Commun. Theor. Phys. **77** (2025) 065107 (10pp)
https://doi.org/10.1088/1572-9494/ada37b
CSTR: 32041.14.CTP.ada37b

Enhanced stabilization performances of an open quantum battery in a photonic band-gap environment

Yan Chen, Jia Tan, Jincheng Lu and Xiang Hao

School of Physical Science and Technology, Suzhou University of Science and Technology, Suzhou 215009, China

E-mail: xhao@mail.usts.edu.cn

Received 10 September 2024, revised 25 December 2024
Accepted for publication 25 December 2024
Published 19 March 2025

Abstract

We investigate the stabilization mechanism of open quantum batteries driven by a classical field in the weak or strong system-reservoir coupling regime. A protocol to improve the steady-state energy storage performance is proposed by engineering the spectral density of a band-gap environment which is described as the superposition of two inhomogeneous Lorentzian spectrums with different weights. We find out that the interplay between the battery-environment-bound state and the reservoir memory effect plays a crucial role in the stabilization performance against energy dissipation. The formation of the bound state and the non-Markovian effect will be strengthened by adjusting the weights of the environment spectral density. In the charging process, the classical field contributes to enhancing the steady ergotropy. Moreover, the manipulation of the spectrum weights results in the speedup scheme of carrying out the energy storage due to the existence of bound states. In the self-discharging process, increasing the spectral weight allows the battery to maintain a higher steady ergotropy. These results provide a practical approach to achieving optimal quantum batteries with better stabilization performance.

Keywords: quantum decoherence, quantum work extraction, quantum battery

(Some figures may appear in colour only in the online journal)

1. Introduction

In recent years, the advancement of quantum technology has sparked great interest in harnessing quantum effects to achieve miniaturization of technology [1–5]. This breakthrough has paved the way for exciting possibilities in the fields of quantum computation [6], quantum sensing [7, 8], and other quantum technologies [9, 10]. Research over the past several decades has shown that many modern technologies based on quantum mechanics [11–16] can outperform classical ones. The design of energy storage devices in the quantum domain, called quantum batteries [17–21], is of increasing interest. The utilization of quantum resources such as quantum coherence and entanglement enables quantum batteries to have superior performance in terms of energy storage density, extractable work, and charging power.

The physical models of quantum batteries have rapidly developed from closed systems [22, 23] to open systems [24–33] due to the fact that practical systems unavoidably interact with their surrounding environments. Quantum decoherence arising from the system-environment couplings leads to energy leakage from quantum batteries to reservoirs. Recent studies [34–36] have demonstrated that the adverse impacts of the environment on quantum battery performance can be significantly mitigated via some specific designs. The non-Markovian effect can contribute to enhancing the charging performance of open quantum batteries. The noise-assisted method was applied to collective charging processes. In the previous works, the environment coupled to quantum batteries was often treated as a single entity described by a simple type of spectral density function. However, the practical environment has so much more degrees of freedom that

the modes of the environment can be referred to as a set of pseudomodes which are identified through evaluating the poles of the complex spectral density function [37, 38]. Among them, one kind of band-gap environment can be modelled by the spectral density function in which both Lorentzians are centered at the same frequency, and one of them is given a negative weighting [39]. This physical setup can be realized by a two-level atom embedded in a photonic crystal cavity with periodic dielectric structures. The present experiments can provide us the possibility to engineer the environment by adjusting the geometrical parameters of photonic crystal structures.

To realize optimal quantum batteries, attention has been paid not only to quantum batteries with high ergotropy of a charging process, but also to ones with low dissipation of a self-discharging process. When quantum batteries are immersed in an environment, another obstacle in achieving high performance is how to efficiently stabilize the charging or self-discharging process against energy dissipation. It is of great value to maintain an open quantum battery in its high and stable ergotropy state. In this paper, we put forward a stabilization mechanism of open quantum batteries in a complex environment like a photonic band-gap structure. The efficient manipulation of the environment spectral density enables the fast generation of battery-reservoir bound states which plays a key role in the stabilization of charging and self-discharging. We find that a classical driving field in resonance with an open quantum battery can result in optimal steady-state charging. We demonstrate that the stabilization performance is dependent on the interplay between the existence of the bound states and environmental memory effects.

This paper is organized as follows. In section 2, we review the energy transfer of an open quantum battery by introducing some figures of merit for charging or self-discharging. In section 3, quantum dynamics of a classical driving battery coupled to a band-gap environment are analyzed. We derive the steady state of the open quantum battery in the charging or self-discharging process. In section 4, the stabilization performance of quantum batteries is determined by the existence of the bound states between the battery and environment. According to the eigenspectrum of the whole system, we derive the condition of the formation of the bound state. The dynamical behavior of the ergotropy is also related to the memory effect of the band-gap environment. Finally, the conclusion is drawn from the present study in section 5.

2. Performance of open quantum batteries

A good open quantum battery needs to exhibit robust energy storage capabilities in the charging or self-discharging process. Equivalently, good performance means two conditions: one is to have a steady and high ergotropy in a short charging time, and the other is the ability to stabilize the stored energy against the energy loss from the interactions between batteries and environments. A quantum battery is quantified by a bounded internal Hamiltonian H_B where a high energy state and a low energy state represent a maximally charged and discharged battery state, respectively. In contrast to a closed quantum battery, an open quantum battery evolves under the influence of the environment. The nature of a quantum battery is a work reservoir where the energy can be extracted in a unitary process. The energy of the battery at time t is represented by $E_B(t) = \mathrm{Tr}[\rho_B(t)H_B]$. The average charging power at time t is given by

$$P_B(t) = \frac{E_B(t) - E_G}{t}, \tag{1}$$

where E_G is the ground state energy of the quantum battery. To characterize the performance of the quantum battery, the maximal amount of the extractable work is defined as the ergotropy,

$$W(t) = \mathrm{Tr}[\rho_B(t)H_B] - \mathrm{Tr}(\sigma_{\rho_B}H_B), \tag{2}$$

where σ_{ρ_B} represents a passive state of the quantum battery, which is the state where no work can be extracted during a unitary operation [40–42]. The evolved state $\rho_B(t)$ of the quantum battery is closely related to the charging protocol and self-discharging process. The passive state σ_{ρ_B} can be expressed by the spectral decompositions of the Hamiltonian and the evolved state of the quantum battery

$$\sigma_{\rho_B} = \sum_j r_j|\varepsilon_j\rangle\langle\varepsilon_j|, \tag{3}$$

where r_j is the eigenvalue of the state ρ_B with the decreasing order of $r_j \geqslant r_{j+1}$. Meanwhile, according to the increasing eigenvalues of $\varepsilon_j \leqslant \varepsilon_{j+1}$, the eigenstates $\{|\varepsilon_j\rangle\}$ of the Hamiltonian H_B are properly ordered.

As another performance, the ratio between the ergotropy and energy value of the battery can be used to evaluate the efficiency of energy storage,

$$\mathcal{L}(t) = \frac{W(t)}{E_B(t) - E_G}. \tag{4}$$

In the self-discharging process, the evolved state of the open quantum battery $\rho_B(t)$ is governed by quantum dynamics, i.e., $\partial_t\rho(t) = -\mathrm{i}[H_B + H_E + H_I, \rho(t)]$ where H_B is the Hamiltonian of the quantum battery, H_E denotes the Hamiltonian of the environment, and H_I represents the interaction between the battery and reservoir. The discharging state of the quantum battery $\rho_B(t) = \mathrm{Tr}_E[\rho(t)]$ is obtained after tracing the degrees of freedom of the environment. The stabilized ergotropy is determined by $W_d = \lim_{t\to\infty} W(t)$ during the self-discharging period. The higher ergotropy in the steady state, the better the stabilization performance in the discharging process.

Moreover, we propose a charging protocol by means of classical field driving. With respect to a two-level system, a quantum battery is generated by the Hamiltonian of $H_B = \omega_0\sigma^+\sigma^-$ where ω_0 denotes the energy transition between an excited state $|e\rangle$ and a ground state $|g\rangle$. The operators $\sigma^\pm$ signify the rising and lowing one, respectively. To realize a charging process, we can utilize a classical coherent field $E(\tau) = E_0\cos(\omega_L\tau)$ to drive the battery. During a charging time interval t, the external driving field exchanges energy with the battery via the dipolar interaction.

In the interaction picture, the charging Hamiltonian of the battery is expressed as $H_B^c = \frac{\Delta}{2}\sigma_z + \mu(\tau)\Omega(\sigma^+ + \sigma^-)$ where $\mu(\tau) = 1$ $(0 \leqslant \tau \leqslant t)$ describes the control function. The Rabi frequency is $\Omega = -d_{eg}E_0$ with d_{eg} being the transition dipole moment. The parameter $\Delta = \omega_0 - \omega_L$ represents the detuning frequency between the system and classical field. At an initial moment of $\tau = 0$, the system is prepared in the ground state, which represents a depleted battery. Through classical field driving, the energy is transferred from classical fields to the quantum battery. Similarly, the charged quantum battery will be stabilized in a steady state with a constant ergotropy under the influence of the environment. The shorter the charging time of the steady state, the better the stabilization performance in the charging process.

With respect to a two-dimensional quantum battery, the expression of the ergotropy is written in the form of the density matrix of the evolved state [31, 43],

$$W(t) = \frac{\omega_0}{2}[\sqrt{(1-2\varrho_{gg})^2 + 4|\varrho_{eg}|^2} + 1 - 2\varrho_{gg}]. \tag{5}$$

Here the density matrix $\rho_B(t) = \sum_{j,k=e,g}\varrho_{jk}(t)|j\rangle\langle k|$ is defined as the evolved state of the quantum battery. The conditions of $\varrho_{gg} + \varrho_{ee} = 1$ and $\varrho_{eg} = \varrho_{ge}^*$ are satisfied due to the Hermitian property of the state. It is obvious that the dynamical behavior of the battery states plays an essential role in the performance of open quantum batteries.

3. Dynamics of open quantum batteries in a band-gap environment

We consider a two-level atom embedded in a photonic band-gap environment as an open quantum battery. The classical field is applied to drive the quantum battery. Through the unitary transformation $U_R = \exp(-\mathrm{i}\omega_L\sigma_z t/2)$ in the rotating reference frame, the Hamiltonian of the whole system including the classical driven battery and the bosonic environment is given as,

$$H = \frac{\Delta}{2}\sigma_z + \Omega\sigma_x + \sum_k \omega_k a_k^\dagger a_k + \left(\sum_k g_k a_k \sigma^+ \mathrm{e}^{\mathrm{i}\omega_L t} + \mathrm{H.c.}\right), \tag{6}$$

where g_k describe the coupling between the battery and the reservoir. $a_k^\dagger$ and a_k represent the creation and annihilation operators of the kth mode with frequency ω_k of the reservoir. To obtain the dynamics of the quantum battery, we transform the total Hamiltonian H into the effective one by using the dressed states of $\{|E\rangle, |G\rangle\}$,

$$\begin{aligned} H_{\mathrm{eff}} = {} & \frac{\omega_D}{2}\tilde{\sigma}_z + \sum_k \omega_k a_k^\dagger a_k \\ & + \cos^2\frac{\eta}{2}\sum_k (g_k a_k \tilde{\sigma}^+ \mathrm{e}^{\mathrm{i}\omega_L t} \\ & + g_k a_k \tilde{\sigma}_z \mathrm{e}^{\mathrm{i}\omega_L t} \\ & + g_k a_k \tilde{\sigma}^- \mathrm{e}^{\mathrm{i}\omega_L t} + \mathrm{H.c.}), \end{aligned} \tag{7}$$

where $\omega_D = \sqrt{\Delta^2 + 4\Omega^2}$ is the dressed frequency, The new operator $\tilde{\sigma}_z = |E\rangle\langle E| - |G\rangle\langle G|$ and the new raising operator $\tilde{\sigma}^+ = |E\rangle\langle G|$. The unitary transformation with respect to the dressed states is expressed as

$$\begin{pmatrix} |E\rangle \\ |G\rangle \end{pmatrix} = \tilde{U}\begin{pmatrix} |e\rangle \\ |g\rangle \end{pmatrix} = \begin{pmatrix} \cos\frac{\eta}{2} & \sin\frac{\eta}{2} \\ -\sin\frac{\eta}{2} & \cos\frac{\eta}{2} \end{pmatrix}\begin{pmatrix} |e\rangle \\ |g\rangle \end{pmatrix}, \tag{8}$$

with the parameter $\eta = \arctan(2\Omega/\Delta)$. By transforming H_{eff} into the interaction picture, we obtain

$$\begin{aligned} H_{\mathrm{eff}}^I = {} & \cos^2\frac{\eta}{2}\sum_k [g_k a_k \tilde{\sigma}^+ \mathrm{e}^{\mathrm{i}(\omega_L+\omega_D-\omega_k)t} + g_k a_k \tilde{\sigma}_z \mathrm{e}^{\mathrm{i}(\omega_L-\omega_k)t} \\ & + g_k a_k \tilde{\sigma}^- \mathrm{e}^{\mathrm{i}(\omega_L-\omega_D-\omega_k)t} + \mathrm{H.c.}]. \end{aligned} \tag{9}$$

In general, $|\omega_D + \omega_k| \gg |\omega_k| \gg |\omega_D - \omega_k|$, so the terms $g_k a_k \tilde{\sigma}^+ \mathrm{e}^{\mathrm{i}(\omega_L+\omega_D-\omega_k)t}$, $g_k a_k \tilde{\sigma}_z \mathrm{e}^{\mathrm{i}(\omega_L-\omega_k)t}$ and their complex conjugates are high-frequency oscillating terms relative to $g_k a_k \tilde{\sigma}^- \mathrm{e}^{\mathrm{i}(\omega_L-\omega_D-\omega_k)t}$ and can be neglected. The above equation can be simplified to

$$H_{\mathrm{eff}}^I = \cos^2\frac{\eta}{2}\sum_k [g_k a_k \tilde{\sigma}^+ \mathrm{e}^{\mathrm{i}(\omega_L+\omega_D-\omega_k)t} + \mathrm{H.c.}]. \tag{10}$$

According to the effective Hamiltonian of equation (10), we demonstrate that there is no more than one excitation in the total system when the environment is initially in the vacuum state. The Hilbert space will be restricted to the following subspace spanned in the dressed bases: $|\psi_0\rangle = |G\rangle \otimes |0\rangle_E$, $|\psi_1\rangle = |E\rangle \otimes |0\rangle_E$, $|\psi_k\rangle = |G\rangle \otimes |1_k\rangle_E$, where $|1_k\rangle_E = \prod_j^{\otimes}|0_j\rangle_E|1_{k\neq j}\rangle_E$ represents the state with one excitation in the kth mode of the environment. $|0\rangle_E = \prod_j^{\otimes}|0_j\rangle_E$ denotes the vacuum state. We assume that the initial state of the total system is

$$|\Psi(0)\rangle = C_0|\psi_0\rangle + C_1(0)|\psi_1\rangle. \tag{11}$$

After a charging time interval t, the evolved state is obtained by,

$$|\Psi(t)\rangle = C_0|\psi_0\rangle + C_1(t)|\psi_1\rangle + \sum_k C_k(t)|\psi_k\rangle, \tag{12}$$

where $|C_0|^2 + |C_1(t)|^2 + \sum_k|C_k(t)|^2 = 1$ satisfies the normalization condition.

To derive the form of equation (12), we need to calculate the closed integral differential equation

$$\dot{C}_1(t) = -\cos^4\frac{\eta}{2}\int_0^t \mathrm{d}t_1 f(t-t_1)C_1(t), \tag{13}$$

where the kernel $f(t-t_1) = \int \mathrm{d}\omega J(\omega)\exp[\mathrm{i}(\omega_D + \omega_L - \omega)(t-t_1)]$ is given by a certain two-point correlation function of the reservoir [44].

We use the limitation $\sum_k|g_k|^2 \to \int J(\omega)\mathrm{d}\omega$ where $J(\omega)$ is the spectral density function characterizing the structured environment. According to pseudomode theory [38, 45, 46], it is feasible to consider complex environments as the superposition of several spectral environments with different weights. In regard to a photonic band-gap environment, we consider a spectral density in the form

$$J(\omega) = \frac{\lambda}{2\pi}\left[\frac{W_1\Gamma_1}{(\omega-\omega_c)^2 + \left(\frac{\Gamma_1}{2}\right)^2} - \frac{W_2\Gamma_2}{(\omega-\omega_c)^2 + \left(\frac{\Gamma_2}{2}\right)^2}\right], \tag{14}$$

where λ the overall coupling strength, Γ_1 the bandwidth of the flat background continuum, and Γ_2 the width of the band-gap. Here W_1 and W_2 represent the relative weight of the background and band-gap, respectively. The condition of $W_1 - W_2 = 1$ is satisfied. Both Lorentzians are centered at the same frequency ω_c. Because the density function must be positive we have $W_1\Gamma_1 > W_2\Gamma_2$. In particular, for a perfect band-gap model, $J(\omega_c) = 0$, i.e., $W_1/\Gamma_1 - W_2/\Gamma_2 = 0$. The relative ratio between the negative Lorentzian and positive one $\beta = W_2/W_1$ can be adjusted by tuning the geometrical parameters of the photonic crystal structure. The effect of the Lorentzian with negative weight is to characterize the dip of the spectral density where the coupling between the battery and the environment will be inhibited.

In this case, the correlation function of the band-gap environment can be written as

$$f(t - t_1) = W_1\lambda \exp[-M_1(t - t_1)] - W_2\lambda \exp[-M_2(t - t_1)], \tag{15}$$

where $M_j = \frac{\Gamma_j}{2} + \mathrm{i}(\omega_c - \omega_D - \omega_L)$. We apply the Laplace transform of equation (13) and then obtain $\tilde{C}_1(s) = \frac{C_1(0)}{s + \cos^4\frac{\eta}{2}\tilde{f}(s)}$ where $\tilde{f}(s) = \lambda(\frac{W_1}{s+M_1} - \frac{W_2}{s+M_2})$ denotes the Laplace transform of the kernel. By using the inverse Laplace transform, we derive that $C_1(t) = C_1(0)\xi(t)$, where $\xi(t) = \sum_{j=1}^{3} K_j \mathrm{e}^{s_j t}$ is determined by the three roots $\{s_j\}$ of the equation $D(s) = sR(s) + \lambda\cos^4\frac{\eta}{2}W_1(s + M_2) - \lambda\cos^4\frac{\eta}{2}W_2(s + M_1) = 0$. Here $R(s) = \prod_{j=1}^{2}(s + M_j)$, $K_j = R(s_j)/D'(s_j)$ and $D'(s) = \partial_s D(s)$ is the first order derivative.

Without loss of generality, the initial state of the total is assumed to be

$$|\Psi(0)\rangle = (\cos\frac{\theta}{2}|g\rangle + \mathrm{e}^{\mathrm{i}\phi}\sin\frac{\theta}{2}|e\rangle) \otimes |0\rangle_E. \tag{16}$$

According to equation (16), the evolved state of the total system can be given by

$$\begin{aligned}|\Psi(t)\rangle = &\left[c_0\cos\tfrac{\eta}{2} + c_1\xi(t)\sin\tfrac{\eta}{2}\right]|g\rangle\ |0\rangle_E \\ &+ \left[-c_0\sin\tfrac{\eta}{2} + c_1\xi(t)\sin\tfrac{\eta}{2}\right]|e\rangle\ |0\rangle_E \\ &+ \left[d(t)\cos\tfrac{\eta}{2}|g\rangle - d(t)\sin\tfrac{\eta}{2}|e\rangle\right]|1_k\rangle_E,\end{aligned} \tag{17}$$

where $c_0 = \cos\frac{\theta}{2}\cos\frac{\eta}{2} - \mathrm{e}^{\mathrm{i}\phi}\sin\frac{\theta}{2}\sin\frac{\eta}{2}$, $c_1 = \cos\frac{\theta}{2}\sin\frac{\eta}{2} - \mathrm{e}^{\mathrm{i}\phi}\sin\frac{\theta}{2}\cos\frac{\eta}{2}$, $d(t) = \sqrt{1 - |c_1\xi(t)|^2 - |c_0|^2}$. The elements of the density matrix of the quantum battery can be expressed as

$$\begin{aligned}\varrho_{gg}(t) = &\cos^2\tfrac{\eta}{2} - |c_1\xi(t)|^2\cos\eta \\ &+ \tfrac{1}{2}[c_0c_1^*\xi^*(t) + c_0^*c_1\xi(t)]\sin\eta, \\ \varrho_{eg}(t) = &\left[-\tfrac{1}{2} + |c_1\xi(t)|^2\right]\sin\eta \\ &- c_0c_1^*\xi^*(t)\sin^2\tfrac{\eta}{2} + c_0^*c_1\xi(t)\cos^2\tfrac{\eta}{2}.\end{aligned} \tag{18}$$

To study the stabilization performance of the open quantum battery, we need to evaluate the steady-state $\rho_B(\infty)$ in the charging or self-discharging process. When the quantum battery is charged via the classical driving field, three roots of the equation $D(s) = 0$ satisfy the relation of $s_j < 0$. Under the condition of $t \to \infty$, $\xi(\infty) = 0$ and the charging steady state is described as

$$\rho_B^c(\infty) = \begin{pmatrix} \sin^2\frac{\eta}{2} & -\sin\frac{\eta}{2}\cos\frac{\eta}{2} \\ -\sin\frac{\eta}{2}\cos\frac{\eta}{2} & \cos^2\frac{\eta}{2}\end{pmatrix}. \tag{19}$$

It is found that the charging steady state is just related to the classical field and independent of the initial states and environment.

If the quantum battery undergoes the self-discharging process where $\Omega = 0$ and $\eta = 0$, three roots of equation $D(s) = 0$ satisfy the relation of $s_1 = 0$ and $s_{j\neq 1} < 0$. Under the condition of $t \to \infty$, $\xi(\infty) = K_1 = \frac{M_1M_2}{s_2s_3}$ and the discharging steady state is written as

$$\rho_B^d(\infty) = \begin{pmatrix} |c_1K_1|^2 & c_0^*c_1K_1 \\ c_0c_1^*K_1^* & 1 - |c_1K_1|^2\end{pmatrix}. \tag{20}$$

It is shown that the self-discharging state can be stabilized and determined by the initial condition and the spectral density of the environment.

4. Stabilization mechanism of open quantum batteries

The recent work has demonstrated that the bound state between the quantum battery and the environment is the fundamental reason for suppressing the self-discharging process in the structured environment [47]. Moreover, the non-Markovian memory effects from the environment can contribute to the energy storage in open quantum batteries in the previous research [36]. Therefore, the energy transfer is dependent not only on the existence of the system-reservoir bound state but also on the non-Markovianity of the environment. These enlightening results have stimulated us to study how the interplay of both factors can have an effect on the stabilization performance of quantum batteries in the charging or self-discharging process.

4.1. Charging stabilization performance

According to the above analysis of quantum dynamics, the stabilized ergotropy for the charging steady state can be given by

$$W_B^c(\infty) = \omega_0\sin^2(\frac{\eta}{2}). \tag{21}$$

It is seen that the stabilized ergotropy in the charging process has no connection with the initial states and the environment. With the classical driving field, the stabilized ergotropy gradually rises to an optimal value in figure 1(a). The optimal value of $W_B^c(\infty) = 0.5\omega_0$ can be achieved in the resonant condition of $\omega_L = \omega_0$ or $\eta = \pi/2$. The physical reason that the

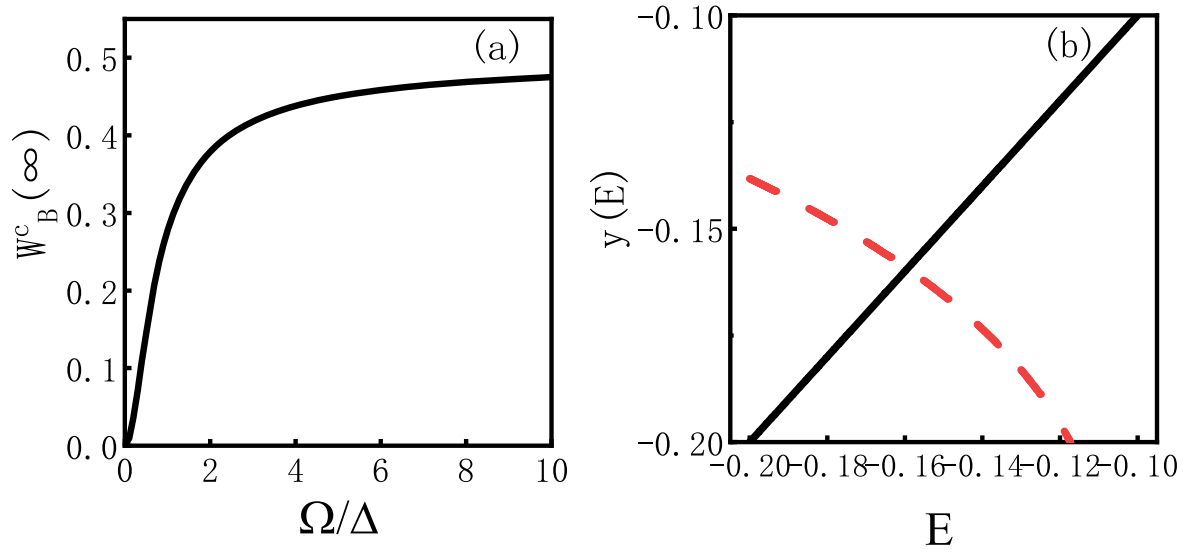

Figure 1. (a) The stabilized scaled ergotropy $W_B^c(\infty)/\omega_0$ in the charging process as a function of the ratio of Rabi frequency to detuning frequency. (b) Diagrammatic solutions of equation (22) in the charging process where $\Omega = 0.1\omega_0$, $\lambda = \omega_0$, $\Gamma_1 = 0.4\omega_0$ and $\beta = 0.6$. The black solid represents the case of $y(E) = E$ and the red dashed line denotes the condition of $\Gamma_1 = 0.4\omega_0$.

ergotropy during the charging period can be stabilized is the formation of the bound state between the battery and environment. To check the spectrum of the total Hamiltonian like equation (6), we first solve the eigenvalue equation $H|\varphi_E\rangle = E|\varphi_E\rangle$ with $|\varphi_E\rangle = c_0^E|\psi_1\rangle + \sum_k c_k^E(t)|\psi_k\rangle$. Substituting H_{eff} into the eigenvalue equation, we can obtain a transcendental equation as,

$$y(E) = \frac{\omega_D}{2} - \cos^4\left(\frac{\eta}{2}\right)\int_0^\infty \frac{J(\omega)}{\omega - \left(E + \frac{\omega_D}{2}\right)}\mathrm{d}\omega = E, \tag{22}$$

where $\omega_D = \sqrt{(\omega_0 - \omega_L)^2 + 4\Omega^2}$ is related to the frequency ω_L of the driving field. A system-environment bound state is an eigenstate with real eigenvalue in the total Hamiltonian of the whole system. When equation (22) has a real resolution for $E < -\omega_D/2$, it is proved that the whole system possesses a bound state [48, 49]. Since $y(E)$ decreases monotonically with the increase of E and $\lim_{E\to-\infty} y(E) = \frac{\omega_D}{2}$, the condition for equation (22) to have at least one root is $E < -\omega_D/2$. In the band-gap environment with the complex spectral density like equation (14), the fact of $\lim_{E\to\left(-\frac{\omega_D}{2}\right)} y(E) = -\infty$ demonstrates that $y(E)$ always satisfies the condition of $y(-\frac{\omega_D}{2}) < -\frac{\omega_D}{2}$. In the regime of $E > -\omega_D/2$, we can see that $y(E)$ is divergent, which means that no real root E can make equation (22) well defined. The point intersection of the plots of $y(E)$ and E is clearly illustrated in figure 1(b). This result means that there exists the battery-environment bound state during the charging period via the classical driving field. As the stationary state of the entire system, the excited state population of the bound state remains unchanged over time, which means that the formation of the bound state can suppress decoherence.

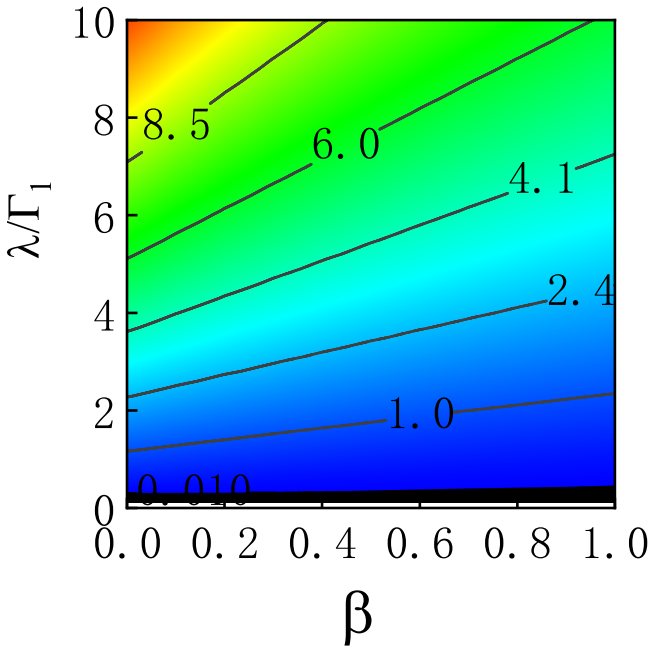

Figure 2. The contour of the non-Markovianity as a function of the coupling strength λ/Γ_1 and the relative ratio between the negative Lorentzian and positive one β.

However, the charging behavior of the ergotropy prior to the stabilization is related to the memory effect of the environment. To quantify the memory effect, we use the non-Markovianity [4, 5],

$$\mathcal{N} = \max_{\rho_1(0),\rho_2(0)} \int_{\sigma>0} \sigma[t, \rho_1(0), \rho_2(0)]\mathrm{d}t, \tag{23}$$

where $\sigma[t, \rho_1(0), \rho_2(0)] = \mathrm{d}D[\rho_1(t), \rho_2(t)]/\mathrm{d}t$ is the rate of change of the trace distance defined by $D[\rho_1(t), \rho_2(t)] = \mathrm{Tr}|\rho_1(t) - \rho_2(t)|/2$. Here the norm of

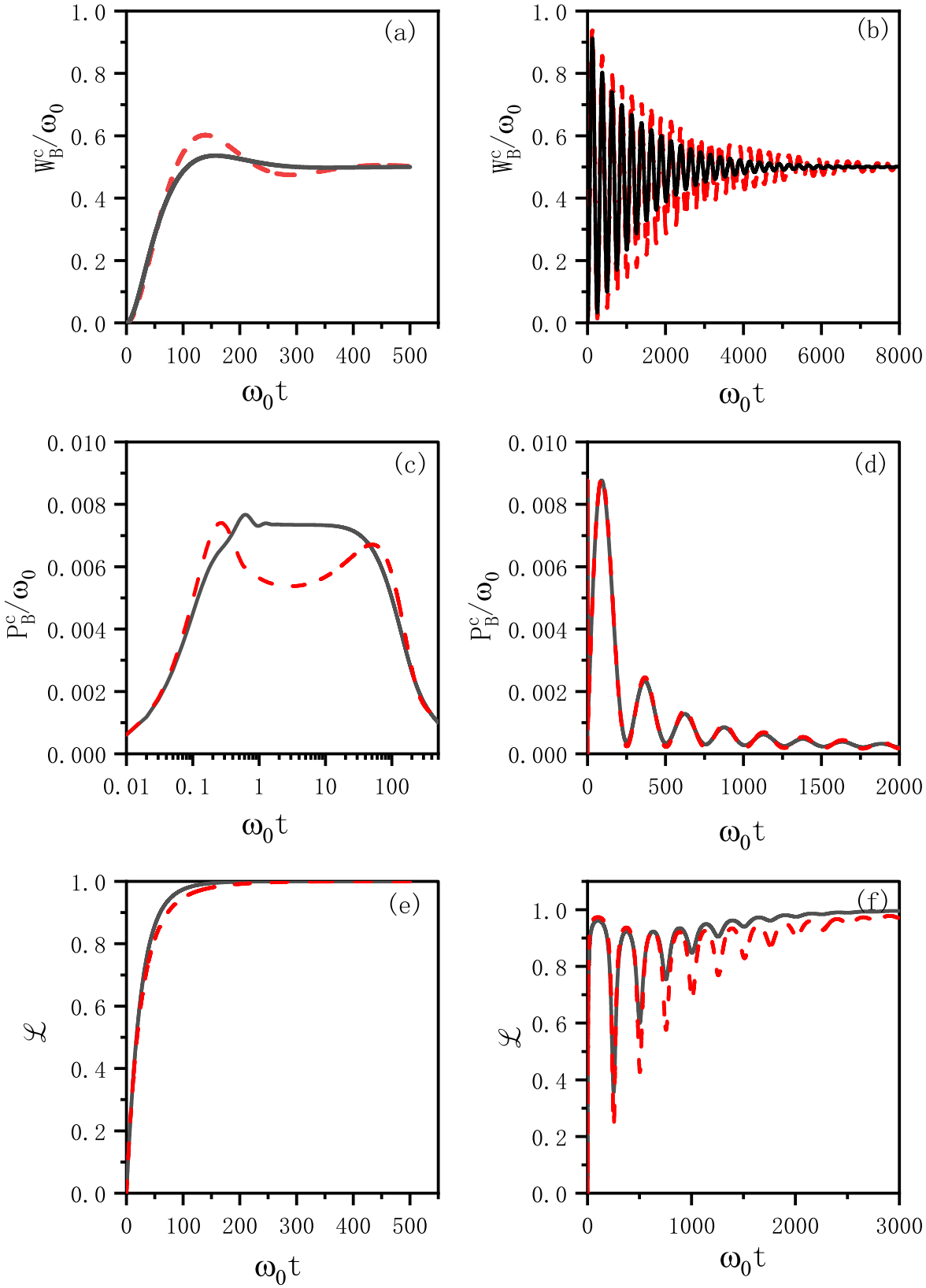

Figure 3. The scaled ergotropy W_B^c/ω_0, average power P_B/ω_0 and efficiency $\mathcal{L}$ in the charging process as a function of $\omega_0 t$: (a,c,e) in the weak coupling regime where $\lambda = \omega_0$ and $\Gamma_1 = 10\omega_0$ (b,d,f) in the strong coupling regime where $\lambda = \omega_0$ and $\Gamma_1 = 0.4\omega_0$. The black solid curves correspond to the case of $\beta = 0.6$ and red dotted curves correspond to the case of $\beta = 0$. The initial state of the battery is prepared at the ground state.

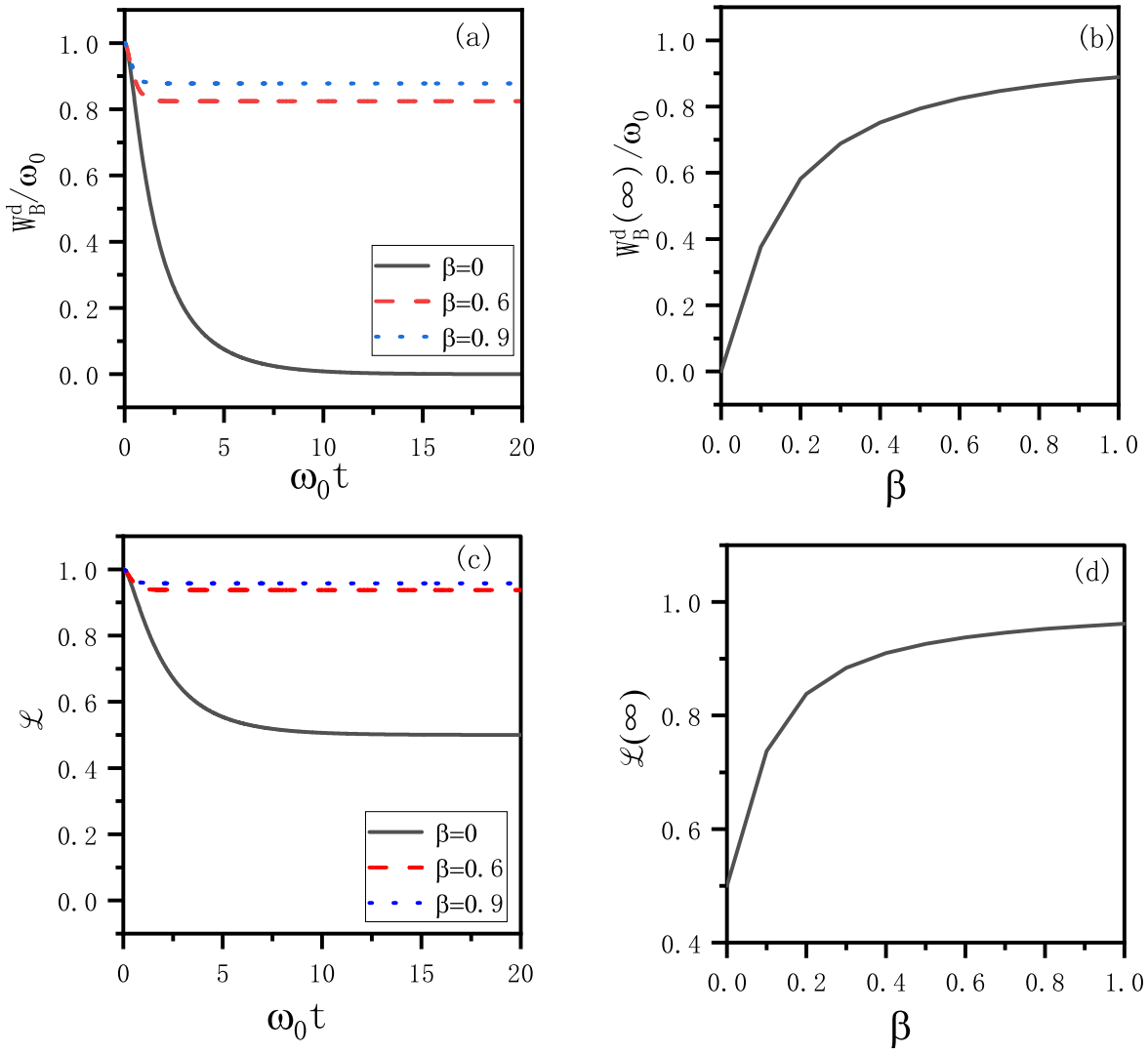

Figure 4. (a) The scaled ergotropy W_B^d/ω_0 and (c) the efficiency $\mathcal{L}$ in the self-discharging process as a function of $\omega_0 t$ with different β in the weak coupling regime with $\lambda = \omega_0$ and $\Gamma_1 = 10\omega_0$. The black solid line denotes the case of $\beta = 0$, the red dashed line represents the case of $\beta = 0.6$, and the blue dotted line corresponds to the case of $\beta = 0.9$. (b) The stabilized scaled ergotropy $W_B^c(\infty)/\omega_0$ and (d) the stabilized efficiency $\mathcal{L}(\infty)$ in the self-charging process as a function of β. The initial state of the battery is prepared at the coherent state of $\frac{1}{\sqrt{2}}(|e\rangle + |g\rangle)$.

$|A| = \sqrt{A^\dagger A}$ is given. The condition of $\sigma[t, \rho_1(0), \rho_2(0)] \leqslant 0$ represents the time-dependent Markovian evolution. In regard to a pair of initial states, the evidence of $\sigma[t, \rho_1(0), \rho_2(0)] > 0$ demonstrates that the dynamical process is non-Markovian and has memory effects. To maximize the non-Markovianity, we can choose the optimal pair of initial states such as $\rho_{1,2}(0) = |\pm\rangle\langle\pm|$ with $|\pm\rangle = (|e\rangle + |g\rangle)/\sqrt{2}$. Then, we plot the variation of the non-Markovianity with the parameters of λ/Γ_1 and β in figure 2. We find that there is no memory effect in the weak coupling case of $\lambda \ll \Gamma_1$. It is also shown that a remarkable non-Markovian evolution occurs in the strong coupling regime of $\lambda > \Gamma_1$. The increase of the negative weight, i.e., the large values of β, can suppress the memory effect. The maximum of the non-Markovianity corresponds to the limiting condition of $\beta \to 0$ in the strong coupling case.

In the weak coupling regime of $\lambda < \Gamma_1$, the dynamical behavior of the ergotropy takes on a gradual increase and then rapidly reaches a stable value, which is shown in figure 3(a). The improvement in the extractable work results from the constant driving by the classical field. The bound-state energy for $\beta = 0.6$ is $-0.1593\omega_0$ while the bound-state energy for $\beta = 0$ is $-0.1005\omega_0$ in the charging process under a Markovian environment. Under the condition of the large bound-state energy, the maximum of the ergotropy is large. Meanwhile, the dissipation from the Markovian environment also impedes energy storage. In contrast to the nearly monotonic increase, the rapid oscillations of the ergotropy are plotted in figure 3(b) in the strong coupling case. The non-Markovian memory effect leads to the continuous revival of the ergotropy. In figures 3(c)–(f), we plot the evolution of the charging power P_B and efficiency $\mathcal{L}$ during the charging process under the corresponding parameters. The results indicate that the maximum charging power $P_{\max}$ of the quantum battery increases with the increase of β under weak coupling

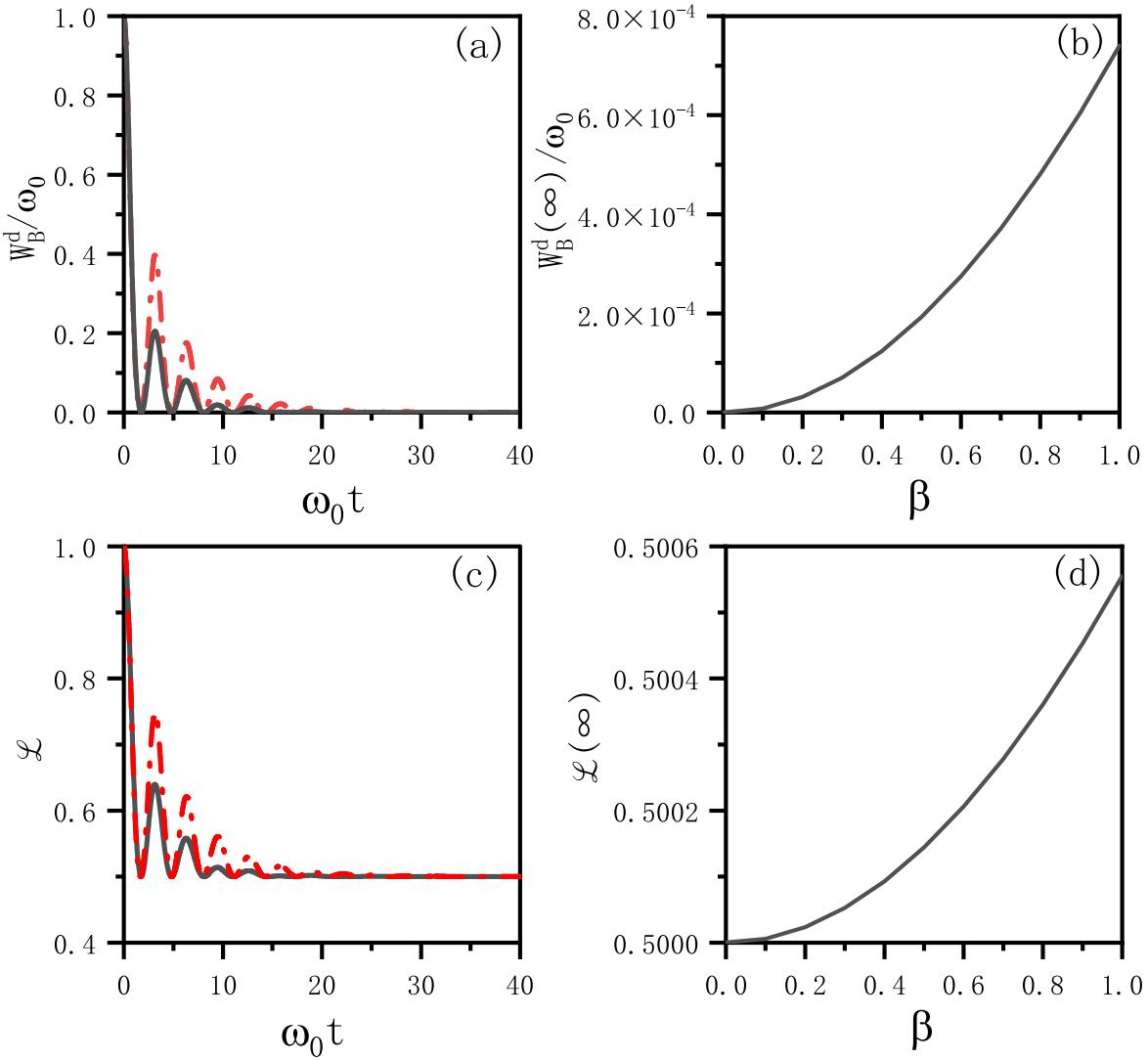

Figure 5. (a) The scaled ergotropy W_B^d/ω_0 and (c) the efficiency $\mathcal{L}$ in the self-discharging process as a function of $\omega_0 t$ with different β in the strong coupling case with $\lambda = \omega_0$ and $\Gamma_1 = 0.4\omega_0$. The black solid line denotes the case of $\beta = 0.6$ and the red dashed line represents the case of $\beta = 0$. (b) The stabilized scaled ergotropy $W_B^c(\infty)/\omega_0$ and (d) the stabilized efficiency $\mathcal{L}(\infty)$ in the self-charging process as a function of β. The initial state of the battery is prepared at the coherent state of $\frac{1}{\sqrt{2}}(|e\rangle + |g\rangle)$.

conditions. In strong coupling conditions, the increase of β has a weak impact on the maximum charging power. Furthermore, under both coupling regimes, the increase of β positively influences the efficiency of the quantum battery. The enhanced stabilization performance can result from two factors: larger β and smaller non-Markovianity can stabilize energy.

4.2. Self-discharging stabilization performance

How to prevent spontaneous discharging and make quantum batteries capable of maintaining so much ergotropy for a long time has been paid attention to in recent years. In this subsection, we focus on the stabilization mechanism of open quantum batteries in the band-gap environment during the self-discharging period. We expect to find a feasible approach to enhancing the self-discharging stabilization performance. In accordance with the steady state described by equation (20), the stabilized ergotropy is dependent on the quantum effects of the environment.

On one hand, the time-dependent behavior of the ergotropy and efficiency can be illustrated in figures 4(a) and (c) when the couplings between the battery and environment are weak. In this case, the initial state of the battery is chosen to be a coherent state of $\frac{1}{\sqrt{2}}(|e\rangle + |g\rangle)$ with the initial ergotropy of $W_0 = 0.5\omega_0$. It is seen that the ergotropy and efficiency quickly decline to a stable value which corresponds to the steady state. The higher the negative Lorentzian weight is, the faster the self-discharging ergotropy and efficiency stabilize. With the increase of the parameter β, the stable values of the ergotropy and efficiency can upgrade to high ones, which is shown in figures 4(b) and (d). According to the pseudomode theory [38], the high value of the negative weight leads to a remarkable inhibiting effect of the decay of quantum batteries in the weak coupling regime. The reason that the self-

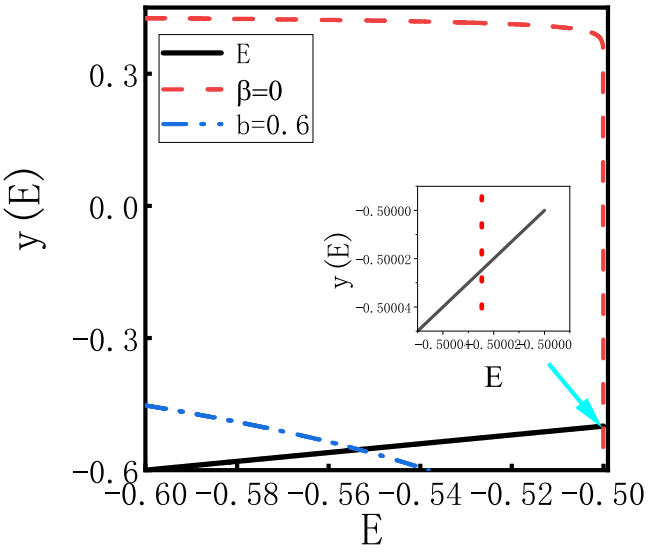

Figure 6. Diagrammatic solutions of equation (22) in the self-discharging process which correspond to $\Gamma_1 = 10\omega_0$(red dashed line) and $\Gamma_1 = 0.4\omega_0$ (blue dash dotted line) when the negative weight is $\beta = 0.6$.

discharging battery can bed stabilized is the formation of the bound state between the battery and environment.

On the other hand, the self-discharging ergotropy and efficiency in the strong coupling regime show the oscillation attenuation with time in figures 5(a) and (c). The non-Markovianity of the environment gives rise to the revivals of the extractable work during the self-charging period. However, the significant memory effect with the large non-Markovianity is considered as one obstacle to stabilizing quantum batteries. Owing to the strong coupling between the battery and environment, the notable energy leakage leads to small values of stabilized ergotropy and efficiency. The stable values of self-discharging ergotropy and efficiency are improved by increasing the negative Lorentzian weight in figures 5(b) and (d), which is similar to the result in the weak coupling regime. The non-Markovianity of the environment can be suppressed by the parameter β.

In a word, the stabilization performance of open quantum batteries is mainly determined by the formation of the bound state between the battery and environment. The stored energy for the steady state is also modulated by the non-Markovianity of the environment. The condition of the existence of the bound state in the self-discharging process is demonstrated in figure 6. It is found that the bound-state energy in the strong coupling regime is lower than that in the weak coupling regime. That is, the faster formation of the bound state is achieved when the quantum battery decays in the weak coupling condition. It is of great value to form a bound state of the entire system, which can inhibit the decoherence of the system to combat the energy loss of the battery. The weak coupling can form the bound state faster in the process of self-discharging, and the weak coupling can effectively inhibit self-discharging. Therefore, the formation speed of the bound state is related to energy storage. The lower bound state energy corresponds to a faster formation rate. Correspondingly, the more energy can be stored in the battery.

5. Discussion

In conclusion, we have investigated the stabilization mechanism of the classical field driven quantum battery in a non-Markovian environment. As an example, the photonic band-gap environment is considered due to the feasible manipulation of the spectral density which is composed of one positive and one negative Lorentzian part. When the quantum battery is charged, the stabilized ergotropy is only dependent on the classical field. In the resonant condition of $\omega_L = \omega_0$, the stable value of the charging ergotropy is optimized to be one-half of the maximal internal energy. The revivals of the charging ergotropy occur due to the non-Markovianity of the environment. During the self-discharging period, the quantum battery is rapidly stabilized to a steady state with a high value of the ergotropy in the weak coupling regime. Similarly, the oscillation declination happens in the strong coupling regime. The ergotropy with respect to the steady states can be adjusted by the negative weight. Such memory effects are remarkable in the strong coupling regime with small values of the negative weight. We derive and numerically demonstrate the conditions of the existence of the bound state between the battery and environment. It is found that the weak coupling condition and large values of the negative weight can contribute to the fast formation of the bound state. The weak battery-environment couplings can suppress the non-Markovianity of the environment and the increase of the negative weight can inhibit the dissipation of the quantum battery. Therefore, the stabilization performance of open quantum batteries is related not only to the non-Markovianity of the environment, but also to the formation of the battery-environment bound state. These results are of both theoretical and experimental interest in exploring the potential of stabilizing open quantum batteries to attain long and steady stored energy under the influence of the environment.

Acknowledgments

We would like to thank Professor Bill Unruh and Professor Philip C. E. Stamp for the discussions on the related work. This work is supported by Postgraduate Research and Practice Innovation Program of Jiangsu Province (Grant No. KYCX24_3424) and funded by SCOAP[3].

References

[1] Baumgratz T, Cramer M and Plenio M B 2014 Quantifying coherence *Phys. Rev. Lett.* **113** 140401
[2] Streltsov A, Adesso G and Plenio M B 2017 Colloquium: quantum coherence as a resource *Rev. Mod. Phys.* **89** 041003

[3] Modi K *et al* 2012 The classical-quantum boundary for correlations: discord and related measures *Rev. Mod. Phys.* **84** 1655

[4] Breuer H P, Laine E M and Piilo J 2009 Measure for the degree of non-Markovian behavior of quantum processes in open systems *Phys. Rev. Lett.* **103** 210401

[5] Laine E M, Piilo J and Breuer H P 2010 Measure for the non-Markovianity of quantum processes *Phys. Rev.* A **81** 062115

[6] Nielsen M A and Chuang I L 2010 *Quantum Computation and Quantum Information* (Cambridge: Cambridge University Press)

[7] Carvalho A R R *et al* 2001 Decoherence, pointer engineering and quantum state protection *Modern Challenges in Quantum Optics: Selected Papers of the First International Meeting in Quantum Optics Held in Santiago* (Berlin: Springer) pp 65–79

[8] Jimnéz Farìas O *et al* 2009 Determining the dynamics of entanglement *Science* **324** 1414–7

[9] Alicki R and Fannes M 2013 Entanglement boost for extractable work from ensembles of quantum batteries *Phys. Rev.* E **87** 042123

[10] Le T P *et al* 2018 Spin-chain model of a many-body quantum battery *Phys. Rev.* A **97** 022106

[11] Hovhannisyan K V *et al* 2013 Entanglement generation is not necessary for optimal work extraction *Phys. Rev. Lett.* **111** 240401

[12] Deffner S and Campbell S 2017 Quantum speed limits: from Heisenbergs uncertainty principle to optimal quantum control *J. Phys. A: Math. Theor.* **50** 453001

[13] Boixo S *et al* 2019 Quantum supremacy using a programmable superconducting processor *Nature* **574** 505–10

[14] Bera M N *et al* 2017 Generalized laws of thermodynamics in the presence of correlations *Nat. Commun.* **8** 2180

[15] Karimi B and Pekola J P 2016 Otto refrigerator based on a superconducting qubit: classical and quantum performance *Phys. Rev.* B **94** 184503

[16] Campisi M and Fazio R 2016 Dissipation, correlation and lags in heat engines *J. Phys. A: Math. Theor.* **49** 345002

[17] Campaioli F *et al* 2017 Enhancing the charging power of quantum batteries *Phys. Rev. Lett.* **118** 150601

[18] Henao I and Serra R M 2018 Role of quantum coherence in the thermodynamics of energy transfer *Phys. Rev.* E **97** 062105

[19] Zhang X L, Song X K and Wang D 2024 Quantum battery in the Heisenberg spin chain models with Dzyaloshinskii–Moriya interaction *Adv. Quant. Tech.* **7** 2400114

[20] Song M L *et al* 2022 Environment-mediated entropic uncertainty in charging quantum batteries *Phys. Rev.* E **106** 054107

[21] Song M L *et al* 2024 Evaluating extractable work of quantum batteries via entropic uncertainty relations *Phys. Rev.* E **109** 064103

[22] Ferraro D *et al* 2018 High-power collective charging of a solid-state quantum battery *Phys. Rev. Lett.* **120** 117702

[23] Zhang Y Y *et al* 2019 Powerful harmonic charging in a quantum battery *Phys. Rev.* E **99** 052106

[24] Santos A C *et al* 2019 Stable adiabatic quantum batteries *Phys. Rev.* E **100** 032107

[25] Santos A C, Saguia A and Sarandy M S 2020 Stable and charge-switchable quantum batteries *Phys. Rev.* E **101** 062114

[26] Rossini D, Andolina G M and Polini M 2019 Many-body localized quantum batteries *Phys. Rev.* B **100** 115142

[27] Xu K *et al* 2023 Enhanced charging in common environments compared to independent environments for two level systems *Annal. Phys.* **535** 2200446

[28] Barra F 2019 Dissipative charging of a quantum battery *Phys. Rev. Lett.* **122** 210601

[29] Bai S Y and An J H 2020 Floquet engineering to reactivate a dissipative quantum battery *Phys. Rev.* A **102** 060201

[30] Zhao F, Dou F Q and Zhao Q 2021 Quantum battery of interacting spins with environmental noise *Phys. Rev.* A **103** 033715

[31] Farina D *et al* 2019 Charger-mediated energy transfer for quantum batteries: an open-system approach *Phys. Rev.* B **99** 035421

[32] Latune C L, Sinayskiy I and Petruccione F 2019 Energetic and entropic effects of bath-induced coherences *Phys. Rev.* A **99** 052105

[33] Mitchison M T, Goold J and Prior J 2021 Charging a quantum battery with linear feedback control *Quantum* **5** 500

[34] Seah S *et al* 2021 Quantum speed-up in collisional battery charging *Phys. Rev. Lett.* **127** 100601

[35] Rosa D *et al* 2020 Ultra-stable charging of fast-scrambling SYK quantum batteries *J. High Energy Phys.* **2020** 1–29

[36] Kamin F H *et al* 2020 Non-Markovian effects on charging and self-discharging process of quantum batteries *New J. Phys.* **22** 083007

[37] Pleasance G and Garraway B M 2017 Application of quantum Darwinism to a structured environment *Phys. Rev.* A **96** 062105

[38] Garraway B M 1997 Nonperturbative decay of an atomic system in a cavity *Phys. Rev.* A **55** 2290

[39] Goban A *et al* 2014 Atom-light interactions in photonic crystals *Nat. Commun.* **5** 3808

[40] Pusz W and Woronowicz S L 1978 Passive states and KMS states for general quantum systems *Commun. Math. Phys.* **58** 273–90

[41] Allahverdyan A E, Balian R and Nieuwenhuizen T M 2004 Maximal work extraction from finite quantum systems *Europhys. Lett.* **67** 565

[42] Perarnau-Llobet M *et al* 2015 Most energetic passive states *Phys. Rev.* E **92** 042147

[43] Hao X *et al* 2023 Quantum work extraction of an accelerated Unruh–DeWitt battery in relativistic motion *Phys. Rev.* A **107** 012207

[44] Breuer H P and Petruccione F 2002 *The Theory of Open Quantum Systems* (Oxford: Oxford University Press)

[45] Pleasance G, Garraway B M and Petruccione F 2020 Generalized theory of pseudomodes for exact descriptions of non-Markovian quantum processes *Phys. Rev. Res.* **2** 043058

[46] Xu K *et al* 2022 Charging performance of quantum batteries in a double-layer environment *Phys. Rev.* A **106** 012425

[47] Xu K, Li H G, Zhu H J and Liu W M 2024 Inhibiting the self-discharging process of quantum batteries in non-Markovian noises *Phys. Rev.* E **109** 054132

[48] Tong Q J *et al* 2010 Mechanism of entanglement preservation *Phys. Rev.* A **81** 052330

[49] Liu H B *et al* 2013 Anomalous decoherence in a dissipative two-level system *Phys. Rev.* A **87** 052139