

## Holography, UV/IR Relation, Causal Entropy Bound, and Dark Energy\*

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(Received December 27, 2008)

**Abstract** *The constraint on the total energy in a given spatial region is given from holography by the mass of a black hole that just fits in that region, which leads to an UV/IR relation: the maximal energy density in that region is proportional to  $M_p^2/L^2$ , where  $M_p$  is the Planck mass and  $L$  is the spatial scale of that region under consideration. Assuming the maximal black hole in the universe is formed through gravitational collapse of perturbations in the universe, then the “Jeans” scale of the perturbations gives a causal connection scale  $R_{CC}$ . For gravitational perturbations,  $R_{CC}^{-2} = \text{Max}(\dot{H} + 2H^2, -\dot{H})$  for a flat universe. We study the cosmological dynamics of the corresponding vacuum energy density by choosing the causal connection scale as the IR cutoff in the UV/IR relation, in the cases of the vacuum energy density as an independently conserved energy component and an effective dynamical cosmological constant, respectively. It turns out that only the case with the choice  $R_{CC}^{-2} = \dot{H} + 2H^2$ , could be consistent with the current cosmological observations when the vacuum density appears as an independently conserved energy component. In this case, the model is called holographic Ricci scalar dark energy model in the literature.*

**PACS numbers:** 95.36.+x, 04.20.-q, 98.80.-k

**Key words:** holography of gravity, dark energy and cosmology

### 1 Introduction

Since the discovery of accelerating expansion of the universe by observing distant supernova,<sup>[1,2]</sup> the nature of dark energy has been one of the hottest issues in cosmology and theoretical physics. Now a lot of astronomical observations indicate that the energy budget of the universe consists of approximately 4% baryon matter, 23% dark matter, 73% dark energy, and negligible radiation. The simplest and economic way to explain the accelerating expansion of the universe is due to a tiny positive cosmological constant introduced first by Einstein himself in 1917. It is well-known that the cosmological constant and the vacuum expectation value of some quantum fields are undistinguished. Thus, the cosmological constant acting as the solution of the dark energy suffers from the so-called fine tuning problem: what is the physical mechanism that sets the value of the cosmological constant to its current observed value, which is 120 orders of magnitude smaller than the naive theoretical expectation. Also there exists the so-called coincidence problem for dark energy: why does the cosmological constant dominate the universe just recently. In other words, the coincidence problem can also be expressed as follows. Why the energy densities of dark energy and dark matter are comparable just recently?

Since the cosmological constant is entangled with the vacuum expectation value of some quantum fields, the cosmological constant problem therefore is essentially an issue of quantum gravity. Indeed, general relativity together

with quantum field theory could shed some lights on the cosmological constant problem.

It is widely believed and precisely tested that particle physics can be accurately described by an effective field theory with an ultraviolet (UV) cutoff less than the Planck mass  $M_p$ , provided that all momenta and field strengths are small compared with this cutoff to the appropriate power.<sup>[3]</sup> This is indeed the case in the absence of gravity. When gravity effect is taken into account, however, something very strange appears. Black hole thermodynamics tells us that a black hole has an entropy proportional to its horizon area. In Einstein gravity theory, black hole entropy satisfies the so-called area formula,  $S = A/4G$ , where  $A$  is the horizon area of the black hole and  $G$  is the Newtonian constant. One learns from statistical physics that entropy of a system describes the number of microscopic degrees of freedom of the system, and that entropy is an extensive quantity and is always proportional to volume of the system. For a black hole, entropy is proportional to its area. This implies that for a gravity system, its effective degrees of freedom are drastically reduced, compared to the same system without gravity. This indicates that the underlying theory describing the nature must be not a local quantum field theory. In other words, there exists a range of validity for a local effective field theory to describe a system with gravity. This leads to it unbelievable that the cosmological constant should be in the order of Planck mass, obtained from the naive estimate of local effective field theory without taking into account the gravity effect.

\*The project supported in part by the Chinese Academy of Sciences under Grant No. KJCX3-SYW-N2 and National Natural Science Foundation of China under Grant Nos. 10821504 and 10525060

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To realize this argument, Cohen *et al.*<sup>[3]</sup> proposed a relation between UV and infrared (IR) cutoffs. To be self-contained, let us briefly repeat some key steps to the relation. For an effective quantum field theory confined in a box of size  $L$  with UV cutoff  $\Lambda$ , the entropy should scale extensively as  $S \sim L^3 \Lambda^3$ . However, black hole thermodynamics leads Bekenstein to argue that the maximal entropy of the box with volume  $L^3$  scales as its area, instead of the volume  $L^3$ . The Bekenstein entropy bound could be satisfied for an effective local quantum field theory if the following inequality is obeyed

$$L^3 \Lambda^3 < S_{\text{BH}} = \pi L^2 M_p^2, \quad (1)$$

where  $S_{\text{BH}}$  is the entropy of a black hole with horizon radius  $L$ . One can see from (1) that the length  $L$  acting as an IR cutoff is no longer independent of the UV cutoff, but scales as  $M_p^2/\Lambda^3$ . However, as argued by Cohen *et al.*, there is evidence that conventional quantum field fails at an entropy well below the bound (1). They gave a stronger constraint on the IR cutoff, which excludes all states residing within their Schwarzschild radius. Note that the maximal energy density in the effective theory is  $\Lambda^4$ , the mass in a box with volume  $L^3$  is  $\Lambda^4 L^3$ . Assuming that the mass is less than the mass of a black hole with radius  $L$  leads to the following constraint

$$\Lambda^4 L^3 \leq LM_p^2, \quad (2)$$

where the IR cutoff scales as  $M_p/\Lambda^2$ . This bound is more restrictive than Eq. (1). To see this, let us consider the case where Eq. (2) is nearly saturated. In that case, the entropy is  $S_{\text{max}} \simeq S_{\text{BH}}^{3/4}$ , which is less than the black hole entropy  $S_{\text{BH}}$ .

The UV/IR relation (2) leads to a very interesting consequence on the cosmological constant problem. If the effective local quantum field theory is valid in an arbitrarily large volume up to the Planck mass, the contribution of the vacuum energy density to the cosmological constant is  $\sim (10^{19} \text{ GeV})^4$ . If SUSY exists and is broken at energy scale  $\sim \text{TeV}$ , then the contribution of the vacuum energy density to the cosmological constant is  $\sim (\text{TeV})^4$ . On the other hand, if the bound (2) plays a role, then the contribution of the vacuum energy density to the cosmological constant is  $M_p^2/L^2$ . If choosing the IR cutoff as the current horizon size of the universe, one has  $\Lambda^4 \sim M_p^2/L^2 \sim (10^{-3} \text{ eV})^4$ . This is exactly in the same order as the observed dark energy scale.

However, as found by Hsu,<sup>[4]</sup> if one takes the current Hubble horizon as the IR cutoff in Eq. (2), although the energy density  $\rho \sim \Lambda^4$  can match the dark energy density of the universe, it cannot make the universe accelerating expansion since its equation of state is the same as the one for dark matter. Li<sup>[5]</sup> found that the particle horizon of the universe also cannot take the job, instead the event horizon of the universe acting as the IR cutoff can derive the universe to accelerating expand, and the vacuum energy density in this case can fit the data well. However, event horizon is a global concept of spacetime, the event horizon of the universe is determined by future

evolution of the universe. As a result, it is not easy to understand why the current dark energy density is determined by future evolution of the universe, rather than the past of the universe.<sup>[6]</sup> In Ref. [6], combining general relativity and uncertainty relation in quantum mechanics, we argued that the energy density of quantum fluctuations of spacetime could act as the dark energy currently observed. The dark energy density is characterized by the age of the universe, and could be consistent with astronomical data if the unique numerical parameter in this model is taken to be a number of order unity. But it was found that it is not consistent with the evolution history of our universe that there is a matter dominated decelerated phase in the past. Several ways out have been proposed such as considering interaction between dark energy and dark matter,<sup>[7]</sup> and replacing the age of the universe by the conformal time of the universe,<sup>[8]</sup> etc. For further considerations see Ref. [9], for example.

In this paper, considering black hole in the universe is formed by gravitational collapse of perturbations of cosmological spacetime and the ‘‘Jeans’’ length of the perturbations sets a causal connection scale, beyond which black hole cannot formed very likely, we study the cosmological dynamics of the vacuum energy density by use of the causal connection scale as the IR cutoff in (2). Here we would like to mention that in fact, the UV/IR relation (2) does not resolve the cosmological constant problem, since as a pure constant energy, black hole cannot form without fluctuations of energy.

## 2 Holography and Causal Entropy Bound

Given a closed system with fixed energy  $E$ , which fits in a sphere with radius  $R$  in three spatial dimensions, what is the maximal entropy of the system? Bekenstein was the first to consider this issue. Based on black hole thermodynamics, he argued there exists an upper bound on the entropy of the system<sup>[10]</sup>

$$S \leq S_B = 2\pi ER = \pi M_p^2 R_g R, \quad (3)$$

where  $R_g = 2E/M_p^2$  is the Schwarzschild radius of the system. This bound is called Bekenstein entropy bound. This bound is believed to be universal valid for a system with limited self-gravity, which means that the gravitational self-energy is negligibly small compared to its total energy  $E$ . However, it is interesting to note that the bound is saturated even for a four dimensional Schwarzschild black hole which is a strongly self-gravitating object (note that it is no longer saturated for higher dimensional ( $D > 4$ ) Schwarzschild black holes). In addition, it is worth mentioning here that although the Bekenstein bound (3) is derived from black hole thermodynamics and generalized second law, it is independent of gravitational theory and spacetime dimensions.<sup>[11]</sup>

When taking into account the effect of gravity, based on the black hole entropy relation with horizon area, the

so-called entropy-area relation in Einstein gravity, it is argued that the maximal entropy of a system is bounded by its area  $A$ ,<sup>[12]</sup>

$$S \leq S_H = \frac{M_p^2 A}{4}. \quad (4)$$

That is, the maximal entropy of a system is given by entropy of the black hole with the same size as the system. The entropy bound (4) is called holographic entropy bound. For a limited self-gravitating system, its Schwarzschild radius  $R_g < R$ , the holographic entropy bound (4) is less restrictive than the Bekenstein bound (3). For both entropy bounds, they are all given by the size of a space-like surface enclosing the system under consideration. Then it is interesting to see whether the entropy bounds (3) and (4) can be applicable to our universe and what consequences can be acquired from those entropy bounds. Bekenstein himself generalized the entropy bound (3) to the cosmological setting by replacing  $R$  by the particle horizon in a FRW universe. On the other hand, Fischler and Susskind<sup>[13]</sup> proposed that the area of the particle horizon should give an bound of matter entropy on the backward-looking light cone in the form (4). However, it is easy to see that this version of entropy bound could be

violated in a closed universe. Several proposals have been suggested in order to remedy this problem, for example, to replace the particle horizon by Hubble horizon or apparent horizon.<sup>[14]</sup> Generalizing the concept of the light-sheet proposed by Fischler and Susskind, Bousso<sup>[15]</sup> suggested the covariant entropy bound, which is applicable to arbitrary spacetimes. The covariant entropy bound gives an entropy bound on a light-like hypersurface. Therefore, in order to give an entropy bound on a space-like region, a “space-like projection” has to be performed.

It is interesting to note that there exists an improved covariant entropy bound, which is applicable to entropy on space-like hypersurfaces and passes several critical tests, proposed by Brustein and Veneziano<sup>[16]</sup> (for a recent review see Ref. [17]). The improved covariant entropy bound is called causal entropy bound. For a system with limited self-gravitating, the Bekenstein bound is the tightest, while in other situations, the causal entropy bound is argued to be a strongest one. The causal entropy bound is given as follows. Consider a generic space-like hypersurface, defined by the equation  $\tau = 0$ , and a compact region lying within it defined by  $\sigma \leq 0$ , the entropy contained in this region,  $S(\tau = 0, \sigma \leq 0)$ , is bounded by  $S_{\text{CEB}}$

$$\begin{aligned} S_{\text{CEB}} &= l_p^{-2} \int_{\sigma < 0} d^4x \sqrt{-g} \delta(\tau) \sqrt{\text{Max}_{\pm} [(G_{\mu\nu} \pm R_{\mu\nu}) \partial^\mu \tau \partial^\nu \tau]} \\ &= l_p^{-1} \int_{\sigma < 0} d^4x \sqrt{-g} \delta(\tau) \sqrt{[\text{Max}_{\pm} [(T_{\mu\nu} \pm T_{\mu\nu} \mp g_{\mu\nu} T/2) \partial^\mu \tau \partial^\nu \tau]]}, \end{aligned} \quad (5)$$

where  $l_p$  is the Planck length,  $G_{\mu\nu}$  and  $R_{\mu\nu}$  are Einstein tensor and Ricci tensor, respectively,  $T_{\mu\nu}$  is stress energy tensor of matter and  $T$  its trace. In the second equality, the Einstein equations  $G_{\mu\nu} = 8\pi G T_{\mu\nu}$  have been used. The causal entropy bound (5) is manifestly covariant and invariant under reparametrization of the hypersurface equation, while the reality of  $S_{\text{CEB}}$  is assured if the source matter obeys the weak energy condition,  $T_{\mu\nu} \partial^\mu \tau \partial^\nu \tau \geq 0$ , since the sum of two combinations in Eq. (5) and thus their maximum, are positive.

Here we are not interested in the causal entropy bound itself, Eq. (5) but the motivation that leads to the causal entropy bound. Note that both the entropy bound (1) and holographic entropy bound (4) are given by assuming the entropy in a given region of space is bounded by entropy of a largest black hole that can fit in that region, while the bound (2) is given by the mass of the largest black hole fitting in that region. The Hubble entropy bound in cosmology<sup>[14]</sup> is given by assuming the largest black hole in the universe is the one with horizon radius of Hubble horizon. However, note that gravitational collapse happens within only “Jeans” length of gravitational fluctuations in a universe, and perturbations with wavelength beyond the “Jeans” length are causally disconnected. The causal entropy bound is just based on the argument that black hole with larger radius than the “Jeans” length cannot formed very likely in the cosmological setting.<sup>[16]</sup> Then

the remained problem is to find out the causal connection (CC) scale  $R_{\text{CC}}$ .

The authors of Ref. [16] identified the causal connection scale  $R_{\text{CC}}$  for a FRW universe as follows. In the Hamiltonian approach,<sup>[18]</sup> the Fourier components of a (normalized) perturbation in a FRW universe and of its (normalized) conjugate momentum satisfy the Schroedinger-like equations

$$\begin{aligned} \hat{\Psi}_k'' + [k^2 - (z^{1/2})'' z^{-1/2}] \hat{\Psi}_k &= 0, \\ \hat{\Pi}_k'' + [k^2 - (z^{-1/2})'' z^{1/2}] \hat{\Pi}_k &= 0, \end{aligned} \quad (6)$$

where  $k$  is the comoving momentum, a prime stands for derivative with respect to conformal time, and  $z^{1/2}$  is the so-called “pump field”, a combination of the various backgrounds, which depends on the special perturbation under study. These perturbation equations clearly indicate a “Jeans-like” CC comoving momentum

$$\begin{aligned} k_{\text{CC}}^2 &= \text{Max}[(z^{1/2})'' z^{-1/2}, (z^{-1/2})'' z^{1/2}] \\ &= \text{Max}[\mathcal{K}' + \mathcal{K}^2, -\mathcal{K}' + \mathcal{K}^2], \end{aligned} \quad (7)$$

where  $\mathcal{K} = (z^{1/2})' z^{-1/2}$ . Since the tensor perturbation is always present in any case, it is therefore natural to consider the tensor perturbation as the “pump field”  $z^{1/2}$ . In that case,  $z^{1/2}$  is given by the scale factor  $a$ , so that one has  $\mathcal{K} = a'/a$ . Note that the comoving momentum  $k$  gives a definition of a proper “Jeans” CC length  $R_{\text{CC}} = ak_{\text{CC}}^{-1}$ ,

and the latter can be further expressed as

$$R_{CC}^{-2} = \text{Max}[\dot{H} + 2H^2, -\dot{H}], \quad (8)$$

where the dot stands for derivative with respect to the cosmic time and  $H$  is the Hubble parameter of the universe. The result (8) is valid for a flat FRW universe. It turns out that for a FRW universe with any spatial curvature, the CC scale (8) is changed to Ref. [16]

$$R_{CC}^{-2} = \text{Max}\left[\dot{H} + 2H^2 + \frac{\kappa}{a^2}, -\dot{H} + \frac{\kappa}{a^2}\right], \quad (9)$$

where  $\kappa$  is the spatial curvature of the universe. Brustein and Veneziano arrived at the causal entropy bound (5) starting from the CC length scale (9). The essence of the causal entropy bound is that the largest black hole in the universe is the one with horizon radius given by  $R_{CC}$  in Eq. (9).

Assuming the matter source in the FRW universe is a perfect fluid with energy-momentum tensor  $T_{\nu}^{\mu} = \text{diag}(\rho_t, p_t, p_t, p_t)$ , and with the help of the 00 components of the Ricci tensor and Einstein tensor,  $R_{00} = -3(\dot{H} + H^2)$  and  $G_{00} = 3(H^2 + \kappa/a^2)$ , the CC scale (9) can be further written as<sup>[16]</sup>

$$\begin{aligned} R_{CC}^{-2} &= \frac{1}{3} \text{Max}_{\mp}(G_{00} \mp R_{00}) \\ &= 4\pi M_p^{-2} \text{Max}(\rho_t/3 - p_t, \rho_t + p_t) \\ &= 4\pi M_p^{-2} \rho_t \text{Max}[(1/3 - \omega_t), (1 + \omega_t)]. \end{aligned} \quad (10)$$

In the third equality, we have used the equation of state of the perfect fluid,  $p_t = \omega_t \rho_t$ . It is clear from (10) that the first term is larger if  $\omega_t < -1/3$ , while the second term larger as  $\omega_t > -1/3$ . For the current universe, astronomical observations indicate that the first term is larger than the second term. This implies that the first term is more suitable for as the IR cutoff for the universe at present. This will be shown indeed the case shortly.

Now we would like to see the consequence by choosing the CC scale (9) as the IR cutoff in the UV/IR relation (2). For simplicity, we will consider the case of a flat universe in what follows and parameterize the vacuum energy density (2) as

$$\rho_{\Lambda} = \frac{3c^2 m_p^2}{R_{CC}^2}, \quad (11)$$

by introducing a parameter  $c^2$ , where  $m_p$  is the reduced Planck mass. Obviously, if  $\dot{H} \ll H^2$ , or  $|\dot{H}| \sim H^2$ , the vacuum energy density (11) gives the current observed dark energy density if the parameter  $c^2$  is of order unity.

### 3 Dynamics of Holographic Vacuum Energy

In this section, for completeness, we will separately discuss the cosmological evolution of the holographic vacuum energy by choosing different CC scales in Eq. (8). Also let us first notice that the vacuum energy density (11) could appear in the Friedmann equation in two different forms. (i) The vacuum energy density obeys the

continuity equation acting as an independent component of energy budget of the universe. That is, it obeys

$$\dot{\rho}_{\Lambda} + 3H(1 + \omega_{\Lambda})\rho_{\Lambda} = 0, \quad (12)$$

where  $\omega_{\Lambda}$  is the equation of state of the vacuum energy density. In that case, there is no interaction between the vacuum energy and other sources like dark matter in the universe. In this case the vacuum energy density will be called independent vacuum energy model. (ii) The vacuum energy density (11) could also appear as a dynamical cosmological constant. That is, its equation of state is always  $\omega_{\Lambda} = -1$ . In this case, due to the Bianchi identity, there must exist some interaction between the vacuum energy and dark (and baryon) matter  $\rho_m$  (in this paper we will neglect the contribution of radiation in the universe). The total energy obeys the continuity equation

$$\dot{\rho}_m + \dot{\rho}_{\Lambda} + 3H\rho_m = 0, \quad (13)$$

where we have used the assumption  $\rho_{\Lambda} + p_{\Lambda} = 0$ . In the following, we will consider separately the two cases.

#### 3.1 IR Cutoff 1: $R_{CC}^{-2} = \dot{H} + 2H^2$

Let us first notice that the Ricci scalar of a flat FRW universe is  $R = 6(\dot{H} + 2H^2)$ . In this case, the vacuum energy density (11) is proportional to the Ricci scalar curvature. Such a holographic dark energy model is introduced first by Gao *et al.*<sup>[19]</sup> without mentioning their motivation. Here we stress that the Ricci scalar curvature gives a causal connection scale of perturbations in the universe. In order to be self-contained, here we give some key results.

(i) *Independent vacuum energy model*

The Friedmann equation reads

$$H^2 = \frac{1}{3m_p^2}(\rho_m + \rho_{\Lambda}). \quad (14)$$

Substituting  $\rho_{\Lambda} = 3c^2 m_p^2 (\dot{H} + 2H^2)$  into Eq. (14), one can obtain

$$h^2 = \Omega_{m0} e^{-3x} + \frac{c^2 \Omega_{m0}}{2 - c^2} e^{-3x} + c_0 e^{-(4-2/c^2)x}, \quad (15)$$

where  $h = H/H_0$ ,  $x = \ln a$ ,  $c_0$  is an integration constant and  $\Omega_{m0}$  is the current fraction dark matter energy density. Clearly the integration constant  $c_0$  has to satisfy the constraint

$$\Omega_{m0} + \frac{c^2 \Omega_{m0}}{2 - c^2} + c_0 = 1. \quad (16)$$

Further, the second and third terms in Eq. (15) can be viewed as the fraction vacuum density

$$\tilde{\rho}_{\Lambda} = \frac{c^2 \Omega_{m0}}{2 - c^2} e^{-3x} + c_0 e^{-(4-2/c^2)x}. \quad (17)$$

Using Eq. (12), one can get the equation of state for the vacuum energy density as

$$\omega_{\Lambda} = -1 - \frac{\tilde{\rho}'_{\Lambda}}{3\tilde{\rho}_{\Lambda}} = -1 + \frac{[c^2/(2 - c^2)]\Omega_{m0} + (1/3)(4 - 2/c^2)c_0 e^{(2/c^2 - 1)x}}{[c^2/(2 - c^2)]\Omega_{m0} + c_0 e^{(2/c^2 - 1)x}}, \quad (18)$$

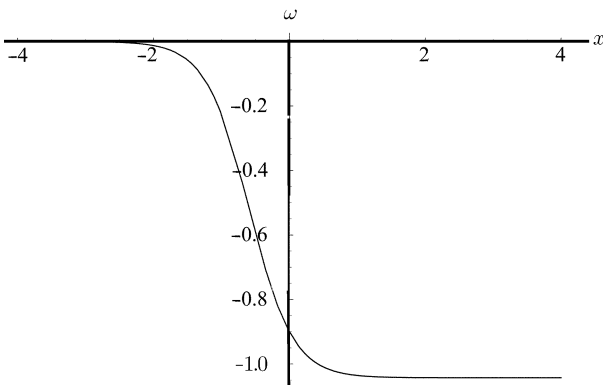
where a prime stands for derivative with respect to  $x$ . The current equation of state is given by

$$\omega_{\Lambda 0} = -1 + \frac{[c^2/(2-c^2)]\Omega_{m0} + (1/3)(4-2/c^2)c_0}{[c^2/(2-c^2)]\Omega_{m0} + c_0}. \quad (19)$$

In Fig. 1 we plot the equation of state for the vacuum energy density. It clearly shows that in early time it behaves as a dust matter, while it behaves like a phantom field at late time. In addition, we can see from Eq. (18) that at infinite future, if  $c^2 < 2$ ,

$$\omega_{\Lambda\infty} = \frac{1}{3} - \frac{2}{3c^2}. \quad (20)$$

$\omega_{\Lambda\infty} < -1$  provided  $c^2 < 1/2$ . It could be a reliable dark energy model. For further discussions on this model, see, for example, Ref. [20]. In addition, let us mention here that the authors of Ref. [21] considered the case of a combination of Hubble parameter, event horizon, particle horizon and the life time of the universe (if finite) as an IR cutoff; Medved in a footnote of Ref. [22] mentioned the possibility of the causal connection scale as the scale of ‘‘causal boundary’’.



**Fig. 1** The equation of state for the vacuum energy density versus  $x = \ln a$ , provided  $\omega_{\Lambda 0} = -0.9$  and  $\Omega_{m0} = 0.3$ . In this case,  $c_0 = 0.604$  and  $c^2 = 0.484$ .

### (ii) Dynamical cosmological constant

In this case, our starting point is the two equations (13) and (14). One has

$$(1 - 2c^2)H^2 = \left(c^2 - \frac{2}{3}\right)\dot{H}. \quad (21)$$

We then have the solution

$$a = a_0(t_0 + \alpha t)^{1/\alpha}, \quad (22)$$

where  $a_0$  and  $t_0$  are two integration constants, and  $\alpha = 3(2c^2 - 1)/(3c^2 - 2)$ . The equation of state for the total energy is

$$\omega_t = -1 + \frac{2}{3}\alpha. \quad (23)$$

Note that in this case, due to  $\rho_m \sim -\dot{H}$ , in order to keep the positivity of the matter energy density, one has to have  $\dot{H} < 0$ , which implies that  $\omega_t > -1$  or  $\alpha > 0$ .

(a) When  $c^2 > 2/3$ , one has  $\alpha > 0$ , but  $1/\alpha < 1$ . In this case, the universe always decelerated expands.

(b) When  $1/2 < c^2 < 2/3$ , one has  $\alpha < 0$  and  $\omega_t < -1$ . This is not a physical allowed case.

(c) When  $1/3 < c^2 < 1/2$ , one has  $\alpha > 0$  and  $1/\alpha > 1$ , the universe always accelerating expands with a power-law form. This implies that there is no decelerated phase in this case. This is not consistent with current observational fact.

(d) When  $c^2 < 1/3$ , one has  $\alpha > 0$  and  $1/\alpha < 1$ . In this case, the universe always in a decelerated phase.

As a result, if acting as an effective dynamical cosmological constant, the vacuum energy density is not consistent with the current observation data.

### 3.2 IR Cutoff 2: $R_{CC}^{-2} = -\dot{H}$

In this case, note that in order  $R_{CC}^{-2}$  to be positive,  $\dot{H}$  should be negative. Let us first discuss the case as an independent energy component.

#### (i) Independent vacuum energy model

In that case, the corresponding Friedmann equation can be rewritten as

$$h^2 = \Omega_{m0}e^{-3x} - \frac{c^2}{2}(h^2)'. \quad (24)$$

Integrating this equation yields

$$h^2 = \Omega_{m0}e^{-3x} + c_0e^{-2x/c^2} - \frac{3c^2}{3c^2 - 2}\Omega_{m0}e^{-3x}, \quad (25)$$

where  $c_0$  is an integration constant, which should obey the constraint

$$\Omega_{m0} + c_0 - \frac{3c^2}{3c^2 - 2}\Omega_{m0} = 1. \quad (26)$$

On the other hand, the fraction vacuum energy density

$$\tilde{\rho}_{\Lambda} = c_0e^{-2x/c^2} - \frac{3c^2}{3c^2 - 2}\Omega_{m0}e^{-3x}, \quad (27)$$

can give its equation of state

$$\omega_{\Lambda} = -1 + \frac{(2c_0/3c^2)e^{-2x/c^2} - [3c^2/(3c^2 - 2)]\Omega_{m0}e^{-3x}}{c_0e^{-2x/c^2} - [3c^2/(3c^2 - 2)]\Omega_{m0}e^{-3x}}. \quad (28)$$

The current equation of state is

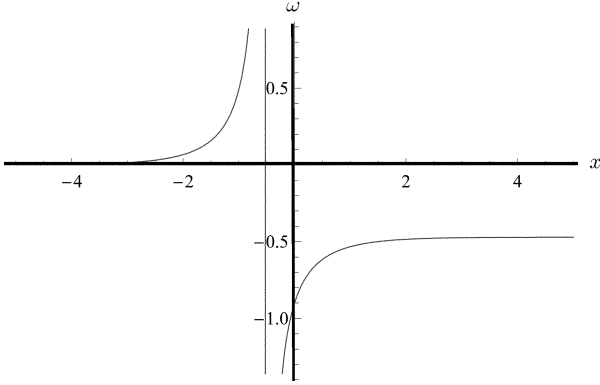
$$\omega_{\Lambda 0} = -1 + \frac{2c_0/3c^2 - [3c^2/(3c^2 - 2)]\Omega_{m0}}{c_0 - [3c^2/(3c^2 - 2)]\Omega_{m0}}, \quad (29)$$

and at infinite future  $x \rightarrow \infty$ ,

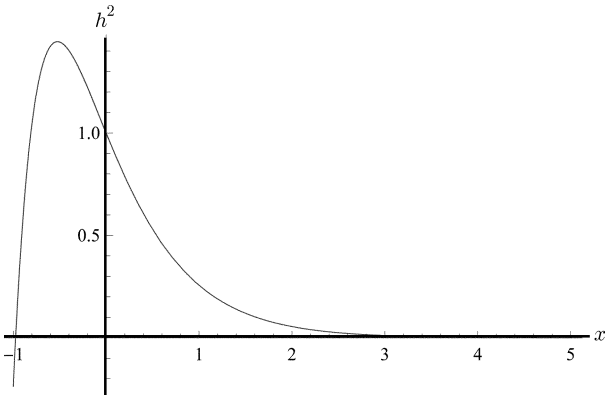
$$\omega_{\Lambda\infty} = -1 + \frac{2}{3c^2}, \quad (30)$$

provided  $c^2 > 2/3$ . In Fig. 2 we plot the equation of state for the vacuum energy density provided  $\Omega_{m0} = 0.3$  and  $\omega_{\Lambda 0} = -0.9$ . The equation of state diverges at some time in the past. Figure 3 plots the fraction Hubble parameter squared  $h^2$ , which turns to be negative at some time in the past. Clearly this is not a physical solution. To see that this case could not be consistent with evolution history of the universe, let us look at the second and third terms in (25). In order to have an accelerating expansion, one has  $c^2 > 1$  from the second term, while one has to have  $c^2 > 2/3$  if requiring the second term is dominant over the

third term currently. Then one has  $3c^2/(3c^2 - 2) > 1$  and the second term would be dominant, which always leads to a negative  $h^2$  in the early time.



**Fig. 2** The equation of state for the vacuum energy density versus  $x = \ln a$ , provided  $\omega_{\Lambda 0} = -0.9$  and  $\Omega_{m0} = 0.3$ .



**Fig. 3** The fraction Hubble parameter squared  $h^2$  versus  $x = \ln a$ , provided  $\omega_{\Lambda 0} = -0.9$  and  $\Omega_{m0} = 0.3$ .

(ii) *Dynamical cosmological constant*

In this case, the Friedmann equation can be cast to

$$H^2 = -\left(c^2 + \frac{2}{3}\right)\dot{H}, \quad (31)$$

which has the solution of the scale factor

$$a = a_0(t_0 + \beta t)^{1/\beta}, \quad (32)$$

where  $1/\beta = c^2 + 2/3$ , while the total equation of state is

$$\omega_t = -1 + \frac{2}{3}\beta. \quad (33)$$

Clearly, in this case, the universe always accelerating (decelerated) expands as  $c^2 > 1/3$  ( $c^2 < 1/3$ ). This is again not consistent with current observational data.

#### 4 Conclusions

From holographical property of gravity, one has the so-called UV/IR relation. The dark energy problem is an IR problem, while the cosmological constant problem is an UV problem. It is therefore natural to make a connection between the UV/IR relation and the dark energy problem. The casual entropy bound for a spatial region in a cosmological setting is given by assuming the maximal black hole in the universe is formed by gravitational collapse with the ‘‘Jeans’’ scale of perturbations, beyond which black hole cannot form very likely. Therefore the ‘‘Jeans’’ scale of perturbations in the universe naturally leads to an IR cutoff in the cosmological setup.

The causal connection scale is given by  $R_{CC}^{-1} = \sqrt{\text{Max}(\dot{H} + H^2, -\dot{H})}$  for gravitational perturbation in a FRW universe.<sup>[16]</sup> We studied the cosmological dynamics of the vacuum energy density by choosing the causal connection scale as the IR cutoff in the UV/IR relation, in the cases of  $R_{CC}^{-2} = \dot{H} + 2H^2$  and  $R_{CC}^{-2} = -\dot{H}$ , respectively. Also we separately considered the cases of the corresponding vacuum density as an independently conserved energy component and as an effective dynamical cosmological constant. It turns out only the case with the choice  $R_{CC}^{-2} = \dot{H} + 2H^2$  could be consistent with current cosmological data if it acts as the observed dark energy. This model is called holographic Ricci scalar model in the literature since  $R_{CC}^{-2}$  is proportional to the Ricci scalar of the FRW spacetime in this case. As a result, it appears that the causal connection scale acts as a new IR cutoff in the cosmological setting. It is of some interesting to investigate other cosmological consequences for this model. Finally let us stress that our discussions do not exclude some interaction between the vacuum energy density and dark matter if  $\Omega_{\Lambda} \neq -1$  in Eq. (13).

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