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Traffic Modelling for Moving-Block Train Control System*

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Abstract *This paper presents a new cellular automaton (CA) model for train control system simulation. In the proposed CA model, the driver reactions to train movements are captured by some updated rules. The space-time diagram of traffic flow and the trajectory of train movement is used to obtain insight into the characteristic behavior of railway traffic flow. A number of simulation results demonstrate that the proposed CA model can be successfully used for the simulations of railway traffic. Not only the characteristic behavior of railway traffic flow can be reproduced, but also the simulation values of the minimum time headway are close to the theoretical values.*

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Key words: cellular automaton, traffic flow, moving-block system

1 Introduction

The modern automatic train control (ATC) systems are based upon the continual communication between trains and wayside systems. The most advanced system of them is the moving-block (MB) train control system. It can free the operation of the railway from the rigid constraint set, and then increase the capacity of the railway and improve the operational flexibility. In practical applications, only a limited number of railway systems have employed the MB signalling system, such as the Docklands Light Railway. The reason is that the application of MB system could bring a number of issues, which are not as significant as fixed-block (FB) system. These issues include the power system loading, the traction and braking control, and the communication system, etc. For example, MB system replaces a protection system with a high-resolution closed-loop position control system. In addition, MB system is confronted with the limitation of ride-comfort.

NaSch model is one of the important traffic simulation model.[1] It has been demonstrated that NaSch model is able to appropriately capture traffic behavior with standard traffic flow in Germany.[2] Since the space, time, and state of NaSch model are discrete, it allows very fast calculations in comparison with other continuous models (it is able to perform several millions updates in a second).[3] Another advantage of NaSch model is that it may be used for simulating large highway networks with thousand of vehicles circulating on them. However, NaSch model is a minimal model which reproduces the basic features of real traffic. For the description of more complex situations, the basic rules outlined in NaSch model have to be modified, such as the multi-lane traffic, bidirectional traffic, and the traffic with different types of vehicles, etc.[4−7]

Computer simulations as a means for evaluating control and management strategies in rail traffic system have gained considerable importance, such as testing control algorithms. In this work, we propose a new CA traffic model to simulate the train control system. Our model has the following features. (i) Using some simple rules, our model can be used to simulate the complex railway traffic; (ii) It is a flexible model, and is easy to simulate different types of railway traffic by modifying the basic rules of the model. The paper is organized as follows. In Sec. 2, we introduce the principle of the moving-block train control system. We develop the CA model in Sec. 3. The numerical and analytical results are presented in Sec. 4. Finally, conclusion of this approach is presented.

2 Moving-Block Train Control System

2.1 *Principle of Moving-Block Signalling System*

Moving-block train control system was proposed to provide more room for headway reduction. In a moving-block system, the separation between two successive trains is governed by their safe stopping distance. On a moving-block equipped railway, the railroad is usually divided into areas or regions, each area is under the control of a computer and has its own radio transmission system. Each train transmits its identity, location, direction, and speed to the area computer. The radio linking between each train and the area computer is continuous so that the computer knows the location of all the trains in its area all the time. It transmits to each train the location of the train in front and gives it a braking curve to enable it to stop before it reaches that train. Theoretical moving-block implies that a train has continuous knowledge of the location of the rear of a train ahead and its braking capability.

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Several types of moving-block scheme have been discussed,[8] i.e., moving space block (MSB), moving time block (MTB), and pure moving block (PMB). Moving space block (MSB) is the simplest scheme, in which the minimum instantaneous distance between two successive trains is

$$d_{\min} = \frac{v_{\max}^2}{2b} + \mathrm{SM}\,, \tag{1}$$

where $v_{\max}$ is the maximum speed, b denotes the deceleration of train, and SM is the safety margin distance. In this scheme, the only information that the following train requires is the position of its leading train and its own position. Equation (1) will be used for train acceleration/deceleration later. From this equation, one can also do a theoretical calculation of scheduled headway between trains. For the purposes of the present paper, only the simplest scheme, MSB, is considered in more detail.

2.2 *Minimum Time Headway*

In automatic train control system, the most critical period occurs at the approach to station, where the leading train must leave in time for the following train to travel into the station following its worst-case braking curve. In general, a station consists of a set of tracks, i.e., the platform tracks, where trains stop to enable passengers to board and alight, the storing tracks, where trains wait after completion of their service, and the tracks, where trains can pass through the station directly.

The time-headway is defined as the time interval that two successive trains pass a same site on a track. In railway traffic, for different routes, it has different formulas. When a train wants to enter a storing track, a route to that track must be free. In this case, the minimum time-headway includes two parts: (i) the reaction time T_r, and (ii) the braking time v_b/b. When a train passes through a station directly, the minimum time-headway includes three parts: (i) the reaction time T_r, (ii) the braking time v_b/b, (iii) the time T_f taken for the leading train to travel the total distance $L_d + L_t$, where L_t is the length of train and L_d is the distance that the leading train has travelled from the station. When a train wants to arrive at a platform track, both a platform track and its route to that track must be free. After its stop at the platform track, the train continues its route through the station. In this case, the minimum time-headway includes four parts: (i) the reaction time T_r, (ii) the braking time v_b/b, (iii) the station dwell time T_d, and (iv) the time taken for the leading train to travel the total distance T_f. When a train has arrived at its platform track, its route to that track is released and only the platform track is occupied during its dwell time.

In this work, we discuss the situation where trains travel on a single railway line, and there is only one platform in the station. Under this condition, we infer the time T_f in detail. For different maximum speed $v_{\max}$, T_f has different formula. If $v_{\max} \geq (2(L_d + L_t)/a)^{1/2}$, train travels the distance $L_d + L_t$ by accelerating, where a denotes the acceleration rate of trains. In the process of acceleration, the time T_f is $(2(L_d + L_t)/a)^{1/2}$. In this case, the minimum headway time is

$$\tau_{\min} = T_r + \frac{v_b}{b} + T_d + \Big(\frac{2(L_d + L_t)}{a}\Big)^{1/2}\,. \tag{2}$$

If $v_{\max} < (2(L_d + L_t)/a)^{1/2}$, the train firstly accelerates, and then maintains a constant speed $v_{\max}$. During the accelerating, the train obtains a maximum speed $v_{\max}$. In the process of acceleration, the distance that the train travels is $v_{\max}^2/(2a)$, and the time that the train takes is $v_{\max}/a$. During the travelling with $v_{\max}$, train travels the surplus distance $L_t + L_d - v_{\max}^2/(2a)$. The time T_f is $[2a(L_t + L_d) - v_{\max}^2]/(2av_{\max})$. In this case, the minimum headway time is then

$$\tau_{\min} = T_r + \frac{v_b}{b} + T_d + \frac{2a(L_t + L_d) + v_{\max}^2}{2av_{\max}}\,. \tag{3}$$

Equations (2) and (3) will not be used directly in the simulation model, but will be used as an analytical benchmark against the simulation model. It should be pointed out that equations (2) and (3) are only valid when there is one platform in the train station. Many high load train systems have two platforms to reduce the conflicts. This case will be discussed in another paper.

3 Proposed CA Model

NaSch model is a one-dimensional probabilistic cellular automaton model for traffic simulation. In NaSch model, the road is divided into L cells numbered by $i = 1, 2, \ldots, L$, and the time is discrete. Each site can be either empty or occupied by a vehicle with integer speed $v = 0, 1, \ldots, v_{\max}$, where $v_{\max}$ is the maximum speed. The underlying dynamics of NaSch model is governed by the updated rules applied at discrete time steps. All sites are simultaneously updated according to four successive steps. (i) Acceleration: increasing v_n by 1 if $v_n < v_{\max}$; (ii) Slowing down: decreasing v_n to $v_n = d$ if necessary (d is the number of empty cells in front of the vehicle); (iii) Randomization: decreasing v_n by 1 with randomization probability p if $v_n > 0$; (iv) Movement: moving vehicle v_n sites forward.

In this work, we use CA model to simulate the railway traffic with the moving-block signalling control system. Our investigation is based on the deterministic NaSch model. The railway line consists of a single-lane, which is divided into L cells numbered by $i = 1, 2, \ldots, L$, and the time is discrete. Each site can be either empty or occupied by a train with integer speed $v_n = 0, 1, \ldots, v_{\max}$. When the site i is occupied by a train, a section from the site $i - L_t$ to the site i will not be allowed to be occupied by another train. This is because that train has a length L_t.

In railway traffic, in order to avoid the collision between two successive trains, the distance between these trains must be larger than or equal to the minimum instantaneous distance. For this purpose, in our method, the acceleration of train n is given by the following step:

$$\begin{aligned}
&\text{if } \Delta x_n > L_s\\
&\quad v_n = \min(v_n + a, v_{\max})\\
&\text{else if } \Delta x_n < L_s\\
&\quad v_n = \max(v_n - b, 0)\\
&\text{else}\\
&\quad v_n = v_n\\
&\text{end}
\end{aligned} \tag{4}$$

where Δx_n is the distance from the n-th train to the $(n+1)$-th train. It represents the distance from the front of the n-th train to the front of the $(n+1)$-th train. L_s is the minimum safety distance, which can be set to be $L_s = d_{\min} + L_t$. In the acceleration rules, when the distance Δx_n is larger than the safety distance L_s, the n-th train accelerates. If the distance Δx_n is less than the safety distance L_s, the n-th train decelerates.

When the n-th train is directly before a station, if the station is occupied by the $(n+1)$-th train or the $(n+1)$-th train has not travelled out of the distance L_d from the station, the minimum instantaneous distance must be maintained between the n-th train and the station. If the station is empty and the $(n+1)$-th train has travelled out of the distance L_d from the station, the n-th train is allowed to travel into the station directly. After the station dwell time T_d, this train leaves the station. During the n-th train travelling into the station, the acceleration of the n-th train is given by the following step:

$$\begin{aligned}
&\text{if } \Delta x_n > d_c\\
&\quad v_n = \min(v_n + a, v_{\max})\\
&\text{else if } \Delta x_n < d_c\\
&\quad v_n = \max(v_n - b, 0)\\
&\text{else}\\
&\quad v_n = v_n\\
&\text{end}
\end{aligned} \tag{5}$$

where Δx_n is the distance from the n-th train to the station, and d_c is the distance that the n-th train can reach the station by deceleration. The distance d_c can be calculated by $d_c = v_n^2/(2b)$.

Comparing the proposed model to NaSch model, step 3 adopted in NaSch model is ignored, i.e., the randomization probability p is $p = 0$. The reason is that in railway traffic, trains travelling on railway line are under the effect of control signals. In principle, drivers are not allowed to drive randomly. So the randomization step (step 3 of NaSch model) must be ignored. The update rules for implementing the train control system are as follows.

Case I The n-th train is behind the $(n+1)$-th train

Step 1 Acceleration:
The update rules are the same as the formula (4)

Step 2 Slowing down:

$$v_n = \min(v_n, \text{gap})$$

Step 3 Movement:

$$x_n = x_n + v_n.$$

Case II The train n is before a station. In this case, the parameters Δx_n and gap are respectively the distance from the n-th train to the station and the empty cell number from the n-th train to the station. L_s is replaced by $d_{\min}$.

(a) The station is occupied by the $(n+1)$-th train or the $(n+1)$-th train has not travelled out of the distance L_d from the station, the update rules are the same as those used in the Case I.

(b) The station is empty and the $(n+1)$-th train has travelled out of the distance L_d from the station.

Step 1 Acceleration:
The update rules are the same as the formula (5).

Step 2 Slowing down:

$$v_n = \min(v_n, \Delta x_n)$$

Step 3 Movement:

$$x_n = x_n + v_n\,.$$

In our method, the boundary condition for CA model is open. It is defined in the following way. (i) When the section from the site 1 to the site L_s is empty, a train with the speed $v_{\max}$ is created. This train immediately moves according to the update rules; (ii) At the site $i = L$, trains simply move out of the system. In order to compare simulation results to field measurements, one cellular automaton iteration roughly corresponds to 1 s, and the length of a cell is about 1 m. This means, for example, that $v_{\max} = 10$ cells/update corresponds to $v_{\max} = 36$ km/h.

4 Simulation Results

We use the proposed CA model to simulate the train control system under moving-block condition. The moving-block scheme adopted in this work is moving space block (MSB). A system of $L = 2000$ is considered. One station is designated at the middle of the system (i.e., the site $i = 1000$). The length of evolution time is $T = 1000$, and the first 500 are discarded to avoid transient. During the numerical simulations, the react time T_r, the acceleration a, and deceleration b are set to be $T_r = 1$, $a = 1$, and $b = 1$. The length of train L_t and the distance L_d are respectively taken as $L_t = 100$ and $L_d = 70$. The safety margin distance SM is written as $\text{SM} = T_r v_{\max}$. It represents the distance that train travels within the react time T_r.

Under moving-block condition, the characteristic behavior of train movements is similar to that observed in road traffic, such as the go-and-stop waves. Figure 1 shows the space-time diagram of railway traffic flow under the MSB condition. Here we plot 2000 sites in 500 consecutive time steps. The horizontal direction indicates the direction in which trains move ahead, and the vertical direction indicates time. In Fig. 1, the positions of trains are indicated by dots. From Fig. 1, we can see that the railway traffic flow behind the station is free, where trains can travel freely. The reason is that at the right boundary, trains can simply move out of the system and without delay. We can also see that before the station, the so-called go-and-stop waves occur. Within the go-and-stop waves, sometimes trains stop, and sometimes trains go.

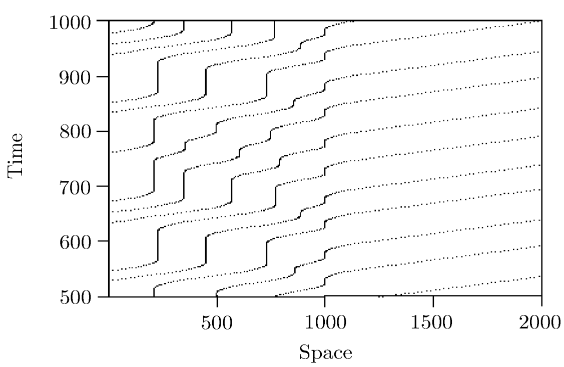

Fig. 1 Space-time diagram of railway traffic flow for $v_{\max} = 20$ and $T_d = 10$.

Speed, distance and time graphs are usually used as basic planning tool by train service planners and signalling design engineers. These graphs give an accurate means of calculating the kinetic performance of trains. Figure 2 presents the local space-time diagram, which displays the positions and speeds of the trains within the go-and-stop waves. Here numbers represent the speeds of trains, and dots correspond to empty sites. From Fig. 2, it is obvious that when the train 1 is delayed, its speed will be decreased, at the next time step, the train 2 that is directly behind the train 1 will also be decreased. As the time proceeds, a number of trains that are behind the train 1 will be delayed.

Within the dwell time T_d, the train which is at the station is delayed. This leads to the formation of go-and-stop waves before the station. When the station is empty, train which is directly before the station can travel into the station by decreasing its speed. The dynamic response of one train which is directly before the station is shown in Fig. 3.

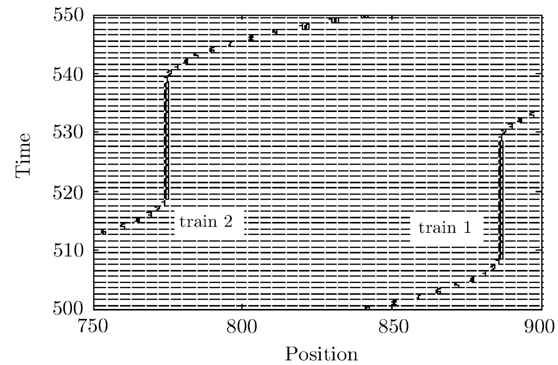

Fig. 2 A local diagram displaying the positions and speeds of trains for $v_{\max} = 10$ and $T_d = 10$.

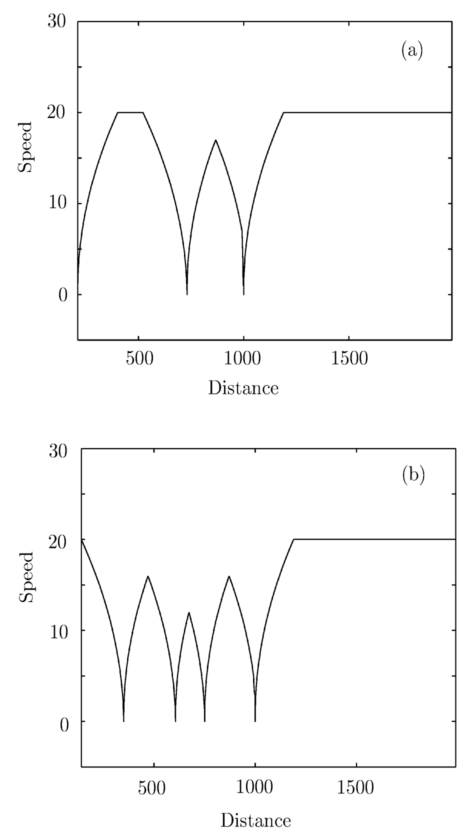

Fig. 3 A diagram displaying the speed and distance of one train. (a) $v_{\max} = 20$, $T_d = 10$; (b) $v_{\max} = 20$, $T_d = 40$.

Figure 3(a) denotes the speeds for the time $T_d = 10$, and figure 3(b) denotes the speeds for the time $T_d = 40$. In Fig. 3, this train is tracked at the time $t = 500$. From Fig. 3, we can see that during this train travelling toward the station, this train adjusts its speed continuously until it stops at the station. After the station dwell time T_d, it accelerates gradually, and then leaves the station. In our method, only trains before the station are delayed. When trains are delayed, as shown in Fig. 2, in order to avoid the collision between two successive trains, the following train must adjust its speed continuously. Sometimes it accelerates, and sometimes it decelerates. After a train passes through the station, the train can travel freely. This is the reason that a train must adjust its speed when it is before the station.

The motion of a train is governed by the maximum speed $v_{\max}$, the acceleration and deceleration rates etc. In order to maintain a safe distance between successive trains on the same track, the following train need to adjust its speed continuously according to the position and speed of its leading train. Figure 4 shows the speed pattern of the following train. Here the following train is tracked at the time $t = 500$. In Fig. 4, the following train decreases or accelerates for several times according to the position and speed of its leading train. Sometimes the train travels with the maximum speed $v_{\max}$, and sometimes the train stops on the track, i.e., $v_n = 0$. The simulation result demonstrates that the speed controlling simulation in this work is close to actual control by the driver. That is to say, simulating train operation like human drivers who operate actual train.

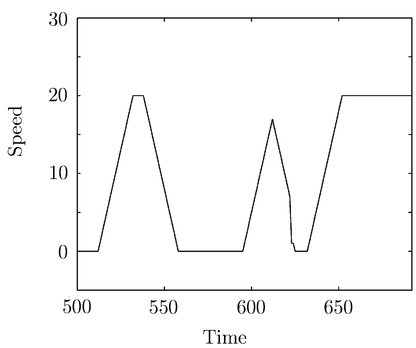

Fig. 4 A diagram displaying the speed and time of one train for $v_{\max} = 20$ and $T_d = 10$.

During simulations, if the distance between successive trains is smaller than the minimum safe distance, these trains will interact through the control signals, and the following train will be forced to brake to a lower speed or stop at a site on the track. The trajectory of one train is shown in Fig. 5. It displays the distance and time of the tracked train. Here the train is tracked at the time $t = 500$. In Fig. 5, the horizontal line denotes that the train stops at a site on the track, where its speed is zero. From Fig. 5, it is obvious that there are several horizontal lines, one is about the station dwell, and the other lines are about the delays of trains within the go-and-stop waves.

Using the proposed CA model to simulate the moving-block control system, we record the time headway of trains at the station. It is obtained by averaging a number of measurements during simulation. In order to obtain the minimum time headway $\tau_{\min}$, the station dwell time T_d is set to be large enough. In this work, since trains travel successively on a single track, the minimum time-headway includes four parts, $\tau_{\min} = T_r + v_b/b + T_d + T_f$. Figure 6 shows how the minimum time headway $\tau_{\min}$ varies with the maximum speed $v_{\max}$. In Fig. 6, the dotted line denotes the simulation values using the proposed model, and the solid line denotes the theoretical values calculated by Eqs. (2) and (3). From Fig. 6, we can see that the simulation values using the proposed model are close to the theoretical values calculated by Eqs. (2) and (3). Apart from these, it can be seen two points of the simulation values: (i) The time headway is not in linear relation with train speed because train braking time increases with its speed; (ii) There is a minimum point of the time headway at the maximum speed, it is at about $v_{\max} = 10$ cells/update, i.e., $v_{\max} = 36$ km/h. These simulation results demonstrate that the proposed model can be successfully used for the simulation of train control system.

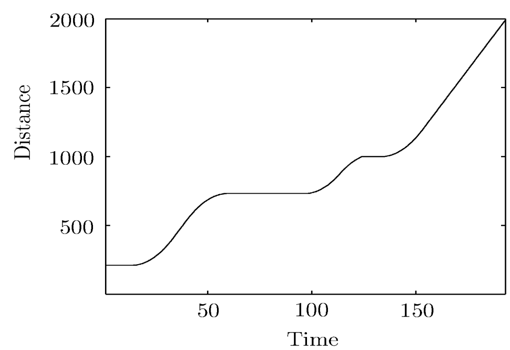

Fig. 5 Trajectory of one train for $v_{\max} = 20$ and $T_d = 10$.

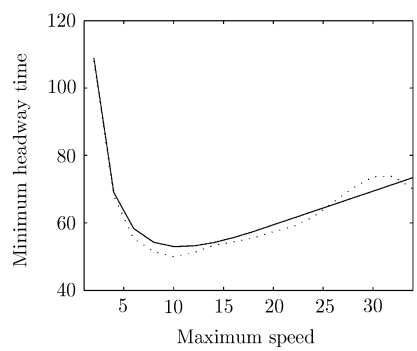

Fig. 6 How the minimum time headway varies with the maximum speed for $T_d = 20$.

In railway traffic, one of the important factors that affect the line capacity is the design speed. With fixed-block control system, the maximum capacity is only achieved when trains travel at the design speed. However, with moving-block control system, every speed can effectively be considered as a design speed. For given train parameters, the relationship between travelling speed and capacity is dynamic and exhibits a maximum capacity at a unique travelling speed. Figure 7 shows how the flux q varies with the maximum speed $v_{\max}$. Here the flux q is defined as $q = (1/L)\sum_{i=1}^{N} v_i$, where v_i is the speed of the i-th train. The simulation values are obtained by averaging over 10 such measurements. From Fig. 7, it is obvious that the flux q increases with increasing the maximum speed $v_{\max}$ in low speed region, and reaches the maximal value, and then decreases continuously with increasing the maximum speed $v_{\max}$. The maximal value $q_{\max}$ is at about $v_{\max}$=12 cells/update, i.e., $v_{\max} = 43$ km/h. On the other hand, the fall-off in low speed region is due to the time taken for a train to travel its own length. When the travelling speed is lower, this time is larger. As the travelling speed is increased, this time reduces, and the capacity increases until the optimum travelling speed is reached.

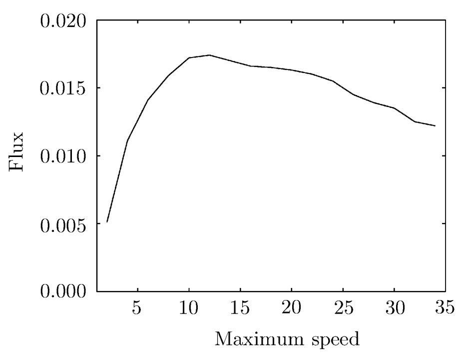

Fig. 7 Capacity versus speed relationship for $T_d = 20$.

In fact, the design and construction of new railway infrastructure is a most time consuming process, it take up to one or two decades. For such a planning horizon, only rough estimates can be provided for the traffic demand. In general, an analytical tool is necessary for railway networks. Such a tool could be most powerful in the first stages of design, to identify bottlenecks in the network, to compare alternative designs in a global way, or to analyze several traffic scenarios. Figure 7 shows how the flux q varies with the maximum speed $v_{\max}$. From Fig. 7, we can find how we can obtain a maximum flux. This is important for our practical building.

5 Conclusions

In conclusions, a new CA model has been used to simulate the train control system under the MSB condition. The simulation results demonstrate that the proposed CA model can well capture the characteristic behavior of train movement. It should be pointed out that the proposed CA model is a basic model, which can be generalized to more complicated traffic conditions by modifying the basic rules. In addition, train movement is not only under the speed restriction, but also is under the constraints of the track geometry and specific external force etc. In our model, these factors have not been taken into account. This leads that the detail of train movement is omitted in the calculation. So we think that it is worthy to study further.

There are two main disadvantages of NaSch model. First, a train can decelerate from a maximum speed $v_{\max}$ to a stopping. The train deceleration is very unrealistic, especially, as the maximum speed $v_{\max}$ is larger. This characteristic of NaSch model originates from the basic rules. Second, as shown in Fig. 6, when the maximum speed $v_{\max}$ of trains is larger, the proposed CA model can not capture the dynamics of train movements well. So an improved CA model should be developed for railway traffic.

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