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Editor's Suggestion

Quantum flutter from the nonlinear Luttinger liquid perspective

Sheng Wang[1,2,3], Zhehao Zhang[1,2] and Xi-Wen Guan[1,4,5,6]

[1] Innovation Academy for Precision Measurement Science and Technology, Chinese Academy of Sciences, Wuhan 430071, China
[2] University of Chinese Academy of Sciences, Beijing 100049, China
[3] Department of Physics and Hong Kong Institute of Quantum Science and Technology, The University of Hong Kong, Hong Kong, China
[4] NSFC-SPTP Peng Huanwu Center for Fundamental Theory, Xi'an 710127, China
[5] Hefei National Laboratory, Hefei 230088, China
[6] Department of Fundamental and Theoretical Physics, Research School of Physics, Australian National University, Canberra ACT 0200, Australia

E-mail: xiwen.guan@anu.edu.au

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Abstract

Quantum flutter is a ubiquitous phenomenon which can be observed from the fast moving impurity injected into a fermionic or bosonic medium of quantum liquid. In this scenario, one usually considers a medium of a fully polarized state and injects a spin-flipped impurity as the initial state. When the initial velocity of the impurity is beyond the intrinsic sound velocity of the medium, the impurity momentum dramatically exhibits a long-lived periodic oscillation with the periodicity remaining invariant with respect to the initial velocity. In this paper, we show that such a novel phenomenon can be explained by a linear Luttinger liquid coupled to a deep hole in the Fermi sea. Once the deep hole excitations are involved and the impurity momentum surpasses the Fermi momentum, the propagator thus displays a periodic oscillation after a quick relaxation decay. The oscillation periodicity is solely determined by the energy of the deepest hole excitation. Our result provides deep insights into the dynamical behavior of quantum impurities immersed into one-dimensional quantum liquids.

Keywords: quantum flutter, Yang–Gaudin model, Luttinger liquid, nonlinear Luttinger liquid

Introduction

In the last few decades, experimental developments of ultracold atomic physics [1–5] have greatly promoted the research on non-equilibrium physics. The progress which has been made continues to introduce new problems and discover rich phenomena at the quantum many-body levels. In this scenario, quantum impurity problems have a long history of study and manifests various forms, such as a charged quasiparticle (polaron) moving in medium [6–8], quantum flutter moving at high speed in a medium of fluid [9, 10], and the lifetime of the quasiparticles induced by impurities [11–13]. Quantum flutter is a dynamic phenomenon observed in a spin-flipped impurity within a spin-polarized gas, regardless of whether the system is bosonic or fermionic. We examine an initial state where the system is in a fully polarized ground state and introduce a spin-flipped impurity. As the system evolves, we measure the momentum of the impurity and observe distinct behaviors depending on the impurity's initial momentum relative to the Fermi momentum. Specifically, when the impurity's initial momentum exceeds the Fermi momentum, its momentum exhibits periodic oscillations over time, a behavior termed quantum flutter. Conversely, if the impurity's initial momentum is below the Fermi momentum, the momentum decays over time without oscillatory behavior. Due to advancements in cold atomic system platforms, research on quantum impurities has rapidly progressed [14–18], now the experiment is capable of not only measuring the effective mass of impurity, but also observing the trajectory of impurity.

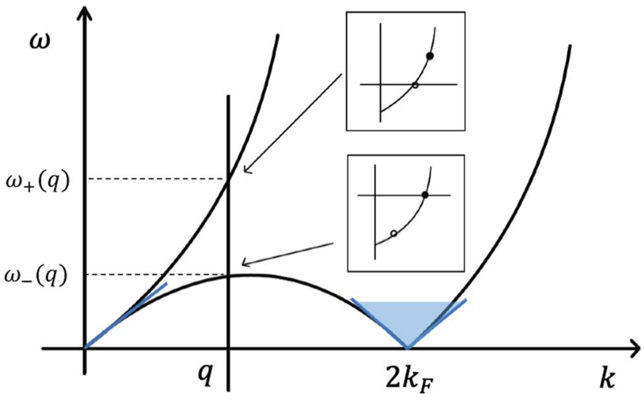

Figure 1. The particle-hole excitation spectra of interacting Fermi gas. For the blue line at small q and the blue area around $2k_F$, the Luttinger liquid description is valid. The upper thresholds correspond to the particle excitations, and the lower thresholds correspond to the hole excitations. They can be identified by $\omega_+(q)$ and $\omega_-(q)$. It should be noted that above the upper thresholds, there exist high excitation spectra contributed to by the string excitations in integrable systems.

However, for non-equilibrium dynamics, a unified method cannot be achieved because different systems and initial states acquire distinct approaches for the calculation of dynamical properties and understanding their dynamical process. Nevertheless, when the system is not far from equilibrium, non-equilibrium dynamics phenomena are often deduced from the properties of the equilibrium states, such as the linear response theory [19]. For systems at low-energy states, reduced degrees of freedom significantly facilitate the theoretical analysis of quantum dynamics. Therefore, it is very inspiring that we transform the non-equilibrium dynamical phenomenon of quantum flutter into the power laws of the propagator in equilibrium state.

A universal effective field theory for one-dimensional systems at low energy is the Luttinger liquid theory [20, 21]. In a one-dimensional interacting Fermi gas, when considering excitations near the Fermi surface, the density fluctuations of the system exhibit as free bosons. Based on the linear Luttinger liquid theory, an impurity with a small momentum usually causes excitations near the Fermi point, i.e. quantum flutter no longer exists. This aligns with the power-law relationship of the propagator with time in the linear Luttinger liquid. However, it is important to note that the Luttinger liquid theory is only valid at low energies, as shown in figure 1, see the blue line for a small q and around $2k_F$. For high-energy excitations, the emergence of band curvatures significantly alters the scenario [22–26]. The particle excitations and hole excitations no longer share the same energy dispersion, thereby the particle-hole symmetry is broken. Therefore, at high energy, the nonlinearity of the dispersion must be considered. This is the main point of this article: the quantum flutter phenomenon originates from the nonlinear spectra and our final results from the correlation functions demonstrate that the eigenstates located in the blue line and $\omega_-(k_F)$ in figure 1 govern the quantum flutter phenomenon.

Nonlinear spectra are essential for the theoretical understanding of the experimental results. Previous theoretical studies on spin-charge separation phenomena have shown that including spectral broadening is necessary for better agreement between experimental data and theoretical results [27]. The same nature applies to the phenomenon of quantum flutter, as its occurrence requires the momentum of the impurity to exceed the Fermi momentum. This implies that the system undergoes high-energy excitations. Therefore, another important question is which high-energy excitations contribute most significantly to the phenomenon of quantum flutter. Examining the spectrum diagram, at the lower and upper thresholds, edge singularities [28] might emerge due to the many-body interaction and it leads to infinite-lifetime of quasiparticles. These provide an alternative way to understand the phenomenon of quantum flutter other than Bethe ansatz exact solution [29]. In this paper, in view of the nonlinear Luttinger liquid theory [25, 26], we combine the linear Luttinger liquid theory and the high-energy deep Fermi sea hole excitations to derive an analytical form of the propagator of the impurity which reveals the essence of quantum flutter.

Yang–Gaudin model

We consider the Yang–Gaudin model which is described by the Hamiltonian [30, 31]

$$h = \int_0^L \mathrm{d}x \sum_{j=\uparrow,\downarrow} \frac{\hbar^2}{2m} \partial_x \hat{\Psi}_j^\dagger(x) \partial_x \hat{\Psi}_j(x) + \sum_{j,l=\uparrow,\downarrow} c \hat{\Psi}_j^\dagger(x) \hat{\Psi}_l^\dagger(x) \hat{\Psi}_l(x) \hat{\Psi}_j(x), \tag{1}$$

where the notation $\uparrow$ ($\downarrow$) shows the spin states with spin-up (-down), $\hat{\Psi}_j(x)$ and $\hat{\Psi}_l(x)$ are field operators and m is the mass of the fermion, and $\hbar$ is Planck's constant. The commutation relation of the field operators satisfy $[\hat{\Psi}_i(x), \hat{\Psi}_j^\dagger(x')]_+ = \delta_{ij}\delta(x - x')$. For convenience, we let $\hbar^2 = 2m = 1$. Then, the dimensionless interaction strength of this system is $\gamma = cL/N$, here N is the total particle number of the system and L is the system size. The model is exactly solvable and exhibits rich many-body physics throughout the temperature regimes.

For a system of a total particle number N with M spin-down fermions and $N - M$ are spin-up fermions. The model was solved by using a nested Bethe Ansatz (BA) [30, 32]. The N-particle eigenstate is characterized by N quasi-momenta $\{k\}_N = \{k_1,\ldots,k_N\}$ and M spin quasi-rapidities $\{\lambda\}_M = \{\lambda_1,\ldots,\lambda_M\}$, respectively. They satisfy the following Bethe ansatz equations (BAE)

$$\mathrm{e}^{\mathrm{i}k_iL} = \prod_{\alpha=1}^{M} \frac{k_i - \lambda_\alpha + \mathrm{i}c/2}{k_i - \lambda_\alpha - \mathrm{i}c/2}, \quad -\prod_{j=1}^{N} \frac{k_j - \lambda_\alpha + \mathrm{i}c/2}{k_j - \lambda_\alpha - \mathrm{i}c/2} = \prod_{\beta=1}^{M} \frac{\lambda_\alpha - \lambda_\beta - \mathrm{i}c}{\lambda_\alpha - \lambda_\beta + \mathrm{i}c}, \tag{2}$$

where $i = 1,\ldots,N$ and $\alpha = 1,\ldots,M$. The solutions of the BAE give all the eigenstates $|\{k\}_N; \{\lambda\}_M\rangle$, which have energy $E = \sum_i k_i^2$ and momentum $K = \sum_i k_i$.

Here, we only wish to introduce the expressions which we need for our calculation. The thermodynamic properties of the spin $1/2$ Fermi gas are described by thermodynamic Bethe Ansatz (TBA) equation [33]. At zero temperature, the TBA equations reduce to the following forms [34, 35]:

$$\varepsilon_c^0(k) = k^2 - \mu - H/2 + \int_{-\lambda_0}^{\lambda_0} a_1(k-\lambda)\phi_s^0(\lambda)\mathrm{d}\lambda, \quad (3)$$

$$\phi_s^0(\lambda) = H + \int_{-k_0}^{k_0} a_1(\lambda - k)\varepsilon_c^0(k)\mathrm{d}k - \int_{-\lambda_0}^{\lambda_0} a_2(\lambda-\lambda')\phi_s^0(\lambda')\mathrm{d}\lambda', \quad (4)$$

where H is the external magnetic field and μ is the chemical potential. The function a_n is given below,

$$a_n(k) = \frac{1}{2\pi}\frac{nc}{(nc)^2/4 + k^2}. \quad (5)$$

Figure 3 illustrates the dressed energy of a spinful system close to the critical point.

There is a phase transition between the full polarized phase and mixture phase. At critical point, $\lambda_0 = 0$ which means $\phi_s^0(0) = 0$. For the full polarized phase, zero-temperature TBA equations become simpler [37],

$$\varepsilon_c^0(k) = k^2 - \mu - H/2,$$
$$\phi_s^0(\lambda) = H + \int_{-k_0}^{k_0} a_1(\lambda-k)\varepsilon_c^0(k)\mathrm{d}k. \quad (6)$$

At the critical point, the Fermi momentum can be obtained from the relation $\varepsilon_c^0(k_0) = 0$, i.e. $k_0 = \sqrt{\mu_c + H_c/2}$. We observe that if the particle density is fixed, the Fermi momentum k_0 is fixed, and we thus have $\phi_s^0(0) = 0$ to determine the relation between the critical magnetic field and chemical potential

$$H_c = \left(\frac{c^2}{2\pi} + \frac{2}{\pi}k_0^2\right)\arctan\left(\frac{2}{c}k_0\right) - \frac{c}{\pi}k_0. \quad (7)$$

Time evolution

At the initial moment, we inject a Fermi impurity into the fully polarized initial state [9]

$$|\Phi_I(\vec{x})\rangle = \int \mathrm{d}x\, \Phi_\downarrow(x)\hat{\Psi}_\downarrow^\dagger(x)|0\rangle \otimes |\Omega\rangle, \quad (8)$$

where $\Phi_\downarrow(x)$ is the wavefunction of the impurity and $|\Omega\rangle$ denotes the ground state of the medium, namely

$$|\Omega\rangle = \int \mathrm{d}\vec{x}\, \Phi_\uparrow(\vec{x})\hat{\Psi}_\uparrow^\dagger(x_{N-1})...\hat{\Psi}_\uparrow^\dagger(x_1)|0\rangle = |\{k\}_{N-1}\rangle. \quad (9)$$

Now we consider the evolution of the initial quantum state under the Hamiltonian (1), and examine the dynamics of the impurity in terms of the time evolution of the momentum of the impurity. Over a repaid decay of relaxation, the impurity's momentum exhibits a periodic oscillation. Such a phenomenon has been known as quantum flutter, see figure 2.

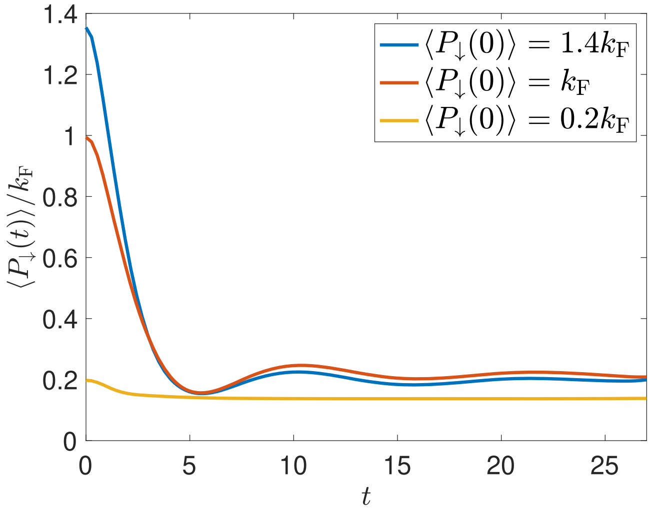

Figure 2. Time evolution of the impurity momentum $P_\downarrow(t)$ from the numerical method based on the determinant representation of the Bethe Ansatz states [29]. We observe that once the initial momentum of the impurity exceeds the Fermi momentum of the medium, i.e. $k \geqslant k_F$, quantum flutter phenomenon remarkably occurs. The oscillation period is invariant with respect to the initial impurity momentum, provided that it exceeds the Fermi momentum. When $k < k_F$, there does not exhibit quantum flutter phenomenon.

Correlation functions

The emergence of the quantum flutter phenomenon indicates that it is primarily dominated by a select subset of eigenstates. In the following discussion, we will demonstrate that the periodic oscillation of the impurity momentum is primarily driven by the momentum transfer between the nonlinear Luttinger liquid and the magnon excitation. To gain deeper insights into the spectrum of the system, we first analyze the correlation function

$$G(x, t) = \langle \hat{\Psi}_\downarrow(x, t)\hat{\Psi}_\downarrow^\dagger(0, 0)\rangle, \quad (10)$$

where the state $\langle\cdot\rangle$ is the average value for the ground state of the fully polarized system. This correlation function provides crucial spectral information that facilitates our investigation of dynamical behavior of the system.

To gain a more intuitive understanding of this phenomenon, we first consider the system in the strong interaction regime. In this regime, the system exhibits spin-charge separation, with the spin sector being equivalent to a Heisenberg spin chain [35, 36]. Consequently, the correlation function under consideration is given by [38]

$$G(x, t) = \langle \hat{\Psi}_\downarrow(x, t)\hat{\Psi}_\downarrow^\dagger(0, 0)\rangle = \langle \hat{\Psi}(x, t)\hat{\Psi}^\dagger(0, 0)\rangle_{\text{charge}}\langle \hat{S}^+(x, t)\hat{S}^-(0, 0)\rangle_{\text{spin}}, \quad (11)$$

where $\langle \hat{\Psi}(x, t)\hat{\Psi}^\dagger(0, 0)\rangle_{\text{charge}}$ represents the correlation function of a spinless Fermi system. For simplicity, we choose the particle density $\rho = \frac{N}{L} = 1$. The interaction of the spinless system is shown in figure 4.

The term $\langle \hat{S}^{+}(x, t)\hat{S}^{-}(0, 0)\rangle_{\text{spin}}$ denotes the correlation function of an antiferromagnetic Heisenberg spin chain with an external magnetic field H_c, described by the Hamiltonian

$$h_{\text{spin}} = 2J\sum_{x} \boldsymbol{S}_x \cdot \boldsymbol{S}_{x+1} - H_c\sum_{x} S_x^z, \tag{12}$$

here, the range of x is from 0 to N. At the critical point of the Yang–Gaudin model, the relationship $H_c = 4J$, see [39]. The ground state of the spin chain is fully spin-polarized. The excitations near this ground state, known as magnons, exhibit the spectrum

$$\varepsilon_s(k_s) = 2J(1 - \cos k_s). \tag{13}$$

Treating magnons as free excitations, the correlation function can be expressed as [38]

$$\langle \hat{S}^{+}(x, t)\hat{S}^{-}(0, 0)\rangle_{\text{spin}} = \int \frac{\mathrm{d}k_s}{2\pi} \mathrm{e}^{\mathrm{i}k_s x - \mathrm{i}\varepsilon_s t}. \tag{14}$$

Consequently, we obtain

$$G(x, t) = \langle \hat{\Psi}(x, t)\hat{\Psi}^{\dagger}(0, 0)\rangle_{\text{charge}} \cdot \int \frac{\mathrm{d}k_s}{2\pi} \mathrm{e}^{\mathrm{i}k_s x - \mathrm{i}\varepsilon_s t}. \tag{15}$$

The above analysis remains valid even for weak interactions, because when we consider only a single magnon excitation, the spectra of the charge sector and the spin sector are decoupled.

Subsequently, we examine the correlation function in the charge sector. Initially, we focus on the low-energy excitations in the Luttinger liquid region in the charge sector, which means $Q \ll k_F$. It can only cause excitations near the Fermi surface. Therefore, we can use the Luttinger liquid theory to calculate the single-particle propagator in the charge sector [25, 40].

$$G(x, t)_{\text{charge}} = \langle \Psi(x, t)\Psi^{\dagger}(0,0)\rangle_{\text{LL}}$$
$$\sim \sum_{n} \frac{\rho \mathrm{e}^{\mathrm{i}(2n+1)k_F x}}{2\mathrm{i}(-1)^n} \frac{C_n}{[\mathrm{i}\rho(vt + x) + 0]^{\mu_L}[\mathrm{i}\rho(vt - x) + 0]^{\mu_R}}. \tag{16}$$

Here, C_n and μ_L, μ_R, v are some parameters depending on the interaction, and ρ is the particle density. Even though we do not rigorously calculate the values of these parameters, we know that after performing a Fourier transformation with respect to the coordinates, the correlation function will not oscillate with respect to time t, but instead, it will exhibit a power-law decay with respect to time t.

Next, we shall consider the nonlinear Luttinger liquid effects. It is sufficient to introduce a single hole excitation with momentum k deeply within the Fermi sea, which contributes to the edge singularities [24, 25].

$$\Psi(x) \to \mathrm{e}^{\mathrm{i}k_F x}\psi_R(x) + \mathrm{e}^{-\mathrm{i}k_F x}\psi_L(x) + \mathrm{e}^{\mathrm{i}kx}\mathrm{d}(x). \tag{17}$$

Then, the mobile-impurity Hamiltonian of the system can be written down [24, 25, 41–54]

$$H_0 = \frac{v}{2\pi}\int \mathrm{d}x \left[K(\nabla\theta)^2 + \frac{1}{K}(\nabla\phi)^2\right],$$
$$H_d = \int \mathrm{d}x d^{\dagger}(x)[\xi(k) - \mathrm{i}v_d\nabla]\mathrm{d}(x). \tag{18}$$

Here, $\xi(k) = \varepsilon(k)$ shown in figure 3, is the single hole excitation dispertion. K is the Luttinger parameter and v is the sound velocity. H_d is the Hamiltonian of the mobile impurity, $\mathrm{d}(x)$ is the annihilation operator, v_d is the velocity of the mobile impurity and ∇ is the gradient operator. Of course, there are interactions between the Luttinger liquid and the mobile impurity,

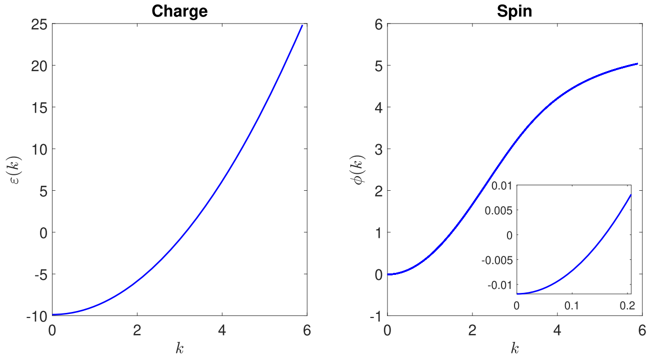

Figure 3. Dressed energy of a spinful system close to critical point. The dressed energy $\varepsilon(k)$ and $\phi(k)$ are correlated to the charge and spin excited energy, respectively.

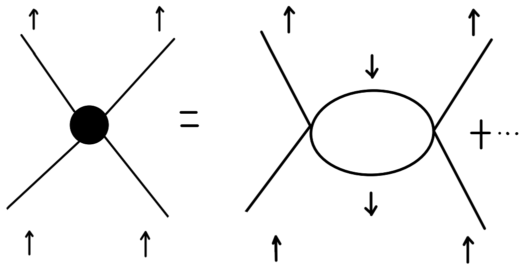

Figure 4. The diagram illustrates the effective interaction among spin-up fermions which is induced by the spin-down impurity.

$$H_{\text{int}} = \int \mathrm{d}x [V_R(k)\rho_R(x) + V_L(k)\rho_L(x)]\mathrm{d}(x)\mathrm{d}^{\dagger}(x)$$
$$= \int \mathrm{d}x \left[V_R(k)\nabla\frac{\theta - \phi}{2\pi} - V_L(k)\nabla\frac{\theta + \phi}{2\pi}\right] dd^{\dagger}. \tag{19}$$

Here, V_L, V_R represent the interaction, and there is a unitary transform

$$U = \exp\left\{\mathrm{i}\int \mathrm{d}x \left(\frac{\delta_+(k)}{2\pi}[\tilde{\theta} - \tilde{\phi}] - \frac{\delta_-(k)}{2\pi}[\tilde{\theta} + \tilde{\phi}]\right) dd^{\dagger}\right\}, \tag{20}$$

where $\tilde{\theta}(x) = \sqrt{K}\theta(x)$, $\quad \tilde{\phi}(x) = \phi(x)/\sqrt{K}$.

$$\delta_-(k) = -\frac{\tilde{V}_L}{v_d + v}, \quad \delta_+(k) = -\frac{\tilde{V}_R}{v_d - v}. \tag{21}$$

$$(V_L - V_R)/\sqrt{K} = \tilde{V}_L - \tilde{V}_R,\ (V_L + V_R)\sqrt{K} = \tilde{V}_L + \tilde{V}_R. \tag{22}$$

After this transformation

$$U^{\dagger}(H_0 + H_d + H_{\text{int}})U = \widetilde{H}_0 + \widetilde{H}_d, \tag{23}$$

where $\widetilde{H}_0$ and $\widetilde{H}_d$ are decoupled in the new representation.

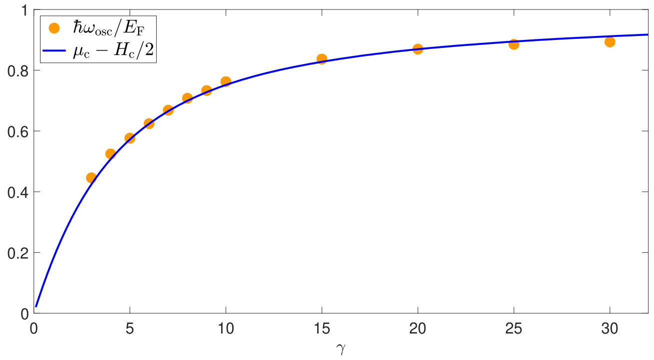

Figure 5. Comparison between the analytical results and numerical results. The orange circles are numerical results obtained by Bethe Ansatz approach [29]. The blue line is the analytical results (25) from the correlation functions.

From the quadratic Hamiltonian, we have the correlation function as the following

$$
\begin{aligned}
G(x,t)_{\rm charge} &= \langle \Psi(x = v_d t, t)\Psi^{\dagger}(0,0)\rangle_{\rm NLL} \\
&= \langle \Psi(v_d t, t)\Psi^{\dagger}(0,0)\rangle_{\rm LL} + \sqrt{\frac{m_d}{-2i\pi}} \\
&\times \sum_n \frac{{\rm e}^{-i\pi\mu_{n,-}/2} A_{n,-}(k_d) {\rm e}^{-i\varepsilon(k_d)t + i(k_d + 2nk_F)x}}{t^{1/2}(vt + x)^{\mu_{n,-,L}}(vt - x)^{\mu_{n,-,R}}}.
\end{aligned} \tag{24}
$$

Here $t \to \infty$, and $A_{n,-}$ is the hole spectral function of a finite system, see [25].

All the parameters depend on the interaction and the single partical spectrum. Our primary interest lies in the correlation function with respect to momentum and its time dependence. To obtain meaningful results, it is straightforward to perform a Fourier transformation on the aforementioned correlation function. Notably, in order to obtain meaningful results, the coordinate x must be able to take arbitrary values which means that the hole velocity v_d goes to zero in the limit $t \to \infty$. The condition $v_d = 0$ corresponds to the deepest hole excitations $k_d = 0$. Therefore, the momentum of the impurity must satisfy $Q \geqslant k_F$. Under these conditions, we observe a time-oscillating term ${\rm e}^{-i\varepsilon(k_{\rm d})t}$ within the correlation function, which contributes to the periodic oscillation of the momentum of the impurity.

By combining the correlation functions of the charge and the spin degrees of freedom, we determine that the oscillation periodicity of the momentum of impurity is governed by the energy difference between the deepest hole excitation in the charge sector and the magnon excitation with the Fermi momentum in the spin sector, namely

$$
\begin{aligned}
\frac{2\pi}{\tau} &= -\varepsilon_c^0(k_d)|_{k_d=0} - \varepsilon_s(k_s)|_{k_s=k_F} \\
&= \mu_c + \frac{H_c}{2} - 4J \\
&= \mu_c - \frac{H_c}{2}.
\end{aligned} \tag{25}
$$

Figure 5 shows the comparison between the analytical results and numerical results, and we find that they agree very well.

Summary

In this paper, we have demonstrated that the quantum flutter phenomenon can be manifested by the propagator of the single fermion correlation function in the interacting Fermi gases. We have investigated the single-particle correlation functions at different energy scales: the Luttinger liquid energy scale and the nonlinear Luttinger liquid energy scale. These correspond to the cases of small and large impurity energies, respectively. Our analysis reveals that the quantum flutter phenomenon does not occur at the Luttinger liquid energy scale. Nevertheless, when there are hole excitations at the deepest part of the Fermi sea, the quantum flutter phenomenon emerges. This study provides a comprehensive understanding of the quantum dynamical phenomenon based on the correlation functions within the framework of nonlinear Luttinger liquid.

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