_c}{\hbar^2} + \frac{\mu^2\omega_0^2}{4\hbar^2}\right)\Psi + \frac{\mu\omega_c}{\hbar}\frac{\partial\Psi}{\mathrm{i}\partial\varphi} + \frac{2\mu\Omega}{\hbar}\frac{\partial\Psi}{\mathrm{i}\partial\varphi}$$
$$- \frac{2\mu\Omega\varphi}{\hbar}\Psi - \frac{\mu\omega_c\varphi}{\hbar}\Psi + \frac{\mu^2\omega_0^2\rho_0^2}{2\hbar^2}\Psi = -\frac{2\mu E}{\hbar^2}\Psi. \quad (7)$$

We suppose the eigenfunctions have the following form [39]

$$\Psi(\rho, \varphi) = f(\rho)\mathrm{e}^{\mathrm{i}m\varphi}, \quad (8)$$

where $m = 0, \pm1, \pm2, \pm3, \ldots$ is the angular momentum quantum number. Via some abbreviations and mathematical calculations, we obtain the radial equation as

$$\frac{1}{\rho}\frac{\mathrm{d}}{\mathrm{d}\rho}\left(\rho\frac{\mathrm{d}F}{\mathrm{d}\rho}\right) - \frac{L_m^2}{\rho^2}F - \frac{\mu^2\eta^2\rho^2}{4\hbar^2}F + k^2F = 0, \quad (9)$$

where $L_m^2 = \sqrt{(m-\varphi)^2 + \frac{\mu^2\omega_0^2\rho_0^4}{4\hbar^2}}$ is the effective angular momentum, $\eta = \sqrt{\omega_c^2 + 4\Omega\omega_c + \omega_0^2}$ is the effective cyclotron frequency, $\omega_c = \frac{e\boldsymbol{B}}{\mu}$ is the cyclotron frequency and $k^2 = \frac{\mu(2\Omega+\omega_c)(m-\varphi)}{\hbar} + \frac{2\mu}{\hbar^2}(\frac{\mu^2\omega_0^2\rho_0^4}{4} + E_{nm})$. By solving equation (9), we calculate the eigenvalues and eigenfunctions as

$$\Psi_{nm}(\rho, \varphi) = \frac{1}{\lambda}\sqrt{\frac{\Gamma(n+L_m+1)}{2^{L_m+1}\quad n!\,[\Gamma(L_m+1)]^2\pi}}\left(\frac{\rho}{\lambda}\right)^{L_m}$$
$$\times\mathrm{e}^{\mathrm{i}m\varphi}\mathrm{e}^{-\frac{\rho^2}{4\lambda^2}}F_1^1\left(-n, L_m+1, \frac{\rho^2}{2\lambda^2}\right), \quad (10)$$

$$E_{nm} = \frac{\hbar^2}{2\mu\lambda^2}(2n + L_m + 1)$$
$$-\frac{\hbar}{2}(2\Omega + \omega_c)(m + \varphi) - \frac{\mu}{4}\omega_0^2\rho_0^2, \quad (11)$$

where $\lambda = \sqrt{\frac{\hbar}{\mu\eta}}$ is the effective magnetic length renormalization by the rotation. We rewrite this more precisely as

$$E_{nm} = \left(n + \frac{1}{2}\sqrt{(m-\varphi)^2 + \frac{\mu^2\omega_0^2\rho_0^4}{4\hbar^2}} + \frac{1}{2}\right)$$
$$\times\hbar\sqrt{\omega_c^2 + 4\Omega\omega_c + \omega_0^2}$$
$$-\frac{\hbar}{2}(2\Omega + \omega_c)(m - \varphi) - \frac{\mu}{4}\omega_0^2\rho_0^2. \quad (12)$$

We obtain the eigenenergy in the absence of rotating effects as follows

$$E_{nm} = \left(n + \frac{1}{2}\sqrt{(m-\varphi)^2 + \frac{\mu^2\omega_0^2\rho_0^4}{4\hbar^2}} + \frac{1}{2}\right)$$
$$\times\hbar\sqrt{\omega_c^2 + \omega_0^2} - \frac{\hbar\omega_c}{2}(m - \varphi) - \frac{\mu}{4}\omega_0^2\rho_0^2. \quad (13)$$

As expected, the above equation recovers the result in [1]. According to the calculated eigenvalues, we obtain the partition function and thermodynamic properties of the considered system. The partition function is deduced by summation over all possible states of the system as below

$$Q = \sum_{n=0}^{\infty}\mathrm{e}^{-\beta E_n}, \quad (14)$$

where $\beta = \frac{1}{k_{\mathrm{B}}T}$, k_{B} is the Boltzmann constant and T is the temperature. With the help of equations (12) and (14), the partition function of the system is deduced as

$$Q = -\frac{\exp\left[-\left(\frac{1}{4}\sqrt{4(m-\varphi)^2 + \frac{\mu^2\omega_0^2\rho_0^4}{4\hbar^2}} + \frac{1}{2}\right)\hbar\sqrt{\omega_c^2 + 4\Omega\omega_c + \omega_0^2} + \frac{\hbar}{2}(2\Omega+\omega_c)(m-\varphi) - \frac{\mu}{4}\omega_0^2\rho_0^2\right]/T}{-1 + \frac{1}{\exp\left(\frac{\hbar}{T}\sqrt{\omega_c^2 + 4\Omega\omega_c + \omega_0^2}\right)}}. \quad (15)$$

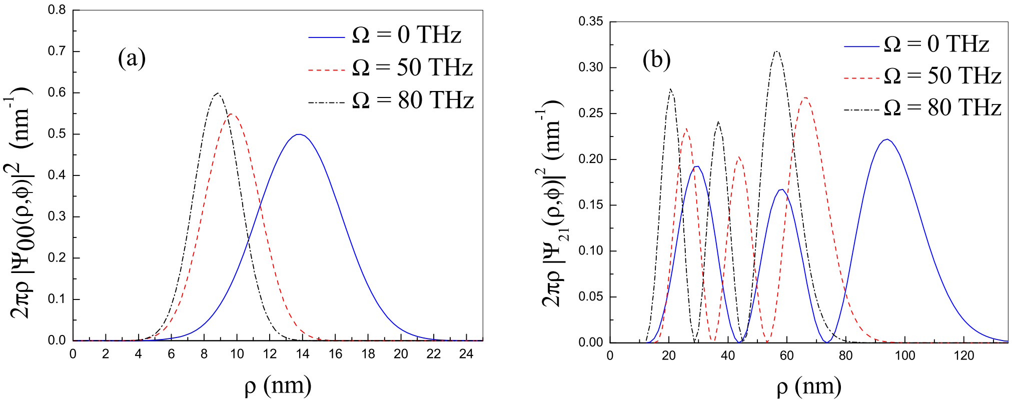

Figure 2. Probability distribution functions for positive rotations: (a) for $n = 0$, $m = 0$ and (b) for $n = 2$, $m = 1$.

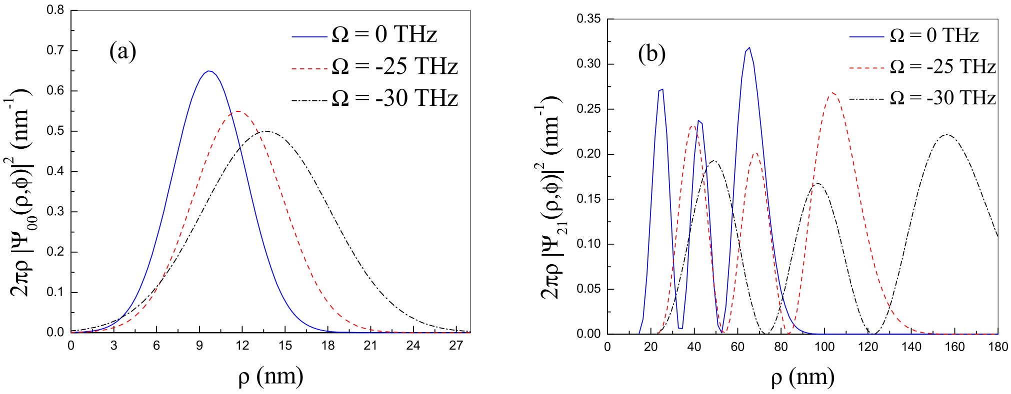

Figure 3. Probability distribution functions considering negative rotations with: (a) $n = 0$, $m = 0$ and (b) $n = 2$, $m = 1$.

Furthermore, the thermodynamic properties of the system can be obtained using the following relations:

- Mean energy $U = -\frac{\partial \ln(Q)}{\partial \beta}$.
- Entropy $S = k_B \ln(Q) - k_B \beta \frac{\partial \ln(Q)}{\partial \beta}$.
- Specific heat $C = -k_B \beta^2 \frac{\partial U}{\partial \beta}$.
- Free energy $F = -\frac{1}{\beta} \ln(Q)$.

3. Results and discussion

In this section, we numerically study the calculated results in section 2 for a 2D quantum ring with $\hbar\omega_0 = 40$ meV and radius of $\rho_0 = 20$ nm. We apply a 2D GaAs heterostructure due to the ease of finding its data. In this regard, we investigate the impact of rotation on the thermodynamic properties of an electron in a 2D quantum ring. To perform the obtained results, we take $\mu = 0.067\mu_0$, where $\mu_e = 9.1094 \times 10^{-31} \mathrm{eV/c^2}$ is the electron mass [1, 40].

We calculated the probability distribution of particles in various states n and m using the normalized eigenfunction, equation (10). This subject allows us to illustrate the behavior of the system and realize its properties in-depth. Figure 2 shows the probability distribution changes according to positive rotation values for the states with ($n = 0$, $m = 0$) and ($n = 2$, $m = 1$). As seen in the figure, by enhancing Ω, the distribution shifts toward lower values. It means that for higher rotations, the particle can be found closer to the inner radius of the ring (inner edge). Also, we can deduce that the probability amplitude becomes more dependent with less spacing among adjacent values as Ω increases. Furthermore, the eigenfunction occupies the edges of the ring by increasing m values and causes a centrifugal effect. Therefore, the

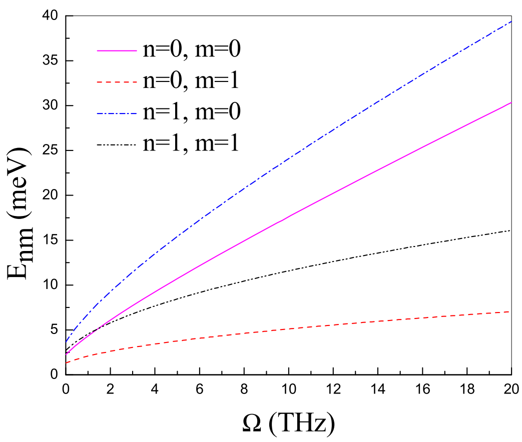

Figure 4. The energy spectrum as a function of rotation quantities for different values of n and m when $\varphi = 0.5(h/e)$ and $B = 15T$.

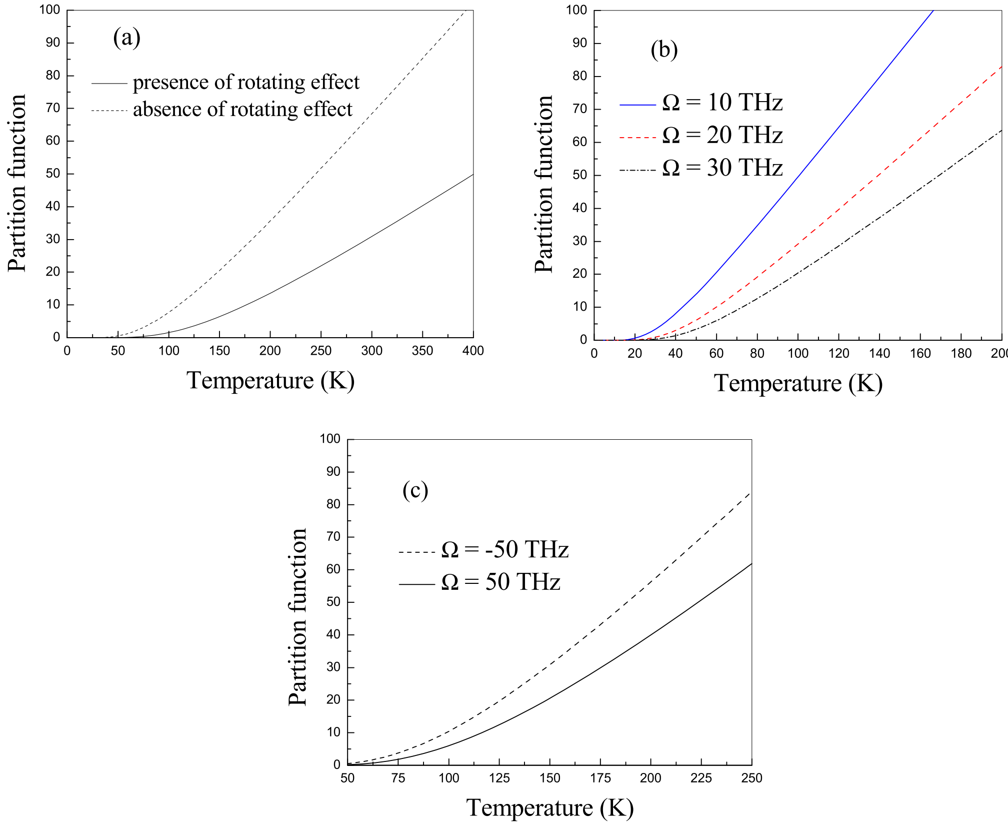

Figure 5. The partition function as a function of temperature for: (a) in the presence and absence of the rotating parameter, (b) different values of the rotating parameter and (c) positive and negative values of the rotating parameter.

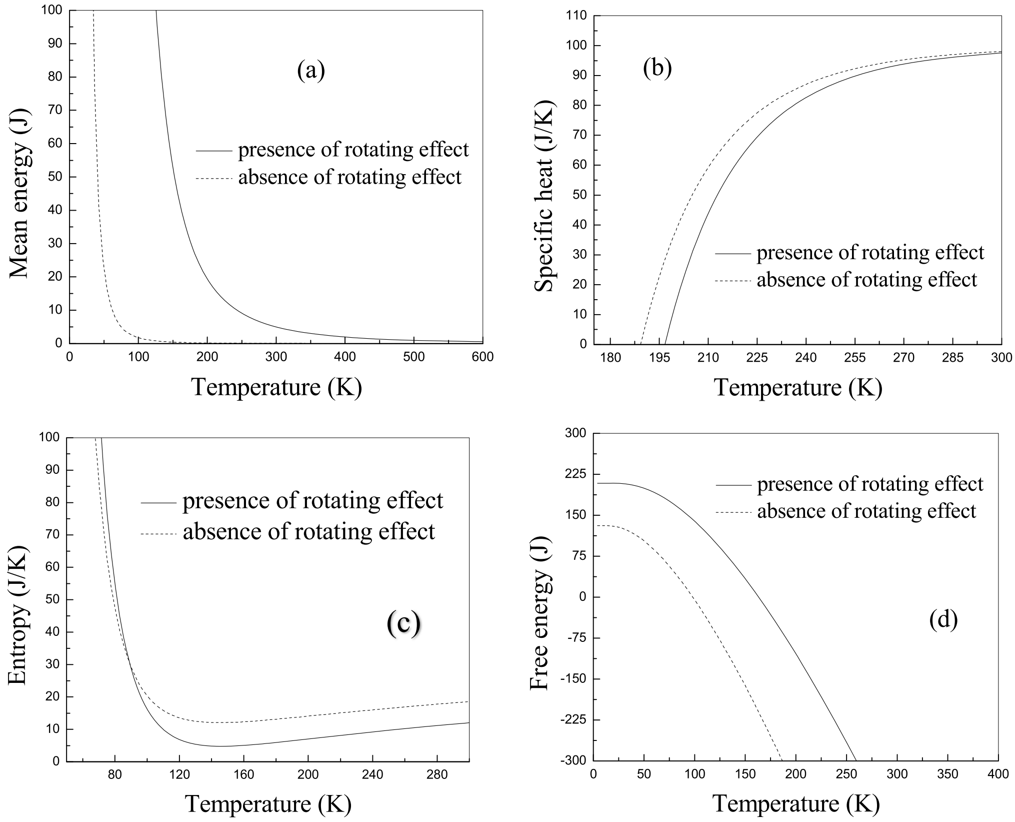

Figure 6. Thermodynamic properties as a function of temperature in the presence and absence of a rotating parameter: (a) mean energy, (b) specific heat, (c) entropy and (d) free energy.

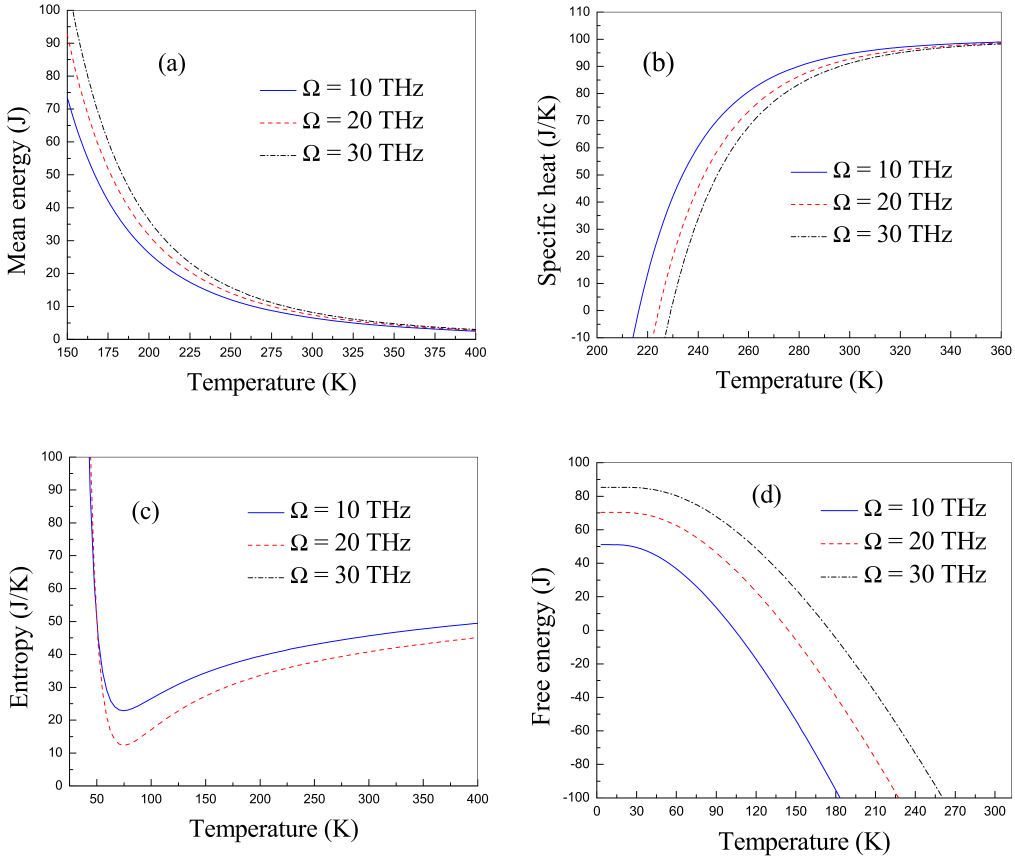

Figure 7. Thermodynamic properties as a function of temperature for different values of the rotating parameter: (a) mean energy, (b) specific heat, (c) entropy and (d) free energy.

particles have a high probability to be found at the outer edge. Additionally, we can see that for $n = 2$, the third wave peak amplitude increases compared to previous ones as it moves along the horizontal axis. Afterward, we observe a symmetrical increase in the amplitude that precedes a decrease. This phenomenon seems to happen for all n values equal to or greater than 1. This is expected behavior because the considered potential in equation (1) approximates that of a pure quantum harmonic oscillator [41] for greater values of ρ (i.e. greater outer radius), see figure 1.

Figure 3, similar to the fact that positive rotations change behavior, represents the probability distribution when negative rotating quantities are used in states with ($n = 0$, $m = 0$) and ($n = 2$, $m = 1$). We can see that by decreasing Ω, the distribution shifts towards higher values. This indicates that the particle is found closer to the outer radius of the ring (outer edge) for lower rotation values. The probability amplitude is less concentrated with larger spacing among adjacent parameters as Ω decreases. As m increases, the eigenfunctions tend to occupy the ring edges with a higher probability to be found at the outer edge. The energy spectrum calculated by equation (12) shows that the electronic states of the considered system have great dependence on the rotation parameter (figure 4). According to the observed states and various rotation values, one can find that the energy levels increase with the strength of the rotation. Also, in the applied rotation range, the energy of the state ($n = 0$, $m = 0$) can surpass states with higher quantum numbers, as Ω increases. For instance, in the positive rotation values, the lowest energy level is not ($n = 0$, $m = 0$). Herein, the higher wave numbers, like ($n = 2$, $m = 1$), have a great probability near the center of the ring. This phenomenon is due to the centrifugal effect caused by the quantum number m. Also, the probability distribution of finding the electrons indicates that the electrons are more likely to appear at the edges of the ring instead of being concentrated at the center of it (see figures 2 and 3). This is expected behavior as we use different physical quantities in our computations. However, it is obvious from figure 4 that the rotation effect has a significant influence on the energy spectrum. In this regard, this effect can change the thermodynamic properties of the considered system due to the rotational effect on the energy spectrum.

Figure 5(a) shows the partition function as a function of temperature in the presence and absence of rotating effects. At low temperatures ($T < 100$ K), the two curves have nearly equal values but as the temperature increases, they move away from each other. Also, we can see that the partition function in the absence of rotating effects has higher values compared to in the presence of rotation effects. In figure 5(b), we have plotted the partition function versus temperature for different values of Ω. It is observed from the figure that as Ω increases, the partition function decreases. Figure 5(c) represents the partition function versus temperature for positive and negative values of Ω. It is obvious that the negative

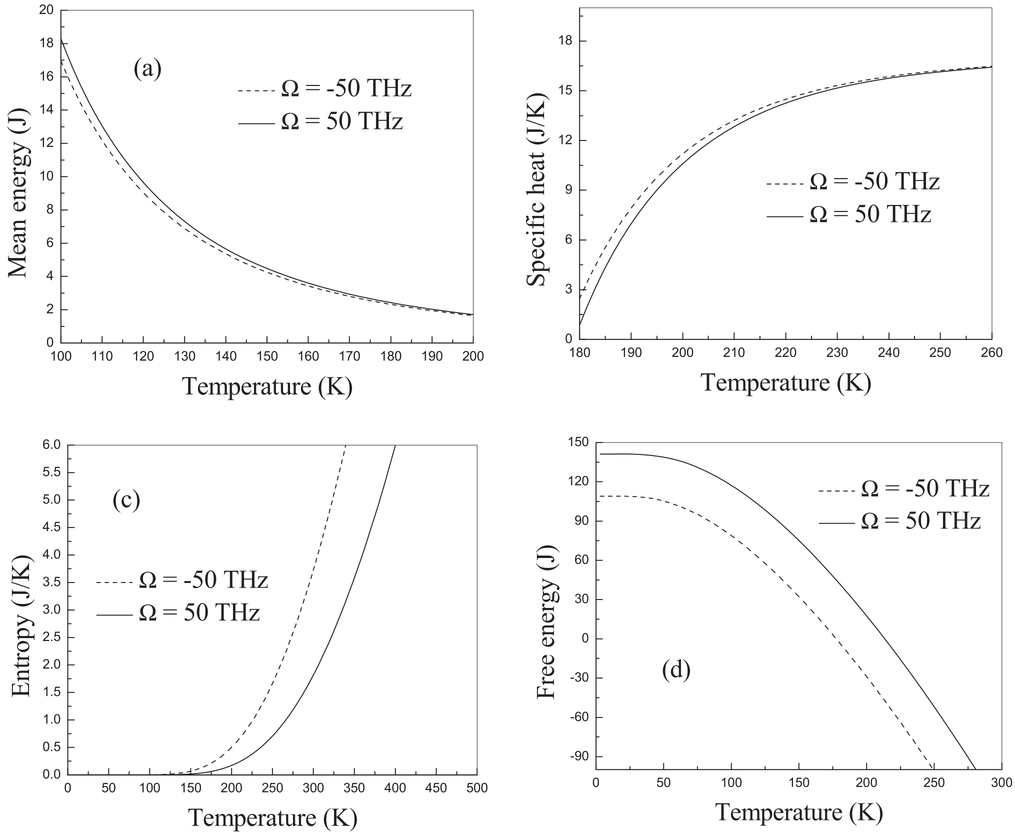

Figure 8. Thermodynamic properties as a function of temperature for positive and negative values of the rotating parameter: (a) mean energy, (b) specific heat, (c) entropy and (d) free energy.

rotating parameter changes the behavior of the partition function. Also, the partition function in negative Ω has higher values related to positive values. Figures 6(a)–(d) represent the mean energy, specific heat, entropy and free energy as a function of temperature in the presence and absence of rotation effects, respectively. It is clear that the rotating parameter has a significant influence on the thermodynamic properties. The mean energy and free energy have higher values in the presence of the rotating parameter. The specific heat has higher values in the absence of the rotating parameter but, as the temperature increases, the two curves show equal values. In figure 6(c), the entropy has equal values in the presence and absence of rotational effects but, as the temperature increases, we can see that the entropy has higher values in the absence of rotating effects. This means that in the absence of rotating effects, we have much more disorder and we are faced with missed information.

In figures 7(a)–(d), we have considered different values of Ω ($\Omega = 10$, 20 and 30 THz). It is obvious from the figures that the mean energy and free energy increase by enhancing Ω but the specific heat and entropy decrease by increasing Ω. Also, we can see that the mean energy and specific heat have equal values for various Ω as the temperature increases. Figures 8(a)–(d) represent the mean energy, specific heat, entropy and free energy versus temperature for positive and negative values of Ω ($\Omega = 50$ Thz and -50 THz). It can be found that the mean energy and free energy have higher values in the positive rotating parameter but the entropy and specific heat show higher values in the negative rotating parameter. Also, we can see that the negative rotating parameter in the mean energy and specific heat has less of an effect, but in the entropy and free energy, it has a greater effect.

4. Conclusions

In this work, we have studied the nonrelativistic quantum motion of a charged particle in a 2D quantum ring under the rotational effect. By calculating the equation of motion by applying the minimal coupling procedure and under the influence of the Aharonov–Bohm effect and a uniform magnetic field, we deduced the eigenvalues and eigenfunctions. According to the obtained results, we derived the probability distribution and partition function of the system. Also, we calculated the thermodynamic properties of the considered system, such as the mean energy, specific heat, entropy and free energy, in the presence and absence of rotation effects. The obtained probability distribution shows a distinct shift toward lower values along the radial coordinates as the rotation parameter increases. This shift reveals a higher possibility of finding the particle near to the outer or inner radii (edges) of the system rather than in the average region ρ_0. Our results are in good agreement with the observed occupation of electrons at the edges of the system and describe the electronic states' dependence on the rotation parameter. Furthermore, we examined the thermodynamic properties of the system using different procedures: (1) in the presence and absence of rotation effects; (2) using different rotation parameter values; and (3) using positive and negative rotation. The significant achievement of this study is understanding the relationship between the rotation and quantum behavior in quantum rings. Also, the presented investigations in this study exhibit important insights into charged-particle behavior in a rotating quantum ring system and reveal the fundamental aspects of quantum mechanics. Finally, we emphasize that our computations considered a rotational effects scale typically of several terahertz (THz). These high rotation rates are considered to better visualize the rotational effects, as discussed in [34]. However, it is clear that in quantum systems such as 2D quantum rings, rotational effects are exhibited even at much lower scales.

Authors' contributions

A. Ghanbari was responsible for the methodology, formal analysis, project administration, software and writing of the original draft.

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