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Covariant Helicity Amplitude Analysis for $J/\psi \to \gamma PP$ *

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Abstract *Covariant helicity amplitude analysis for the process of $J/\psi \to \gamma PP$ is discussed. Starting from the S-matrix elements of decay process, we deduce the formulae of helicity coupling amplitudes for two-body decay process. These formulae are used to analyze intermediate resonance states in the process of J/ψ decay to $\gamma\pi\pi$, $\gamma K\bar{K}$, $\gamma\eta\eta'$ etc.*

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1 Introduction

It is generally believed that quantum chromodynamics (QCD) which has $SU(3)_c$ symmetry is the most prospective theory for strong interactions. QCD predicts the existence of gluon and bound states of gluon, such as glueball and hybrid states. The most direct method to prove the validity of QCD is to search for the evidence of the existence of glueball and hybrid states in experiments. According to theoretical calculation, glueballs will be abundantly produced in J/ψ radiative decays. So, it is an ideal place for us to search for the evidence of the existence of glueball in J/ψ radiative decays. Many J/ψ radiative decay channels belong to the type of γPP, where P stands for a pseudoscalar meson, such as J/ψ decays to $\gamma\pi\pi$, $\gamma K\bar{K}$, $\gamma\eta\eta$, $\cdots$, etc. Some important glueball candidates or states which contain large gluon content, such as $f_0(1500)$, $f_J(1710)$ and $\xi(2230)$, appear in these channels. In this paper, we give out the covariant helicity amplitude analysis for $J/\psi \to \gamma PP$ which is proved to be a powerful tool in experimental data analysis. We have used this method in BES physics analyses and have already obtained some meaningful results.

2 S-Matrix for Two-Body Decay Process

First, let us discuss the following two-body decay process

$$a \to b + c\,. \tag{1}$$

The S-matrix element for this process is[1−6]

$$\langle\, \vec{p}_b\vec{p}_c\lambda_b\lambda_c|S|p_a JM\rangle\,, \tag{2}$$

where $\vec{p}_b$, $\vec{p}_c$ are space momenta of two final state particles, p_a is the four-momentum of the initial state particle. The state $|p_a JM\rangle$ is the direct product state of the plane wavefunction and the angular momentum wavefunction of the particle a. λ_b and λ_c are helicities of two daughter particles b and c, J and M are spin and magnetic quantum numbers along Z-axis of mother particle a. The normalization for two-particle direct product state is

$$\langle\, \vec{p}_b{}'\vec{p}_c{}'\lambda_b'\lambda_c'|\vec{p}_b\vec{p}_c\lambda_b\lambda_c\rangle = (2\pi)^6 2E_b 2E_c \delta^3(\vec{p}_b{}' - \vec{p}_b)\delta^3(\vec{p}_c{}' - \vec{p}_c)\delta_{\lambda_b'\lambda_b}\delta_{\lambda_c'\lambda_c}\,, \tag{3a}$$

and for the sake of simplicity, the normalization for initial state wavefunction is selected as

$$\langle\, p_a' J'M'|p_a JM\rangle = \delta^4(p_a' - p_a)\delta_{JJ'}\delta_{MM'}\,. \tag{3b}$$

Because S matrix is translation-invariant, we get

$$\langle\, \vec{p}_b\vec{p}_c\lambda_b\lambda_c|S|p_a JM\rangle = \frac{(2\pi)^4}{\Omega}\delta^4(p_a - p_b - p_c)\langle\vec{p}_b\vec{p}_c\lambda_b\lambda_c|S|p_a JM\rangle\,, \tag{4}$$

where Ω is the volume of infinite four-dimensional Minkowski space. For the final state two-particle system, we can separate the center of mass motion from the relative motion[7,8]

$$|\,\vec{p}_b\vec{p}_c\lambda_b\lambda_c\rangle = (2\pi)^3\sqrt{\frac{4\sqrt{s}}{|\vec{p}\,|}}|\theta\phi\lambda_b\lambda_c\rangle|P\rangle\,, \tag{5}$$

where s is the invariant mass of the final state two-particle system, $|P\rangle$ stands for the plane wave of center of mass motion. The renormalizations of the states $|P\rangle$ and $|\theta\phi\lambda_b\lambda_c\rangle$ respectively are

$$\langle P'|P\rangle = \delta^4(P' - P)\,, \tag{6}$$

$$\langle\theta'\phi'\lambda_b'\lambda_c'|\theta\phi\lambda_b\lambda_c\rangle = \delta(\cos\theta' - \cos\theta) \times \delta(\phi' - \phi)\delta_{\lambda_b'\lambda_b}\delta_{\lambda_c'\lambda_c}\,. \tag{7}$$

P_μ is the momentum of center of mass system

$$P_\mu = (p_b + p_c)_\mu\,. \tag{8}$$

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In the center of mass system, the space momentum of particle b is $\vec{p}$ and the space momentum of particle c is $-\vec{p}$. The magnitude of momentum $\vec{p}$ is denoted as $|\vec{p}|$. According to Eq. (5), the S-matrix element will be changed into

$$\langle \vec{p}_b\vec{p}_c\lambda_b\lambda_c|S|p_aJM\rangle = \frac{(2\pi)^7}{\Omega}\sqrt{\frac{4\sqrt{s}}{|\vec{p}|}}\delta^4(P-p_a)\langle P\theta\phi\lambda_b\lambda_c|S|p_aJM\rangle\,. \quad (9)$$

The state $|P\theta\phi\lambda_b\lambda_c\rangle$ is a reducible state. We can write it into another form

$$|P\theta\phi\lambda_b\lambda_c\rangle = \sum_{jm}\sqrt{\frac{2j+1}{4\pi}}D^j_{m\lambda}(\phi\theta 0)|Pjm\lambda_b\lambda_c\rangle\,, \quad (10)$$

$$|Pjm\lambda_b\lambda_c\rangle = \sqrt{\frac{2j+1}{4\pi}} \times\int \sin\theta\mathrm{d}\theta\mathrm{d}\phi D^{j*}_{m\lambda}(\phi\theta 0)|P\theta\phi\lambda_b\lambda_c\rangle\,, \quad (11)$$

where $\lambda = \lambda_b - \lambda_c$. The renormalization of the state $|Pjm\lambda_b\lambda_c\rangle$ is

$$\langle P'j'm'\lambda'_b\lambda'_c|Pjm\lambda_b\lambda_c\rangle = \delta^4(P'-P)\delta_{j'j}\delta_{m'm}\delta_{\lambda'_b\lambda_b}\delta_{\lambda'_c\lambda_c}\,. \quad (12)$$

The set of all states $|Pjm\lambda_b\lambda_c\rangle$ is a complete set, they satisfy the following completeness relation

$$\int \mathrm{d}^4P\sum_{jm}\sum_{\lambda_b\lambda_c}|Pjm\lambda_b\lambda_c\rangle\langle Pjm\lambda_b\lambda_c| = 1\,. \quad (13)$$

Using all these relations, we can rewrite the S-matrix as

$$\langle \vec{p}_b\vec{p}_c\lambda_b\lambda_c|S|p_aJM\rangle = (2\pi)^3\sqrt{\frac{4\sqrt{s}}{|\vec{p}|}}\sqrt{\frac{2j+1}{4\pi}}\delta^4(P-p_a)D^{J*}_{M\lambda}(\phi\theta 0)F^J_{\lambda_b\lambda_c}\,, \quad (14)$$

where

$$\frac{\Omega}{(2\pi)^4}F^J_{\lambda_b\lambda_c} = \langle PJM\lambda_b\lambda_c|S^J(P_\mu)|p_aJM\rangle\,. \quad (15)$$

Relation (14) is the S-matrix element for two-body decay process. In the above relation, $F^J_{\lambda_b\lambda_c}$ is called the helicity coupling amplitude. The part of S-matrix which depends on angular argument is completely contained in the D-function. The helicity coupling amplitude $F^J_{\lambda_b\lambda_c}$ is independent of angular argument, it only depends on the mass of center of mass system, the relative momentum of two final state particles and their helicities.

3 Helicity Coupling Amplitude

Helicity coupling amplitude $F^J_{\lambda_b\lambda_c}$ can be constructed from spin wavefunction of all three particles and relative orbital angular momentum wavefunction of two final state particles.[5–8] It is known that the spin wavefunction of a spin-0 particle is a scalar, the spin wavefunction of a spin-1 particle is a four-vector, and the spin wavefunction of a spin-n particle is an nth-order tensor. So, we always set the spin wavefunction of a spin-0 particle to 1. In canonical rest frame, the spin-1 wavefunction is usually selected as

$$\phi^\alpha(\pm) = \mp\frac{1}{\sqrt{2}}(0,1,\pm\mathrm{i},0)\,, \quad (16)$$

$$\phi^\alpha(0) = (0,0,0,1)\,. \quad (17)$$

The spin wavefunction for higher spin can be constructed from spin-1 wavefunction by using Clebsch–Gordan coefficient. Such as, the spin-2 wavefunction is

$$\phi^{\alpha\beta}(m) = \sum_{m_1m_2}(1m_11m_2|2m)\phi^\alpha(m_1)\phi^\beta(m_2)\,, \quad (18)$$

where $(1m_11m_2|2m)$ is the Clebsch–Gordon coefficient. And the spin-$(n+1)$ wavefunction is

$$\phi^{\alpha_1\alpha_2\cdots\alpha_{n+1}}(m) = \sum_{m_1m_2}(nm_11m_2|(n+1)m) \times\phi^{\alpha_1\alpha_2\cdots\alpha_n}(m_1)\phi^{\alpha_{n+1}}(m_2)\,. \quad (19)$$

It can be strictly proved that the spin-n wavefunction is a symmetric and traceless tensor, and it is orthogonal to the momentum of the particle

$$\phi^{\alpha_1\alpha_2\cdots\alpha_n}(m)=\phi^{\alpha_2\alpha_1\cdots\alpha_n}(m) = \cdots = \phi^{\alpha_n\alpha_2\cdots\alpha_1}(m)\,, \quad (20)$$

$$g_{\alpha_1\alpha_2}\phi^{\alpha_1\alpha_2\cdots\alpha_n}(m) = 0\,, \quad (21)$$

$$p_{\alpha_1}\phi^{\alpha_1\alpha_2\cdots\alpha_n}(m) = 0\,. \quad (22)$$

These states are orthonormal and complete states

$$\phi^*_{\alpha_1\alpha_2\cdots\alpha_n}(m)\phi^{\alpha_1\alpha_2\cdots\alpha_n}(m') = (-1)^n\delta_{mm'}\,, \quad (23)$$

$$\sum_m \phi_{\alpha_1\alpha_2\cdots\alpha_n}(m)\phi^*_{\beta_1\beta_2\cdots\beta_n}(m) = P^{(n)}_{\alpha_1\alpha_2\cdots\alpha_n\beta_1\beta_2\cdots\beta_n}\,, \quad (24)$$

where $P^{(n)}_{\alpha_1\alpha_2\cdots\alpha_n\beta_1\beta_2\cdots\beta_n}$ is a projection operator for nth-order symmetric and traceless operator. The above spin wavefunctions are only for rest particles. After making a Lorentz transformation, we can obtain the spin wavefunctions for moving particles.

The total spin angular momentum is constructed from two spin angular momenta of final state particle system, it is

$$\phi^{(n)\alpha_1\alpha_2\cdots\alpha_{n_b+n_c}}(P,m) = \sum_{m_1m_2}(n_bm_1n_cm_2|nm) \times\omega^{\alpha_1\alpha_2\cdots\alpha_{n_b}}(p_b,m_1)\varepsilon^{\alpha_{n_b+1}\alpha_{n_b+2}\cdots\alpha_{n_b+n_c}}(p_c,m_2)\,, \quad (25)$$

where $|n_b-n_c|\le n\le n_b+n_c$ (n is the total spin quantum number). Because particles b and c have different momenta, $\phi^{(n_b+n_c)\alpha_1\alpha_2\cdots\alpha_{n_b+n_c}}(P,m)$ is not a completely symmetric and traceless tensor. Therefore, this total spin angular momentum wavefunction cannot be obtained from projection operator for completely symmetric and traceless tensor. According to quantum mechanics, the projection operator is

$$P^{(n)}_{\alpha_1\alpha_2\cdots\alpha_{n_b+n_c}\beta_1\beta_2\cdots\beta_{n_b+n_c}}(p_b p_c) = \sum_m \phi^{(n)}_{\alpha_1\alpha_2\cdots\alpha_{n_b+n_c}}(P,m)\phi^{(n)*}_{\beta_1\beta_2\cdots\beta_{n_b+n_c}}(P,m) \,. \quad (26)$$

If $n = n_b + n_c$ and $\vec{p}_b = \vec{p}_c = 0$, this projection operator will become the projection operator for $(n_b + n_c)$th-order symmetric and traceless tensor. The normalization of the spin angular momentum wavefunction $\phi^{(n_b+n_c)\alpha_1\alpha_2\cdots\alpha_{n_b+n_c}}(P,m)$ is

$$\phi^{(n_b+n_c)*}_{\alpha_1\alpha_2\cdots\alpha_{n_b+n_c}}(P,m)\phi^{(n_b+n_c)\alpha_1\alpha_2\cdots\alpha_{n_b+n_c}}(P,m') = (-1)^{n_b+n_c}\delta_{mm'} \,. \quad (27)$$

Generally speaking, this spin wavefunction is not orthogonal to the four-momentum $P_\mu = p_{b\mu} + p_{c\mu}$, but its first n_b indices are orthogonal to four-momentum p_b, and the last n_c indices are orthogonal to four-momentum p_c,

$$p_b^{\alpha_1}\phi^{(n)}_{\alpha_1\alpha_2\cdots\alpha_{n_b+n_c}}(P,m) = 0 \,, \quad (28)$$

$$p_c^{\alpha_{n_b+1}}\phi^{(n)}_{\alpha_1\alpha_2\cdots\alpha_{n_b+n_c}}(P,m) = 0 \,. \quad (29)$$

Besides, the first n_b indices are traceless and the last n_c indices are traceless

$$g^{\alpha_1\alpha_2}\phi^{(n)}_{\alpha_1\alpha_2\cdots\alpha_{n_b+n_c}}(P,m) = 0 \,, \quad (30)$$

$$g^{\alpha_{n_b+1}\alpha_{n_b+2}}\phi^{(n)}_{\alpha_1\alpha_2\cdots\alpha_{n_b+1}\alpha_{n_b+2}\cdots\alpha_{n_b+n_c}}(P,m) = 0 \,. \quad (31)$$

The orbital angular momentum wavefunction in momentum space is also a tensor. Generally speaking, if the orbital angular momentum quantum number is l, the corresponding wavefunction is represented by an lth-order tensor. For S-wave, its l equals zero. Its orbital angular momentum wavefunction in coordinate space is homogeneous. The orbital angular momentum wavefunction in momentum space is obtained after a Fourier transformation, it is independent of relative momentum $\vec{r}$. For P-wave, $l = 1$. Its wavefunction is a four-vector. In momentum space, its wavefunction is similar to spin-1 wavefunction. They are

$$t^{(1)\alpha}(\pm) = \mp\frac{1}{\sqrt{2}}(0,1,\pm\mathrm{i},0) \,, \quad (32)$$

$$t^{(1)\alpha}(0) = (0,0,0,1) \,. \quad (33)$$

The orbital angular momentum wavefunction with higher quantum numbers can be constructed from $l = 1$ wavefunction, such as the $l = n+1$ orbital angular momentum wavefunction can be constructed from $l = n$ orbital angular momentum wavefunction,

$$t^{(n+1)\alpha_1\alpha_2\cdots\alpha_{n+1}}(m) = \sum_{m_1 m_2}(nm_1 1m_2|(n+1)m) \times t^{(n)\alpha_1\alpha_2\cdots\alpha_n}(m_1)t^{(1)\alpha_{n+1}}(m_2) \,. \quad (34)$$

In the canonical rest frame, the relative momentum is along the Z-axis, so the orbital angular momentum is orthogonal to the Z-axis. It means that the orbital angular momentum in canonical rest frame should be $t^{\alpha_1\alpha_2\cdots\alpha_n}(m = 0)$. The orbital angular momentum wavefunction can also be obtained by using the projection operator for completely symmetric and traceless tensor. In this way, it is defined by

$$\tilde{t}^{(n)\alpha_1\alpha_2\cdots\alpha_n} = P^{(n)\alpha_1\alpha_2\cdots\alpha_n\beta_1\beta_2\cdots\beta_n} r_{\beta_1} r_{\beta_2}\cdots r_{\beta_n} \,, \quad (35)$$

where r^α is the relative momentum four-vector and $P^{(n)\alpha_1\alpha_2\cdots\alpha_n\beta_1\beta_2\cdots\beta_n}$ is the projection operator for symmetric traceless tensor. Two methods give the same results, because

$$r^n t^{(n)\alpha_1\alpha_2\cdots\alpha_n}(m = 0) = \tilde{t}^{(n)\alpha_1\alpha_2\cdots\alpha_n} \,, \quad (36)$$

where r is the magnitude of relative momentum.

The helicity coupling amplitude is a scalar which can be constructed from spin wavefunction, orbital angular momentum wavefunction, energy-momentum four-vector of initial and final state particles, metric tensor and Levi–Civita tensor. Now, we will use the technique of covariance analysis to construct the helicity coupling amplitude for the two-body decays which will be used in $J/\psi \to \gamma PP$. We will use the following notations to represent a definite decay process

$$J \to s + \sigma \,, \qquad \eta_J\eta_s\eta_\sigma \,, \quad (37)$$

where J, s and σ represent the spins of initial and final state particles, η_J, η_s and η_σ represent parity of the corresponding particles. Suppose that parity is conserved in the decay process, then the helicity coupling amplitude will have the following symmetry

$$F^J_{\lambda\nu} = \eta_J\eta_s\eta_\sigma(-1)^{J-s-\sigma}F^J_{-\lambda-\nu} \,, \quad (38)$$

and orbital angular momentum quantum number l must satisfy the following relation

$$(-1)^l\eta_J\eta_s\eta_\sigma = +1 \,. \quad (39)$$

The helicity coupling amplitudes which will be used in the analyses of the process $J/\psi \to \gamma PP$ are respectively discussed below.

(i) $\mathbf{1 \to 0 + 1}$, $\boldsymbol{\eta_J\eta_s\eta_\sigma = +1}$

The second particle is photon. Because of parity conservation, the orbital angular momentum quantum number l must be 0 or 2, and $F^1_{0\lambda} = F^1_{0\,-\lambda}$. Because the helicities of photon can only be ± 1, there is only one independent helicity coupling amplitude, it is F^1_{01}. Denote the polarization four-vectors for J/ψ and photon as ϕ and ω respectively. The helicity coupling amplitudes for $l=0$ and $l=2$ are $A_0 = \tilde{\omega}\phi^*$ and $A_2 = (\omega \tilde{t}^{(2)}\phi^*)$ respectively. The total helicity coupling amplitude is the liner combination of them

$$F^1_{0\lambda} = g_0 A_0(\lambda) + g_2 A_2(\lambda)\,.$$

In canonical rest frame, it is

$$F^{(1)}_{01} = g_0 - g_2 r^2/3\,,$$

where g_0 is a scalar.

(ii) $\mathbf{1 \to 2 + 1}$, $\boldsymbol{\eta_J \eta_s \eta_\sigma = +1}$

The second particle is photon. Because of parity conservation, the orbital angular momentum quantum number l must be 0, 2 or 4, and $F^1_{\lambda\nu} = F^1_{-\lambda\ -\nu}$. There are three independent helicity coupling amplitudes, they are F^1_{21}, F^1_{11} and F^1_{01}. Denote the total spin wavefunction as $\psi^{(J)}(J=1,2,3)$, which is constructed from spin-1 and spin-2 wavefunctions. $\psi^{(J)}$ is a third-order tensor. There are five covariant amplitudes

$$A_1 = \phi^{\alpha *}\psi^{(1)\ \beta}_{\beta\alpha}\,, \qquad A_2 = \phi^{\alpha *}\psi^{(1)}_{\alpha\beta\gamma}\tilde{t}^{(2)\beta\gamma}\,,$$
$$A_3 = \phi^{\alpha *}\psi^{(2)}_{\alpha\beta\gamma}\tilde{t}^{(2)\beta\gamma}\,, \qquad A_4 = \phi^{\alpha *}\psi^{(3)}_{\alpha\beta\gamma}\tilde{t}^{(2)\beta\gamma}\,,$$
$$A_5 = \phi^{\alpha *}\tilde{t}^{(4)}_{\alpha\beta\gamma\sigma}\psi^{(3)\beta\gamma\sigma}\,.$$

The total helicity coupling amplitude is the linear combination of them

$$F^1_{\lambda\nu} = g_1 A_1(\lambda\nu) + g_2 A_2(\lambda\nu) + g_3 A_3(\lambda\nu) + g_4 A_4(\lambda\nu) + g_5 A_5(\lambda\nu)\,.$$

In canonical rest frame, they are

$$F^1_{21} = -\frac{7}{10}g_1 + \frac{7}{30}g_2 r^2 + \frac{1}{18}g_3 r^2 + \frac{2}{45}g_4 r^2 - \frac{2}{525}g_5(3+4\gamma_s^2)r^4\,,$$
$$F^1_{11} = \frac{1}{525\sqrt{2}}\gamma_s(-315 g_1 + 105 g_2 r^2 + 70 g_4 r^2 + 48 g_5 r^4)\,,$$
$$F^1_{01} = \frac{1}{1050\sqrt{6}}(-735 g_1 + 245 g_2 r^2 - 175 g_3 r^2 + 280 g_4 r^2 - 72 g_5 r^4 - 96 g_5 \gamma_s^2 r^4)\,,$$

where g_1, g_2, g_3, g_4 and g_5 are scalars.

(iii) $\mathbf{1 \to 4 + 1}$, $\boldsymbol{\eta_J \eta_s \eta_\sigma = +1}$

The second particle is photon. Because of parity conservation, the orbital angular momentum quantum number l must be 2, 4 or 6, and $F^1_{\lambda\nu} = F^1_{-\lambda\ -\nu}$. There are three independent helicity coupling amplitudes, they are F^1_{21}, F^1_{11} and F^1_{01}. Denote the total spin wavefunction as $\psi^{(J)}(J=3,4,5)$, which is constructed from spin-1 and spin-4 wavefunctions. There are five covariant amplitudes

$$A_1 = \phi^{\alpha *}\psi^{(3)\beta}_{\beta\alpha\rho\sigma}\tilde{t}^{(2)\rho\sigma}\,, \qquad A_2 = \phi^{\alpha *}\psi^{(3)}_{\alpha\beta\gamma\rho\sigma}\tilde{t}^{(4)\beta\gamma\rho\sigma}\,,$$
$$A_3 = \phi^{\alpha *}\psi^{(4)}_{\alpha\beta\gamma\rho\sigma}\tilde{t}^{(4)\beta\gamma\rho\sigma}\,, \qquad A_4 = \phi^{\alpha *}\psi^{(5)}_{\alpha\beta\gamma\rho\sigma}\tilde{t}^{(4)\beta\gamma\rho\sigma}\,,$$
$$A_5 = \phi^{\alpha *}\tilde{t}^{(6)}_{\alpha\beta\gamma\rho\sigma\delta}\psi^{(5)\beta\gamma\rho\sigma\delta}\,.$$

The total helicity coupling amplitude is the linear combination of them

$$F^1_{\lambda\nu} = g_1 A_1(\lambda\nu) + g_2 A_2(\lambda\nu) + g_3 A_3(\lambda\nu) + g_4 A_4(\lambda\nu) + g_5 A_5(\lambda\nu)\,.$$

In canonical rest frame, they are

$$F^1_{21} = \frac{1}{69300\sqrt{7}}(-13475 g_1 r^2 - 26950 g_1 \gamma_s^2 r^2 + 3465 g_2 r^4 + 13860 g_2 \gamma_s^2 r^4 + 891 g_3 r^4 + 3564 g_3 \gamma_s^2 r^4 + 1584 g_4 r^4 + 6336 g_4 \gamma_s^2 r^4 - 400 g_5 r^6 - 2880 g_5 \gamma_s^2 r^6 - 640 g_5 \gamma_s^4 r^6)\,,$$
$$F^1_{11} = \frac{1}{10395\sqrt{7}}\gamma_s(-1925 g_1 r^2 - 3850 g_1 \gamma_s^2 r^2 + 495 g_2 r^4 + 1980 g_2 \gamma_s^2 r^4 + 396 g_4 r^4 + 1580 g_4 \gamma_s^2 r^4 + 480 g_5 r^4 + 640 g_5 \gamma_s^2 r^6)\,,$$
$$F^1_{01} = \frac{1}{6930\sqrt{70}}\gamma_s(-2695 g_1 r^2 - 5390 g_1 \gamma_s^2 r^2 + 693 g_2 r^4 + 2772 g_2 \gamma_s^2 r^4 - 297 g_3 r^4 - 1188 g_3 \gamma_s^2 r^4 + 792 g_4 r^4 + 3168 g_4 \gamma_s^2 r^4 - 200 g_5 r^6 - 1440 g_5 \gamma_s^2 r^6 - 320 g_5 \gamma_s^4 r^6)\,,$$

where g_1, g_2, g_3, g_4 and g_5 are scalars.

(iv) $\mathbf{0 \to 0 + 0}$, $\boldsymbol{\eta_J \eta_s \eta_\sigma = +1}$

The orbital angular momentum quantum number is 0. There is only one independent helicity coupling amplitude, it is $F^J = F^J_{00} = g$, where g is a scalar.

(v) $\mathbf{2 \to 0 + 0}$, $\boldsymbol{\eta_J \eta_s \eta_\sigma = +1}$

The orbital angular momentum quantum number is 2. There is only one independent helicity coupling amplitude. The only one covariant amplitude is $A = \phi^{\alpha\beta *}\tilde{t}^{(2)}_{\alpha\beta}$.

The helicity coupling amplitude is proportional to the above covariant amplitude. In the rest frame, it is $F^J = F^J_{00} = g r^2$, where g is a scalar.

(vi) $\mathbf{4 \to 0 + 0}$, $\boldsymbol{\eta_J \eta_s \eta_\sigma = +1}$

The orbital angular momentum quantum number is 4. There is only one independent helicity coupling amplitude.

The only one covariant amplitude is $A = \phi^{\alpha\beta\gamma\delta *}\tilde{t}^{(2)}_{\alpha\beta\gamma\delta}$. The helicity coupling amplitude is proportional to the above covariant amplitude. In the rest frame, it is $F^J = F^J_{00} = gr^4$, where g is a scalar.

4 Sequential Decay

There are three final state particles in the process of $J/\psi \to \gamma PP$. Suppose that this radiative decay is realized through two-body sequential decays

$$\begin{aligned} J/\psi &\to V+X \\ &\qquad \hookrightarrow P+P\,. \end{aligned} \tag{40}$$

The helicity coupling amplitudes which describe the above process is generally written as

$$A_s = cM^{S_{J/\psi}}_{\lambda_\gamma \lambda_X} BW_X(s,m,\Gamma) M^{S_X}_{0\ 0}\,, \tag{41}$$

where c is a constant, S_X is the spin of the resonance, $M^{S_{J/\psi}}_{\lambda_\gamma\lambda_X}$ is the decay amplitude for the process $J/\psi \to VX$, $M^{S_X}_{0\ 0}$ is the decay amplitude for the process $X \to PP$, $BW_X(s,m,\Gamma)$ is the Breit–Wigner propagator for the resonance X,

$$BW_X(s,m,\Gamma) = \frac{-m\Gamma_{el}}{s-m^2+\mathrm{i}m\Gamma}\,. \tag{42}$$

In the above relation, m and Γ are the mass and width of the resonance X, Γ_{el} is the partial width of the resonance, s is the invariant mass of two final state particles. The $M^{S_a}_{\lambda_b\lambda_c}$ is used to represent the decay amplitude of the following process

$$\begin{aligned} a &\to b+c\,, \\ S_a^{\eta_a} &\to S_b^{\eta_b} + S_c^{\eta_c}\,. \end{aligned} \tag{43}$$

It is

$$M^{S_a}_{\lambda_b\lambda_c} = F^{S_a}_{\lambda_b\lambda_c} D^{S_a}_{\lambda_a(\lambda_b-\lambda_c)}\,, \tag{44}$$

where λ_a, λ_b and λ_c are helicities of the corresponding particles.

In this process, there usually are several resonances which have different spins and parities. Besides, there are background events. The total differential cross section which describes the whole process is

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi} = \sum_{\lambda_{J/\psi}\lambda_\gamma} |A_0+A_2+A_4+\cdots|^2 + \mathrm{BG}\,, \tag{45}$$

where A_s represents the total amplitude of spin s, Φ represents all angular arguments and BG represents the contribution from background.

Maximum likelihood method is used to fit the whole mass spectrum. The normalized probability density function which is used to describe the whole decay process is

$$f(x,\alpha) = \frac{\mathrm{d}\sigma}{\mathrm{d}\Phi} \Big/ \sigma\,, \tag{46}$$

where x represents a set of quantities which are measured by experiments, α represents some unknown parameters which are needed to be determined. σ is the total cross section

$$\sigma = \int W(\Phi)\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi}\mathrm{d}\Phi\,, \tag{47}$$

where $W(\Phi)$ is the integration weight. The total cross section can be determined by Monte Carlo integration

$$\sigma = \frac{1}{N_{\mathrm{mc}}}\sum_{i=1}^{N_{\mathrm{mc}}}(|A_0+A_2+A_4+\cdots|^2+\mathrm{BG})\,, \tag{48}$$

where N_{mc} is the total number of Monte Carlo events. It is required that these Monte Carlo events are obtained through real detector simulation and they have passed all cut conditions which are used to obtain the data sample of this process.

Maximum likelihood function is given by the adjoint probability density for all data. It is

$$\mathcal{L} = \prod_{i=1}^{N_{\mathrm{event}}} f(x,\alpha)\,. \tag{49}$$

Then we define

$$S = -\ln\mathcal{L}\,. \tag{50}$$

In our data analysis, the goal is to find the set of values, α, which minimize S.

5 Conclusion

In the global helicity analysis, not only the information about the mass spectrum, but also the information about the angular distribution are used. Because more information is used, the results obtained by this method are more reliable. This method is used in BES physics analysis. It has the following three advantages:

(i) Supposed that there are some peaks in the mass spectrum which are not real resonances. If we use the conventional Breit–Wigner function to fit them, we cannot differentiate them from the real resonances. But if we use the global helicity analysis to fit the whole spectrum (Including mass spectrum and angular spectrum), we can differentiate them by mass and width scan. Those peaks caused by statistical fluctuations have no structure in the mass and width scan.

(ii) In the traditional Breit–Wigner fitting, we usually need to divide the whole mass spectrum into several bins. And the results sometimes depend on the manner how we separate the mass spectrum. But in the global helicity analysis, we do global fitting to the whole spectrum. So, our results are independent of the manner how we separate the mass spectrum and are not affected by small statistical fluctuations.

(iii) In the global helicity analysis, the interference between various resonances is considered in the fitting. And because of destructive interference, a resonance sometimes shows a dip, rather than a peak, in the mass spectrum. This phenomenon is found in our fitting.[9,10]

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