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Production of the massless dark photon $\tilde{\gamma}$ associated with the photon γ from charged lepton flavor violating decay processes

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Abstract

The massless dark photon $\tilde{\gamma}$ can only interact with the Standard Model (SM) sector via higher-dimensional operators. In this letter, we investigate its production associated with the ordinary photon γ from the lepton flavor violation (LFV) process $\ell_i \to \ell_j\tilde{\gamma}\gamma$ and di-production from the LFV process $\ell_i \to \ell_j\tilde{\gamma}\tilde{\gamma}$ induced by dipole operators. Comparing the obtained numerical results with the corresponding experimental measurements, we obtain the constraints on the effective couplings of $\tilde{\gamma}$ with the SM charged leptons. The upper limit of the effective coupling $|D_L^{\mu e}|^2 + |D_R^{\mu e}|^2$ coming from the process $\mu \to e\tilde{\gamma}\gamma$ is looser than the process $\mu \to e\tilde{\gamma}$ by about one order of magnitude.

Keywords: massless dark photon, lepton flavor violation process, effective couplings

1. Introduction

The Standard Model (SM) of particles successfully describes three fundamental interactions and its predictions have been verified with great accuracy. Nevertheless, it is widely considered to be a low-energy effective theory since it contains no gravitational interaction and does not explain the dark components of the Universe, etc. One of the minimal and interesting extensions of the SM is the $U(1)$ extension in the gauge sector, which includes an extra Abelian boson called a dark or hidden photon [1, 2].

The dark photon as a hypothetical Abelian gauge boson is currently known as a possible candidate for new physics beyond the SM, which can be probed by various experiments and observations. The current and prospective constraints on dark photons have been obtained, see [3, 4] for reviews.

The dark photon may be massive or massless, depending on whether the dark $U(1)$ symmetry is spontaneously broken or unbroken. For the massless dark photon denoted by $\tilde{\gamma}$, it does not directly interact with the SM particles but could arise at loop-level via higher-dimensional operators, which may translate into detectable effects at current or future experiments. For example, production of the massless dark photon $\tilde{\gamma}$ from the flavor changing (FC) processes $f \to f'\,\tilde{\gamma}$ and its possible signals at present and future collider experiments have been extensively studied in [5, 6]. In this paper, we will focus our attention on the production of the massless dark photon $\tilde{\gamma}$ associated with the ordinary photon γ from the lepton flavor violation (LFV) process $\ell_i \to \ell_j\tilde{\gamma}\gamma$ and di-production from the LFV process $\ell_i \to \ell_j\tilde{\gamma}\tilde{\gamma}$, where ℓ_i and ℓ_j stand for the SM leptons e, μ or τ with $i \neq j$. Experimentally, compared to the two-body decay $\ell_i \to \ell_j\tilde{\gamma}$, the three-body decays emitting an extra gauge boson from the initial or final state lepton, ℓ_i or ℓ_j, can provide a better final state to be detected with a more robust background discrimination[3]. Thus, we consider the three-body decays $\ell_i \to \ell_j\tilde{\gamma}\gamma$ and $\ell_i \to \ell_j\tilde{\gamma}\tilde{\gamma}$, and expect that they can give more severe constraints on the effective couplings of $\tilde{\gamma}$ with the SM leptons.

2. The LFV decay processes $\ell_i \to \ell_j\tilde{\gamma}\gamma$ and $\ell_i \to \ell_j\tilde{\gamma}\tilde{\gamma}$

The massless dark photon $\tilde{\gamma}$ can interact with the SM particles via higher-dimensional operators. The operators of the lowest

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[3] Based on this fact, [7, 8] have studied the possibilities of detecting the light axion-like particle a at MEG II via the decay processes $\mu \to ea$ and $\mu \to ea\gamma$.

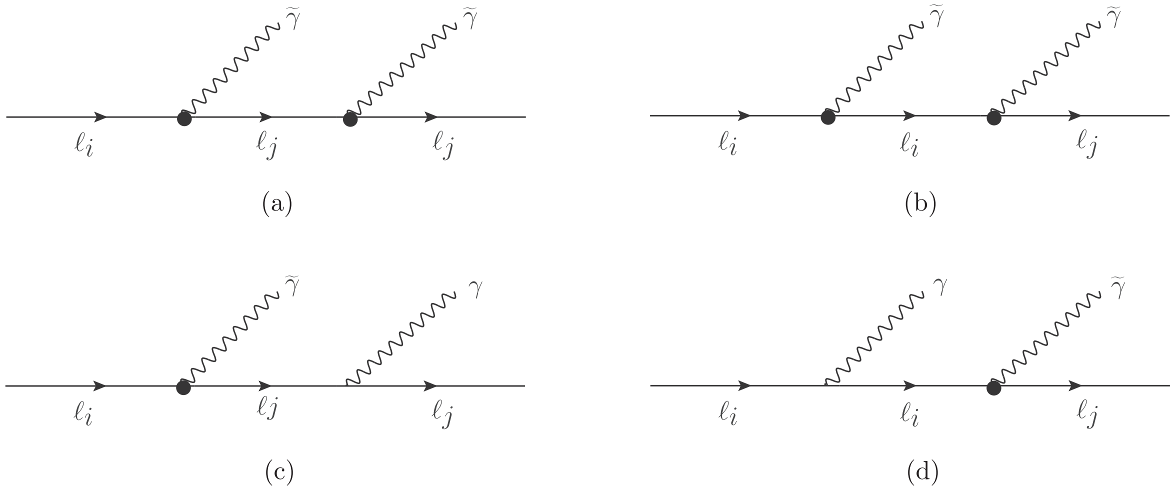

Figure 1. Feynman diagrams contributing to $\ell_i \to \ell_j\tilde{\gamma}\tilde{\gamma}$ (a), (b) and $\ell_i \to \ell_j\tilde{\gamma}\gamma$ (c), (d). Black dots denote the effective interactions of $\tilde{\gamma}$ with charged leptons and γ represents the ordinary photon.

dimension are magnetic dipole operators of dimension five [9], which can appear at one-loop level in a UV completed model [5]. In this paper, we consider the charged-lepton dipole interactions in a model-independent way, which are related the LFV processes $\ell_i \to \ell_j\tilde{\gamma}\gamma$ and $\ell_i \to \ell_j\tilde{\gamma}\tilde{\gamma}$, and are described by the following effective Lagrangian

$$\mathcal{L}_{DP}^{\rm dipole} = -\frac{1}{2}\sum_{\ell,\ell'=e,\mu,\tau}\overline{\ell'}(D_L^{\ell\ell'}P_L + D_R^{\ell\ell'}P_R)\sigma^{\mu\nu}\ell F'_{\mu\nu}, \quad (1)$$

where $D_L^{\ell\ell'}$ and $D_R^{\ell\ell'}$ are coupling constants of dimension -1 and there is $D_R^{\ell\ell'} = D_L^{\ell\ell'*}$ due to the Hermiticity of the effective Lagrangian[4]. $P_{R/L} = (1 \pm \gamma_5)/2$, $\sigma^{\mu\nu} = {\rm i}[\gamma^\mu, \gamma^\nu]/2$ and $F'_{\mu\nu} = \partial_\mu\tilde{F}_\nu - \partial_\nu\tilde{F}_\mu$ is a field-strength tensor of the massless dark photon $\tilde{\gamma}$.

From equation (1), we can see that the LFV decays $\ell_i \to \ell_j\tilde{\gamma}\tilde{\gamma}$ and $\ell_i \to \ell_j\tilde{\gamma}\gamma$ can be induced via the Feynman diagrams as shown in figure 1. The partial decay widths can be written as

$$\Gamma(\ell_i \to \ell_j\tilde{\gamma}\tilde{\gamma}) = \frac{m_i^5}{768\pi^3}(|D_L^{ij}|^2 + |D_R^{ij}|^2)\,|D_V|^2\lambda\left(\frac{E_{\tilde{\gamma}}^{\rm cut}}{m_i}\right), \quad (2)$$

$$\Gamma(\ell_i \to \ell_j\tilde{\gamma}\gamma) = \frac{\alpha m_i^3}{192\pi^2}(|D_L^{ij}|^2 + |D_R^{ij}|^2)\lambda\left(\frac{E_{\tilde{\gamma}}^{\rm cut}}{m_i}\right). \quad (3)$$

Here we have neglected the mass of the final lepton and m_i stands for the mass of the lepton ℓ_i. We have assumed $D_R^{ii} = D_L^{ii} = D_R^{jj} = D_L^{jj} = D_V$, just as the couplings of photon γ with charged leptons in equation (2). $E_{\tilde{\gamma}}^{\rm cut}$ is the minimal energy cut of the massless dark photon $\tilde{\gamma}$ to regularize the collinear and infrared divergences. The parameter $\lambda(x)$ is given by [11]

$$\begin{aligned}\lambda(x) \approx\, & 6 + 2\pi^2 + 6\log^2 2 + 21\log(2x) + 6\log(x)\log(4x)\\ & + 18x[2\log(2x) + 1] + 6x^2[8\log(2x) - 29]\\ & + \mathcal{O}(x^3).\end{aligned} \quad (4)$$

The charged-lepton dipole interactions described by equation (1) can also induce the LFV decay process $\ell_i \to \ell_j\tilde{\gamma}$. Its decay width is given by

$$\Gamma(\ell_i \to \ell_j\tilde{\gamma}) = \frac{m_i^3}{16\pi}(|D_L^{ij}|^2 + |D_R^{ij}|^2)\left(1 - \frac{m_j^2}{m_i^2}\right)^3. \quad (5)$$

Searching for the LFV processes $\ell_i \to \ell_j + {\rm inv.}$, such as $\mu \to e + {\rm inv.}$ and $\tau \to \mu(e) + {\rm inv.}$ has gathered considerable attention by various experiments for a long time, where inv. represents a missing massless boson or missing energy. An experimental upper limit on the branching ratio $Br(\mu \to e + X^0)$ has been studied in [12] for both massless and massive bosons. For the boson X^0 with a mass of less than 13 MeV, the upper limit of $Br(\mu \to e + X^0)$ is up to 5.8×10^{-5}, whereas for the massless boson, in combination with the results from [13], the limit on the branching ratio is smaller than that for the massive boson by about one order of magnitude. To obtain constraints on the effective couplings of the massless dark photon $\tilde{\gamma}$ with charged leptons, we use $Br(\mu \to e + {\rm inv.}) < 2.6 \times 10^{-6}$ at 90% confidence level (CL) [13] and $Br(\tau \to \mu(e) + inv.) < 0.59(0.94) \times 10^{-3}$ given recently by the Belle II Collaboration [14] and take $E_\gamma = E_{\tilde{\gamma}} = 7(50)$ MeV for $\mu(\tau)$ decays in our numerical estimations. Using these experimental upper limits and equation (5), [6] has given the constraints on the effective couplings $|D_L^{ij}|^2 + |D_R^{ij}|^2$,

$$\begin{aligned}&|D_L^{\mu e}|^2 + |D_R^{\mu e}|^2 \leqslant 3.31 \times 10^{-14}{\rm TeV}^{-2},\\ &|D_L^{\tau\mu}|^2 + |D_R^{\tau\mu}|^2 \leqslant 1.21 \times 10^{-8}{\rm TeV}^{-2},\\ &|D_L^{\tau e}|^2 + |D_R^{\tau e}|^2 \leqslant 1.93 \times 10^{-8}\,{\rm TeV}^{-2},\end{aligned} \quad (6)$$

for the LFV processes $\mu \to e\tilde{\gamma}$ and $\tau \to \mu(e)\tilde{\gamma}$, respectively.

[4] The operators of dimension seven and eight can also contribute to the LFV decays $\ell_i \to \ell_j\tilde{\gamma}\tilde{\gamma}$ [10, 11]. However, we only consider the lowest-dimension operators to calculate the branching ratios $Br(\ell_i \to \ell_j\tilde{\gamma}\tilde{\gamma}(\gamma))$ in this paper.

Table 1. The SM prediction and experimental measured values for the branching ratios $Br(\ell_i \to \ell_j \nu_{\ell_i} \bar{\nu}_{\ell_j} \gamma)$.

Br	SM (NLO) [15]	Exp. [16]
$\mu \to e\bar{\nu}\nu\gamma$	$5.850(1) \times 10^{-8}$	$(6.0 \pm 0.5) \times 10^{-8}$
$\tau \to \mu\bar{\nu}\nu\gamma$	$3.571(1) \times 10^{-3}$	$(3.67 \pm 0.08) \times 10^{-3}$
$\tau \to e\bar{\nu}\nu\gamma$	$1.645(1) \times 10^{-2}$	$(1.83 \pm 0.05)\%$

Using the numerical results of [6], we can obtain from equation (2), the lower limits of the effective diagonal coupling D_V. Certainly, we can also give the constraints on D_V by combining equation (2) with equation (5). The numerical results generated via the two ways are approximately equal to each other, which are $|D_V|^2 \geqslant 1.421 \times 10^{10}\text{TeV}^{-2}$, $1.204 \times 10^7\,\text{TeV}^{-2}$ and $1.216 \times 10^7\,\text{TeV}^{-2}$ for $\mu \to e\tilde{\gamma}\tilde{\gamma}$, $\tau \to \mu\tilde{\gamma}\tilde{\gamma}$ and $\tau \to e\tilde{\gamma}\tilde{\gamma}$, respectively. These constraints are relatively looser than the astrophysical and cosmological constraints on the effective diagonal couplings $D^{ii}_{L,R}$ [3, 6].

Certainly, if we compare the branching ratios $Br(\ell_i \to \ell_j\tilde{\gamma}\tilde{\gamma})$ with the upper limits of $Br(\ell_i \to \ell_j + \text{inv.})$ directly, one can give the constraints on the combined effective couplings $(|D^{ij}_L|^2 + |D^{ij}_R|^2)\,|D_V|^2$, which can be written as

$$\begin{aligned}(|D^{\mu e}_L|^2 + |D^{\mu e}_R|^2)\,|D_V|^2 &\leqslant 4.73 \times 10^{-4}\,\text{TeV}^{-4},\\ (|D^{\tau\mu}_L|^2 + |D^{\tau\mu}_R|^2)\,|D_V|^2 &\leqslant 1.46 \times 10^{-1}\,\text{TeV}^{-4},\\ (|D^{\tau e}_L|^2 + |D^{\tau e}_R|^2)\,|D_V|^2 &\leqslant 2.33 \times 10^{-1}\,\text{TeV}^{-4}.\end{aligned} \quad (7)$$

Reference [6] has recently investigated the constraints on the combined effective couplings $(|D^{ij}_L|^2 + |D^{ij}_R|^2)\,|D^{jj}_R|^2$ from the LFV processes $\mu^+ \to e^+e^-e^+$ and $\tau^+ \to \ell^+\ell^-\ell^+$ with ($\ell = e$ or μ). Obviously, the constraints given by [6] are tighter than our results from the LFV processes $\ell_i \to \ell_j + inv$.

Radiative leptonic decays $\ell_i^+ \to \ell_j \nu_{\ell_i}\bar{\nu}_{\ell_j}\gamma$ can be used to precisely determine the SM input parameters and can be seen as an irreducible background to search for the LFV decays $\ell_i \to \ell_j\gamma$, which provide an additional promising tool to search for new physics beyond the SM. The SM prediction values of the branching ratios $Br(\ell_i \to \ell_j \nu_{\ell_i}\bar{\nu}_{\ell_j}\gamma)$ have been calculated in [15] at next-to-leading order and the corresponding experimental values are summarized in [16], which is shown in table 1.

If the massless dark photon $\tilde{\gamma}$ escapes detectors and therefore is reconstructed as missing energy in experiments, then the decays $\ell_i \to \ell_j\tilde{\gamma}\gamma$ can contribute to the SM leptonic decays $\ell_i \to \ell_j \nu_{\ell_i}\bar{\nu}_{\ell_j}\gamma$. We will assume that the decays $\ell_i \to \ell_j\tilde{\gamma}\gamma$ are solely responsible for the observed difference $Br(\ell_i \to \ell_j \nu_{\ell_i}\bar{\nu}_{\ell_j}\gamma)^{\text{EXP.}} - Br(\ell_i \to \ell_j \nu_{\ell_i}\bar{\nu}_{\ell_j}\gamma)^{\text{SM}}$ at 90% CL, which demands $0 \leqslant Br(\mu \to e\tilde{\gamma}\gamma) \leqslant 1.071 \times 10^{-8}$, $2.961 \times 10^{-5} \leqslant Br(\tau \to \mu\tilde{\gamma}\gamma) \leqslant 2.961 \times 10^{-4}$ and $2.138 \times 10^{-3} \leqslant Br(\tau \to e\tilde{\gamma}\gamma) \leqslant 3.948 \times 10^{-3}$, respectively. Then we can obtain the constraints on the effective couplings $|D^{ij}_L|^2 + |D^{ij}_R|^2$ at 90% CL,

$$\begin{aligned}&0 \leqslant |D^{\mu e}_L|^2\\ &\quad + |D^{\mu e}_R|^2 \leqslant 2.37 \times 10^{-13}\,\text{TeV}^{-2},\\ &2.53 \times 10^{-7}\,\text{TeV}^{-2} \leqslant |D^{\tau\mu}_L|^2\\ &\quad + |D^{\tau\mu}_R|^2 \leqslant 2.53 \times 10^{-6}\,\text{TeV}^{-2},\\ &1.83 \times 10^{-5}\,\text{TeV}^{-2} \leqslant |D^{\tau e}_L|^2\\ &\quad + |D^{\tau e}_R|^2 \leqslant 3.37 \times 10^{-5}\,\text{TeV}^{-2}.\end{aligned} \quad (8)$$

From the above equations, we can see that the constraint on the effective coupling $|D^{\mu e}_L|^2 + |D^{\mu e}_R|^2$ for process $\mu \to e\tilde{\gamma}\gamma$ is looser than for the process $\mu \to e\tilde{\gamma}$ by about one order of magnitude. While the constraints from the decays $\tau \to \mu(e)\tilde{\gamma}\gamma$ on the corresponding effective couplings are much loose than those from the decays $\tau \to \mu(e)\tilde{\gamma}$.

3. Conclusions

The dark photon may interact with the SM sector through gauge-invariant kinetic mixing, the massive dark photon can also be coupled to ordinary particles through mass mixing, while the massless dark photon can only interact with ordinary particles via higher-dimensional operators. So, dark photons can generate rich phenomenology at low-energy and high-energy experiments, which might be probed by various experiments and observations.

In this paper, we consider the interactions between the massless dark photon $\tilde{\gamma}$ and the ordinary charged leptons through higher-dimensional operators and have investigated its production associated with the photon γ from the charged LFV processes $\ell_i \to \ell_j\tilde{\gamma}\gamma$ and di-production from the charged LFV process $\ell_i \to \ell_j\tilde{\gamma}\tilde{\gamma}$ induced by dipole operators. Comparing the obtained numerical results with the corresponding experimental measurements, we obtain the constraints on the effective couplings of $\tilde{\gamma}$ with the SM charged leptons. Our results show that the constraints on the effective diagonal coupling D_V from the processes $\ell_i \to \ell_j\tilde{\gamma}\tilde{\gamma}$ are relatively looser than the astrophysical and cosmological constraints on the effective diagonal couplings $D^{ii}_{L,R}$ and the constraints on the combined effective couplings $(|D^{ij}_L|^2 + |D^{ij}_R|^2)|D_V|^2$ are also looser than those from the processes $\ell_i^+ \to \ell_j^+\ell_j^-\ell_j^+$ induced by the massless dark photon $\tilde{\gamma}$. However, if we assume that the deviations between the SM predictions and the experimental measurements for the branching ratios $Br(\ell_i \to \ell_j \nu_{\ell_i}\bar{\nu}_{\ell_j}\gamma)$ come from the processes $\ell_i \to \ell_j\tilde{\gamma}\gamma$, then we can obtain constraints on the effective couplings $|D^{ij}_L|^2 + |D^{ij}_R|^2$, which are relatively loose for processes $\ell_i \to \ell_j\tilde{\gamma}$. We expect that our results will be helpful in the detection of massless dark photons in future experiments.

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