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Worldvolume fermions as baryons in holographic quantum chromodynamics with instantons

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Abstract
In this study, we investigated worldvolume fermions on the flavor brane in the D0–D4/D8 model, which is holographically equivalent to four-dimensional quantum chromodynamics with instantons or equivalently with a theta angle. The action involving the worldvolume fermions was obtained by the T-duality rules in string theory, and we accordingly derived their effective five-dimensional and canonical four-dimensional forms by using the systematic dimensional reduction and decomposition of the spinor. Subsequently, we used the AdS/CFT dictionary to evaluate the two-point correlation function as the spectral function for the worldvolume fermions and interpreted the fermions as baryons by analyzing their quantum number with the baryon vertex in holography. In this sense, the interacted action involving the worldvolume fermions and gauge field on the flavor brane was finally derived in holography, which describes the various interactions of mesons and baryons with instantons in the large-N limit. Therefore, this study provides a holographic picture to describe baryons and their interactions based on string theory, particularly in the presence of instantons or a theta angle.

Keywords: gauge–gravity duality, AdS/CFT, holographic QCD

(Some figures may appear in colour only in the online journal)

1. Introduction

In the theory of quantum chromodynamics (QCD), it is known that instanton is the nontrivial topological excitation of the vacuum [1–3], which contributes to the thermodynamics of QCD and interactions of quarks and hadrons, and also relates to the spontaneous parity violation or breaking of chiral symmetry. In particular, there has been a significant amount of time to study the spontaneous parity violation and breaking of chiral symmetry with the running of the relativistic heavy-ion collision [4, 5]. In gauge theory, the instantonic vacuum can be characterized by a nonvanished theta term in QCD or Yang–Mills action as

$$S = -\frac{1}{4g_{\mathrm{YM}}^2}\int \mathrm{d}^4x\,\mathrm{Tr}\,F_{\mu\nu}F^{\mu\nu} + \frac{\theta}{8\pi^2}\mathrm{Tr}\int F\wedge F, \qquad (1)$$

where g_{YM} is the Yang–Mills coupling constant, and θ refers to the concerned theta angle. Although the exact experimental value of the theta angle may be very small ($|\theta| \leqslant 10^{-10}$), in the last two decades, it has attracted great interest in the theoretical and phenomenological investigations in the Yang–Mills theory or QCD, e.g. the deconfinement phase transition [6, 7], glueball spectrum [8], and large N behavior [9] with the theta angle. A summary of the theta term in the Yang–Mills theory or QCD can be reviewed in detail in the excellent literature [10]. The chiral magnetic effect in heavy-ion collisions has also become an important focus for confirming the theta dependence in QCD in recent years [11–15]. However, because asymptotic freedom is one of the characteristic features of QCD, it implies that QCD is strongly coupled and nonanalytical in the low-energy region. This means that the standard analytical technique in quantum field theory (QFT) based on the perturbation method is powerless for analyzing QCD matter, e.g. mesons and baryons, in the low-energy region. Fortunately, the framework of AdS/CFT and

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gauge–gravity duality based on string theory [16, 17] could offer an alternative analytical approach to investigate the aspects of the strongly coupled gauge theory. Significantly, in 2004, Sakai and Sugimoto proposed a concrete model [18] (i.e. the D4/D8 model or named as Witten–Sakai–Sugimoto model) by using the construction of the D4-brane in Witten's [19], which successfully includes almost all the elementary ingredients of QCD, e.g. quark, gluon, meson [20–22], baryon [23–28], glueball [29–34], and chiral/deconfinement transitions [35–37]. Moreover, to include the instanton configuration or theta term presented in (1) in dual theory of the D4/D8 model, the authors of [38] suggested the introduction of N_0 smeared D0-branes into the background geometry produced by N_c D4-branes in the D4/D8 model. By keeping the ratio of N_0/N_c fixed and $N_0/N_c \ll 1$ in the large N_c limit, the background geometry determined by D0- and D4-branes together can be obtained by solving type IIA supergravity, in which the number density of D0-branes (as we will see in the following sections) corresponds to the instanton density or theta angle in QCD [39–41]. Therefore, it is possible to use this D0–D4/D8 system to systematically study the properties of QCD with a theta term in the holograph, e.g. [41–45].

Although the above framework of gauge–gravity duality has achieved many successes, there may be an issue in the approach of the D4/D8 or D0–D4/D8 system by imposing the project of the compactification in [19]. That is, the D8-branes (as the flavor branes) remain supersymmetric in principle in the low-energy theory because Project of the compactification as the mechanism to break down the supersymmetry in the model works only for the D4-branes instead of for the D8-branes [46]. Therefore, the remaining supersymmetry on the flavor branes leads to the existence of the superpartner of the bosonic mesons (named mesinos) in the dual theory, which, however, is always absent in QCD and hadron physics.[1] In this sense, dual theory is less realistic. In addition, there is no reason to neglect these worldvolume fermions as mesinos on the flavor branes without a mechanism to further break down the supersymmetry in principle.

Motivated by this issue, in this study, we attempt to interpret the supersymmetric fermions on the flavor branes as baryons instead of mesinos to improve the dual field theory in the D4/D8 or D0–D4/D8 model for realistic QCD. To this end, we systematically studied the worldvolume fermions on the flavor branes in the D0–D4/D8 model by analyzing its action, dimensional reduction, spectrum strictly through string theory, and gauge–gravity duality, and then explored how to interpret these fermions in terms of as baryons. Our numerical evaluation of the fermionic spectrum illustrates that even if the worldvolume fermions are identified as superpartners of bosonic mesons, they are too heavy to arise in low-energy theory. Thus, below the compactified energy scale, the meson sector of dual theory must be purely bosonic without mesinos. Moreover, when the baryon vertex described in [23, 24] is introduced in this model, our analysis of the associated quantum numbers implies worldvolume fermions, and its dual operator may be interpreted as baryons through gauge–gravity duality, which leads to a nicely natural description of fermionic baryons in holography. Noticeably, the baryon vertex is the key to make the open strings on the flavor brane become baryonic. Accordingly, we finally derive the interacted action of mesons and baryons in holography using the dimensional reduction of the coupling terms in the worldvolume action involving the fermions. Because the existing studies, e.g. [42, 43, 48–50], have never revealed exactly that baryons in this model are fermions, this study may fill this gap. On the other hand, investigation of the baryonic correlation function with instantons in holography is also an extension of the existing QFT framework [1, 2]. In addition, our numerical evaluation also displays the metastable states of baryons in the presence of the theta angle, which is in agreement with the existing studies [42, 43, 48–50] describing metastabilization in the instantonic or theta-dependent QCD [4, 5]. Altogether, we believe that this study provides a holographic framework for field theory to describe baryons and their interactions with instantons based on string theory.

The outline of this paper is as follows. In section 2, we review the D0–D4/D8 model as a four-dimensional (4D) QCD with a theta angle in holography. In section 3, we derive the five-dimensional (5D) effective action and the associated 4D canonical form for worldvolume fermions, and then numerically evaluate the fermionic spectrum by analyzing the holographic correlation function. In section 5, we specify how to interpret worldvolume fermions as baryons with the baryon vertex, and then derive the interacted action for the various interactions of mesons and baryons in holography. A summary and discussion are given in section 6.

2. D0–D4/D8 model as theta-dependent QCD in holography

2.1. Color sector

In this section, we briefly review the D0–D4/D8 model as holographic QCD with instantons or a theta term, and the details can be found in [38–41]. In this model, the gravity background is produced by N_c coincident D4-branes as colors with N_0 smeared D0-branes as D-instantons. In the large N_c limit, the dynamics of the gravity background is described by the type IIA supergravity whose bosonic action is given as

$$S_{\mathrm{IIA}}^{10\mathrm{d}} = \frac{1}{2\kappa_{10}^2}\int \mathrm{d}^{10}x\sqrt{-G}\,\mathrm{e}^{-2\phi}[\mathcal{R} + 4\partial_\mu\phi\partial\phi] - \frac{1}{4\kappa_{10}^2}\int \mathrm{d}^{10}x\sqrt{-G}\,(|F_2|^2 + |F_4|^2), \tag{2a}$$

where $\mathcal{R}$, ϕ, and G refer to the ten-dimensional (10D) scalar curvature, dilaton, and determinant of the metric, respectively, and $2\kappa_{10}^2 = 16\pi G_{10} = (2\pi)^7 l_s^8 g_s^2$ is the 10D gravity coupling constant. $F_{2,4} = \mathrm{d}C_{1,3}$ denotes the field strength of the Ramond–Ramond one- and three-form $C_{1,3}$. To take into account the back reaction of the D0-branes, in the large N_c limit, we keep $N_0/N_c \ll 1$ but is finite because, as we will see,

[1] In the top-down approach of holographic QCD, breaking down the supersymmetry in the low-energy region is a common issue (see a similar discussion in the D3/D7 approach [47]).

Table 1. D-brane configuration of the D0–D4/D8 model. – represents that the D-brane extends along this direction. □ denotes the smeared directions of the D0-branes inside the N_c D4-branes.

	0	1	2	3	4	5(*U*)	6	7	8	9
N_c D4-branes	—	—	—	—	—					
N_0 D0-branes	□	□	□	□	—					
N_f D8/$\overline{\text{D8}}$ -branes	—	—	—	—		—	—	—	—	—
Baryon vertex (D4)	—						—	—	—	—

N_0 relates to the theta term of QCD in this model. In this sense, the equations of motion obtained by action (2*a*) can be solved using a D4 bubble solution with N_0 smeared D0-branes [38–41], as it is in the D4/D8 model [18]. Taking the near-horizon limit, in string frame, the supergravity solution is given as [41]

$$\mathrm{d}s^2 = \left(\frac{U}{R}\right)^{3/2}[H_0^{1/2}\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + H_0^{-1/2}f(U)(\mathrm{d}x^4)^2] + H_0^{1/2}\left(\frac{R}{U}\right)^{3/2}\left[\frac{\mathrm{d}U^2}{f(U)} + U^2\mathrm{d}\Omega_4^2\right], \tag{2b}$$

and

$$\mathrm{e}^\phi = \left(\frac{U}{R}\right)^{3/4}H_0^{3/4},\ F_2 = \frac{(2\pi l_s)^7 g_s N_0}{\Omega_4 V_4} \times \frac{1}{U^4 H_0^2}\mathrm{d}U\wedge \mathrm{d}x^4,\ F_4 = \frac{(2\pi l_s)^3 N_c g_s}{\Omega_4}\epsilon_4, \tag{2c}$$

where

$$H_0 = 1 + \frac{U_{Q_0}^3}{U^3},\ f(U) = 1 - \frac{U_{KK}^3}{U^3},$$
$$U_{Q_0}^3 = \frac{1}{2}(-U_{KK}^3 + \sqrt{U_{KK}^6 + ((2\pi)^5 l_s^7 g_s \kappa N_c)^2}),$$
$$R^3 = \pi g_s l_s^3 N_c,\ \kappa = \frac{N_0}{N_c V_4}. \tag{2d}$$

Note that U is the radial coordinate perpendicular to the N_c D4-branes; thus, the holographic boundary is located at $U\to\infty$. The parameter l_s refers to the length of the string, and V_4 denotes the worldvolume of the D4-brane. Ω_4 refers to the volume of a unit S^4, which means $\Omega_4 = 8\pi^2/3$; κ relates to the density of the D0-branes presented in the worldvolume of the D4-branes. Because the N_0 D0-branes are considered to be homogeneously smeared, κ is also a constant. Overall, the supergravity solutions (2*b*)–(2*d*) describe the bubble geometry produced by N_c coincident D4-branes, in which the N_0 D0-branes are homogeneously smeared along the direction x^4, as illustrated in table 1. The bubble geometry means that there is no event horizon located at $U = U_{KK}$; instead, the bulk shrinks at $U = U_{KK}$, which implies that it must be defined in $U > U_{KK}$. To obtain a dual theory close to QCD, we must further eliminate the supersymmetry on the worldvolume of the D4-branes in the low-energy theory. A simple way to achieve this goal is to follow Witten's [19] as it is used in the D4/D8 model, that is, to compactify the x^4 direction on a circle S^1, then impose periodic and antiperiodic boundary conditions on the gauge field and supersymmetric fermions, respectively. Hence, below the energy scale $M_{KK} = 2\pi/\beta$, where β refers to the size of S^1, the dual theory on the D4-brane is effectively 4D pure Yang–Mills theory. In addition, because the wrap factor $(U/R)^{3/2}H_0^{1/2}$ in equation (2*a*) can never go to zero, the dual theory would also exhibit confinement due to the behavior of the Wilson loop in this geometry [19, 40].

Next, let us take a closer look at dual theory. First, to avoid conical singularity in dual theory, we impose the following condition:

$$\beta = \frac{4\pi}{3}U_{KK}^{-1/2}R^{3/2}b^{1/2},\ b = H_0(U_{KK}). \tag{2e}$$

Yang–Mills coupling constant g_{YM} for the dual theory must then be given by following the dimensional reduction as

$$g_{\mathrm{YM}}^2 = \frac{g_5^2}{\beta} = \frac{4\pi^2 g_s l_s}{\beta}, \tag{2f}$$

where g_5 refers to the 5D Yang–Mills coupling, and g_s is the string coupling constant. Accordingly, the relation of b and R^3 is evaluated as

$$b = \frac{1}{2}[1 + (1 + \mathcal{C}\beta^2)^{1/2}],$$
$$\mathcal{C} \equiv (2\pi l_s^2)^6\lambda\kappa^2/U_{KK}^6,\ R^3 = \frac{\beta\lambda l_s^2}{4\pi}, \tag{2g}$$

where λ is the 't Hooft coupling constant given by $\lambda = g_{\mathrm{YM}}^2 N_c$ and $b \geqslant 1$. Subsequently, the dual theory can be examined by taking into account a probe D4-brane at the holographic boundary, and its action is given as

$$S_{\mathrm{D4}} = -g_s^{-1}\mu_4\,\mathrm{Tr}\int \mathrm{d}^4x\mathrm{e}^{-\phi}\sqrt{-\det(G + 2\pi\alpha'\mathcal{F})} + \mu_4\int C_5 + \frac{1}{2}(2\pi\alpha')^2\mu_4\int C_1\wedge\mathcal{F}\wedge\mathcal{F}, \tag{2h}$$

where $\alpha' = l_s^2$, and G is the induced metric on the D4-brane. $\mathcal{F}$ refers to the Yang–Mills gauge field strength on the D4-brane. $C_{1,5}$ is the Romand–Romand one- and five-form. While the field strength of C_1 is given in equation (2*c*), C_5 satisfies ${}^\star\mathrm{d}C_5 = \mathrm{d}C_3 = F_4$, where F_4 is given in equation (2*c*). Keeping these in hand, we can find at the holographic boundary $U\to\infty$ and in the low-energy limit $\alpha'\to 0$, the leading-order action of the first term in equation (2*h*) is the 4D Yang–Mills action, the second term in equation (2*h*) is a constant by inserting the solution for C_4, and the last term reduces to a theta term of the Yang–Mills theory. Altogether,

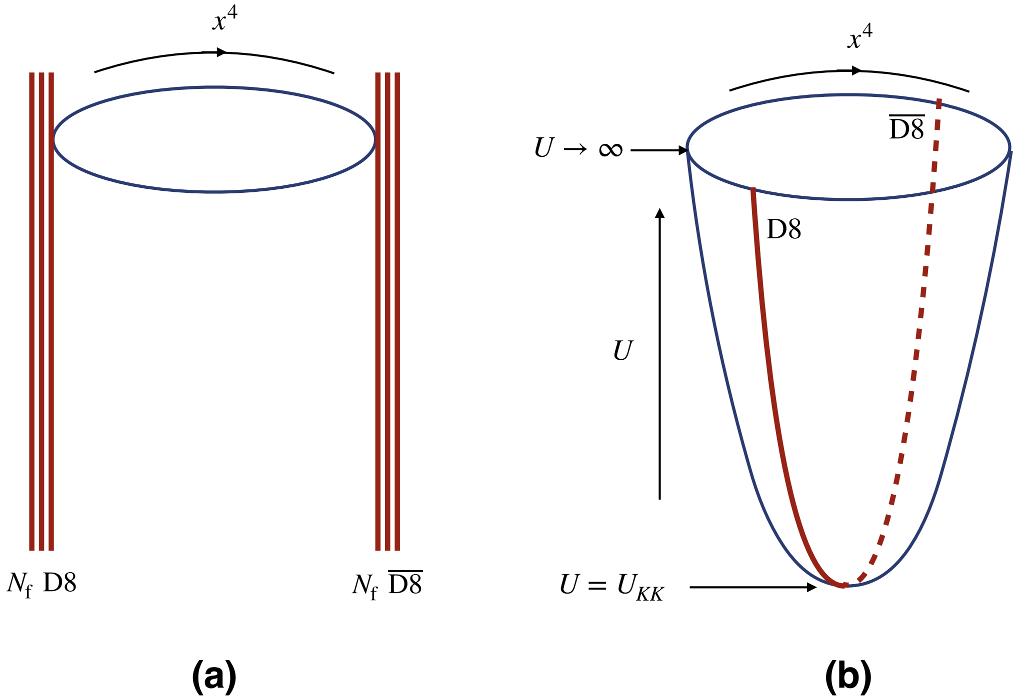

Figure 1. Configuration of the D8/$\overline{\text{D8}}$-branes in the D0–D4 background. (a) D8/$\overline{\text{D8}}$-branes are located at the antipodal position of S^1. (b) This configuration in the large N_c limit, i.e. in the D4 bubble background with D0-branes.

action (2*h*) reduces to

$$S_{D4} \simeq -\frac{N_c}{4\lambda}\mathrm{Tr}\int d^4x\mathcal{F}^2 + \frac{\theta}{8\pi^2}\mathrm{Tr}\int\mathcal{F}\wedge\mathcal{F} + O(\mathcal{F}^4), \tag{2i}$$

where

$$C_1 = \sqrt{\frac{b-1}{b}\lambda}\frac{f(U)}{H_0(U)}dx^4,\ \theta + 2\pi k = l_s^{-1} \times \int_{S^1} C_1|_{U\to\infty} = \frac{\beta}{l_s}\sqrt{\frac{b-1}{b}\lambda},\ k\in\mathbb{Z}. \tag{2j}$$

Therefore, we can see in a given branch that if the density of D0-branes vanishes, i.e. $N_0 = 0$ and $b = 1$, the theta angle θ vanishes as well. In this sense, we can obtain a confining Yang–Mills theory with a theta term that relates to the number density of the instantons by all the holographic constructions in the D0–D4 system. It is possible to evaluate the glue condensate $\langle\mathrm{Tr}\mathcal{F}\wedge\mathcal{F}\rangle$ in this model as $\langle\mathrm{Tr}\mathcal{F}\wedge\mathcal{F}\rangle = 8\pi^2 N_c\kappa$ [41].

2.2. Flavor sector

Because QCD also has flavors, in the D0–D4 background, it is possible to introduce a stack of coincident N_f pairs of probe D8- and (anti-D8) $\overline{\text{D8}}$-branes as flavors by following the discussion of the D4/D8 model. The N_f pairs of the probe D8/$\overline{\text{D8}}$-branes were located at the antipodal position of S^1, perpendicular to the N_c D4-branes. The relevant configurations of the D8/$\overline{\text{D8}}$-branes are listed in table 1 and illustrated in figure 1. The fundamental fermions in the low-energy theory are identified as the fermionic zero modes of the 4–8 or $4-\bar{8}$ string in the Ramond sector[2] because such strings take both colors and flavors whose fermionic zero mode is in the fundamental representation of $U(N_c)$ and $U(N_f)$. In string theory, the Gliozzi–Scherk–Olive projection removes fundamental fermions with one of the chiralities; therefore, it is possible to choose the fundamental fermions with positive and negative chirality as the massless fermionic modes of $4-8$ and $4-\bar{8}$ string, respectively. Thus, the flavor symmetry on the D8/$\overline{\text{D8}}$-branes can be denoted as $U(N_f)_L\times U(N_f)_R$ as the chiral symmetry. Note that these chiral fermions are all complex spinors because the 4–8 and $4-\bar{8}$ strings have two orientations.

As in the case of the D4/D8 model, the disconnected and connected configurations of the D8/$\overline{\text{D8}}$-branes represent the chirally symmetric and broken phases in the dual theory, respectively. To test the dual theory, let us introduce a probe D4-brane located at $U = U_\Lambda$ as before, and the effective action for the fundamental fermions denoted by $q_{L,R}$ on the D4-branes intersected with N_f D8/$\overline{\text{D8}}$-branes is

$$S = \int_{D4} d^4x\sqrt{-g}\,[\delta(x^4 - X_L)q_L^\dagger\bar{\sigma}^\mu(i\nabla_\mu + A_\mu)q_L + \delta(x^4 - X_R)q_R^\dagger\bar{\sigma}^\mu(i\nabla_\mu + A_\mu)q_R], \tag{2k}$$

where $X_{L,R}$ denotes the intersection of the D4- and D8-branes and D4- and $\overline{\text{D8}}$-branes, respectively. A_μ refers to the gauge field on the D4-branes, i.e. the gluon field. As all the fields depend on $\{x^\mu, x^4\}$, it means q_L would be identified as q_R if $X_L = X_R$, which leads to an action with a single flavor group $U(N_f)$. Therefore, for the connected configuration of the D8/$\overline{\text{D8}}$-branes shown in figure 1, we can see that the D8- and $\overline{\text{D8}}$-branes are separated at a very high energy ($U_\Lambda \to \infty$, $X_L \neq X_R$), which leads to an approximated $U(N_f)_L\times U(N_f)_R$ chiral symmetry. However, at a low energy ($U_\Lambda \to U_{KK}$, $X_L \to X_R$), the flavored D8- and $\overline{\text{D8}}$-branes are joined into a

[2] The 4–8 string denotes the open string connecting the N_c D4-brane and N_f D8-branes. It is similar for, e.g. the 8–8 or $4-\bar{8}$ string.

single pair of D8-branes at $U_\Lambda = U_{KK}$ ($X_L = X_R$), which means that the $U(N_f)_L \times U(N_f)_R$ symmetry breaks down to a single $U(N_f)$. Accordingly, this configuration of D8/$\overline{\text{D8}}$-branes displays a geometric interpretation of chiral symmetry in holography.

2.3. Bosonic meson tower

The mesons in this model are identified as the zero modes of the bosonic states created by the open strings on the flavor branes, because these states are the gauge fields in the adjoint representation of the flavor group $U(N_f)_L \times U(N_f)_R$. Accordingly, let us consider the bosonic action of the gauge fields on the flavor D8-branes as

$$\begin{aligned} S_{\rm D8} &= -T_{\rm D8}\int_{\rm D8} {\rm d}^9x {\rm e}^{-\phi}\,{\rm Tr}\sqrt{-\det[g_{\rm ab} + (2\pi\alpha')\mathcal{F}_{\rm ab}]} + S_{\rm WZ},\\ &= -T_{\rm D8}\int_{\rm D8} {\rm d}^9x\sqrt{-g}\,{\rm e}^{-\phi}\Big[1 + \frac{1}{4}(2\pi\alpha')^2\mathcal{F}_{\rm MN}\mathcal{F}^{\rm MN}\\ &\quad + \mathcal{O}(\mathcal{F}^4)\Big], \end{aligned} \tag{2l}$$

where $S_{\rm WZ}$ refers to the Wess–Zumino term for the D8-brane, and we expanded the action up to the leading order of α'. Following the discussion in [18, 22] and assuming that the nonzero components of the gauge field are denoted as $\mathcal{A}_{\rm M} = \{\mathcal{A}_\mu(x, z), \mathcal{A}_z(x, z)\}$, $\mu = 0, 1 \ldots 3$, the Yang–Mills part of action (2*l*) becomes

$$\begin{aligned} S_{\rm YM} &= -T_{\rm D8}\int_{\rm D8} {\rm d}^9x\sqrt{-g}\,{\rm e}^{-\phi}\frac{1}{4}(2\pi\alpha')^2\mathcal{F}_{\rm MN}\mathcal{F}^{\rm MN}\\ &= -T(2\pi\alpha')^2\int {\rm d}^4x{\rm d}ZH_0^{1/2}\,{\rm Tr}\\ &\quad\times\left(\frac{1}{2}K^{-1/3}\eta^{\mu\rho}\eta^{\nu\sigma}\mathcal{F}_{\mu\nu}\mathcal{F}_{\rho\sigma} + KM_{KK}^2\eta^{\mu\nu}\mathcal{F}_{\mu Z}\mathcal{F}_{\nu Z}\right), \end{aligned} \tag{2m}$$

where

$$T = \frac{2}{3}R^{3/2}U_{KK}^{1/2}T_{\rm D8}\Omega_4,\ K(Z) \equiv 1 + Z^2 = \frac{U^3}{U_{KK}^3}, \tag{2n}$$

and we have used the induced metric for the antipodal D8-branes, which is computed as follows:

$$\begin{aligned} {\rm d}s_{\rm D8}^2 &= \left(\frac{U}{R}\right)^{3/2}H_0^{1/2}\eta_{\mu\nu}{\rm d}x^\mu{\rm d}x^\nu\\ &\quad + H_0^{1/2}\left(\frac{R}{U}\right)^{3/2}\left[\frac{{\rm d}U^2}{f(U)} + U^2{\rm d}\Omega_4^2\right]\\ &= \frac{2}{3}M_{KK}R^3b^{1/2}H_0^{1/2}\frac{4}{9}M_{KK}^2b\eta_{\mu\nu}K^{1/2}{\rm d}x^\mu{\rm d}x^\nu\\ &\quad + \frac{4}{9}K^{-5/6}{\rm d}Z^2 + K^{1/6}{\rm d}\Omega_4^2. \end{aligned} \tag{2o}$$

To obtain a 4D canonical action for mesons, we expand $\mathcal{A}_\mu(x, z)$, $\mathcal{A}_z(x, z)$ by a complete set of basis functions $\{\psi_n(z), \phi_n(z)\}$ as

$$\begin{aligned} \mathcal{A}_\mu(x, z) &= \sum_n B_\mu^{(n)}(x)\psi_n(z),\\ \mathcal{A}_z(x, z) &= \sum_n \varphi^{(n)}(x)\phi_n(z), \end{aligned} \tag{2p}$$

where the basis functions are expected to satisfy the normalization condition

$$\begin{aligned} &T(2\pi\alpha')^2R^3\int {\rm d}ZH_0^{1/2}K^{-1/3}\psi_n\psi_m = \delta_{mn},\\ &T(2\pi\alpha')^2R^3M_{KK}^2H_0(U_{KK})U_{KK}^2\int {\rm d}ZH_0^{1/2}K\phi_n\phi_m = \delta_{mn}, \end{aligned} \tag{2q}$$

and the eigenvalue equation

$$-H_0^{-1/2}K^{1/3}\partial_Z(H_0^{1/2}K\partial_Z\psi_m) = \Lambda_m\psi_m. \tag{2r}$$

These conditions imply that the eigenfunctions can be chosen as

$$\phi_n = \frac{1}{M_nU_{KK}}\partial_Z\psi_n,\ M_n = \Lambda_n^{1/2}M_{KK}H_0^{1/2}, \tag{2s}$$

for $n > 0$, and for $n = 0$

$$\phi_0 = \frac{c}{H_0^{1/2}K}, \tag{2t}$$

where c is a numerical number given by

$$c = \left[T(2\pi\alpha')^2R^3M_{KK}^2H_0(U_{KK})U_{KK}^2\int {\rm d}ZH_0^{-1/2}K^{-1}\right]^{-1/2}. \tag{2u}$$

Then, by imposing equations (2*p*)–(2*u*) into equation (2*m*), we obtain the canonical form of the action as

$$\begin{aligned} S_{\rm YM} &= -\int {\rm d}^4x\frac{1}{2}\partial_\mu\varphi^{(0)}\partial^\mu\varphi^{(0)}\\ &\quad + \sum_{n=1}^{\infty}\left[\frac{1}{4}F_{\mu\nu}^{(n)}F^{(n)\mu\nu} + \frac{1}{2}M_n^2V_\mu^{(n)}V^{(n)\mu}\right], \end{aligned} \tag{2v}$$

where

$$\begin{aligned} F_{\mu\nu}^{(n)} &= \partial_\mu V_\nu^{(n)} - \partial_\nu V_\mu^{(n)},\\ V_\mu^{(n)} &= B_\mu^{(n)} - M_n^{-1}\partial_\mu\varphi^{(n)}, \end{aligned} \tag{2w}$$

giving the infinite meson tower. We note that there is an alternative gauge condition $A_z = 0$, which can be obtained by a gauge transformation

$$\mathcal{A}_M \to \mathcal{A}_M - \partial_M\Lambda. \tag{2x}$$

In this case, the components of A_M become

$$\begin{aligned} \mathcal{A}_\mu(x, z) &= -\partial_\mu\varphi^{(0)}(x)\psi_0(z)\\ &\quad + \sum_{n=1}^{\infty}[B_\mu^{(n)}(x) - M_n^{-1}\partial_\mu\varphi^{(n)}]\psi_n(z)\\ &= -\partial_\mu\varphi^{(0)}(x)\psi_0(z) + \sum_{n=1}^{\infty}V_\mu^{(n)}(x)\psi_n(z),\\ \mathcal{A}_z(x, z) &= 0. \end{aligned} \tag{2y}$$

3. Flavored fermionic spectroscopy on the worldvolume of the D8-branes

While the supersymmetry on the D4-branes is broken down due to the compactification on S^1 and the method used in [19], the flavored D8-branes remain supersymmetric because the D8/$\overline{\text{D8}}$-branes are perpendicular to S^1 and thus are not compactified. The same issue arises in the D4/D8 model [46]. This means that supersymmetric fermions in addition to the gauge bosons will also arise in the low-energy theory, and there is no reason to neglect them in principle. Therefore, we investigated the spectroscopy of the worldvolume fermions on the D8-branes first, and then attempted to find a reasonable interpretation in terms of hadron physics in holography. The holographic investigation with instantons may also be an interesting extension to the framework of QFT with instantons.

3.1. Fermionic action and dimensional reduction

In string theory, the action for the worldvolume field on a D-brane is, in principle, obtained under the rule of T-duality [22], which includes supersymmetrically the bosonic and fermionic parts. The bosonic action of a D-brane can be reviewed in many textbooks, e.g. [21, 51, 52]. In particular, the bosonic features of the D0–D4/D8 model can be completely reviewed in [38–42]. However, the full formula of the action of worldvolume fermions on D-brane is quite complex in general. Because our concern is fermionic spectroscopy in holography, let us focus on the quadratic part of the fermionic action, which can be collected as [22, 53, 54]

$$S_{\mathrm{f}}^{\mathrm{D_p}} = \frac{\mathrm{i}T_{\mathrm{p}}}{2}\int \mathrm{d}^{p+1}x\mathrm{e}^{-\phi}\sqrt{-(g+f)}\,\bar{\Psi}(1-\Gamma_{\mathrm{D_p}}) \times(\Gamma^{\alpha}\hat{D}_{\alpha}-\Delta+L_{\mathrm{D_p}})\Psi, \tag{3a}$$

where for type IIA string theory

$$\begin{aligned}
\hat{D}_{\alpha} &= \nabla_{\alpha}+\frac{1}{4\cdot 2!}H_{\alpha NK}\Gamma^{NK}\bar{\gamma}+\frac{1}{8}\mathrm{e}^{\phi}\\
&\quad\times\left(\frac{1}{2!}F_{NK}\Gamma^{NK}\Gamma_{\alpha}\bar{\gamma}+\frac{1}{4!}F_{KLNP}\Gamma^{KLNP}\Gamma_{\alpha}\right),\\
\Delta &= \frac{1}{2}\left(\Gamma^{M}\partial_{M}\phi+\frac{1}{2\cdot 3!}H_{MNK}\Gamma^{MNK}\bar{\gamma}\right)\\
&\quad+\frac{1}{8}\mathrm{e}^{\phi}\left(\frac{3}{2!}F_{MN}\Gamma^{MN}\bar{\gamma}+\frac{1}{4!}F_{KLNP}\Gamma^{KLNP}\right),\\
\Gamma_{\mathrm{D_p}} &= \frac{1}{\sqrt{-(g+f)}}\sum_{q}\\
&\quad\times\frac{\epsilon^{\alpha_1\ldots\alpha_{2q}\beta_1\ldots\beta_{p-2q+1}}}{q!2^q(p-2q+1)!}\\
&\quad f_{\alpha_1\alpha_2}\ldots f_{\alpha_{2q-1}\alpha_{2q}}\Gamma_{\beta_1\ldots\beta_{p-2q+1}}\bar{\gamma}^{\frac{p-2q+2}{2}},\\
L_{\mathrm{D_p}} &= \sum_{q}\frac{\epsilon^{\alpha_1\ldots\alpha_{2q}\beta_1\ldots\beta_{p-2q+1}}}{q!2^q(p-2q+1)!}\\
&\quad\times\frac{(-\bar{\gamma})^{\frac{p}{2}-q+1}}{\sqrt{-(g+f)}}f_{\alpha_1\alpha_2}\ldots f_{\alpha_{2q-1}\alpha_{2q}}\Gamma_{\beta_1\ldots\beta_{p-2q+1}}{}^{\lambda}\hat{D}_{\lambda}.
\end{aligned}\tag{3b}$$

Let us clarify the notations used in equations (3*a*) and (3*b*). First, Ψ denotes the worldvolume fermions on the $\mathrm{D_p}$-brane, and T_{p} denotes the tension of the $\mathrm{D_p}$-brane, which is given as $T_{\mathrm{p}}=g_s^{-1}(2\pi)^{-p}l_s^{-(p+1)}$. The capital letters K, L, M, N... denote the index run over the 10D spacetime, and the lowercase letters a, b, ... denote the index run over the tangent space of the 10D spacetime. The Greek letters α, β, and λ refer to the indices running over the worldvolume of the $\mathrm{D_p}$-brane. For gravity theory with fermions, the metric should be written in terms of elfbein e_M^a as $g_{MN}=e_M^a\eta_{ab}e_N^b$, and the gamma matrices are given as follows:

$$\{\gamma^a,\gamma^b\}=2\eta^{ab},\ \{\Gamma^M,\Gamma^N\}=2g^{MN}, \tag{3c}$$

with $e_M^a\Gamma^M=\gamma^a$. Note that $\omega_{\alpha ab}$ refers to the spin connection, and the covariant derivative for fermions is given by $\nabla_{\alpha}=\partial_{\alpha}+\frac{1}{4}\omega_{\alpha ab}\gamma^{ab}$. By alternately ranking the indices antisymmetrically and symmetrically, we can define a gamma matrix with multiple indices, e.g.

$$\begin{aligned}
\gamma^{ab} &= \frac{1}{2}[\gamma^a,\gamma^b],\ \gamma^{abc}=\frac{1}{2}\{\gamma^a,\gamma^{bc}\},\\
\gamma^{abcd} &= \frac{1}{2}[\gamma^a,\gamma^{bcd}]\ldots
\end{aligned}\tag{3d}$$

and $\gamma^{abc\ldots}$ shares the same definition with $\Gamma^{MNK\ldots}$; $\bar{\gamma}$ is $\bar{\gamma}=\gamma^{01\ldots9}$, and σ_2 is the associated Pauli matrix. The worldvolume field f is a sum as $f=B+(2\pi\alpha')\mathcal{F}$, where $\mathcal{F}$ is the Yang–Mills field strength and B refers to the NS–NS two-form in type IIA string theory with its field strength $H=\mathrm{d}B$. All the fields denoted by F, e.g. F_M, F_{MN}, F_{KLM}..., refer to the field strength of the R–R forms. Note that p should be chosen as $p=8$ in the presented formulas for the D8-brane.

For convenience in the following discussion by holography, let us simplify the kinetic part of action (3*a*) to be a 5D form by setting $f=0$ and $p=8$. In this case, action (3*a*) becomes

$$\begin{aligned}
S_{\mathrm{f}}^{\mathrm{D_8}} &= \frac{\mathrm{i}T_8}{2}\int \mathrm{d}^9x\sqrt{-g}\,\bar{\Psi}\Big[\mathrm{e}^{-\phi}\Gamma^{\alpha}\nabla_{\alpha}\\
&\quad+\frac{1}{8\cdot 4!}F_{KLNP}(\Gamma^{\alpha}\Gamma^{KLNP}\Gamma_{\alpha}-\Gamma^{KLNP})\\
&\quad-\frac{1}{2}\mathrm{e}^{-\phi}\Gamma^{M}\partial_{M}\phi+\frac{1}{8\cdot 2!}F_{NK}(\Gamma^{\alpha}\Gamma^{NK}\Gamma_{\alpha}-3\Gamma^{MN})\bar{\gamma}\Big]\Psi.
\end{aligned}\tag{3e}$$

Metric (2*o*) is plugged into equation (3*e*). After some straightforward calculations, we obtain the Dirac operator as follows:

$$\begin{aligned}
\Gamma^{\alpha}\nabla_{\alpha} &= \left(\frac{2}{3}b^{1/2}M_{KK}R\right)^{-3/2}H_0^{-1/4}\\
&\quad\times\Bigg[K^{-1/4}\gamma^{\mu}\partial_{\mu}+\frac{2}{3}M_{KK}b^{1/2}K^{-1/12}\not{D}_{S^4}\\
&\quad+M_{KK}b^{1/2}\gamma^{Z}\left(ZK^{-7/12}+\frac{1}{2}K^{5/12}\frac{H_0'}{H_0}+K^{5/12}\partial_Z\right)\Bigg],
\end{aligned}\tag{3f}$$

where $\not{D}_{S^4}=\gamma^m D_m$ is the covariant derivative operator for a spinor on S^4. Further substituting solution (2*c*) for the R–R fields and dilaton leads to

$$\frac{1}{8\cdot 2!}F_{NK}(\Gamma^{\alpha}\Gamma^{NK}\Gamma_{\alpha}-3\Gamma^{MN})\bar{\gamma}\Psi$$
$$=\left(\frac{2}{3}b^{1/2}M_{KK}R\right)^{-3/2}H_0^{-1/4}$$
$$\times\left(\frac{2}{3}b^{1/2}M_{KK}R\right)^{-15/2}(2\pi l_s)^4\kappa M_{KK}b^{1/2}H_0^{-7/4}K^{-4/3}\gamma^Z\Psi, \quad (3g)$$

and

$$\frac{1}{2}\Gamma^M\partial_M\phi=\left(\frac{2}{3}b^{1/2}M_{KK}R\right)^{-3/2}H_0^{-1/4}M_{KK}b^{1/2}\gamma^Z \times\left(\frac{1}{4}ZK^{-7/12}+\frac{3}{8}K^{5/12}\frac{H_0'}{H_0}\right), \quad (3h)$$

where we have imposed the project $\bar{\gamma}\Psi=\gamma^4\Psi=\Psi$. Note that the contribution of the R–R C_4 form vanishes in the presented setup. Then, the fermionic action of the D8-branes can be written as

$$S_{\mathrm{f}}^{\mathrm{D_8}}=\frac{\mathrm{i}\mathcal{T}}{(2\pi\alpha')^2\Omega_4}b^{11/4}\int \mathrm{d}^4x\mathrm{d}Z\mathrm{d}\Omega_4H_0^{5/4}\bar{\Psi}P_-$$
$$\times\left\{K^{5/12}\gamma^{\mu}\partial_{\mu}+\frac{2}{3}M_{KK}b^{1/2}K^{7/12}\not{D}_{S^4}\right.$$
$$+M_{KK}b^{1/2}\gamma^Z\left[\sqrt{\lambda b(b-1)}H_0^{-1}K^{-5/12}+\frac{3}{4}ZK^{1/12}\right.$$
$$\left.\left.+\frac{1}{4}ZK^{1/12}\frac{b-1}{Z^2+b}+K^{13/12}\partial_Z\right]\right\}\Psi, \quad (3i)$$

where

$$\mathcal{T}=\frac{1}{2}\left(\frac{2}{3}\right)^{13/2}T_8(2\pi\alpha')^2\Omega_4(M_{KK}R)^{11/2}R^5,$$
$$P_-=\frac{1}{2}(1-\Gamma_{\mathrm{D8}}). \quad (3j)$$

Keeping these in hand, let us impose the decomposition for the spinor in this model by following the steps in [18, 22, 46] on equation (3*i*). Specifically, we first decompose the worldvolume nine-dimensional spinor into a $1+3$-dimensional part $\psi(x,Z)$, an S^4 part φ, and a remaining two-dimensional part β as[3]

$$\Psi=\psi(x,Z)\otimes\varphi(S^4)\otimes\beta. \quad (3k)$$

Second, the associated 10D gamma matrices can be chosen as

$$\gamma^{\mu}=\sigma_1\otimes\boldsymbol{\gamma}^{\mu}\otimes\mathbf{1},\ \mu=0,1,2,3$$
$$\gamma^Z=\sigma_1\otimes\boldsymbol{\gamma}\otimes\mathbf{1},$$
$$\gamma^4=\sigma_2\otimes\mathbf{1}\otimes\tilde{\gamma},$$
$$\gamma^m=\sigma_2\otimes\mathbf{1}\otimes\gamma^m,\ m=6,7,8,9,$$
$$\boldsymbol{\gamma}=\mathrm{i}\boldsymbol{\gamma}^0\boldsymbol{\gamma}^1\boldsymbol{\gamma}^2\boldsymbol{\gamma}^3,$$
$$\tilde{\gamma}=\mathrm{i}\gamma^6\gamma^7\gamma^8\gamma^9, \quad (3l)$$

where bold font denotes the 4×4 gamma matrices and γ^m, where $m=6,7,8,9$ refers to the gamma matrix on the tangent space of S^4. Note that the 10D chirality matrix has a simple form as $\bar{\gamma}=\sigma_3\otimes\mathbf{1}\otimes\mathbf{1}$ in this decomposition. By choosing the representation of σ_3, β can be decomposed by the eigenstates of σ_3 as

$$\sigma_3\beta_{\pm}=\beta_{\pm},\ \sigma_1\beta_{\pm}=\beta_{\mp},\ \sigma_2\beta_{\pm}=\pm\mathrm{i}\beta_{\mp}, \quad (3m)$$

where $\beta_{\pm}$ denotes the two eigenstates of σ_3. Because the condition $\bar{\gamma}\Psi=\Psi$ is fixed by the kappa symmetry, we need to choose $\beta=\beta_+$ on the D8-brane. In addition, the spinor φ as the component on S^4 must satisfy the Dirac equation on S^4, and it is possible to use the spherical harmonic function with the eigenstates of $\Gamma^{\underline{m}}\nabla_m^{S^4}$ as [55, 56]

$$\gamma^m\nabla_m^{S^4}\varphi^{\pm l,s}=\mathrm{i}\Lambda_l^{\pm}\varphi^{\pm l,s};\ \Lambda_l^{\pm}=\pm(2+l),\ l=0,1\ldots, \quad (3n)$$

where the angular quantum number is s, and l represents the angular quantum numbers on S^4. Altogether, by imposing the decomposition (3*k*)–(3*n*) into action (3*i*) and rescaling $\psi\to(2\pi\alpha')H_0^{-5/8}K^{-13/24}\psi$, we can obtain the following 5D effective action ($\Lambda_l\equiv\Lambda_l^+$):

$$S_{\mathrm{f}}^{\mathrm{D_8}}=\mathrm{i}\mathcal{T}b^{11/4}\int \mathrm{d}^4x\mathrm{d}Z\bar{\psi}\{K^{-2/3}\gamma^{\mu}\partial_{\mu}$$
$$-\frac{2}{3}M_{KK}b^{1/2}\Lambda_lK^{-1/2}$$
$$+M_{KK}b^{1/2}\gamma\partial_Z+M_{KK}b^{1/2}\gamma$$
$$\times\left[\frac{\sqrt{b(b-1)\lambda}}{H_0K^{3/2}}+\frac{3}{2}\frac{(b-1)Z}{H_0K^2}\right]\Big\}\psi. \quad (3o)$$

In the following sections, we will study the fermionic spectroscopy in this model with this 5D fermionic action (3*o*) and attempt to find its holographic interpretation in terms of hadron physics.

3.2. Canonical 4D action

To obtain the mass term in 4D dual theory, we need to rewrite action (3*o*) in canonical form. To this end, we work with the Weyl basis by

$$\psi=\begin{pmatrix}\psi_+\\ \psi_-\end{pmatrix}, \quad (3p)$$

with the choice of gamma matrices

$$\boldsymbol{\gamma}^{\mu}=\mathrm{i}\begin{pmatrix}0 & \sigma^{\mu}\\ \bar{\sigma}^{\mu} & 0\end{pmatrix},\ \boldsymbol{\gamma}=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}, \quad (3q)$$

where $\sigma^{\mu}=(1,-\sigma^i)$, $\bar{\sigma}^{\mu}=(1,\sigma^i)$. Then, we decompose the spinor by the basis functions $\{f_{\pm}^{(n)}(Z)\}$, $n = 0,1,2\ldots$, as a complete set as

$$\psi_+=\sum_n\psi_+^{(n)}(x)f_+^{(n)}(Z),\ \psi_-=\sum_n\psi_-^{(n)}(x)f_-^{(n)}(Z), \quad (3r)$$

where the functions $\{f_{\pm}^{(n)}(Z)\}$ are real eigenfunctions for the coupled eigenequations

$$\pm M_{KK}b^{1/2}\partial_Zf_{\pm}^{(n)}+M_{KK}b^{1/2}V_{\pm}f_{\pm}^{(n)}=M_n^fK^{-2/3}f_{\mp}^{(n)}, \quad (3s)$$

with

$$V_{\pm}=-\frac{2}{3}\Lambda_lK^{-1/2}\pm\left[\frac{\sqrt{b(b-1)\lambda}}{H_0K^{3/2}}+\frac{3}{2}\frac{(b-1)Z}{H_0K^2}\right], \quad (3t)$$

[3] In D dimension, a Dirac spinor usually has $[D/2]$ components, where $[D/2]$ refers to the integer part of $D/2$ [51, 52].

and satisfy the normalization condition

$$\mathcal{T}b^{11/4}\int dZK^{-2/3}f_{\pm}^{(m)}f_{\pm}^{(n)}=\delta^{mn}. \tag{3u}$$

Here, $M_n^{\rm f}$ refers to the nth eigenvalue of equation (3*s*). By imposing equations (3*s*)–(3*u*) into action (3*o*), it reduces to the following 4D canonical action as

$$S_{\rm f}^{\rm D_8}=-\sum_n\int d^4x[i\psi_-^{(n)\dagger}\sigma^\mu\partial_\mu\psi_-^{(n)}+i\psi_+^{(n)\dagger}\bar{\sigma}^\mu\partial_\mu\psi_+^{(n)} + M_n^f\psi_+^{(n)\dagger}\psi_-^{(n)}+M_n^f\psi_-^{(n)\dagger}\psi_+^{(n)}], \tag{3v}$$

which can be further written as

$$S_{\rm f}^{\rm D_8}=i\sum_n\int d^4x[\bar{\psi}^{(n)}\gamma^\mu\partial_\mu\psi^{(n)}+M_n^f\bar{\psi}^{(n)}\psi^{(n)}], \tag{3w}$$

once the Dirac spinor

$$\psi^{(n)}=\begin{pmatrix}\psi_+^{(n)}\\ \psi_-^{(n)}\end{pmatrix}, \tag{3x}$$

is introduced. Equation (3*s*) can be rewritten as two decoupled second-order differential equations in the Sturm–Liouville form as

$$\begin{aligned}&-\partial_Z\left[\frac{\partial_Z f_+^{(n)}+V_+f_+^{(n)}}{K^{-2/3}}\right]+V_-\left[\frac{\partial_Z f_+^{(n)}+V_+f_+^{(n)}}{K^{-2/3}}\right]\\ &=m_n^2K^{-2/3}f_+^{(n)}\\ &\partial_Z\left[\frac{-\partial_Z f_-^{(n)}+V_-f_-^{(n)}}{K^{-2/3}}\right]+V_+\left[\frac{-\partial_Z f_-^{(n)}+V_-f_-^{(n)}}{K^{-2/3}}\right]\\ &=m_n^2K^{-2/3}f_-^{(n)},\end{aligned} \tag{3y}$$

where $m_n=\frac{M_n^f}{M_{KK}b^{1/2}}$ and their eigenvalues can be evaluated numerically. Because $V_\pm$ depends on the density of the D0-branes, the eigenvalue m_n also relates to the charge of the D0-brane, which means that it depends on the theta term in the language of QCD. In the next section, we numerically evaluate the mass spectrum of fermions using the two-point Green function in AdS/CFT as its spectral function.

3.3. Holographic Green function as spectral function

In this section, we numerically evaluate the mass spectrum of the fermions on the D8-branes using the prescription for the correlation function in the AdS/CFT dictionary. Let us take into account a fermionic operator χ in the dual theory described by the QCD action (2*k*), in which the bulk operator of χ is a worldvolume fermion ψ on the D8-branes $\mathcal{M}$ presented in action (3*o*). Recall the AdS/CFT dictionary with spinor [57, 58]. It is known that the partition function associated with ψ in the bulk is equivalent to the average value of the generating function associated with χ as

$$\exp\left\{\int_{\mathcal{M}}\mathcal{L}_{f,\rm ren}^{\rm D_8}[\bar{\psi},\psi]d^{D+1}x\right\} =\left\langle\exp\left\{\int_{\partial\mathcal{M}}(\bar{\chi}\psi_0+\bar{\psi}_0\chi)d^Dx\right\}\right\rangle, \tag{3z}$$

where ω, $\vec{p}$ denotes the frequency and three-momentum of the associated Fourier modes, $\mathcal{L}_{f,\rm ren}^{\rm D_8}$ refers to the renormalized onshell Lagrangian associated with action (3*o*), ψ_0 denotes the boundary value of ψ, and D refers to the dimension of the dual theory. Hence, the retarded two-point correlation function $G_{\rm R}(\omega,\vec{p})$ of χ can be obtained by

$$\langle\chi(\omega,\vec{p})\rangle=G_{\rm R}(\omega,\vec{p})\psi_0(\omega,\vec{p}). \tag{3aa}$$

On the other hand, we have

$$\langle\bar{\chi}(\omega,\vec{p})\rangle=-\frac{\delta S_{f,\rm ren}^{\rm D_8}}{\delta\psi_0}=\Pi_0(\omega,\vec{p}),\quad S_{\rm f,ren}^{\rm D_8}=\int_{\mathcal{M}}\mathcal{L}_{\rm f,ren}^{\rm D_8}[\bar{\psi},\psi]d^{D+1}x. \tag{3bb}$$

Therefore, it is possible to evaluate the two-point correlation function by imposing equations (3*aa*) and (3*bb*) on action (3*o*).

To achieve our goal, we need to first evaluate the renormalized onshell action $S_{\rm f,ren}^{\rm D_8}$by solving the Dirac equation associated with equation (3*o*), which is derived as

$$\left\{K^{-2/3}\gamma^\mu\partial_\mu-\frac{2}{3}M_{KK}b^{1/2}\Lambda_lK^{-1/2}+M_{KK}b^{1/2}\gamma\partial_Z + M_{KK}b^{1/2}\gamma\left[\frac{\sqrt{b(b-1)\lambda}}{H_0K^{3/2}}+\frac{3}{2}\frac{(b-1)Z}{H_0K^2}\right]\right\}\psi=0. \tag{3cc}$$

Using the ansatz of the Fourier mode of equation (3*p*) on Weyl basis

$$\psi=e^{ip\cdot x}\begin{pmatrix}\psi_+\\ \psi_-\end{pmatrix},\ p\cdot x=p_\mu x^\mu=-\omega t+\vec{p}\cdot\vec{x}. \tag{3dd}$$

Equation (3*cc*) can be rewritten as

$$\begin{aligned}&\partial_Zf_++V_+f_+-K^{-2/3}\frac{(\sigma\cdot p)}{M_{KK}b^{1/2}}f_-=0,\\ &-K^{-2/3}\frac{(\bar{\sigma}\cdot p)}{M_{KK}b^{1/2}}f_+-\partial_Zf_-+V_-f_-=0,\end{aligned} \tag{3ee}$$

where

$$V_\pm=-\frac{2}{3}\Lambda_lK^{-1/2}\pm\left[\frac{\sqrt{b(b-1)\lambda}}{H_0K^{3/2}}+\frac{3}{2}\frac{(b-1)Z}{H_0K^2}\right]. \tag{3ff}$$

Equation (3*ee*) can be further written as two decoupled second-order differential equations, which are simply equation (3*y*). With simplification, they are

$$\begin{aligned}&\partial_Z^2f_++\left(V_+-V_-+\frac{4}{3}\frac{Z}{K}\right)\partial_Zf_+ +\left(\frac{4}{3}\frac{Z}{K}V_+-V_+V_-+\partial_ZV_+-\frac{p^2}{K^{4/3}M_{KK}^2b}\right)f_+=0,\\ &\partial_Z^2f_-+\left(V_+-V_-+\frac{4}{3}\frac{Z}{K}\right)\partial_Zf_- -\left(\frac{4}{3}\frac{Z}{K}V_-+V_+V_-+\partial_ZV_--\frac{p^2}{K^{4/3}M_{KK}^2b}\right)f_-=0,\end{aligned} \tag{3gg}$$

which can be solved analytically at the holographic boundary, i.e. $Z\to\pm\infty$ because, in AdS/CFT, their boundary values contribute to ψ_0 (the boundary value of the bulk fermions ψ).

Here, we can assume that the N_f D8-branes stretch to the boundary $Z \to \infty$, and the N_f anti-D8-branes stretch to the boundary $Z \to -\infty$ while they are connected at $Z = 0$. Therefore, although we will discuss the solution for equation (3*gg*) for the D8-branes, it would be the same as the solution for the anti-D8-branes. Thus, we can obtain the analytical solution at $Z \to \infty$ as

$$f_+ = AZ^{\frac{2}{3}\Lambda_l} + BZ^{-\frac{1}{3}-\frac{2}{3}\Lambda_l},$$
$$f_- = CZ^{-\frac{1}{3}+\frac{2}{3}\Lambda_l} + DZ^{-\frac{2}{3}\Lambda_l}. \tag{3hh}$$

Due to $\Lambda_l \geqslant 2$, the finite boundary value ψ_0 of ψ can be defined as [57, 58]

$$\psi_0 = \lim_{Z\to+\infty} Z^{-\frac{2}{3}\Lambda_l}\psi = \begin{pmatrix} A \\ 0 \end{pmatrix}. \tag{3ii}$$

Then, the onshell value of action (3*o*) is computed as

$$\begin{aligned} S_f^{D_8} &= i\mathcal{T}b^{11/4}\int d^4x dZ\bar{\psi}\{K^{-2/3}\gamma^\mu\partial_\mu \\ &\quad - \frac{2}{3}M_{KK}b^{1/2}\Lambda_l K^{-1/2} \\ &\quad + M_{KK}b^{1/2}\gamma\partial_Z + M_{KK}b^{1/2}\gamma \\ &\quad \times \left[\frac{\sqrt{b(b-1)\lambda}}{H_0 K^{3/2}} + \frac{3}{2}\frac{(b-1)Z}{H_0 K^2}\right]\}\psi. \\ &= i\mathcal{T}b^{11/4}\int d^4x(\bar{\psi}M_{KK}\gamma\psi)|_0^{+\infty} \\ &\quad - i\mathcal{T}_c\int d^4x dZ\bar{\psi}\{-K^{-2/3}\gamma^\mu\overleftarrow{\partial_\mu} \\ &\quad + \frac{2}{3}M_{KK}b^{1/2}\Lambda_l K^{-1/2} \\ &\quad + M_{KK}b^{1/2}\gamma\overleftarrow{\partial_Z} + M_{KK}b^{1/2}\gamma \\ &\quad \times \left[\frac{\sqrt{b(b-1)\lambda}}{H_0 K^{3/2}} + \frac{3}{2}\frac{(b-1)Z}{H_0 K^2}\right]\}\psi \\ &\equiv \int d^4x(\Pi\psi)|_0^{+\infty} = \int d^4x\Pi_+\psi_+|_{Z\to+\infty} + \dots, \end{aligned} \tag{3jj}$$

where we have imposed the conjugate equation associated to equation (3*o*). Using solution (3*hh*), the boundary action in equation (3*jj*) is given as

$$S_f^{D_8} \supseteq -\mathcal{T}M_{KK}b^{13/4}\int d^4x[C^\dagger AZ^{-\frac{1}{3}+\frac{4}{3}\Lambda_l} + D^\dagger A]|_{Z\to+\infty}, \tag{3kk}$$

which leads to the holographic boundary counterterm S_{ct} to the fermionic action as

$$S_{ct} = \mathcal{T}M_{KK}b^{13/4}\int d^4x[C^\dagger AZ^{-\frac{1}{3}+\frac{4}{3}\Lambda_l}]|_{Z\to+\infty}. \tag{3ll}$$

Therefore, the renormalized onshell action for the fermions on the D8-branes is obtained as

$$\begin{aligned} S_{f,ren}^{D_8} &= S_f^{D_8} + S_{ct} \\ &= -\mathcal{T}M_{KK}b^{13/4}\int d^4x D^\dagger A, \end{aligned} \tag{3mm}$$

and the Green function is given by

$$\mathcal{T}M_{KK}b^{13/4}D = G_R(\omega, \vec{p})A, \tag{3nn}$$

using equations (3*aa*), (3*bb*), (3*ii*), and (3*mm*). To numerically evaluate the Green function, we can write the concerned function $f_\pm$ as

$$f_+ = \begin{pmatrix} F_+^{(1)} \\ F_+^{(2)} \end{pmatrix},\ f_- = \begin{pmatrix} F_-^{(1)} \\ F_-^{(2)} \end{pmatrix}, \tag{3oo}$$

and without loss of generality, set momentum along x^1 as $p_\mu = (-\omega, p, 0, 0)$, and then define the ratio

$$\xi_1 = \frac{F_-^{(1)}}{F_+^{(1)}},\ \xi_2 = \frac{F_-^{(2)}}{F_+^{(2)}}. \tag{3pp}$$

According to equation (3*hh*), we can see that the boundary value of $f_\pm$ gives the boundary spinor D, A, so using equations (3*ii*) and (3*nn*), the Green function can be rewritten as

$$G_R^{(1,2)} = \mathcal{T}M_{KK}b^{13/4}\lim_{Z\to\infty} Z^{\frac{4}{3}\Lambda_l}\xi_{1,2}(Z). \tag{3qq}$$

The ratios satisfy the equation ($'$ is the derivative with respect to Z)

$$\begin{aligned} \xi_{1,2}' &= (V_+ + V_-)\xi_{1,2} + \frac{K^{-2/3}}{M_{KK}b^{1/2}} \\ &\quad \times [(\omega + p\cdot h)\xi_{1,2}^2 + (\omega - p\cdot h)], \end{aligned} \tag{3rr}$$

which is derived from the Dirac equation (3*ee*). Here

$$\frac{F_+^{(2)}}{F_+^{(1)}} = \frac{F_-^{(2)}}{F_-^{(1)}} = h, \tag{3ss}$$

can be simply set as $h = \pm 1$ for the periodic and antiperiodic fermions, also as the boundary condition of $\xi_{1,2}$. Altogether, it is possible to numerically solve equation (3*rr*) with incoming wave boundary condition $\xi_{1,2}(0) = \pm 1$.

3.4. Numerical analysis

In this section, we numerically analyze the fermionic spectrum by solving equation (3*rr*). As a first overlook, we numerically plotted the Green function as a dense function of ω, p, as illustrated in figure 2. We can see that the peaks in the Green function basically display the dispersion curves for the onshell relation as $\omega^2 - k^2 = (M_n^f)^2$. Because fermionic action (3*o*) can be written in the canonical 4D form (equation (3*w*)), the general form of the fermionic propagator for the operator χ in the dual theory should be

$$G_R(\omega, \vec{p}) = \frac{1}{ip_\mu\gamma^\mu - M_n^f}. \tag{3tt}$$

Therefore, the poles in the Green function denote the onshell energy of the states created by χ, and, in this sense, the two-point Green function is the spectral function of χ. In addition, recall the relationship given in equations (2*j*), (3*s*), and (3*u*), which means that M_n^f must depend on b, which is related to the instanton density in the Yang–Mills theory. Thus, we also

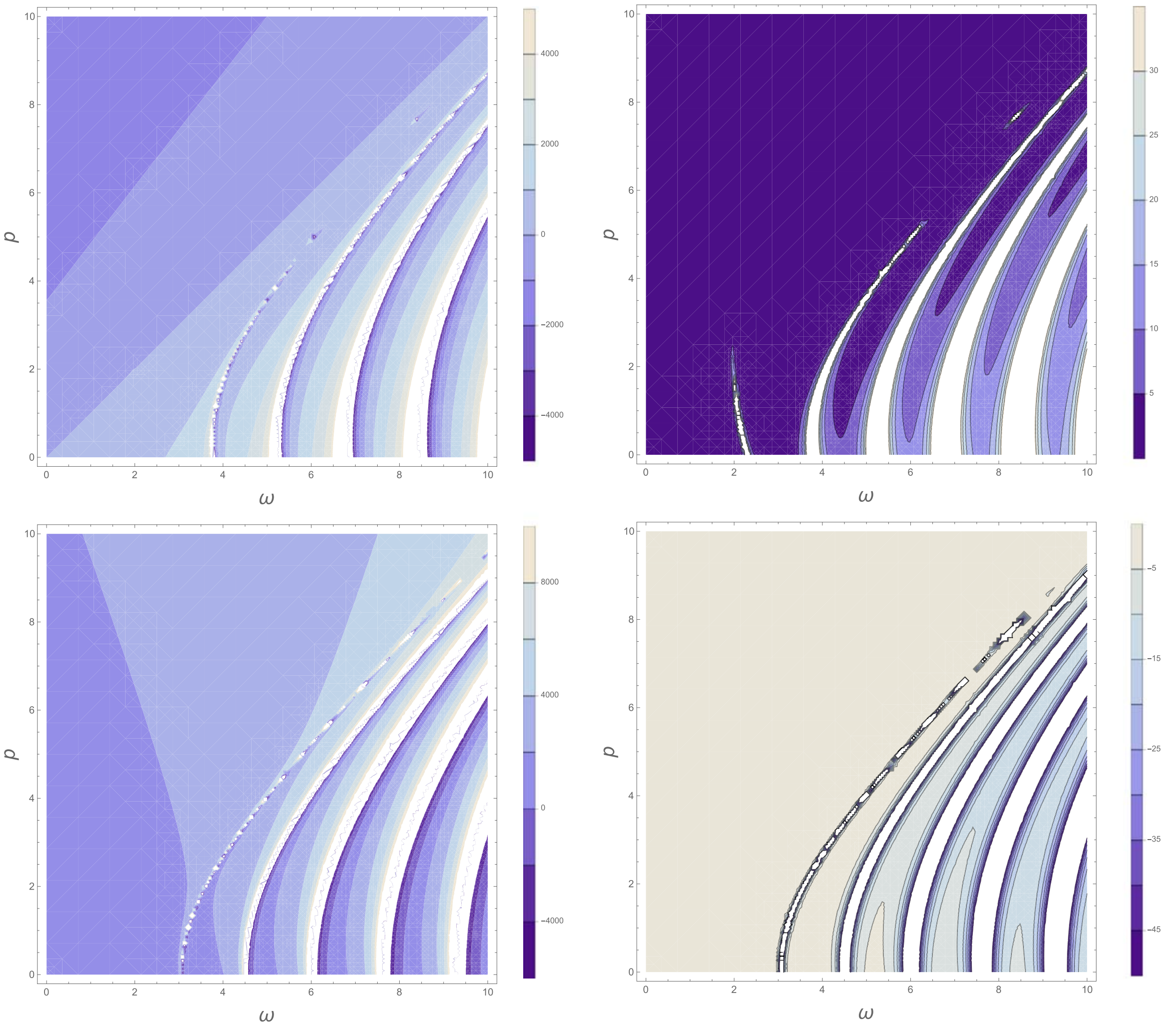

Figure 2. Real and imaginary parts of the fermionic Green function $G_{\mathrm{R}}(\omega, p)$ as the spectral function in the D0–D4/D8 model. The parameters chosen were $\Lambda_1 = 2$, $b = 2$, and $M_{KK} = 1$. The peak of the Green function is represented by white corrugation.

plotted the relationship between M_n^f and b (figure 3) to confirm this relationship by setting $p = 0$. According to our numerical calculation, the relationship between M_n and b can be simply fitted as

$$M_n^f \propto M_{KK} b^{1/2} \simeq \sqrt{\frac{\lambda}{\lambda - \frac{l_s^2}{\beta^2}\theta^2}} M_{KK}, \tag{3uu}$$

leading to a fermionic mass spectrum, as shown in figures 4 and 5 and table 2. The presented formula indicates that the fermionic mass spectrum increases by b (i.e. it relates to the instanton density). Hence, it should describe the metastable fermionic states for $b > 1$. In this sense, this holographic framework provides an alternative way to investigate baryonic correlation, which is additional to the framework of perturbative QFT as [1, 2]. These conclusions are also in agreement with several studies [42, 43, 48–50], which revealed that the hadron could be metastable in the presence of instantons.

4. Worldvolume fermions as baryons

4.1. Holographic interpretation

As we have discussed, although the N_c D4-branes are non-supersymmetric by following the proposal of the compactification in Witten's study [19], there is no mechanism to break down the supersymmetry on the N_f D8-branes in principle, so they remain supersymmetric, and its low-energy theory contains supersymmetric fermions in addition to the gauge bosons. Usually, these supersymmetric fermions are interpreted as mesinos (the supersymmetric partner of mesons) in terms of hadron physics [46] because their bosonic partner (the gauge field) is identified as mesons in low-energy theory, as illustrated in section 2. However, such mesinos are always absent in QCD and hadron physics.

In this study, we attempted to interpret supersymmetric fermions as baryons by including a baryon vertex in this model for the following reasons. First, it is well known that all baryons are fermionic. Second, the supersymmetric fermions

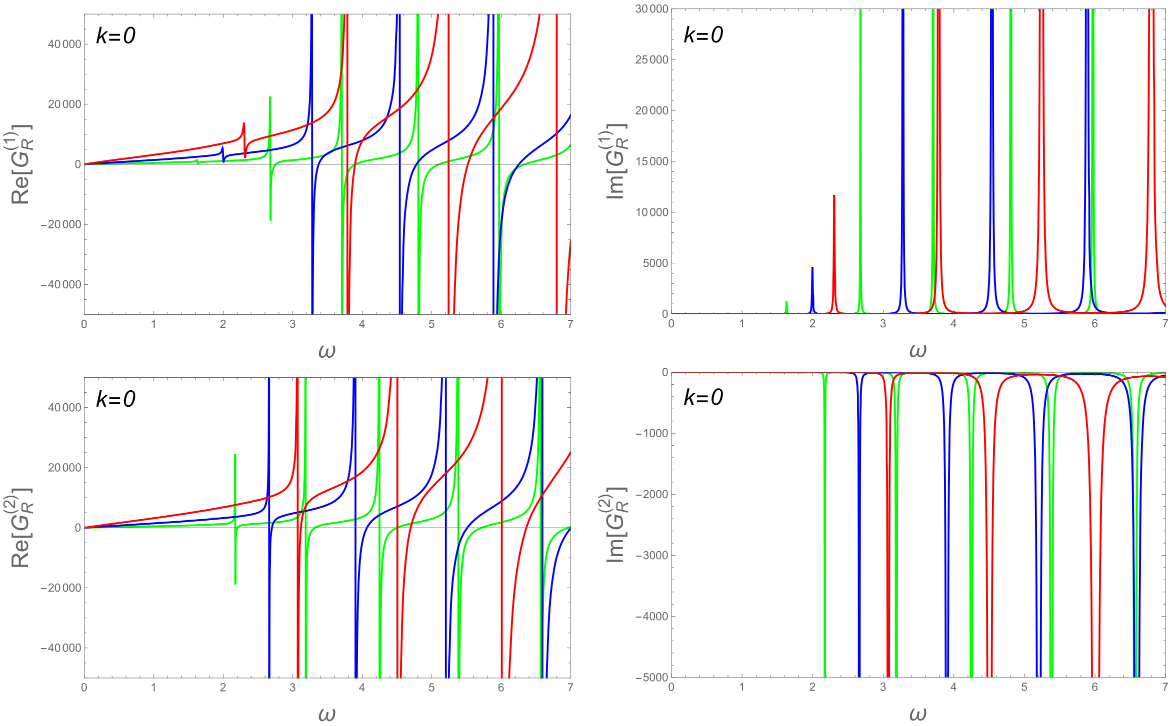

Figure 3. Real and imaginary parts of the fermionic Green function $G_{\mathrm{R}}(\omega, p)$ as the spectral function in the D0–D4/D8 model with $p = 0$. The position of the peaks refers to the onshell energy of a fermionic bound state, and the green, blue, and red colors represent the Green functions with $b = 1$, 1.5, and 2, respectively. The other parameters were fixed at $\Lambda_l = 2$ and $M_{KK} = 1$.

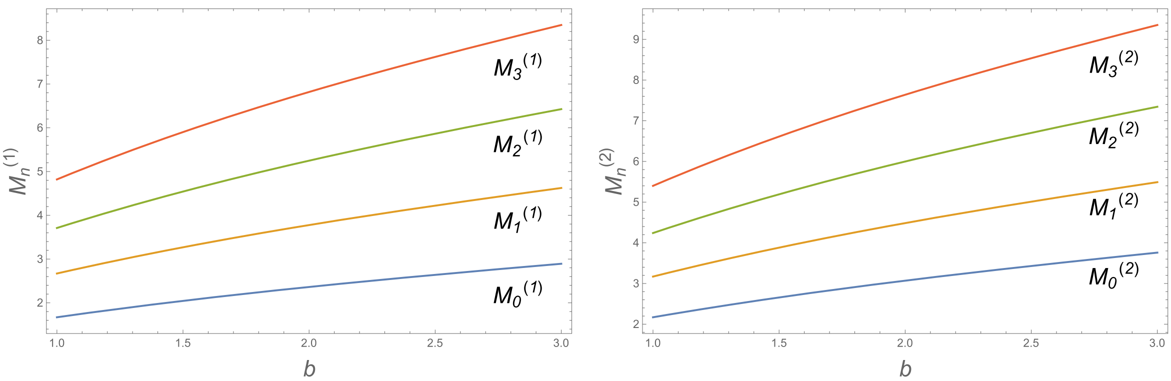

Figure 4. Lowest spectrum with $n = 0$, 1, 2, and 3 and $\Lambda_l = 2$ of fermions as a function of b from the position of the poles in the spectral function $G_{\mathrm{R}}(\omega, \vec{p})$. Indices (1) and (2) indicate that the mass spectrum is obtained from $G_{\mathrm{R}}^{(1,2)}$.

on the worldvolume of D8-branes take flavors and are color singlets because they are also adjoint representations of the flavor group and singlet of the color group. The features of worldvolume fermions on the D8-branes are in agreement with the current understanding of baryons. However, the baryon number is usually conserved when the baryons are concerned; thus, the baryon vertex is essential in our holographic construction. Recall that the baryon vertex in AdS/CFT is identified as the D-brane wrapped on the spherical part of the bulk geometry with N_{c} open strings ending on it stretching to the holographic boundary [23, 24]. Therefore, in this model, the baryon vertex is a D4-brane wrapped on S^4 with N_{c} open strings, as illustrated in table 1 and figure 6.

Keeping these in hand, when the N_{f} D8-branes are introduced into this model, the baryon vertex lives totally inside the N_{f} D8-branes, as shown in table 1 and figure 6. Hence, the N_{c} open strings ending on the baryon vertex as 8–8 strings are flavored and take the baryon number from the baryon vertex. The worldvolume fermions ψ created by these open strings are flavored and also take the baryon number. On the side of the boundary theory that lives on a probe D4-brane, the N_{c} open strings play as 4–8 strings as the fundamental quarks, i.e. the fundamental representation of the color group. Therefore, the fermionic color singlet operator χ must be obtained by the decomposition of direct products of irreducible representations of the color and flavor symmetry group, i.e. it could be a baryon field consisting of N_{c} quarks. Thus, the worldvolume fermionic field ψ created by the N_{c} open strings is the dual field of χ, which must share exactly the same quantum numbers. This is consistent with the AdS/CFT dictionary and the

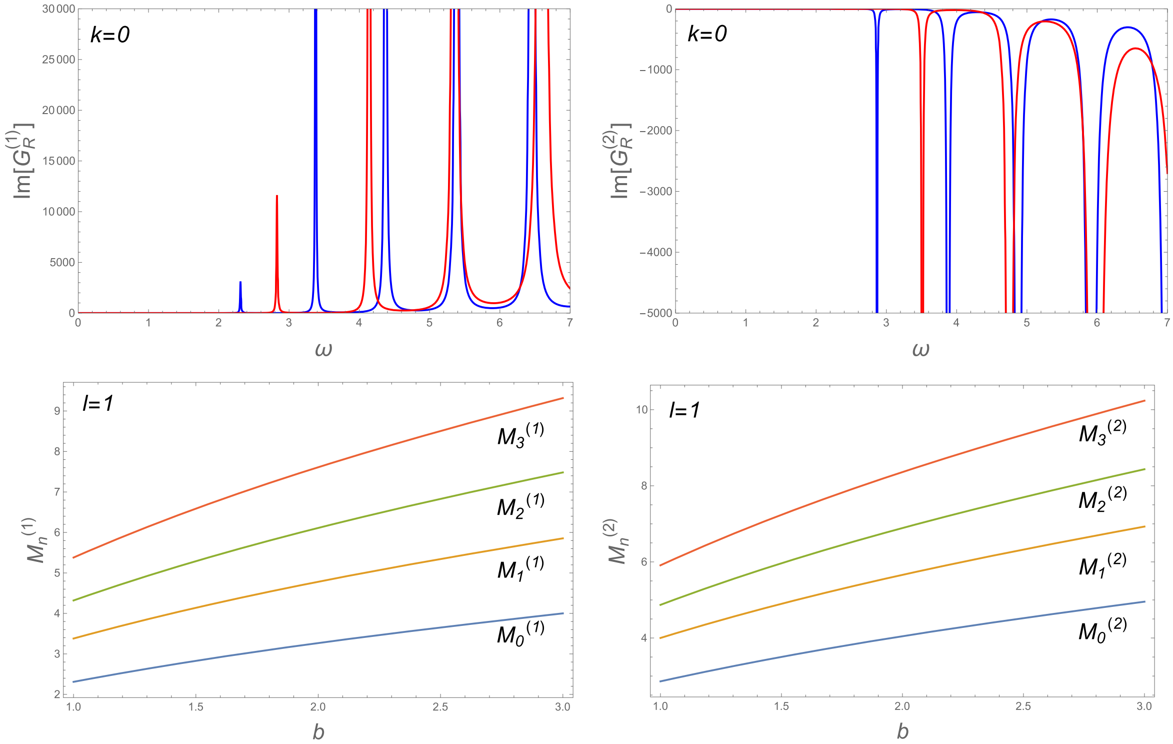

Figure 5. Imaginary part of the Green function and lowest mass spectrum with $l = 1$, $\Lambda_l = 3$ in the unit of $M_{KK} = 1$. The blue and red lines in the upper figures denote the Green function with $b = 1$ and 1.5, respectively.

Table 2. Fermionic mass spectrum $M_n^{(\alpha)}(\Lambda_l)$ by numerically fitting the spectral function $G_{\mathrm{R}}^{(\alpha)}$ with $\Lambda_l = 2$ and 3 in the unit of $M_{KK}b^{1/2}$.

$M_n^{(\alpha)}(\Lambda_l = 2)$	$n = 0$	$n = 1$	$n = 2$	$n = 3$
$\alpha = 1$	1.67	2.67	3.71	4.82
$\alpha = 2$	2.17	3.17	4.24	5.40

$M_n^{(\alpha)}(\Lambda_l = 3)$	$n = 0$	$n = 1$	$n = 2$	$n = 3$
$\alpha = 1$	2.31	3.38	4.32	5.38
$\alpha = 2$	2.86	4.00	4.87	5.91

analysis of the symmetries in the D0–D4/D8 system. In this sense, we constructed a holographic correspondence of a baryon field χ in boundary theory and the worldvolume fermionic field ψ as the bulk field dual to χ, which is a holographic interpretation of equation (3z) in terms of hadron physics. While the prior discussion may be less clear regarding the decomposition of the unitary group in the large N_c limit with generic N_f, it could be easy to understand when we take into account the realistic case of baryons in QCD with N_c, $N_f = 3$. Recall the $SU(3)$ decomposition of direct products of irreducible representations

$$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{10} \otimes \mathbf{8} \otimes \mathbf{8}^{*} \otimes \mathbf{1}. \tag{4a}$$

As is known, in the color sector, a baryon is a color singlet; thus, it is the $\mathbf{1}$ of $SU(3)_c$. In the flavor sector, because baryons usually consist of three flavored quarks, it could be $\mathbf{10}$ (decuplet), $\mathbf{8}$ (octet), or $\mathbf{8}^*$ (anti-octet) of $SU(3)_f$. Keeping these in mind, let us turn to the D0–D4/D8 model with N_c, $N_f = 3$. On the bulk side, the worldvolume fermions ψ take the baryon number from the baryon vertex, which is also the adjoint representation of $SU(3)_f$ and the color singlet. Thus, it could be baryonic $\mathbf{8}$ (octet) or $\mathbf{8}^*$ (anti-octet) of $SU(3)_f$ and $\mathbf{1}$ of $SU(3)_c$[4]. In boundary theory, the dual operator χ must share the same quantum numbers of ψ according to AdS/CFT [21, 22, 52]; therefore, it implies that χ is holographically a baryonic field of octet or anti-octet in QCD due to its quantum numbers. This is how holographic correspondence works in the D0–D4/D8 model.

In addition, to include the contribution of the N_c open strings to the worldvolume field ψ as a baryonic field (i.e. in holography, its dual field χ is a baryon field of N_c quarks), we need to further rescale ψ by $\psi \to \sqrt{N_c}\,\psi$, as is usually done in large N_c theory [59], so that the fermionic mass in equation (3w) rescales with an overall factor N_c to be $M_f \to M_f N_c$, as expected as the baryon

[4] Note that baryons in $\mathbf{10}$ (decuplet) of the flavor group are usually spin 3/2, which is not our concern.

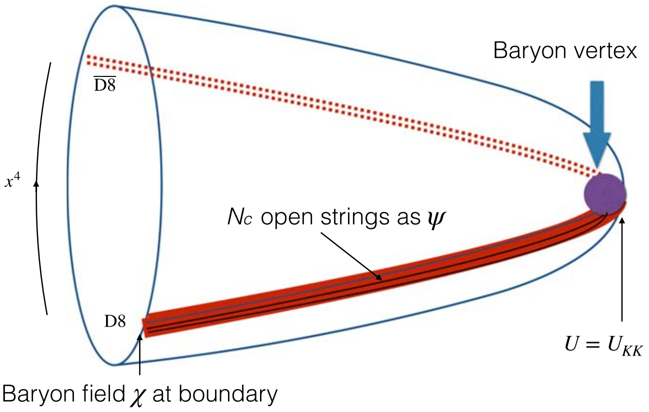

Figure 6. D0–D4/D8 model with baryon vertex. On the bulk side, the N_c open strings inside the flavor branes are 8–8 strings, creating ψ as a color singlet (gauge invariant operator) in the adjoint representation of the flavor group on the worldvolume of D8-branes. In boundary theory, N_c open strings are 4–8 strings, and their endpoints produce the baryon field χ as the color singlet in the adjoint representation of the flavor group. Therefore, χ is the dual operator of ψ because it has the same quantum numbers.

mass in the large N_c limit. In this sense, the fermionic spectrum discussed in section 3 could therefore be identified as baryon states, and the quantum numbers of the angular momentum l, s presented in equation (3*n*) can represent the isospin and spin of a baryon. The baryon spectrum from $G_R^{(1,2)}$ corresponds to baryons with different parities. Altogether, it is possible to compare the fermionic spectrum with the experimental data. For example, the lowest octets with the same parity can be identified to proton, $N(1440)$ and $N(1710)$. In our model, their mass data can be read from figure 5 for $l=1$, which leads to a numerical evaluation as $M_{N(1440)}/M_{\text{proton}} \simeq 1.49$; $M_{N(1710)}/M_{\text{proton}} \simeq 1.87$, which are very close to the existing experimental data $M^{\text{exp}}_{N(1440)}/M^{\text{exp}}_{\text{proton}} \simeq 1.53$, $M^{\text{exp}}_{N(1440)}/M^{\text{exp}}_{\text{proton}} \simeq 1.82$ [60].

4.2. Interaction of mesons and baryons

By interpreting worldvolume fermions as baryons, this model naturally includes the various interactions of mesons and baryons with the influence of the theta angle. Recall action (3*a*) for the worldvolume fermions on the flavor brane. We can see the coupling terms of gauge bosons and fermions, which refer to the interaction of mesons and baryons. Let us expand action (3*a*) up to the linear order of $\mathcal{F}_{MN}$; it includes the interaction terms in the action as

$$S_{\text{int}} = \mathrm{i}\frac{T_8}{4}\int \mathrm{d}^9x\sqrt{-g}\,\mathrm{e}^{-\phi}\bar{\Psi}\frac{1}{2}\gamma^Z\tilde{\gamma}\Gamma^{mn}\mathcal{F}_{mn}(\Gamma^\rho\hat{D}_\rho - \Delta)\Psi - \mathrm{i}\frac{T_8}{4}\int \mathrm{d}^9x\sqrt{-g}\,\mathrm{e}^{-\phi}\bar{\Psi}(1-\gamma^Z)\gamma^Z\tilde{\gamma}\Gamma^m\mathcal{F}_{mn}g^{nl}\hat{D}_l\Psi, \tag{4b}$$

where indices m and n run over 0, 1, 2, 3, and Z. When the decomposition project of the spinor, as discussed in section 3.1, is imposed, the term in the first line of equation (4*b*) vanishes because it is the Dirac equation that the basis functions $f_\pm^{(n)}$ satisfy. Thus, only the second line contributes to S_{int}, which can be written as

$$S_{\text{int}} = -\mathrm{i}\frac{T_8}{4}\int \mathrm{d}^9x\sqrt{-g}\,\mathrm{e}^{-\phi}\bar{\Psi}(1-\gamma^Z)\gamma^Z\tilde{\gamma} \times (\Gamma^\mu\mathcal{F}_{\mu Z}g^{ZZ}\hat{D}_Z + \Gamma^\rho\mathcal{F}_{\rho\nu}g^{\nu\mu}\hat{D}_\mu + \Gamma^Z\mathcal{F}_{Z\nu}g^{\nu\mu}\hat{D}_\mu)\Psi, \tag{4c}$$

where the indices run over 0, 1, 2, and 3. Plugging the induced metric (2*o*), the solution for dilaton, and the R–R fields (2*c*), then after a series of straightforward calculation, action (4*c*) becomes

$$\begin{aligned} S_{\text{int}} = &-\mathrm{i}\frac{27\pi}{4\lambda}\mathcal{T}b^{9/4}\int \mathrm{d}^4x\mathrm{d}ZH_0^{-1/2}K^{1/6}\bar{\psi}\gamma^\mu\mathcal{F}_{\mu Z} \\ &\times[\partial_Z + \mathcal{U}_1(Z,b)]\psi \\ &- \mathrm{i}\frac{27\pi}{4\lambda}\frac{\mathcal{T}b^{5/4}}{M_{KK}^2}\int \mathrm{d}^4x\mathrm{d}ZH_0^{-1/2}K^{-7/6}\bar{\psi}\gamma^\mu\mathcal{F}_{\mu\nu} \\ &\times[\partial^\nu + \mathcal{U}_2(Z,b)\gamma^Z\gamma^\nu]\psi \\ &- \mathrm{i}\frac{27\pi}{4\lambda}\frac{\mathcal{T}b^{7/4}}{M_{KK}}\int \mathrm{d}^4x\mathrm{d}ZH_0^{-1/2}K^{-1/2}\bar{\psi}\mathcal{F}_{Z\mu}\gamma^Z \\ &\times[\partial^\mu + \mathcal{U}_2(Z,b)\gamma^Z\gamma^\mu]\psi, \end{aligned} \tag{4d}$$

where we have imposed the dimensional reduction as it is discussed in section 3.1, and

$$\begin{aligned} \mathcal{U}_1(Z,b) &= -\frac{\sqrt{(b-1)b\lambda}}{4H_0K^{3/2}} - \frac{13Z}{12K} - \frac{5H_0'}{8H_0} + \frac{1}{4K^{1/2}}, \\ \mathcal{U}_2(Z,b) &= -\frac{M_{KK}b^{1/2}}{8H_0K^{1/3}}(2ZH_0 + KH_0') \\ &\quad - \frac{M_{KK}b\sqrt{(b-1)\lambda}}{4H_0K^{5/6}} + \frac{1}{4}M_{KK}b^{1/2}K^{1/6}. \end{aligned} \tag{4e}$$

Recalling further the meson tower given in equation (2*y*), it is easy to obtain

$$\begin{aligned} \mathcal{F}_{\mu Z} &= -\partial_Z\mathcal{A}_\mu = \partial_\mu\pi\phi_0' - \sum_{n=1}^{\infty}V_\mu^{(n)}\psi_n', \\ \mathcal{F}_{\mu\nu} &= \partial_\nu\mathcal{A}_\mu - \partial_\mu\mathcal{A}_\nu + \mathrm{i}[A_\mu, A_\nu] \\ &= \sum_{n=1}^{\infty}W_{\mu\nu}^{(n)}\psi_n + \mathrm{i}[\partial_\mu\pi, \partial_\nu\pi]\phi_0 + \mathrm{i}\phi_0\sum_{n=1}^{\infty}\psi_n \\ &\quad\times([V_\mu^{(n)}, \partial_\nu\pi] + [\partial_\mu\pi, V_\nu^{(n)}]) \\ &\quad + \mathrm{i}\sum_{n,m=1}^{\infty}\psi_n\psi_m[V_\mu^{(n)}, V_\nu^{(m)}], \end{aligned} \tag{4f}$$

where

$$W_{\mu\nu}^{(n)} = \partial_\mu V_\nu^{(n)} - \partial_\nu V_\mu^{(n)}, \tag{4g}$$

and we denote the scalar $\varphi^{(0)}$ as π meson. Then, we substitute equation (4*f*) into equation (4*d*). Action (4*d*) indicates that the nth baryon decays to light mesons, e.g. scalar meson π and vector meson V_μ (ρ meson). Specifically, after a series of tedious but straightforward calculations, the exact form of S_{int}

(up to the linear term of light mesons) is given as

$$\begin{aligned} S_{\rm int} = &\int d^4x [a_{+,+}^{(m,n,0)}\psi_+^{(m)\dagger}\bar{\sigma}^\mu\partial_\mu\pi\psi_+^{(n)} \\ &- a_{+,+}^{(m,n,l)}\psi_+^{(m)\dagger}\bar{\sigma}^\mu V_\mu^{(l)}\psi_+^{(n)}] \\ &+ \int d^4x [a_{-,-}^{(m,n,0)}\psi_-^{(m)\dagger}\sigma^\mu\partial_\mu\pi\psi_-^{(n)} \\ &- a_{-,-}^{(m,n,l)}\psi_-^{(m)\dagger}\sigma^\mu V_\mu^{(l)}\psi_-^{(n)}] \\ &+ \int d^4x \sum_{m,n} [b_{-,+}^{(m,n,0)}\psi_-^{(m)\dagger}\partial_\mu\pi\partial^\mu\psi_+^{(n)} \\ &+ b_{+,-}^{(m,n,0)}\psi_+^{(m)\dagger}\partial_\mu\pi\partial^\mu\psi_-^{(n)}] \\ &+ \int d^4x \sum_{m,n} [c_{-,-}^{(m,n,0)}\psi_-^{(m)\dagger}\sigma^\mu\partial_\mu\pi\psi_-^{(n)} \\ &- c_{+,+}^{(m,n,0)}\psi_+^{(m)\dagger}\bar{\sigma}^\mu\partial_\mu\pi\psi_+^{(n)}] \\ &- \int d^4x \sum_{m,n,l} [b_{-,+}^{(m,n,l)}\psi_-^{(m)\dagger}V_\mu^{(l)}\partial^\mu\psi_+^{(n)} \\ &+ b_{+,-}^{(m,n,l)}\psi_+^{(m)\dagger}V_\mu^{(l)}\partial^\mu\psi_-^{(n)}] \\ &- \int d^4x \sum_{m,n,l} [c_{-,-}^{(m,n,l)}\psi_-^{(m)\dagger}\sigma^\mu V_\mu^{(l)}\psi_-^{(n)} \\ &- c_{+,+}^{(m,n,l)}\psi_+^{(m)\dagger}\bar{\sigma}^\mu V_\mu^{(l)}\psi_+^{(n)}] \\ &+ \int d^4x \sum_{m,n,l} [d_{+,+}^{(m,n,l)}\psi_+^{(m)\dagger}\bar{\sigma}^\mu W_{\mu\nu}^{(l)}\partial^\nu\psi_+^{(n)} \\ &+ d_{-,-}^{(m,n,l)}\psi_-^{(m)\dagger}\sigma^\mu W_{\mu\nu}^{(l)}\partial^\nu\psi_-^{(n)}] \\ &+ \int d^4x \sum_{m,n,l} [f_{-,+}^{(m,n,l)}\psi_-^{(m)\dagger}\sigma^\mu\bar{\sigma}^\nu W_{\mu\nu}^{(l)}\psi_+^{(n)} \\ &- f_{+,-}^{(m,n,l)}\psi_+^{(m)\dagger}\bar{\sigma}^\mu\sigma^\nu W_{\mu\nu}^{(l)}\psi_-^{(n)}], \end{aligned} \tag{4h}$$

where we have imposed the rescaling $\psi \to \sqrt{N_c}\psi$, and the associated coupling constants are given by the following numerical integrals with the basis functions ψ_n, ϕ_0, and $f_\pm^{(n)}$ as

$$\begin{aligned} g &= \frac{3^{9/2}\pi^{5/2}N_c^{1/2}}{2\lambda^{3/2}}, \mathcal{N} = \mathcal{T}b^{11/4}T^{1/2}(2\pi\alpha')R^{3/2} \\ a_{\pm,\pm}^{(m,n,l)} &= \frac{g}{b^{3/4}}\mathcal{N}\int d^4x dZ H_0^{-1/2}K^{1/6} \\ &\times [f_\pm^{(m)}\partial_Z f_\pm^{(n)} + \mathcal{U}_1(Z,b) f_\pm^{(m)} f_\pm^{(n)}]\psi_l', \\ b_{\pm,\pm}^{(m,n,l)} &= \frac{ig}{M_{KK}b^{5/4}}\mathcal{N} \\ &\times \int d^4x dZ H_0^{-1/2}K^{-1/2} f_\pm^{(m)} f_\pm^{(n)}\psi_l', \\ c_{\pm,\pm}^{(m,n,l)} &= -\frac{g}{M_{KK}b^{5/4}}\mathcal{N} \\ &\times \int d^4x dZ H_0^{-1/2}K^{-1/2}\mathcal{U}_2(Z,b) f_\pm^{(m)} f_\pm^{(n)}\psi_l' \\ d_{\pm,\pm}^{(m,n,l)} &= \frac{g}{M_{KK}^2 b^{7/4}}\mathcal{N} \\ &\times \int dZ H_0^{-1/2}K^{-7/6} f_\pm^{(m)} f_\pm^{(n)}\psi_l, \\ f_{\pm,\pm}^{(m,n,l)} &= -\frac{ig}{M_{KK}^2 b^{7/4}}\mathcal{N} \\ &\times \int dZ H_0^{-1/2}K^{-7/6}\mathcal{U}_2(Z,b) f_\pm^{(m)} f_\pm^{(n)}\psi_l. \end{aligned} \tag{4i}$$

We note that the function ψ_l is the basis function given in equations (2p)–(2r), and we have defined $\psi_0 = \phi_0$ to write equation (4i) compactly. $\mathcal{N}$ is the combination of the normalization factors of the basis function given in equations (2q) and (3u); therefore, all the integrals in equation (4i) with factor $\mathcal{N}$ are purely numerical numbers. It is easy to verify that all integrand functions in equation (4i) converge at $Z \to \infty$ due to the asymptotics of the basis functions

$$\psi_0 \sim \frac{1}{Z^2}, \psi_n \sim \frac{1}{Z}, f_+^{(m)} \sim Z^{\frac{4}{3}}, f_-^{(n)} \sim Z, \tag{4j}$$

according to the eigenequations (2r) and (3y). Therefore, all the coupling constants listed in equation (4i) are finite and depend on parameter b, i.e. they are affected by the presence of the theta angle in QCD, according to equation (2j). In addition, the unit of the coupling constant g presented in equation (4i) is of $\mathcal{O}(N_c^{1/2})$, with large N_c behavior that agrees with coupling of the mesons and baryons in the large N_c theory [59]. Altogether, the holographic action (4h) with the coupling constants listed in equation (4i) describes various interactions of baryons and mesons in the D0–D4/D8 model.

5. Summary and discussion

In this study, we first demonstrated dimensional reduction with respect to the fermionic action for the flavor brane, which was obtained by the T-duality rules in string theory used in the D0–D4/D8 model, and derived its 5D effective form and 4D canonical action, which illustrates the essential conditions for dimensional reduction. We then computed the two-point Green function for the dual operators of the worldvolume fermions on the flavor brane using the standard technique in AdS/CFT to evaluate the fermionic mass spectrum. Afterward, we holographically analyzed the quantum number of the bulk field and its dual field in the D0–D4/D8 system and accordingly interpreted the worldvolume fermions as baryons with a baryon vertex. In this sense, our fermionic spectrum is recognized as the baryon spectrum, and we found that it fits well with the experimental data of the lowest baryon spectrum. Finally, we derived the linear interaction terms involving the gauge field in the fermionic action and interpreted them as various interactions of baryons and light mesons. In the presence of instantons, the mass spectrum and interacting coupling constants all depend on the instanton density, which implies the metastabilization in QCD with instantons, as discussed in [42, 43, 48–50]. Overall, this study investigated the worldvolume fermions on a D-brane and found a holographic method to describe the interaction of baryons and mesons; hence, it is also an extension and supplement to our previous study [61].

Finally, we provide comments to close this study. First, below the energy-scale M_{KK}, as the D0–D4/D8 model is holographically dual to 4D QCD with a theta term; therefore, if we interpret the worldvolume fermions without a baryon vertex on the flavor branes as supersymmetric mesons (mesinos), they remain absent in the low-energy theory due to their over-heavy mass. The lowest mass of the fermionic meson is about $1.67M_{KK}$; thus, it is larger than M_{KK} according to our numerical calculation. In this sense, the issue about the D4/D8 model proposed in [46] was automatically determined. Second, the baryon vertex is indispensable when the fundamental quark and baryons are concerned in this model because the baryon number would not be conserved without the baryon vertex, as commented in [23]. In this sense, baryon vertex is the essential element to make that open strings on the flavor branes behave like baryons, so that our study is also a supplement to [62]. Finally, the classical mass of baryons in this model can be obtained by evaluating the total energy of a wrapped D4-brane as a baryon vertex as $m_{\rm B} = \frac{\lambda N_c}{27\pi} b^{3/2}$ [42, 43], which deviates slightly from our numerical evaluation, $m_{\rm B} \propto b^{1/2}$. To match the mass of the baryon vertex exactly, we can further rescale the basis functions presented in equation (3*r*) as $f_{\pm}^{(m)} \to b^{1/2} f_{\pm}^{(m)}$. As a result, all coupling constants given in equation (4*i*) will pick up a factor b. This may completely determine the dependence of b and the associated basis functions in this model. Thus, interpreting worldvolume fermions as baryons with a baryon vertex in this study is seemingly reasonable and workable; thus, it could provide a new way to study baryons in holography.

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