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# Spherical quantum dot described by a scalar exponential potential under the influence of a linear electric field

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**Abstract**

We study the confinement of a spinless charged particle to a spherical quantum dot under the influence of a linear electric field. The spherical quantum dot is described by a short-range potential given by the power-exponential potential. Then, by analysing the region near the spherical quantum dot centre, we discuss two cases where the energy levels can be obtained for *s*-waves and how the linear electric field modifies the spectrum of energy of the spherical quantum dot.

Keywords: spherical quantum dot, power-exponential potential, short-range potential, linear electric field

## 1. Introduction

The idea of describing a quantum dot through a short-range potential brings the possibility of dealing with a confining potential with finite depth and range [1–5]. Quantum dots have a great interest in condensed matter physics with possible applications in nanotechnology [1, 2, 6]. This kind of mesoscopic device allows us to search for quantum effects such as the Aharonov–Bohm effect [7] and persistent currents [8–13]. An interesting example of a short-range potential is the attractive Gaussian potential [1]. With the aim of extending this idea of working with short-range potentials, Ciurla *et al* [2] proposed the power-exponential potential. In this mathematical model of a quantum dot, Ciurla *et al* [2] showed the attractive Gaussian potential as a particular case of it. In this way, the power-exponential potential can be an adequate mathematical model to work with quantum dots, where we can cite the works [14–25] as part of this quantum dot model.

In this work, we consider a uniform distribution of electric charges inside the spherical quantum dot. We assume that the quantum dot is a non-conducting medium. This electric charge distribution yields a linear electric field which, in turn, interacts with a spinless charged particle. Our interest is inspired by several studies in which have considered the presence of a linear electric field and raised a discussion about its influence. For instance, Edery and Audin [26] showed that the degeneracy of the Landau levels [27] are modified by a linear electric field applied in the $z$-direction. In another context, the interaction between the linear electric field and a spinless charged particle has been studied in the presence of disclination and spiral dislocation [28–30]. Linear electric fields have also been used to achieve the Landau quantization for neutral particles [31, 32] and in studies of the confinement of neutral particles to a quantum ring [33, 34]. Therefore, by confining a spinless charged particle to a spherical quantum dot, our interest is to search for the influence of a linear electric field near the spherical quantum dot centre. In this region, we assume that the short-range potential dominates. From the energy eigenvalues for $s$-waves, we thus discuss the electric field influence.

The structure of this paper is as follows: in section 2, we introduce the spherical quantum dot model proposed by Ciurla *et al* [2]. Then, we consider the case $p = 1$ of the Ciurla *et al* model to study the confinement of a spinless charged particle to a quantum dot. By including the interaction between a linear electric field and a spinless charged particle, we analyse the case in which the spherical quantum dot dominates; in section 3, we present our conclusions.

## 2. Influence of a linear electric field

In this section, we deal with the confinement of a spinless charged particle to a spherical quantum dot under the influence of a linear electric field. Our analysis is made based on the spherical symmetry. Hence, the Schrödinger equation is given by [35]:

$$\mathcal{E}\psi = -\frac{1}{2m}\left[\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}}{\mathrm{d}r}\right) - \frac{\hat{L}^2}{r^2}\right]\psi + V(\vec{r})\psi, \quad (1)$$

where $\hat{L}^2$ is the angular momentum squared operator and $V(\vec{r})$ is the potential energy of the system.

We shall work with the power-exponential potential [2] with the purpose of describing the confinement of the particle to a spherical quantum dot. In this model, the quantum dot consists in a short-range potential, whose mathematical expression is

$$V_p(r) = -V_0\,\mathrm{e}^{-\left(\frac{r}{r_0}\right)^p}. \quad (2)$$

In equation (2), we have $p \geqslant 1$ and $V_0 > 0$ is the confining potential depth. In addition, $r$ corresponds to the radial coordinate and $r_0$ is the range of the confining potential. The potential energy (2) can describe several attractive short-range potentials. For instance, the attractive Gaussian potential [1, 14–18] is defined by $p = 2$, and the rectangular potential for $p \to \infty$ [2]. The simplest case of the quantum dot model (2) is for $p = 1$ [23, 24]. In this work, our focus is on the spherical quantum dot model defined by $p = 1$ in equation (2), i.e.

$$V_1(r) = -V_0\,\mathrm{e}^{-\frac{r}{r_0}}. \quad (3)$$

Recently, the power-exponential potential (2) has been explored in search of global monopole topological effects [23–25, 36].

Next, let us consider the electric field produced by a uniform charge density ($\rho$) inside a non-conducting material. This electric field is given by $\vec{E} = \frac{\rho\, r}{2}\,\hat{r}$, which gives us the potential energy:

$$V_q(r) = -q\int_0^r \vec{E}\cdot\mathrm{d}\vec{r} = -\frac{q\,\rho\,r^2}{4}, \quad (4)$$

where $q$ is the electric charge of the particle. From now on, we are considering $q = -|q|$ and $\rho > 0$. It is worth mentioning that the interaction of a charged particle with the radial electric field $\vec{E} = \frac{\rho\, r}{2}\,\hat{r}$ has been studied under the influence of the topological effects of a disclination and a spiral dislocation in [28–30].

Henceforth, we consider a spinless charged particle confined to the spherical quantum dot (3) where it interacts with the radial electric field as described by the potential energy (4). With the aim of writing the radial equation, we should take into account that $V_1(r)$ and $V_q(r)$ depend only on the radial coordinate. In this way, the solution to the Schrödinger equation (1) is given by $\psi(r,\theta,\varphi) = R(r)\mathrm{Y}_\ell^{m_\ell}(\theta,\varphi) = \frac{u(r)}{r}\mathrm{Y}_\ell^{m_\ell}(\theta,\varphi)$ [35]. Note that the eigenvalues of the operator $\hat{L}^2$ are $\ell = 0$, 1, 2, 3, …, while the eigenvalue of the operator $\hat{L}_z$ is $m_\ell$. Hence, the radial equation is

$$\frac{\mathrm{d}^2u}{\mathrm{d}r^2} - \frac{\ell(\ell+1)}{r^2}\,u + 2m\,V_0\,\mathrm{e}^{-r/r_0}\,u - \frac{m|q|\,\rho}{2}\,r^2\psi + 2m\mathcal{E}\,u = 0. \quad (5)$$

Let us search for bound states for $s$-waves. Therefore, we take $\ell = 0$ in equation (5). Moreover, let us define $x = r/r_0$, thus, equation (5) becomes

$$\frac{\mathrm{d}^2u}{\mathrm{d}x^2} + 2m\,V_0\,r_0^2\,\mathrm{e}^{-x}\,u - \frac{m|q|\,\rho\,r_0^4}{2}\,x^2\psi + 2m\mathcal{E}\,r_0^2\,u = 0. \quad (6)$$

In this work, our aim is to analyse the region $x \ll 1$, i.e. the region near the quantum dot centre [2]. We should observe that the spherical quantum dot (3) is a short-range potential, hence we can assume that the spherical quantum dot dominates in the region $x \ll 1$. In this way, we have $|V_1| \gg |V_q|$ when $x \ll 1$. We also assume that $\mathcal{E} < 0$ and define the parameter:

$$\beta = \sqrt{-2m\mathcal{E}\,r_0^2}. \quad (7)$$

Besides, let us use the approximation $x \approx 1 - \mathrm{e}^{-x}$. Thereby, we have in equation (6):

$$\frac{\mathrm{d}^2u}{\mathrm{d}x^2} + 2m\,V_0\,r_0^2\,\mathrm{e}^{-x}\,u - \frac{m|q|\,\rho\,r_0^4}{2} \times (1-\mathrm{e}^{-x})^2\psi - \beta^2\,u = 0. \quad (8)$$

By performing the change of variables:

$$y = \sqrt{2m|q|\,\rho\,r_0^4}\;\mathrm{e}^{-x}, \quad (9)$$

we obtain in equation (8):

$$\frac{\mathrm{d}^2u}{\mathrm{d}y^2} + \frac{1}{y}\frac{\mathrm{d}u}{\mathrm{d}y} - \frac{\lambda^2}{y^2}\,u + \frac{\delta}{y}\,u - \frac{u}{4} = 0. \quad (10)$$

The parameters $\lambda$ and $\delta$ are defined as

$$\lambda^2 = \beta^2 + \frac{m|q|\,\rho\,r_0^4}{2};$$

$$\delta = \sqrt{\frac{m|q|\,\rho\,r_0^4}{2}} \times \left[\frac{2\,V_0}{|q|\,\rho\,r_0^2} + 1\right]. \quad (11)$$

Therefore, the solution to equation (10) is given by

$$u(y) = \frac{c_1}{\sqrt{y}}\,M_{\delta,\lambda}(y) + \frac{c_2}{\sqrt{y}}\,W_{\delta,\lambda}(y), \quad (12)$$

where the functions $M_{\delta,\lambda}(y)$ and $W_{\delta,\lambda}(y)$ are the Whittaker functions of first kind and second kind, respectively [37, 38].

From equation (9) we have that when $r \to \infty \Rightarrow x \to \infty \Rightarrow y = 0$. Then, a regular solution to equation (10) at $y = 0$ is obtained by taking $c_2 = 0$ in equation (12). Thus, we have

$$u(y) = \frac{c_1}{\sqrt{y}}\,M_{\delta,\lambda}(y). \quad (13)$$

Furthermore, from equation (9) we have that when $r = 0 \Rightarrow x = 0 \Rightarrow y \to y_0 = \sqrt{2m|q|\,\rho\,r_0^4}$. Thereby, we have the boundary condition:

$$u(y_0) = 0. \tag{14}$$

With the purpose of exploring the boundary condition (14), let us write equation (13) in terms of the confluent hypergeometric function. According to [37, 38], we can write the function $M_{\delta,\lambda}(y)$ in terms of the confluent hypergeometric function ${}_1F_1(a,\,b;\,y)$. Thereby, equation (13) becomes

$$u(y) = c_1\, y^{\lambda} \times e^{-y/2} \times \,{}_1F_1\left(\frac{1}{2} + \lambda - \delta,\, 1 + 2\lambda;\, y\right). \tag{15}$$

Note that $a = \frac{1}{2} + \lambda - \delta$ and $b = 1 + 2\lambda$. By substituting equation (15) into equation (14), we find

$$u(y_0) = c_1\, y_0^{\lambda} \times e^{-y_0/2} \times \,{}_1F_1\left(\frac{1}{2} + \lambda - \delta,\, 1 + 2\lambda;\, y_0\right) = 0. \tag{16}$$

We can explore equation (16) in two different cases. The first one is to consider $a \to -\infty$ ($y_0$, $b$ fixed). In this case, the confluent hypergeometric function ${}_1F_1(a,\,b;\,y_0)$ can be written as follows [37]:

$${}_1F_1(a,\,b;\,y_0) \propto \cos\left(\sqrt{[2b - 4a]\,y_0} - \frac{b\pi}{2} + \frac{\pi}{4}\right). \tag{17}$$

Thereby, by substituting equation (17) into equation (16) we find the relation:

$$\beta^2 = \left[\frac{\sqrt{4\delta y_0}}{\pi} - n - \frac{3}{4}\right]^2 - \frac{m|q|\,\rho\, r_0^4}{2}, \tag{18}$$

where $n \gg 1$ (with $n = 0,\ 1,\ 2,\ 3,\ \ldots$) is the radial quantum number. Since $\beta^2 > 0$, equation (18) is valid only if $\left[\frac{\sqrt{4\delta y_0}}{\pi} - n - \frac{3}{4}\right]^2 > \frac{m\,|\,q\,|\,\rho\, r_0^4}{2}$, otherwise, no bound states exist. As a consequence, we have an upper limit to the radial quantum number given by

$$n_{\max} < \frac{\sqrt{4\delta y_0}}{\pi} - \frac{3}{4} - \sqrt{\frac{m|q|\,\rho\, r_0^4}{2}}. \tag{19}$$

For $n > n_{\max}$, there are no bound states. Furthermore, after substituting equation (7) into equation (18), we obtain the energy levels:

$$\mathcal{E}_n = -\frac{1}{2m\, r_0^2}\left[\frac{\sqrt{4\delta y_0}}{\pi} - n - \frac{3}{4}\right]^2 + \frac{|q|\,\rho\, r_0^2}{4}. \tag{20}$$

Hence, we have a discrete spectrum of energy for $s$-waves in the region $x \ll 1$, i.e. in the region where the spherical quantum dot dominates. The influence of the linear electric field modifies the energy eigenvalues of the spherical quantum dot as shown in [23]. Besides, the maximal value of the radial quantum number obtained in [23] is modified, and thus, it is now given by that of equation (19).

The second case that can be explored from the boundary condition (16) is by considering $y_0 \gg 1$ ($a$, $b$ fixed). In this case, we have that the confluent hypergeometric function ${}_1F_1(a,\,b;\,y_0)$ diverges when $y_0 \to \infty$. For this reason, we must impose that ${}_1F_1(a,\,b;\,y_0)$ becomes a polynomial of degree $n$ ($n = 0,\ 1,\ 2,\ 3,\ \ldots$). This occurs when $a = -n$ [37, 38]. This condition guarantees that the boundary condition (14) is satisfied. In this way, we have $\frac{1}{2} + \lambda - \delta = -n$, which yields the relation:

$$\beta^2 = \left[\delta - n - \frac{1}{2}\right]^2 - \frac{m|q|\,\rho\, r_0^4}{2}. \tag{21}$$

Note that $n$ remains the radial quantum number. Due to $\beta^2 > 0$, we also have an upper limit to the radial quantum number. In this case, the maximal value of $n$ is determined by

$$n_{\max} < \delta - \frac{1}{2} - \sqrt{\frac{m|q|\,\rho\, r_0^4}{2}}. \tag{22}$$

Thus, there are bound states for $n = 0,\ 1,\ 2,\ 3, \ldots, n_{\max}$. Let us go further by substituting equation (7) into equation (21). Then, we obtain the energy levels:

$$\mathcal{E}_n = -\frac{1}{2m\, r_0^2}\left[\delta - n - \frac{1}{2}\right]^2 + \frac{|q|\,\rho\, r_0^2}{4}. \tag{23}$$

Hence, by considering $y_0 \gg 1$, we also obtain a discrete spectrum of energy for $s$-waves in the region $x \ll 1$. In this region where the spherical quantum dot dominates, the linear electric field also influences the energy eigenvalues for the case $y_0 \gg 1$. The presence of the linear electric field also modifies both the energy levels of the spherical quantum dot and the maximal value of $n$ obtained in [23]. Note that equation (23) differs from equation (20). Moreover, the maximal value of the radial quantum number established in equation (22) differs from that of equation (19).

## 3. Conclusions

We have studied the confinement of a spinless charged particle to a spherical quantum dot in the presence of a linear electric field. This electric field is produced by a uniform charge density inside a non-conducting medium. We have considered a spherical quantum dot described by an attractive short-range potential [2] which has a great interest in semiconductor devices. In this study, we have focused on the case $p = 1$ of the power-exponential potential [2] and dealt with $s$-waves. Thereby, we have analyzed the region where spherical quantum dot dominates and discussed two cases where the energy levels can be achieved. In both cases studied, we have shown that a discrete spectrum of energy can be obtained, and the radial quantum number possesses a maximal value. Therefore, the presence of the linear electric field modifies the energy levels of the spherical quantum dot (3) in contrast to those obtained in [23].

An interesting perspective is to explore the short-range potential (3) in an elastic medium, but dealing with cylindrical symmetry. In this perspective, we can consider an elastic medium with disclinations and dislocations [39, 40], and then, search for analogues of the Aharonov–Bohm effect [41–47].

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## Competing interests

We have no competing interests.

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