Commun. Theor. Phys. (Beijing, China) **47** (2007) pp. 270–274
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Vol. 47, No. 2, February 15, 2007

Symbolic Computation and q-Deformed Function Solutions of (2+1)-Dimensional Breaking Soliton Equation*

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(Received March 1, 2006; Revised May 18, 2006)

Abstract *In this paper, by using symbolic and algebra computation, Chen and Wang's multiple Riccati equations rational expansion method was further extended. Many double soliton-like and other novel combined forms of exact solutions of the (2+1)-dimensional Breaking soliton equation are derived by using the extended multiple Riccati equations expansion method.*

PACS numbers: 02.30.Ik, 05.45.Yv
Key words: q-deformed hyperbolic functions, symbolic computation, Riccati equation, soliton-like solution

1 Introduction

The appearance of solitary wave solutions of nonlinear partial differential equations (NPDEs) in nature is quite common. In the past several decades, both mathematicians and physicists have devoted considerable effort to the study of solitons and related issue of the construction of solutions to NPDEs. It is known that there exist many kinds of powerful methods to obtain the solutions of NPDEs.[1−10]

With the rapid development of computer science, many symbolic computation softwares appear. So computer algebra method[9−15] also becomes a powerful method to solve NPDEs. More recently, Chen and Wang[13,14] presented multiple Riccati equations rational expansion method and obtained many new exact solutions of integrable soliton equations. The aim of this research is to further improve the multiple Riccati equations method. The paper is organized as follows. In Sec. 2, some solutions of Riccati equation are listed in the form of q-deformed hyperbolic and triangular functions. Subsequently, the extended multiple Riccati equations algorithm is described. The general idea of Sec. 2 is applied to the (2+1)-dimensional breaking soliton equation in Sec. 3 and many new double soliton-like wave solutions and other novel exact solutions are successfully obtained. Finally, some conclusions are given in Sec. 4.

2 Further Extended Multiple Riccati Equations Expansion Method

In this section, we first review some knowledge about the q-deformed hyperbolic functions, which were first introduced by Arai,[16]

$$\begin{aligned}
&\sinh_q(x)=\frac{\mathrm{e}^{x}-q\,\mathrm{e}^{-x}}{2}\,,\quad \cosh_q(x)=\frac{\mathrm{e}^{x}+q\,\mathrm{e}^{-x}}{2}\,,\\
&\tanh_q(x)=\frac{\sinh_q(x)}{\cosh_q(x)}\,,\quad \coth_q(x)=\frac{\cosh_q(x)}{\sinh_q(x)}\,,\\
&\mathrm{sech}_q(x)=\frac{1}{\cosh_q(x)}\,,\quad \mathrm{csch}_q(x)=\frac{1}{\sinh_q(x)}\,,\\
&x\in C\,.
\end{aligned}\tag{1}$$

It is straightforward to see that the following formulas hold,

$$\begin{aligned}
&(\sinh_q(x))'=\cosh_q(x)\,,\quad (\cosh_q(x))'=\sinh_q(x)\,,\\
&\cosh_q^2(x)-\sinh_q^2(x)=q\,,\quad (\tanh_q(x))'=q\,\mathrm{sech}_q^2(x)\,,\\
&(\mathrm{sech}_q(x))'=-\tanh_q(x)\,\mathrm{sech}_q(x)\,,\\
&\tanh_q^2(x)=1-q\,\mathrm{sech}_q^2(x)\,.
\end{aligned}$$

The relationships between q-deformed hyperbolic functions and the classical hyperbolic functions are as follows:

$$\sinh_q(x)=\frac{1+q}{2}\sinh(x)+\frac{1-q}{2}\cosh(x)\,,\tag{2}$$

$$\cosh_q(x)=\frac{1-q}{2}\sinh(x)+\frac{1+q}{2}\cosh(x)\,,\tag{3}$$

$$\tanh_q(x)=\frac{(1+q)\sinh(x)+(1-q)\cosh(x)}{(1-q)\sinh(x)+(1+q)\cosh(x)}\,.\tag{4}$$

We know that

$$\sin(x)=\frac{\mathrm{e}^{\mathrm{i}x}-\mathrm{e}^{-\mathrm{i}x}}{2\mathrm{i}}\,,\quad \cos(x)=\frac{\mathrm{e}^{\mathrm{i}x}+\mathrm{e}^{-\mathrm{i}x}}{2}\,.\tag{5}$$

Correspondingly, we can define q-deformed triangular functions as follows:

$$\sin_q(x)=\frac{\mathrm{e}^{\mathrm{i}x}-q\,\mathrm{e}^{-\mathrm{i}x}}{2\mathrm{i}}\,,\quad \cos_q(x)=\frac{\mathrm{e}^{\mathrm{i}x}+q\,\mathrm{e}^{-\mathrm{i}x}}{2}\,,\tag{6}$$

$$\tan_q(x)=\frac{\sin_q(x)}{\cos_q(x)}\,,\quad \sec_q(x)=\frac{1}{\cos_q(x)}\,.\tag{7}$$

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*The project partially supported by National Natural Science Foundation of China under Grant No. 10471143 and the State 973 Project under Grant No. 2004CB318001

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They satisfy the following formulas:

$$(\sin_q(x))' = \cos_q(x)\,, \quad (\cos_q(x))' = -\sin_q(x)\,,$$

$$(\tan_q(x))' = q\sec_q^2(x)\,, \tag{8}$$

$$\sin_q^2(x) + \cos_q^2(x) = q\,, \quad 1 + \tan_q^2(x) = q\sec_q^2(x)\,. \tag{9}$$

The following formulas are the relationships between q-deformed triangular functions and the classical triangular functions:

$$\sin_q(x) = -\frac{\mathrm{i}(1-q)}{2}\cos(x) + \frac{(1+q)}{2}\sin(x)\,, \tag{10}$$

$$\cos_q(x) = \frac{(1+q)}{2}\cos(x) + \frac{\mathrm{i}(1-q)}{2}\sin(x)\,. \tag{11}$$

We know that some q-deformed hyperbolic function solutions of several NPDEs have been constructed by Li and Liu *et al.* with projective Riccati equations method and $\sec_q$-$\tanh_q$-method in Refs. [9] and [15], respectively. In the following, we will derive the q-deformed hyperbolic and triangular function solutions of Riccati equation,[13,14,17,18]

$$\phi' = \frac{\mathrm{d}}{\mathrm{d}x}\phi = q + p\phi^2\,, \quad \phi = \phi(x)\,, \tag{12}$$

to construct new exact solutions of a type of NPDEs. Riccati equation (12) is a very important auxiliary equation to construct exact solutions of NPDEs. To find the solutions of this equation is a significant and important job. Authors[13,14,17,18] have analyzed and given the solutions of Eq. (12). In the following part, we will give all the real q-deformed hyperbolic and triangular function solutions and a complex solution of it. Without loss of generality, here we only consider the case of $q > 0$.

It is known that equation (12) can be changed into a linear ordinary differential equation by a special reversible transformation. Solving this linear ordinary differential equation, the following five solutions of Eq. (12) are given. They are all the real solutions of Eq. (12).

(i) When $pq = -1$,

$$\phi_1 = q\tanh_q(x)\,, \tag{13}$$

$$\phi_2 = q\coth_q(x)\,; \tag{14}$$

(ii) When $pq = 1$,

$$\phi_1 = q\tan_q(x)\,, \tag{15}$$

$$\phi_2 = q\cot_q(x)\,; \tag{16}$$

(iii) When $q = 0$,

$$\phi_5 = -\frac{1}{px}\,. \tag{17}$$

The following solution of Eq. (12) is a complex solution and it is very helpful to constructing explicitly exact solutions of complex NPDEs.

(iv) When $pq = -1/4$,

$$\phi_6 = 2\,q[\tanh_q(x) + \mathrm{i}\,\sqrt{q}\,\mathrm{sech}_q(x)]\,. \tag{18}$$

In the following, we would like to outline the main steps of extended multiple Riccati equations expansion algorithm.

Let us consider a given NPDEs with three independent variables x, y, t, and a dependent variable $u = u(x,y,t)$:

$$F(u_t, u_x, u_y, u_{xt}, u_{yt}, u_{tt}, u_{xy}, \ldots) = 0\,. \tag{19}$$

Step 1 Firstly, we assume that equation (19) has the following form of solution:

$$u(x,y,t) = A_0 + \sum_{k=1}^{n}\sum_{i+j=k} A_i^j \phi^i(\xi)\psi^j(\eta)\,, \tag{20}$$

where ξ, η, A_0, A_i^j, $(i,j = 1,2,\ldots,n)$ are the functions of (x,y,t) to be determined, and n is an undetermined integer, and $\phi = \phi(\xi)$, $\psi = \psi(\eta)$ satisfy

$$\phi' = \frac{\mathrm{d}}{\mathrm{d}\xi}\phi = q_1 + p_1\phi^2\,, \quad \psi' = \frac{\mathrm{d}}{\mathrm{d}\eta}\psi = q_2 + p_2\psi^2\,, \tag{21}$$

where q_1, $p_1 \neq 0$, q_2, $p_2 \neq 0$ are constants.

Step 2 By balancing the highest-order derivative term and the highest-order nonlinear term, we can determine the value of n in Eq. (20).

Step 3 With the aid of *Maple*, substituting Eq. (20) with Eq. (21) into Eq. (19) yields a partial differential equation about $\phi^i(\xi)\psi^j(\eta)$ $(i = 0,1,2,\ldots; j = 0,1,2,\ldots)$.

Step 4 Setting the coefficients of the terms $\phi^i(\xi)\psi^j(\eta)$ $(i = 0,1,2,\ldots; j = 0,1,2,\ldots)$ of the above partial differential equation to zero, we can obtain a group of over-determined partial differential equations about the unknown variables ξ, η, A_0, A_i^j $(i,j = 1,2,\ldots,n)$.

Step 5 Solving the nonlinear parameterized over-determined partial differential equations by using *Maple* software, we can determine the values of ξ, η, A_0, A_i^j $(i,j = 1,2,\ldots,n)$.

Step 6 Substituting the results in Step 5 along with the explicit solutions of Eq. (21) into Eq. (20), we can derive the exact non-travelling wave solutions of Eq. (19). Finally, check the solutions by substituting them into initial Eq. (19).

Remark 1 The key of the above algorithm is the special transformation Eq. (20). Chen and Wang's multiple Riccati equations method[13,14] is a powerful tool to solve NPDEs, but it is very complicated and need plenty of computation. Compared with their method, the above method is simpler and more powerful and can obtain the double soliton-like solutions, combined non-traveling wave solutions of q-deformed hyperbolic and triangular functions, and even multiple soliton-like solutions of NPDEs when we let Eq. (20) be a more general form.

3 Exact Solutions of (2+1)-Dimensional Breaking Soliton Equations

(2+1)-dimensional breaking soliton equations[18−22]

$$u_t + \beta u_{xxy} + 4\beta(uv)_x = 0\,, \quad u_y = v_x\,, \tag{22}$$

describe the (2+1)-dimensional interaction of a Riemann wave propagating along the y-axis with a long wave along the x-axis, and they seem to have been generated.[22] Li and Zhang[19] have constructed infinitely many symmetries for Eq. (22) via the infinitesimal version of the dressing method. Recently, Yan[20] and Chen *et al.*[18] obtained the exact soliton-like solutions of Eq. (22) by using symbolic computation. In this letter, with the algorithm in Sec. 2, we derive some other novel exact non-travelling wave solutions of Eq. (22) which cannot be found before.

By balancing the highest-order partial derivative term and the nonlinear term in Eq. (22), we obtain $n = 2$. So we suppose that equation (22) has the solution in the following form:

$$\begin{aligned} u &= A_0 + A_1\phi(\xi) + B_1\psi(\eta) + A_2\phi^2(\xi) \\ &\quad + B_2\psi(\eta)\phi(\xi) + A_3\psi^2(\eta)\,, \\ v &= D_0 + D_1\phi(\xi) + E_1\psi(\eta) + D_2\phi^2(\xi) \\ &\quad + E_2\phi(\xi)\psi(\eta) + D_3\psi^2(\eta)\,, \end{aligned} \tag{23}$$

where for convenience, we assume that $\xi = \alpha_1 x + \beta_1$, $\eta = \alpha_2 x + \beta_2$, A_0, A_1, B_1, A_2, B_2, D_0, D_1, E_1, D_2, E_2, A_3, D_3, β_1, and β_2 are the functions of (y, t), while α_1 and α_2 are functions of t to be determined and $\phi = \phi(\xi)$ and $\psi = \psi(\eta)$ satisfy Eq. (21).

According to the algorithm in Sec. 2, we can determine the concrete values of A_0, A_1, B_1, A_2, B_2, D_0, D_1, E_1, D_2, E_2, A_3, D_3, β_1, and β_2 as follows:

$$\begin{aligned} &A_1 = B_1 = B_2 = D_1 = E_1 = 0\,, \quad A_0 = \frac{C_4C_1 - 8\beta {C_2}^3C_3p_2q_2 - 8C_2\beta {C_1}^2p_1C_3q_1}{8C_3\beta C_2}\,, \\ &E_2 = 0\,, \quad \alpha_2 = C_2\,, \quad \alpha_1 = C_1\,, \quad A_3 = -\frac{3{C_2}^2{p_2}^2}{2}\,, \quad D_3 = \frac{3C_3{C_2}^2{p_2}^2}{2C_1}\,, \\ &A_2 = -\frac{3{C_1}^2{p_1}^2}{2}\,, \quad D_2 = -\frac{3C_1{p_1}^2C_3}{2}\,, \quad \beta_2 = \frac{C_2F_2(t) + (C_4t + C_5)C_1 - C_3C_2y}{C_1}\,, \\ &\beta_1 = C_3y + F_2(t)\,, \quad D_0 = \frac{2C_2(\mathrm{d}/\mathrm{d}t)F_2(t) + C_4C_1 - 8\beta {C_2}^3C_3p_2q_2 + 8C_2\beta {C_1}^2p_1C_3q_1}{-8C_2\beta C_1}\,, \end{aligned} \tag{24}$$

where C_1, C_2, C_3, C_4, and C_5 are arbitrary constants, and $F_2(t)$ is an arbitrary function of t.

So from Eqs. (23) and (24) we can obtain the general form of solutions of the (2+1)-dimensional breaking soliton equation:

$$\begin{aligned} u &= A_0 - \frac{3{C_1}^2{p_1}^2}{2}\phi^2(C_1x + C_3y + F_2(t)) - \frac{3{C_2}^2{p_2}^2}{2}\psi^2(C_2x + \beta_2)\,, \\ v &= D_0 - \frac{3C_1{p_1}^2C_3}{2}\phi^2(C_1x + C_3y + F_2(t)) + \frac{3C_3{C_2}^2{p_2}^2}{2C_1}\psi^2(C_2x + \beta_2)\,, \end{aligned} \tag{25}$$

where A_0, D_0, and β_2 satisfy Eq. (24).

If we properly select special values of q_1, p_1, q_2, and p_2 in Eq. (21), we can construct families of non-travelling wave solutions of Eq. (22) from general form of solution (25). Here we let q_1 and q_2 in Eq. (21) satisfy $q_1 > 0$, $q_2 > 0$.

Family 1 When we let $p_1q_1 = -1$, $p_2q_2 = -1$, we can obtain the following double soliton-like solution of Eq. (22):

$$\begin{aligned} u_1 &= A_0 - \frac{3{C_1}^2{p_1}^2q_1^2}{2}\tanh^2_{q_1}(C_1x + C_3y + F_2(t)) - \frac{3{C_2}^2{p_2}^2q_2^2}{2}\tanh^2_{q_2}(C_2x + \beta_2)\,, \\ v_1 &= D_0 - \frac{3C_1{p_1}^2C_3q_1^2}{2}\tanh^2_{q_1}(C_1x + C_3y + F_2(t)) + \frac{3C_3{C_2}^2{p_2}^2q_2^2}{2C_1}\tanh^2_{q_2}(C_2x + \beta_2)\,, \end{aligned} \tag{26}$$

where A_0, D_0, and β_2 satisfy Eq. (24).

The following three solutions are combined hyperbolic function-like solutions of Eq. (22):

$$\begin{aligned} u_2 &= A_0 - \frac{3{C_1}^2{p_1}^2q_1^2}{2}\tanh^2_{q_1}(C_1x + C_3y + F_2(t)) - \frac{3{C_2}^2{p_2}^2q_2^2}{2}\coth^2_{q_2}(C_2x + \beta_2)\,, \\ v_2 &= D_0 - \frac{3C_1{p_1}^2C_3q_1^2}{2}\tanh^2_{q_1}(C_1x + C_3y + F_2(t)) + \frac{3C_3{C_2}^2{p_2}^2q_2^2}{2C_1}\coth^2_{q_2}(C_2x + \beta_2)\,; \end{aligned} \tag{27}$$

$$\begin{aligned} u_3 &= A_0 - \frac{3{C_1}^2{p_1}^2q_1^2}{2}\coth^2_{q_1}(C_1x + C_3y + F_2(t)) - \frac{3{C_2}^2{p_2}^2q_2^2}{2}\tanh^2_{q_2}(C_2x + \beta_2)\,, \\ v_3 &= D_0 - \frac{3C_1{p_1}^2C_3q_1^2}{2}\coth^2_{q_1}(C_1x + C_3y + F_2(t)) + \frac{3C_3{C_2}^2{p_2}^2q_2^2}{2C_1}\tanh^2_{q_2}(C_2x + \beta_2)\,; \end{aligned} \tag{28}$$

$$\begin{aligned} u_4 &= A_0 - \frac{3{C_1}^2{p_1}^2q_1^2}{2}\coth^2_{q_1}(C_1x + C_3y + F_2(t)) - \frac{3{C_2}^2{p_2}^2q_2^2}{2}\coth^2_{q_2}(C_2x + \beta_2)\,, \\ v_4 &= D_0 - \frac{3C_1{p_1}^2C_3q_1^2}{2}\coth^2_{q_1}(C_1x + C_3y + F_2(t)) + \frac{3C_3{C_2}^2{p_2}^2q_2^2}{2C_1}\coth^2_{q_2}(C_2x + \beta_2)\,, \end{aligned} \tag{29}$$

where A_0, D_0, and β_2 satisfy Eq. (24).

Family 2 When we let $p_1q_1 = 1$, $p_2q_2 = 1$, we can obtain the following combined triangular function-like solution of Eq. (22):

$$\begin{aligned}
u_5 &= A_0 - \frac{3{C_1}^2{p_1}^2q_1^2}{2}\tan_{q_1}^2(C_1x + C_3y + F_2(t)) - \frac{3{C_2}^2{p_2}^2q_2^2}{2}\tan_{q_2}^2(C_2x + \beta_2)\,,\\
v_5 &= D_0 - \frac{3C_1{p_1}^2C_3q_1^2}{2}\tan_{q_1}^2(C_1x + C_3y + F_2(t)) + \frac{3C_3{C_2}^2{p_2}^2q_2^2}{2C_1}\tan_{q_2}^2(C_2x + \beta_2)\,;
\end{aligned}\tag{30}$$

$$\begin{aligned}
u_6 &= A_0 - \frac{3{C_1}^2{p_1}^2q_1^2}{2}\tan_{q_1}^2(C_1x + C_3y + F_2(t)) - \frac{3{C_2}^2{p_2}^2q_2^2}{2}\cot_{q_2}^2(C_2x + \beta_2)\,,\\
v_6 &= D_0 - \frac{3C_1{p_1}^2C_3q_1^2}{2}\tan_{q_1}^2(C_1x + C_3y + F_2(t)) + \frac{3C_3{C_2}^2{p_2}^2q_2^2}{2C_1}\cot_{q_2}^2(C_2x + \beta_2)\,;
\end{aligned}\tag{31}$$

$$\begin{aligned}
u_7 &= A_0 - \frac{3{C_1}^2{p_1}^2q_1^2}{2}\cot_{q_1}^2(C_1x + C_3y + F_2(t)) - \frac{3{C_2}^2{p_2}^2q_2^2}{2}\tan_{q_2}^2(C_2x + \beta_2)\,,\\
v_7 &= D_0 - \frac{3C_1{p_1}^2C_3q_1^2}{2}\cot_{q_1}^2(C_1x + C_3y + F_2(t)) + \frac{3C_3{C_2}^2{p_2}^2q_2^2}{2C_1}\tan_{q_2}^2(C_2x + \beta_2)\,;
\end{aligned}\tag{32}$$

$$\begin{aligned}
u_8 &= A_0 - \frac{3{C_1}^2{p_1}^2q_1^2}{2}\cot_{q_1}^2(C_1x + C_3y + F_2(t)) - \frac{3{C_2}^2{p_2}^2q_2^2}{2}\cot_{q_2}^2(C_2x + \beta_2)\,,\\
v_8 &= D_0 - \frac{3C_1{p_1}^2C_3q_1^2}{2}\cot_{q_1}^2(C_1x + C_3y + F_2(t)) + \frac{3C_3{C_2}^2{p_2}^2q_2^2}{2C_1}\cot_{q_2}^2(C_2x + \beta_2)\,,
\end{aligned}\tag{33}$$

where A_0, D_0, and β_2 satisfy Eq. (24).

Family 3 When we let $p_1q_1 = -1$, $p_2q_2 = 1$, we can obtain the following combined non-travelling wave solutions of hyperbolic functions and triangular functions of Eq. (22):

$$\begin{aligned}
u_9 &= A_0 - \frac{3{C_1}^2{p_1}^2q_1^2}{2}\tanh_{q_1}^2(C_1x + C_3y + F_2(t)) - \frac{3{C_2}^2{p_2}^2q_2^2}{2}\tan_{q_2}^2(C_2x + \beta_2)\,,\\
v_9 &= D_0 - \frac{3C_1{p_1}^2C_3q_1^2}{2}\tanh_{q_1}^2(C_1x + C_3y + F_2(t)) + \frac{3C_3{C_2}^2{p_2}^2q_2^2}{2C_1}\tan_{q_2}^2(C_2x + \beta_2)\,;
\end{aligned}\tag{34}$$

$$\begin{aligned}
u_{10} &= A_0 - \frac{3{C_1}^2{p_1}^2q_1^2}{2}\tanh_{q_1}^2(C_1x + C_3y + F_2(t)) - \frac{3{C_2}^2{p_2}^2q_2^2}{2}\cot_{q_2}^2(C_2x + \beta_2)\,,\\
v_{10} &= D_0 - \frac{3C_1{p_1}^2C_3q_1^2}{2}\tanh_{q_1}^2(C_1x + C_3y + F_2(t)) + \frac{3C_3{C_2}^2{p_2}^2q_2^2}{2C_1}\cot_{q_2}^2(C_2x + \beta_2)\,;
\end{aligned}\tag{35}$$

$$\begin{aligned}
u_{11} &= A_0 - \frac{3{C_1}^2{p_1}^2q_1^2}{2}\coth_{q_1}^2(C_1x + C_3y + F_2(t)) - \frac{3{C_2}^2{p_2}^2q_2^2}{2}\tan_{q_2}^2(C_2x + \beta_2)\,,\\
v_{11} &= D_0 - \frac{3C_1{p_1}^2C_3q_1^2}{2}\coth_{q_1}^2(C_1x + C_3y + F_2(t)) + \frac{3C_3{C_2}^2{p_2}^2q_2^2}{2C_1}\tan_{q_2}^2(C_2x + \beta_2)\,;
\end{aligned}\tag{36}$$

$$\begin{aligned}
u_{12} &= A_0 - \frac{3{C_1}^2{p_1}^2q_1^2}{2}\coth_{q_1}^2(C_1x + C_3y + F_2(t)) - \frac{3{C_2}^2{p_2}^2q_2^2}{2}\cot_{q_2}^2(C_2x + \beta_2)\,,\\
v_{12} &= D_0 - \frac{3C_1{p_1}^2C_3q_1^2}{2}\coth_{q_1}^2(C_1x + C_3y + F_2(t)) + \frac{3C_3{C_2}^2{p_2}^2q_2^2}{2C_1}\cot_{q_2}^2(C_2x + \beta_2)\,,
\end{aligned}\tag{37}$$

where A_0, D_0, and β_2 satisfy Eq. (24).

If we let $p_1q_1 = 1$, $p_2q_2 = -1$, we will derive some other combined non-travelling wave solutions of hyperbolic functions and triangular functions of Eq. (22). In additions, if we let $p_1q_1 = -1/4$, $p_2q_2 = -1/4$, $p_1q_1 = -1$, $p_2q_2 = -1/4$, or $p_1q_1 = 1$, $p_2q_2 = -1/4$, we will obtain the complex non-travelling wave solutions of Eq. (22). Here, we cancel these cases for convenience.

Remark 2 It is necessary to point that all the exact solutions of the (2+1)-dimensional breaking soliton equation (22) constructed in this paper have been checked by *Maple* software. Because of the special transformation (20) in our algorithm, we can not only get new q-deformed hyperbolic function solutions and q-deformed triangular function solutions, but also get the combined non-travelling wave solutions of q-deformed hyperbolic and triangular function of a class of NPDEs. All the solutions (namely Eqs. (26) $\sim$ (37)) of (2+1)-dimensional breaking soliton equation (22) obtained in this paper include two independent variables

$$\xi = C_1x + C_3y + F_2(t)\,, \quad \eta = C_2x + \frac{C_2F_2(t) + (C_4t + C_5)C_1 - C_3C_2y}{C_1}\,. \tag{38}$$

In these solutions the arbitrary function $F_2(t)$ about t implies that equation (22) has rich local physical structures. To our knowledge, these solutions have not been reported in other literatures.

4 Conclusions

In conclusion, by use of symbolic and algebra computation, Chen and Wang's multiple Riccati equations rational expansion method is further extended to construct kinds of explicitly exact non-travelling wave solutions of NPDEs. The veracity of the algorithm has been tested by successfully applying it to the (2+1)-dimensional breaking soliton equation (22). The extended method is simpler and more powerful than Chen and Wang's multiple Riccati equations rational expansion method. And we can obtain the double soliton-like solutions, combined non-travelling wave solutions of q-deformed hyperbolic and triangular function, and other novel exact solutions of NPDEs, which cannot be found in the existing literatures. In addition, the extended method is also applicable to many other NPDEs, such as the (2+1)-dimensional asymmetric version of the Nizhnik–Novikov–Veselov equation, the (2+1)-dimensional Burgers equation, and so on.

Acknowledgments

The authors are very grateful to Prof. Hong-Bo Li, Yong Chen, Zhen-Ya Yan, and Zhuo-Sheng Lü for their kind help and valuable suggestions. They also thank Prof. En-Gui Fan and Prof. Chun-Ping Liu for their constructive suggestions about the solutions of Riccati equation.

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