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Test of the Possible Application of the Half-Way Bounce-Back Boundary Condition for Lattice Boltzmann Methods in Complex Geometry*

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Abstract *The way of handling boundary conditions with simple bounce-back rule in the lattice gas and lattice Boltzmann method had been considered as one of the advantage compared with other numerical schemes. The half-way bounce-back rule inherits the advantage of the bounce-back rule and improves the accuracy to the second-order on flat boundaries. In this paper, we test the possible application of the half-way bounce-back rule to the system with complex geometry. Our simulation results show that the half-way bounce-back rule is a good boundary condition in the problems without emphasis on accuracy.*

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1 Introduction

In recent years, the lattice Boltzmann method (LBM)[1,2] has been proposed as a new numerical scheme for simulating fluid flows governed by Navier–Stokes (NS) equations. Unlike the traditional numerical scheme, the LBM is based on the microscopic kinetic equation for the particle distribution function. This scheme is distinguished from other methods for its linear convection operator (or streaming process) in phase space. The incompressible Navier–Stokes equations can be obtained in the nearly-incompressible limit in the LBM.

Till now, much progress has been achieved in the application of the LBM.[3] Many models have been proposed to simulate a wide variety of physical systems such as thermal flow,[4] viscoelastic media,[5] multi-phase flow[6] and flow in the blood vessels.[7] And also the LBM for irregular grids has been developed.[8]

Boundary condition is very important to obtain accurate physical results, especially for the systems with complex geometry or multi-component fluids. In the early period the boundary conditions in LBM have been directly adopted from the lattice gas automaton method.[9] For example, a distribution function bounce-back scheme is used at walls to obtain no-slip velocity condition. By the bounce-back scheme we mean that when a particle reaches a wall node, the particle will scatter back to the fluid nodes just along its coming direction. In recent years, it has been noticed that the bounce-back scheme actually only gives first-order accuracy in boundaries.[10,11] At the same time, several boundary treatments have been proposed for achieving second-order accuracy for no-slip velocity boundary conditions.[10,12−18] The numerical demonstration of these methods has been quite successful in simulating flows with flat-wall boundaries. However, they are difficult to implement for general geometries, because there is a need to distinguish distribution functions according to their orientation to the wall, and there are additional or different treatments at corner nodes. On the other hand, the bounce-back boundary condition seems to provide a computational efficient method for a curved surface. In 1997, Martha[11] used the bounce-back boundary condition to study the flow through a two-dimensional periodic array of infinite parallel cylinders. They find that the error inherent in the bounce-back method is higher than that associated with the consistent hydrodynamic boundary condition and the rate of convergence is slower. Ziegler[19] understood that the bounce-back scheme with the wall located half-way between a flow node and a bounce-back node (It will be called half-way wall bounce-back thereafter) produces the results of the second-order accuracy for the simple flows considered. The half-way bounce-back rule inherits the advantage of the bounce-back rule that it is very easy to be implemented in a computer code. A direct question is that whether the half-way bounce-back rule can be applied to the system with complex geometry.

In this paper, we will take the fluid flow in an inclined tube as an example to test the accuracy by making use of the half-way bounce-back boundary conditions in the system with complex geometry. The simulation results can fit analytical results approximately. However, the accuracy is not high and the real boundary corresponding to the zero velocity cannot be predetermined. Consider the simplicity of the technique, we conclude that the half-way bounce-back rule is a good boundary condition to be applied in the problems without emphasis in accuracy.

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This paper is organized as follows. Section 2 is devoted to basic theory we used. The simulations on the flows in an inclined tube are carried out in Sec. 3. Conclusion is presented in Sec. 4.

2 Basic Theory

The LBM is a kinetic-theory-based technique for modeling fluid flow and is formulated in terms of the probability of the existence of a fluid particle in the vicinity of a given location and time that is moving in one of a number of discrete directions. We take the case on a square lattice in two dimensions as an example. Let $f_i(\boldsymbol{x}, t)$ be a non-negative real number describing the distribution function of the fluid density at site $\boldsymbol{x}$ at time t moving in direction $\boldsymbol{e}_i$. Here $\boldsymbol{e}_0 = (0, 0)$, $\boldsymbol{e}_i = (\cos\pi(i-1)/2, \sin\pi(i-1)/2)$, $i = 1, 2, 3, 4$, and $\boldsymbol{e}_i = (\cos\pi(i-4-\frac{1}{2})/2, \sin\pi(i-4-\frac{1}{2})/2)$, for $i = 5, 6, 7, 8$ are the nine possible velocity vectors (See Fig. 1). The distribution functions evolve according to a Boltzmann equation that is discrete in both space and time,

$$f_i(\boldsymbol{x} + \boldsymbol{e}_i, t + 1) - f_i(\boldsymbol{x}, t) = \Omega_i(\boldsymbol{x}, t)\,. \tag{1}$$

The most convenient choice for $\Omega_i(\boldsymbol{x}, t)$ is a single time relaxation form[1,2]

$$\Omega_i(\boldsymbol{x}, t) = -\frac{1}{\tau}(f_i - f_i^{\rm eq})\,. \tag{2}$$

The density ρ and macroscopic velocity $\boldsymbol{u}$ are defined by

$$\rho = \sum_i f_i, \qquad \rho\boldsymbol{u} = \sum_i f_i\boldsymbol{e}_i\,, \tag{3}$$

and the equilibrium distribution functions $f_i^{\rm eq}$ are usually supposed to be dependent only on the local flow velocity $\boldsymbol{u}$. A suitable equilibrium distribution function has been proposed with

$$f_i^{\rm eq} = \rho\alpha_i\Big[1 + 3\boldsymbol{e}_i\cdot\boldsymbol{u} + \frac{9}{2}(\boldsymbol{e}_i\cdot\boldsymbol{u})^2 - \frac{3}{2}u^2\Big]\,, \tag{4}$$

where $\alpha_0 = 4/9$, $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 1/9$, and $\alpha_5 = \alpha_6 = \alpha_7 = \alpha_8 = 1/36$. The macroscopic equations recover the Navier–Stokes equations by a Chapman–Enskog procedure.[1,2]

Now let us recall the basic idea of the half-way bounce-back boundary conditions. In Fig. 2, the node A is on the physical boundary, nodes B, C and D are on the first layer above the boundary. Particles will arrive at node A from nodes B, C and D. Whenever a particle going in direction-8 arrives from node B, a direction-6 particle is send back to node B in the following time step, and similarly for direction-4 particles from node C and direction-7 particles from node D. Consequently, the time average of the population at node A has an equal number of direction-8 and direction-6 particles, direction-4 and direction-2 particles and direction-7 and direction-5 particles, and so the average velocity at node A is zero. This result is the basic of the logic of using direct reflection at the wall. But it is clearly different from the intended result of having a no-slip wall at A.

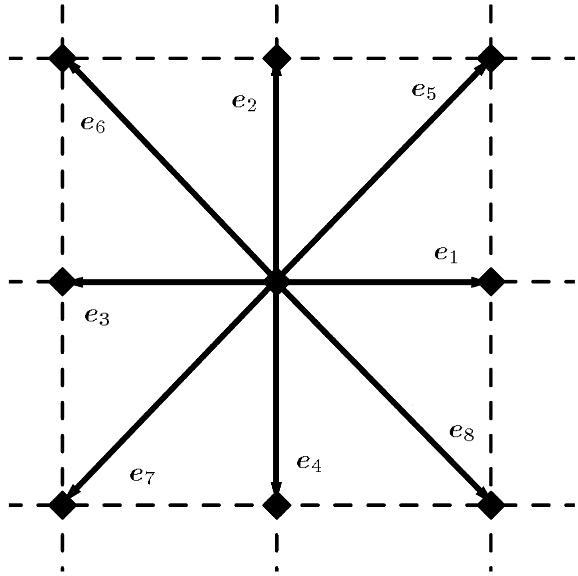


Fig. 1 Basic cell for the two-dimensional "9-speed" lattice Boltzmann model.

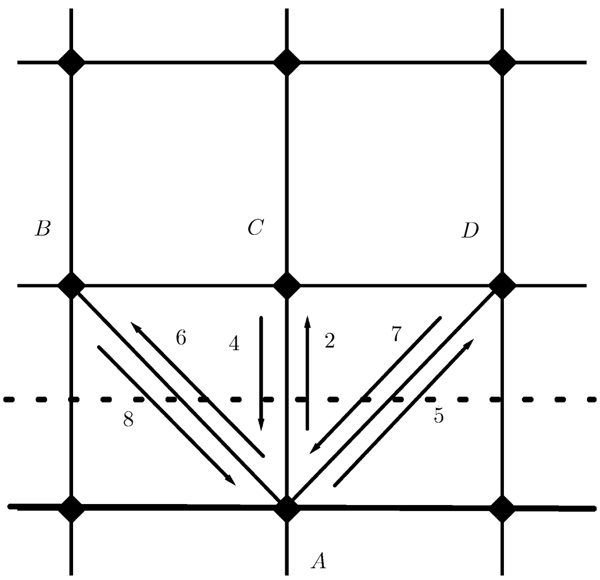


Fig. 2 Sketch map for bounce-back and half-way bounce-back boundary conditions.

The alternate interpretation is that the "bounce-back" condition corresponds to a zero-velocity boundary condition that is applied at a wall half-way between nodes A and B-C-D (the dotted line in Fig. 2). The velocity at A in the opposite direction is the symmetric reflection of the state at B-C-D through the no-slip wall half-way between. This is a major improvement in the understanding of these simulations that permits use of the same general algorithm and gives much improved accuracy in the interpretation of the flow near the walls.

3 Numerical Simulations

In order to exploit the accuracy of the half-way bounce-back boundary conditions on complex geometry, we perform simulation on viscous flow in inclined tubes, figure 3 is an example for $\tan\theta = 1/2$. We can see that if we apply the half-way bounce-back boundary condition, the boundaries are the staircase boundaries as shown in Fig. 3. The analytical solution for the velocity profile is known and given by

$$u_j = u_0\Big(1 - \frac{j^2}{d^2}\Big)\,, \tag{5}$$

where u_j is the component of the velocity vector along the flow direction at a distance j from the center of the tube, d is the radius of the tube, and u_0 is the maximal velocity, which relates to the pressure difference Δp between the inlet and outlet.

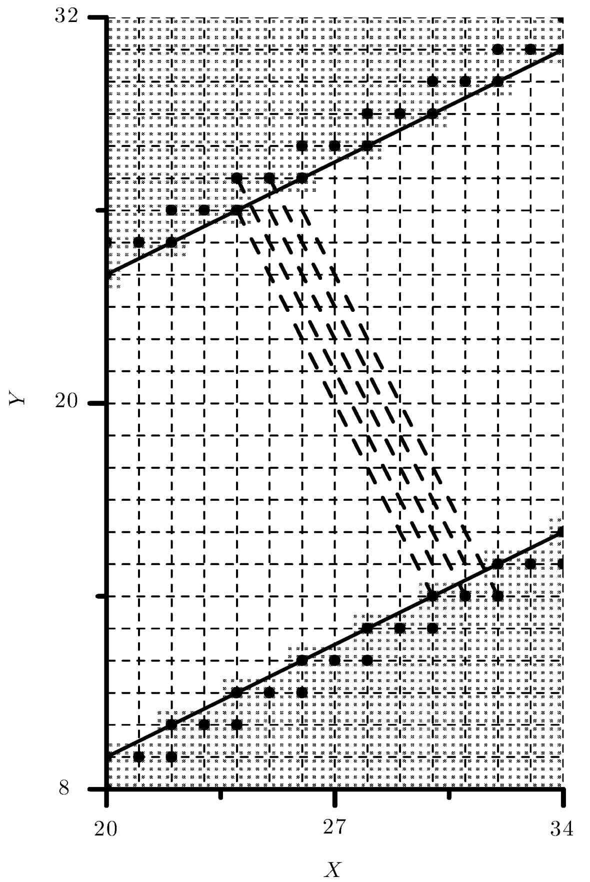


Fig. 3 Part of the inclined tube for an inclination angle $\theta = \arctan(1/2)$. The solid lines are the boundaries of the tube. The black dots are the boundary nodes. If we apply the half-way bounce-back rule, the unshaded part represents the tube filled with fluid. The heavy dashed line is a reference line for measuring the velocity.

In this paper, we will take the viscous flow in an inclined tube as shown in Fig. 3 for example to study the accuracy of the half-way bounce-back boundary condition. In order to minimize the error due to the inlet and outlet, we consider the inclined tube with $\theta = \tan^{-1}(k/l)$, where k is a non-negative integer, l is a positive integer, and $l \le k$. We assume that the relative distributions f_i/f_0 for $i = 1, 2, \cdots, 8$ at a node $\boldsymbol{x}$ at inlet (or outlet) are equal to those at the node $\boldsymbol{x} + (l, k)$ (or $\boldsymbol{x} - (l, k)$) in the fluid domain, respectively.

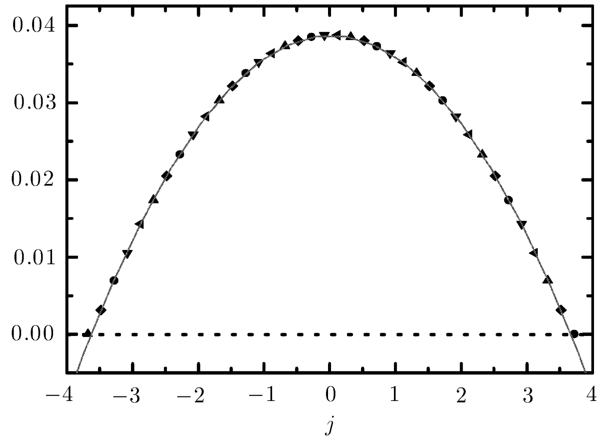
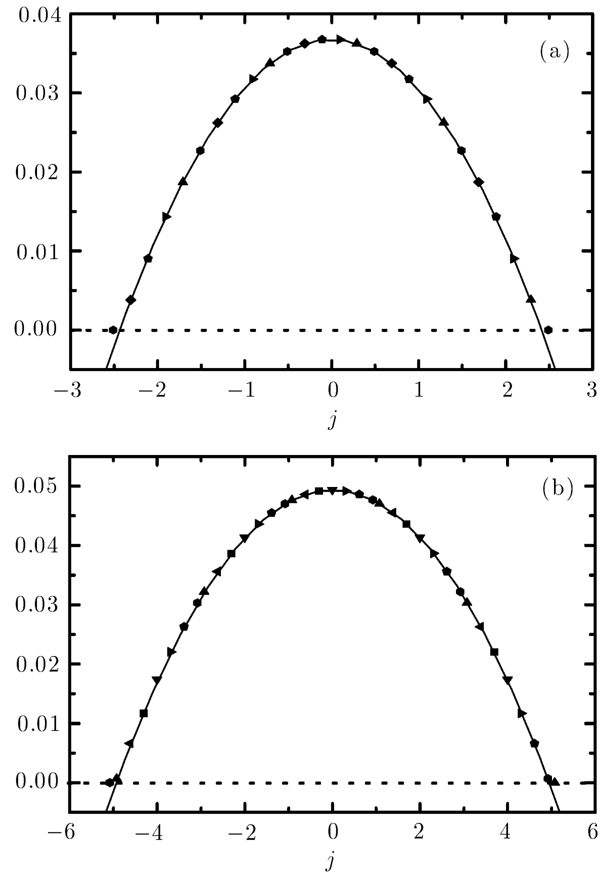


Fig. 4 The velocity profile of the inclined tube for an inclination angle $\theta = \arctan(1/2)$ obtained by applying the half-way bounce-back boundary condition. The solid line is the analytical result. The filled squares, circles, diamonds, uptriangles and the downtriangles are simulation results for the nodes on the heavy dashed lines shown in Fig. 3, from left to right, respectively.



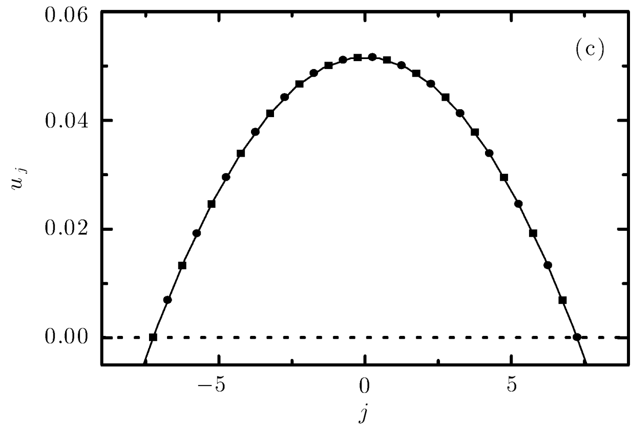


Fig. 5 The same as Fig. 4 except $\theta = \arctan(1/3)$, $\theta = \arctan(2/3)$, $\theta = \arctan(1/1)$ for (a), (b), (c), respectively.

We have performed simulation on the viscous flow in the inclined tube with $\tan\theta = \frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{1}{1}$, the results are shown in Fig. 4 and Figs 5a, 5b, 5c, the solid lines are the best fit parabola to the simulated results. It is clear that the numerical results are consistent with the parabola approximately. However, the situation is different for different inclined angles. Take the case with $\tan\theta = 1/2$ for example, we have indicated five vertical lines in Fig. 3. Any other fluid node can be mapped to one of the fluid node on these lines with the same distance j from the center of the tube. Consequently, we only show the velocities of all the fluid nodes on these lines in Fig. 4. It is clear that some of the points, especially the point with $u_j = 0$ departure from the parabola considerably. Moreover, the real boundary corresponding to the zero velocity cannot be predetermined as that on a flat boundary. Similar behavior can be observed for other angle θ except $\tan\theta = \frac{1}{1}$, for which all the numerical data can be fit to the parabola quite well. This can be seen from the intersection point of the parabola and the dashed line in these figures.

4 Conclusion

The way of handling boundary conditions with single bounce-back rule in the lattice gas and LBM had been considered as one of the advantage compared with other numerical schemes. This technique has been used as a major argument to support the idea that the lattice gas and lattice Boltzmann models are ideal models for simulating fluid flows in complicated geometries. The half-way bounce-back rule inherits the advantage of the bounce-back rule and improves the accuracy to the second-order on flat boundaries. In this paper, we test the possible application of the half-way bounce-back rule to the system with complex geometry. We take the viscous flow in an inclined tube as an example. The numerical results can fit to the parabola approximately. However, the accuracy is not high and the real boundary corresponding to the zero velocity cannot be predetermined. Consider the simplicity of the technique, we conclude that the half-way bounce-back rule is a good boundary condition to be applied in the problems without emphasis in accuracy.

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