Commun. Theor. Phys. (Beijing, China) **49** (2008) pp. 133–136
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Vol. 49, No. 1, January 15, 2008

Hawking Tunneling Radiation of a Particle with Electric and Magnetic Charge from Kerr–Newman–Kasuya Black Hole*

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(Received January 15, 2007; Revised March 26, 2007)

Abstract *Extending the Parikh's quantum tunneling method of an uncharged particle, we investigate the quantum radiation characteristics of a particle with electric and magnetic charge via tunneling from the event horizon of the Kerr–Newman–Kasuya black hole. The derived result supports the Parikh's opinion and the correction to the thermal spectrum is of precisely the form that satisfies the underlying unitary quantum theory, and finally provides a might explanation to the black hole information puzzle.*

PACS numbers: 04.70.Dy, 04.62.+v, 03.65.Sq
Key words: black hole, tunneling rate, quantum theory, Bekenstein–Hawking entropy

At the beginning of 1970s, B. Carter and D.C. Robinson put forward the no hair theorem for black holes, which implies that all information about the collapsing body is lost from the outside region apart from three conserved quantities: the mass, the angular momentum, and the electric charge.[1] In 1975, the loss of information was first introduced by S.W. Hawking. After that, a great number of arguments were continued for years without any resolution either way. Finally, it was settled in favor of conservation of information by a conjectured duality between string theory in anti-de Sitter [ADS] space and a conformal field theory [CFT] on the boundary of anti-de Sitter space at infinity.[2] But it still was not clear how information could get out of a black hole.

In 2000, Parikh and Wilczek treated the Hawking radiation as a tunneling process, where a particle moves in dynamic geometry, and derived the Hawking radiation spectrum formula for the static Schwarzshild and Reissner–Nordstrom black holes.[3,4] The result shows that the exact Hawking radiation spectrum has corrections of higher order in ω, the presence of which ensures the Hawking radiation is not pure thermal. This is exciting news because arguments that information is lost during the black hole evaporation rely in part on the assumption of strict thermality for the radiate spectrum. The non-thermal spectrum can open the way to looking for the lost information. In this method, the energy conservation and the particle's self- gravitation are taken into account, and a coordinate system well-behaved at the event horizon is introduced into the tunneling frame. Following this method, Hemming and Keski–Vakkuri have investigated Hawking radiation from AdS black holes,[5] and Medved found that from a de Sitter cosmological horizon,[6] both the derived results support the Parikh's opinion.

In 2005, Zhang and Zhao extended the Parikh's method to stationary black hole, and obtained the tunneling radiation spectrum of an uncharged particle from the Kerr–Newman and Kerr black holes,[7] in which they introduced general Painleve coordinate transformation to eliminate the coordinate singularity, and perform the dragging coordinate transformation to make the event horizon and the infinite red-shift surface coincide with each other. The results still support the Parikh's opinion. In this paper, the quantum tunneling radiation of a particle with electric and magnetic charge from the Kerr–Newman–Kasuya black hole is discussed. Different from that of an uncharged particle, the geodesics followed by a particle with electric and magnetic charge is not light-like, but decided by the phase velocity and the group velocity. The derived result shows that the Hawking pure thermal spectrum derived in the fixed space-time need to be modified. Taking the particle's self-gravitation interaction into account, the true radiation spectrum deviates from the pure thermal one.

According to Ref. [8], the line element of the stationary axisymmetric Kerr–Newman–Kasuya black hole is

$$\begin{aligned}\mathrm{d}s^2 = &-\Big(1-\frac{2mr-Q^2-\Phi^2}{\Sigma}\Big)\mathrm{d}t_{\scriptscriptstyle\mathrm{KNK}}^2+\frac{\Sigma}{\Delta}\mathrm{d}r^2\\ &+\Big[\frac{(2mr-Q^2-\Phi^2)a^2\sin^2\theta}{\Sigma}+(r^2+a^2)\Big]\sin^2\theta\mathrm{d}\varphi^2\\ &+\Sigma\mathrm{d}\theta^2-\frac{2a\sin^2\theta}{\Sigma}(2mr-Q^2-\Phi^2)\mathrm{d}t_{\,\scriptscriptstyle\mathrm{KNK}}\mathrm{d}\varphi\,,\quad(1)\end{aligned}$$

where

$$\Sigma = r^2+a^2\cos^2\theta\,,\quad \Delta = r^2+a^2-2mr+Q^2+\Phi^2\,,$$

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*The project supported by the Fundamental Research Project of Sichuan Province of China under Grant No. 05JY029-092

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and t_{KNK} is the coordinate time of the black hole. The electrical potential and magnetic potential can be written as

$$A_\mu = (A_t, 0, 0, A_\varphi), \quad B_\mu = (B_t, 0, 0, B_\varphi)\,, \tag{2}$$

respectively. Here

$$A_t = -\frac{Qr}{\Sigma}\,, \quad A_\varphi = \frac{Qra\sin^2\theta}{\Sigma}\,,$$

$$B_t = -\frac{\Phi r}{\Sigma}\,, \quad B_\varphi = \frac{\Phi ra\sin^2\theta}{\Sigma}\,.$$

In Eq. (1), as the existence of the rotation freedom, it is not convenient for us to investigate the Hawking radiation at the event horizon. (i) The infinite red-shift surface and the black hole horizon are not coincident with each other, so the geometrical optics limit is not reliable. (ii) The existence of rotation contributes the dragging effect of the coordinate system in spacetime and that of the matter field in the energy layer near the horizon. Obviously, a rational and physical picture should be described in the dragging coordinate system. Performing the dragging coordinate transformation

$$\frac{\mathrm{d}\varphi}{\mathrm{d}t} = -\frac{g_{03}}{g_{33}} = \frac{a(r^2+a^2-\Delta)}{(r^2+a^2)^2-\Delta a^2\sin^2\theta}\,, \tag{3}$$

to Eqs. (1) and (2), the metric, and electrical potential as well as magnetic potential can be rewritten as

$$\mathrm{d}s^2 = \hat{g}_{00}\mathrm{d}t^2_{\text{KNK}} + \frac{\Sigma}{\Delta}\mathrm{d}r^2 + \Sigma\mathrm{d}\theta^2\,, \tag{4}$$

$$A'_t = -\frac{(r^2+a^2)Qr}{(r^2+a^2)^2-\Delta a^2\sin^2\theta}\,,$$

$$B'_t = -\frac{(r^2+a^2)\Phi r}{(r^2+a^2)^2-\Delta a^2\sin^2\theta}\,. \tag{5}$$

In Eq. (4), there still exists a coordinate singularity at the black hole horizon. Now, performing general Painlevé coordinate transformation to Eq. (4), that is,[9]

$$\mathrm{d}t_{\text{KNK}} = \mathrm{d}t + G(r,\theta)\mathrm{d}r + F(r,\theta)\mathrm{d}\theta\,, \tag{6}$$

and demanding that constant-time slices are flat Euclidean spaces in radial yields

$$\begin{aligned}\mathrm{d}s^2 &= \hat{g}_{00}\mathrm{d}t^2 + 2\sqrt{\hat{g}_{00}\Big(1-\frac{\Sigma}{\Delta}\Big)}\mathrm{d}t\mathrm{d}r + \mathrm{d}r^2 \\ &\quad + [\hat{g}_{00}G^2(r,\theta)+\Sigma]\mathrm{d}\theta^2 + 2\hat{g}_{00}G(r,\theta)\mathrm{d}t\mathrm{d}\theta \\ &\quad + 2\sqrt{\hat{g}_{00}\Big(1-\frac{\Sigma}{\Delta}\Big)}\,G(r,\theta)\mathrm{d}r\mathrm{d}\theta\,.\end{aligned} \tag{7}$$

According to the Landau's condition of coordinate clock synchronization, the integrability condition $\partial_\theta G(r,\theta) = \partial_r F(r,\theta)$ can be also derived by the condition of coordinate clock synchronization. Obviously, the Painlevé–Kerr–Newman–Kasuya line element has the following attractive features: Firstly, it does not have coordinate singularity. Secondly, the event horizon and the infinite red-shift surface are coincident with each other. Finally, it satisfies Landau's condition of the coordinate clock synchronization. All of these are necessary for us to investigate the tunneling radiation characteristics.

From Eq. (7), we can obtain the null geodesics of the uncharged massless particle in radial near the event horizon,

$$\dot{r} = \frac{\mathrm{d}r}{\mathrm{d}t} = \sqrt{-\frac{\Sigma}{\Delta}\hat{g}_{00}} - \sqrt{\hat{g}_{00}\Big(1-\frac{\Sigma}{\Delta}\Big)}\ . \tag{8}$$

But, the particle via tunneling from a charged and magnetic black hole should be charged and magnetic, so the null geodesics of the uncharged massless particle is not applicable here for describing the true tunneling behavior at the event horizon. de Broglie's hypothesis implies that the outgoing particle corresponds to a kind of wave, and its phase velocity and group velocity satisfy

$$v_p = \frac{1}{2}v_g\,. \tag{9}$$

Since the tunneling process is an instantaneous effect, and the metric in Eq. (7) satisfies Landau's theory of the coordinate clock synchronization. So the difference of coordinate times of two events taking place simultaneously in different places is

$$\mathrm{d}t = -\frac{g_{01}}{g_{00}}\mathrm{d}r_c\,, \quad (\mathrm{d}\theta = \mathrm{d}\varphi = 0)\,, \tag{10}$$

where r_c is the location of the tunneling particle. So the phase velocity (the radial geodesics) is

$$\begin{aligned}\dot{r} = \nu_p &= \frac{1}{2}\frac{\mathrm{d}r_c}{\mathrm{d}t} = -\frac{1}{2}\frac{g_{00}}{g_{01}} \\ &= \frac{\Delta}{2}\sqrt{\frac{\Sigma}{(\Sigma-\Delta)[(r^2+a^2)^2-\Delta a^2\sin^2\theta]}}\ .\end{aligned} \tag{11}$$

Next, we will study the tunneling radiation characteristics of a particle with electric and magnetic charge at the event horizon of the Kerr–Newman–Kasuya black hole. Considering energy conservation, charge conservation, magnetic conservation, and angular momentum conservation, when a particle with energy ω, charge q, and magnet ϕ tunnels out of the event horizon, the mass, charge, and magnet parameters of the black hole will be replaced by $m-\omega$, $Q-q$, and $\Phi-\phi$ in Eqs. (7) and (11). Applying the WKB approximation, we obtain the tunneling rate[10]

$$\Gamma \sim \mathrm{e}^{-2\,\mathrm{Im}\,S}\,, \tag{12}$$

where

$$\mathrm{Im}\,S = \mathrm{Im}\int_{t_i}^{t_f} L\mathrm{d}t\,, \tag{13}$$

L is the Lagrangian function of the matter-gravity system. When a particle with electric and magnetic charge tunnels out, the effect of the electromagnetic field should be taken into account. So the matter-gravity system consists of the black hole and the electromagnetic field outsides the hole. As the Lagrangian function of the electromagnetic field corresponding to the generalized coordinates described by Eq. (5) is $-(1/4)F_{\mu\nu}F^{\mu\nu}$, we can find that the generalized coordinate is an ignorable coordinate. In addition, the line element (7) is obtained in the dragging coordinate system, so the coordinate φ is also an ignorable coordinate. In order to eliminate the freedoms, the imaginary part of the action should be written as

$$\mathrm{Im}\ S = \mathrm{Im}\int_{t_i}^{t_f}(L - P_\varphi\dot{\varphi} - P_{A'_t}\dot{A}'_t - P_{B'_t}\dot{B}'_t)\mathrm{d}t = \mathrm{Im}\int_{r_i}^{r_f}\int_{(0,0,0)}^{(P_r,P_\varphi,P_t)}\Big[\mathrm{d}P'_r - \frac{\dot{\varphi}}{\dot{r}}\mathrm{d}P'_\varphi - \frac{\dot{A}'_t}{\dot{r}}\mathrm{d}P_{A'_t} - \frac{\dot{B}'_t}{\dot{r}}\mathrm{d}P_{B'_t}\Big]\mathrm{d}r\,, \tag{14}$$

where P_φ and P_t are the canonical momentum conjugate to the coordinates φ and A'_t, and r_i and r_f represent the locations of the event horizon before and after the particle with electric and magnetic charge emission, they are often regarded as the two turning points of the tunneling potential hill, the distance between them depends on the energy, charge and magnet of the outgoing particle. According to Hamilton canonical equation, we have

$$\begin{aligned}
&\dot{r} = \frac{\mathrm{d}H}{\mathrm{d}P_r}\Big|_{(r;\varphi,P_\varphi;A'_t,P_{A'_t};B'_t,P_{B'_t})}, \quad \mathrm{d}H|_{(r;\varphi,P_\varphi;A'_t,P_{A'_t};B'_t,P_{B'_t})} = \mathrm{d}(m-\omega)\,,\\
&\dot{\varphi} = \frac{\mathrm{d}H}{\mathrm{d}P_\varphi}\Big|_{(\varphi;A'_t,P_{A_t};B'_t,P'_B;r,P_r,)}, \quad \mathrm{d}H|_{(\varphi;A'_t,P_{A_t};B'_t,P'_B;r,P_r,)} = a\Omega_+\mathrm{d}(m-\omega)\,,\\
&\dot{A}'_t = \frac{\mathrm{d}H}{\mathrm{d}P_{A'_t}}\Big|_{(A'_t;B'_t,P_{B'_t};r,P_r;\varphi,P_\varphi)}, \quad \mathrm{d}H|_{(A'_t;B'_t,P_{B'_t};r,P_r;\varphi,P_\varphi)} = \frac{(r^2+a^2)r(Q-q)}{(r^2+a^2)^2-\Delta a^2\sin^2\theta}\mathrm{d}(Q-q)\,,\\
&\dot{B}'_t = \frac{\mathrm{d}H}{\mathrm{d}P_{B'_t}}\Big|_{(B'_t;r,P_r;\varphi,P_\varphi;A'_t,P_{A'_t})}, \quad \mathrm{d}H|_{(B'_t;r,P_r;\varphi,P_\varphi;A'_t,P_{A'_t})} = \frac{(r^2+a^2)r(\Phi-\phi)}{(r^2+a^2)^2-\Delta a^2\sin^2\theta}\mathrm{d}(\Phi-\phi)\,,
\end{aligned} \tag{15}$$

where Ω_+ is the dragging angular velocity at the event horizon. Substituting the above formula into Eq. (14), we have

$$\begin{aligned}
\mathrm{Im}\ S =&\ \mathrm{Im}\int_{r_i}^{r_f}\int_{(m,Q,\Phi)}^{(m-\omega,Q-q,\Phi-\phi)}\Big[\frac{\mathrm{d}H|_{(r;\varphi,P_\varphi;A'_t,P_{A'_t};B'_t,P_{B'_t})}}{\dot{r}} - \frac{\mathrm{d}H|_{(\varphi;A'_t,P_{A_t};B'_t,P'_B;r,P_r,)}}{\dot{r}}\\
&-\frac{\mathrm{d}H|_{(A'_t;B'_t,P_{B'_t};r,P_r;\varphi,P_\varphi)}}{\dot{r}} - \frac{\mathrm{d}H|_{(B'_t;r,P_r;\varphi,P_\varphi;A'_t,P_{A'_t})}}{\dot{r}}\Big]\mathrm{d}r\,,\\
=&\ \mathrm{Im}\int_{r_i}^{r_f}\int_{(m,Q,\Phi)}^{(m-\omega,Q-q,\Phi-\phi)}\frac{2\sqrt{(\Sigma-\Delta')[(r^2+a^2)^2-\Delta' a^2\sin^2\theta]}}{\sqrt{\Sigma}\,\Delta'}\Big[(1-a\Omega'_+)\mathrm{d}(m-\omega')\\
&-\frac{(r^2+a^2)(Q-q')r}{(r^2+a^2)^2-\Delta' a^2\sin^2\theta}\mathrm{d}(Q-q') - \frac{(r^2+a^2)(\Phi-\phi)r}{(r^2+a^2)^2-\Delta' a^2\sin^2\theta}\mathrm{d}(\Phi-\phi')\Big]\mathrm{d}r,
\end{aligned} \tag{16}$$

where

$$\begin{aligned}
&\Delta' = r^2 + a^2 - 2(m-\omega')r + (Q-q')^2 + (\Phi-\phi')^2 = (r-r'_-)(r-r'_+)\,,\\
&r_i = m + \sqrt{m^2-a^2-Q^2-\Phi^2}\,, \quad r_f = (m-\omega) + \sqrt{(m-\omega)^2-a^2-(Q-q)^2-(\Phi-\phi)^2}\,,\\
&r'_\pm = (m-\omega') \pm \sqrt{(m-\omega')^2-a^2-(Q-q')^2-(\Phi-\phi')^2}\,, \quad \Omega'_+ = \frac{a}{r'^2_+ + a^2}\,.
\end{aligned} \tag{17}$$

As $r = r'_+$ is a pole, the integral can be evaluated by deforming the contour around the pole. Doing the r integral firstly, we have

$$\mathrm{Im}\ S = \pi\int_{(m,Q,\Phi)}^{(m-\omega,Q-q,\Phi-\phi)}\frac{2(r'^2_+ + a^2)}{r'_+ - r'_-}\Big[(1-a\Omega'_+)\mathrm{d}(m-\omega') - \frac{(Q-q')r'_+}{r'^2_+ + a^2}\mathrm{d}(Q-q') - \frac{(\Phi-\phi')r'_+}{r'^2_+ + a^2}\mathrm{d}(\Phi-\phi')\Big]\,. \tag{18}$$

Finishing the integral, one obtains

$$\begin{aligned}
\mathrm{Im}\ S =&\ -\frac{\pi}{2}\Big[2(m-\omega)^2 - (Q-q)^2 - (\Phi-\phi)^2 + 2(m-\omega)\sqrt{(m-\omega)^2-a^2-(Q-q)^2-(\Phi-\phi)^2}\\
&-2m^2 + Q^2 + \Phi^2 - 2M\sqrt{m^2-a^2-Q^2-\Phi^2}\Big] = -\frac{\pi}{2}(r_f^2 - r_i^2)\,.
\end{aligned} \tag{19}$$

The areas of the event horizon before and after the particle tunneling out are

$$A_i = \int \sqrt{-g}\,\mathrm{d}\theta\mathrm{d}\varphi = 4\pi(r_i^2 + a^2)\,,$$

$$A_f = \int \sqrt{-g}\,\mathrm{d}\theta\mathrm{d}\varphi = 4\pi(r_f^2 + a^2)\,. \tag{20}$$

So the tunneling rate can be expressed as

$$\Gamma \sim \mathrm{e}^{-2\ \mathrm{Im}\ S} = \mathrm{e}^{\pi(r_f^2 - r_i^2)} = \mathrm{e}^{\Delta S_{\mathrm{BH}}}\,, \tag{21}$$

where

$$\Delta S_{\mathrm{BH}} = S_{\mathrm{BH}}(M-\omega, Q-q, \Phi-\phi) - S_{\mathrm{BH}}(M, Q, \Phi)$$

is the change of Bekenstein–Hawking entropy. Obviously, the true radiation spectrum deviates from the purely thermal one, and is connected with the change of Bekenstein–Hawking entropy.

Note that equation (21) satisfies the underlying unitary theory. Since the tunneling rate in quantum mechanics is obtained by

$$\Gamma(i \to f) \sim |A_{fi}|^2 \quad (\text{phase space factor})\,, \tag{22}$$

where $|A_{fi}|^2$ is the square of the amplitude for the tunneling action. The phase space factor is derived by averaging the number of the initial states and the number of the final states, and the number of the initial and final states are the exponent of the initial and final entropies

$$\Gamma \sim \frac{\mathrm{e}^{S_{\mathrm{final}}}}{\mathrm{e}^{S_{\mathrm{initial}}}} = \mathrm{e}^{\Delta S}\,. \tag{23}$$

Obviously, equation (23) is consistent with our result obtained by applying the Parikh's tunneling method. So, equation (21) satisfies the underlying unitary theory in quantum mechanics, and then provides a might explanation to the black hole information puzzle.

Acknowledgments

We would like to thank Qing-Quan Jiang for his helpful discussions.

References

[1] J.M. Maldacena, Adv. Theor. Math. Phys. **2** (1998) 231.

[2] A.M. Polyakov, Phys. Lett. B **428** (1998) 105; E. Witten, Adv. Theor. Math. Phys. **2** (1998) 253.

[3] M.K. Parikh and F. Wilczek, Phys. Rev. Lett. **85** (2000) 5042.

[4] M.K. Parikh, Int. J. Mod. Phys. D **13** (2004) 2351; Phys. Lett. B **546** (2002) 189.

[5] S. Hemming and E. Keski-Vakkuri, Phys. Rev. D **64** (2001) 044006.

[6] A.J.M. Medved, Phys. Rev. D **66** (2002) 124009.

[7] J.Y. Zhang and Z. Zhao, Mod. Phys. Lett. A **20** (2005) 1673; Phys. Lett. B **549** (2005) 1.

[8] M. Kasuya, Phys. Rev. D **25** (1982) 995.

[9] P. Painleve and C.R. Hebd, Seances. Acad. Sci. **173** (1921) 677.

[10] P. Kraus and E. Keski-Vakkuri, Nucl. Phys. B **491** (1997) 219; R. Parentani, Nucl. Phys. B **575** (2000) 333.