

Synchronization in a Mutualism Ecosystem Induced by Noise Correlation*

ZHONG Wei-Rong,^{1,2} SHAO Yuan-Zhi,^{1,†} BIE Meng-Jie,¹ and HE Zhen-Hui¹

¹School of Physics & Engineering, Sun Yat-sen University, Guangzhou 510275, China

²Centre for Nonlinear Studies, Hong Kong Baptist University, Kowloon Tong, Hong Kong, China

(Received October 29, 2007)

Abstract Understanding the cause of the synchronization of population evolution is an important issue for ecological improvement. Here we present a Lotka–Volterra-type model driven by two correlated environmental noises and show, via theoretical analysis and direct simulation, that noise correlation can induce a synchronization of the mutualists. The time series of mutual species exhibit a chaotic-like fluctuation, which is independent of the noise correlation, however, the chaotic fluctuation of mutual species ratio decreases with the noise correlation. A quantitative parameter defined for characterizing chaotic fluctuation provides a good approach to measure when the complete synchronization happens.

PACS numbers: 87.23.Cc, 05.40.-a, 02.50.Ey

Key words: environmental noise, correlation, synchronous fluctuation, chaos, population dynamics, mutualism

Synchronization is a common phenomenon in nature,^[1] e.g., in physical systems,^[2] chemical systems^[3] and biosystems.^[4] Specially, the population synchrony has become an important issue in population biology.^[5,6] Additionally, throughout the twentieth century, a dominant study has focused on the stochastic fluctuation in population abundance.^[7,8] Interestingly, synchronization and stochasticity regarded as two independent phenomena attract more and more concerns.^[9–11] Recently, Benton *et al.*^[5] suggested that environmental noise plays a determined role in the population synchrony.

Regrettably, due to the difficulties in mathematical analyses and experimental demonstrations, the study of relationship between noise correlation and population synchrony is still insufficient. However, surprising phenomena emerge sometimes just because there exists correlation or collaboration; one of the most well-known examples is the self-organized behavior.^[12] We know that two stochastic processes in ecosystems may originate from the same source like nature disasters and epidemics. For example, epidemics can lead one kind of species in a mutualism system as well as another to death synchronously, thus it is reasonable to suppose that the population fluctuations of the two species of mutualism system are correlated.

Mutualism is an association between two species that benefits both. For example, Van der Heijden *et al.*^[13] have argued that mycorrhizal fungi determine plant species diversity. As the number of fungal species per system is increased, the collective biomass of shoots and roots increases, so does the diversity of plant community. Namely the number of two species has a positive linear relationship, indicating a synchronous fluctuation. For this reason, we focus on the Lotka–Volterra-type two-species model with correlated stochastic components. The noise

correlation we will consider here induces some novel synchronous phenomena in a mutualism ecosystem that are not found before. Our main methods will be expected to have applicability to other ecosystems, since they have been also of similar correlations of two populations.

Additionally, noise-induced chaos is observed in many experimental and theoretical situations.^[14–16] Can noise induce chaos in a mutualism ecosystem? How does this happen? When is the chaos synchronization? In this paper, we will give physical explanations for these problems.

Mutualistic relationships can be modeled with equations similar to the Lotka–Volterra competition equations,

$$\frac{dx}{dt} = r_1x - k_1x^2 + \gamma_1xy, \quad (1)$$

$$\frac{dy}{dt} = r_2y - k_2y^2 + \gamma_2xy, \quad (2)$$

where γ_1 and γ_2 are the positive effects of species y on species x and of species x on species y , respectively. r_1 and r_2 are respectively growth rate of species x and that of species y . k_1 and k_2 are the parameters related to carrying capacities while one of the mutualists is alone. Due to environmental noise like epidemics, natural disasters and adventitious species, realistic mutualism systems do not follow the solutions of the above deterministic differential equations exactly. The simplest way to include these environmental noises is to add stochastic terms to the above deterministic differential equations and the equivalent stochastic differential equations are

$$\frac{dx}{dt} = r_1x - k_1x^2 + \gamma_1xy + x\xi(t), \quad (3)$$

$$\frac{dy}{dt} = r_2y - k_2y^2 + \gamma_2xy + y\eta(t), \quad (4)$$

*The project partially supported by National Natural Science Foundation of China under Grant No. 60471023

†Corresponding author, E-mail: stssyz@mail.sysu.edu.cn

where $\xi(t)$ and $\eta(t)$ are referred to as the birth rate fluctuations of species x and those of species y , respectively. Considering the synchronization of the two fluctuations, we define them as correlated noises. They are Gaussian white noises satisfying

$$\langle \xi(t)\xi(t') \rangle = 2M_x\delta(t-t'), \quad (5)$$

$$\langle \eta(t)\eta(t') \rangle = 2M_y\delta(t-t'), \quad (6)$$

$$\langle \xi(t)\eta(t') \rangle = 2\lambda\sqrt{M_xM_y}\delta(t-t'), \quad (7)$$

in which M_x and M_y are the intensities of noises, λ ranges from zero to one, denoting the correlation coefficient between $\xi(t)$ and $\eta(t)$, i.e., the correlation degree between species x and species y . When λ equals zero, the growth rate fluctuations of species x and those of species y are independent. When λ equals 1, their growth rate fluctuations are completely correlated. $\delta(t-t')$ is a Dirac delta function under different moments.

In the absence of noises, it can be shown that the system has a stable equilibrium state (x_0, y_0) when $t \rightarrow \infty$ for $\gamma_1\gamma_2 < k_1k_2$, where $x_0 = (k_2r_1 + \gamma_1r_2)/(k_1k_2 - \gamma_1\gamma_2)$ and $y_0 = (k_1r_2 + \gamma_2r_1)/(k_1k_2 - \gamma_1\gamma_2)$, describes a limit point of system.^[17] For the sake of simplicity, we set $k_1 = k_2 = r_1 = r_2 = 1$ and $\gamma_1 = \gamma_2 = 0.5$. Figure 1 shows that the system can lead to a stable point where the isoclines cross. In the presence of noises, we define quantitative parameters, i.e., the mean population densities of the mutualists, which are useful to displaying the influences of noises on the mutualism ecosystem. The mean population densities of the mutualists are respectively written as

$$\langle x \rangle = \int_0^\infty \int_0^\infty xp(x, y) dx dy, \quad (8)$$

$$\langle y \rangle = \int_0^\infty \int_0^\infty yp(x, y) dx dy, \quad (9)$$

and their variances are $\sigma_x^2 = \langle x^2 \rangle - \langle x \rangle^2$ and $\sigma_y^2 = \langle y^2 \rangle - \langle y \rangle^2$, respectively. Here $p(x, y)$ is the joint stationary probability distribution of species x and y satisfying $\int_0^\infty \int_0^\infty p(x, y) dx dy = 1$.

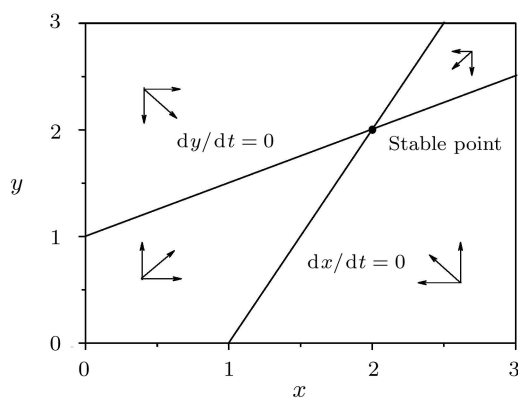


Fig. 1 Graphic model of the mutualism ecosystem is based on modified Lotka–Volterra equations for $k_1 = k_2 = r_1 = r_2 = 1$ and $\gamma_1 = \gamma_2 = 0.5$. There is a stable equilibrium point.

The equivalent stationary Fokker–Planck equations of Eqs. (3) and (4) can be derived and the joint stationary probability distributions can be obtained for two independent noises.^[18] However, they are far difficult to be solved for correlated noises, here we obtain the joint stationary probability distributions of the population densities of the mutualists via a direct simulation of Eqs. (3) and (4).^[19–22]

To describe noise-induced chaos and the relationship between chaos and synchrony, we need to define chaos carefully. For example, given the time series $\{x(t)\}$, an m -dimensional phase portrait is reconstructed with delay coordinates, i.e., a point on the attractor is given by $\{x(t), x(t+\tau), \dots, x(t+(m-1)\tau)\}$, with m being the embedding dimension and τ being the delay time (here we choose $m = 2$ and $\tau = 2$).^[16,23] We denote the initial distance between these two points $L(t_0)$. At a later time t_1 , the initial length will have evolved to length $L'(t_1)$. Using Wolf algorithm,^[23] we can give Lyapunov exponent as follows:

$$\Lambda_x = \left\langle \frac{1}{t_N - t_0} \sum_{k=1}^N \ln \frac{L'(t_k)}{L(t_{k-1})} \right\rangle, \quad (10)$$

in which N is total number of replacement steps.

It is comprehensible that noises can induce instability. Figure 2(a) illustrates that the populations of species x and those of species y driven by environmental noise are instable. However, when the noise correlation increases, shown in Figs. 2(b) and 2(c), more and more populations of species x and those of species y concentrate on a line, in which the ratio of species y to species x is constant. When the correlation equals 1, the ratio of species y to species x is independent of the noise, i.e., the population fluctuations of species y and those of species x are synchronization. This extreme situation can be analyzed from Eqs. (3) and (4). If the noise correlation equals 1, $\xi(t)$ of Eq. (3) can be regarded as same as $\eta(t)$ of Eq. (4) for the same noise intensity. If considering

$$\frac{d(y/x)}{dt} = \frac{1}{x} \frac{dy}{dt} - \frac{y}{x^2} \frac{dx}{dt},$$

and substituting Eqs. (3) and (4) for it, we obtain

$$\frac{d(y/x)}{dt} = (k_1 + \gamma_2)y - (k_2 + \gamma_1)\frac{y^2}{x}. \quad (11)$$

Considered a steady solution $d(y/x)dt = 0$, then equation (11) can be written as

$$\frac{y}{x} = \frac{k_1 + \gamma_2}{k_2 + \gamma_1} = 1,$$

i.e., the populations of species x have a positive linear relationship with those of species y , shown in Fig. 2(c).

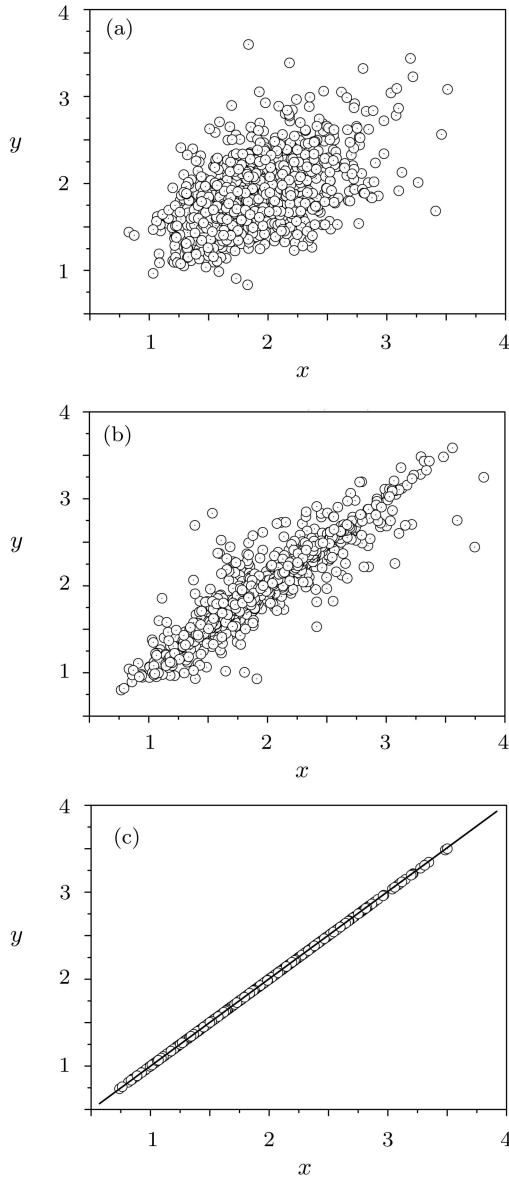


Fig. 2 Distributions of the mutualists driven by environmental noise with different correlations. (a) $\lambda = 0.0$; (b) $\lambda = 0.8$; (c) $\lambda = 1.0$, including simulated results (circle) and theoretical analysis (solid line) results. The parameter are $M_x = M_y = 0.8$.

The linear relationships between the variance of y/x and the noise correlation are given in Fig. 3, indicating the variance of y/x decreases with the noise correlation. When the correlation equals 1, the fluctuations of the ratio of species y to species x are extinct.

Figure 4 shows that the Lyapunov exponents of species y and those of species x fluctuate between 0.2 and 0.35 when the noise correlation increases, however, the Lyapunov exponent of the ratio of species y to species x drops

to zero. Generally speaking, when Lyapunov exponent is positive, the time series are regarded as disorder or chaotic characteristic. On the contrary, if Lyapunov exponent is non-positive, the time series are of order or periodic characteristic. Figure 4 indicates that two disorder time series with correlation can exhibit an order behavior, and the synchronization in a mutualism ecosystem happens at the Lyapunov exponent equals zero.

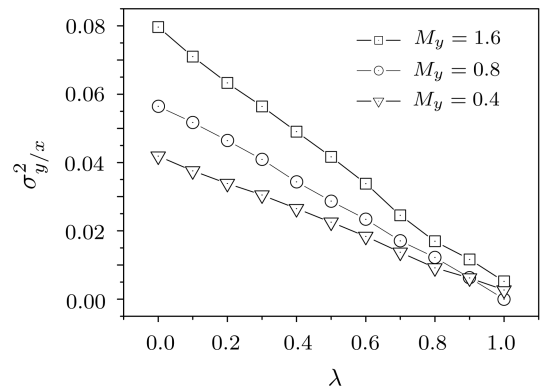


Fig. 3 Variances of the ratio of species y and species x vs. noise correlations under different noise intensities. The remaining parameters are the same in Fig. 2.

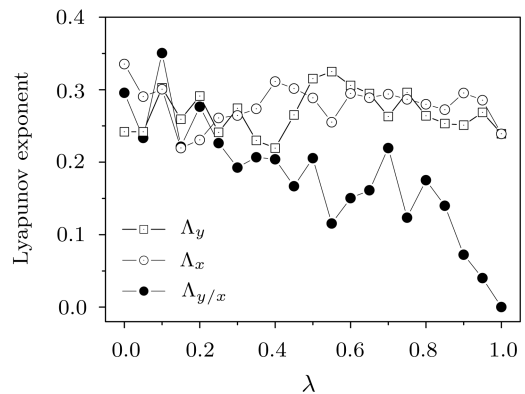


Fig. 4 Lyapunov exponents of species y , species x and their ratio vary with noise correlations. The completely synchronous fluctuation point corresponds to zero-Lyapunov exponent.

In summary, the environmental noise correlation can cause a synchronous fluctuation in mutualism ecosystems, and this synchronous fluctuation can be characterized by the Lyapunov exponent. If environmental noise correlation originates from the correlation degree of the mutualists, our main results illustrate that the mutualism ecosystems with closer relationship have more stable evolutions. This is a helpful illumination for ecologists to study ecosystem balance.

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