Commun. Theor. Phys. (Beijing, China) **54** (2010) pp. 578–582
Vol. 54, No. 3, September 15, 2010

Dynamic Evolution with Limited Learning Information on a Small-World Network*

DONG Lin-Rong (董林荣)†

Oujiang College, Wenzhou University, Wenzhou 325035, China

(Received December 22, 2009; revised manuscript received March 10, 2010)

Abstract *This paper investigates the dynamic evolution with limited learning information on a small-world network. In the system, the information among the interaction players is not very lucid, and the players are not allowed to inspect the profit collected by its neighbors, thus the focal player cannot choose randomly a neighbor or the wealthiest one and compare its payoff to copy its strategy. It is assumed that the information acquainted by the player declines in the form of the exponential with the geographical distance between the players, and a parameter V is introduced to denote the inspect-ability about the players. It is found that under the hospitable conditions, cooperation increases with the randomness and is inhibited by the large connectivity for the prisoner's dilemma; however, cooperation is maximal at the moderate rewiring probability and is chaos with the connectivity for the snowdrift game. For the two games, the acuminous sight is in favor of the cooperation under the hospitable conditions; whereas, the myopic eyes are advantageous to cooperation and cooperation increases with the randomness under the hostile condition.*

PACS numbers: 87.23.Kg, 02.50.Le, 87.23.Ge, 89.75.Fb
Key words: inspect-ability, limited learning information, small-world network, prisoner's dilemma, snowdrift game

1 Introduction

Cooperative phenomena are essential in natural and human systems and the emergence of cooperation remains a challenge to scientists to date, who often resort to Evolutionary Game Theory.[1−2] The classical theoretical framework for studying cooperation of unrelated individuals is the prisoner's dilemma (PD) game,[3] in which two players simultaneously decide whether to cooperate or defect. If both partners cooperate, they both receive a higher fitness R than the one P if both partners defect, but a defector gets the highest fitness T when its opponent cooperates, in which the cooperator is left with the sucker's payoff S, such that $T > R > P > S$ and $T + S < 2R$. As a result, in a single round of the PD it is best to defect regardless of the opponent's decision. Defectors can invade and destroy cooperation in a cooperative population while cooperators cannot spread in a defective population. However, mutual cooperation would be preferable for both of individuals. Thus, the dilemma is caused by the selfishness of the individuals. Since cooperation is abundant and robust in nature, considerable efforts have been concentrated on exploration of the origin and persistence of cooperation. During the last decades, some rules, such as, kin selection,[4] direct reciprocity,[5] indirect reciprocity,[6] network reciprocity,[7] and group selection,[8] have been found to benefit the evolution of cooperation in creatural societies.

In order to provide better explanations for the emergence of cooperation, the proposal of the snowdrift game (SG)[9−10] was generated to be an alternative to the PD. The SG is defined by $T > R > S > P$, which is equivalent to the hawk-dove game. To illustrate this game, consider two drivers on their way home that are caught in a blizzard and trapped on either side of a snowdrift. Each driver has the option to remove the snowdrift and start shoveling or to remain in the car. In contrast to the PD, the best choice now clearly depends on the other driver. If the other cooperates and starts shoveling, it pays to defect and remain in the car, but if the other defects, it is better to shovel and get home than to wait for snow thawing.

The PD game and the SG game have been widely studied in different versions, as two standard models. They are usually implemented in a zero-dimensional system, where every player can interact with any other. They have also been studied on a regular lattice,[10−11] where a player can interact with its nearest neighbors in an array. However, social situations are rarely well described by such extreme network. The topology of social communities is much better described by what has been called small-world network,[12−14] which is analogy with the small-world phenomenon (popularly known as six degrees of separation).

Most of the existing models about evolutionary dynamics are implemented by an imitation behavior, in which the players are allowed to inspect the profit collected by its neighbors, and the focal individual adopts the strategy of the wealthiest among them[14] or the one of a randomly choosing neighbor[10−11] by comparing their payoff. References [15–17] present the dynamic preferential learning rules. In Ref. [15], the more frequently the focal player adopted a neighbor's strategy in the previous rounds, the more likely it will be chosen in the subsequent rounds. Nevertheless, in Ref. [16] one of the neighbors is chosen with a probability denoting the attractiveness of the neighbor, which is proportional to the payoff. The above theories are all based on two key assumptions: (i)

*Supported by Natural Science Foundation of China under Grant No. 10974146
†Corresponding author, E-mail: dlingrong@wzu.edu.cn

perfect rationality of the players and (ii) there is common information among the players. Perfect rationality means that the players have well-defined functions, and they are fully aware of their own and opponents' strategy options and payoff values.[18] However, bounded rationality becomes a natural concept when the goal of the theory is to understand the behavior about creatural societies. In real situations, the information among the interaction players is not very lucid and limited, the players are not allowed to inspect the profit collected by its neighbors, and the players' ability to inspect the information is restricted. Actually, some similar approaches have been proposed in the economics literatures.[19−21] Based on the observation, we consider that the players may only know the information of the adjacent neighbors and the information may decay quickly with the geographical distance between the players. Of course, the extent of the information depends on the inspect-ability of the player. Therefore, it is reasonable to consider that the information acquisition is limited in real world. In the following, we study the dynamic evolution with limited learning information on a small-world network with different inspect-abilities.

2 Model

A small-world network is set up as follows: Starting from a ring lattice with N vertices and k edges per vertex, in which the distance between two nearest vertices is the same and equal to the arc length between them, which is defined as one unit. We then run sequentially through each of the vertices, rewiring $k/2$ of the links with probability p. This construction allows us to "tune" the graph between regularity ($p = 0$) and disorder ($p = 1$). The intermediate region $0 < p < 1$ produces a continuous spectrum of the small-world networks. Double connections between vertices, as well as the connection of a vertex with itself, are avoided in the construction of the network. A player is arranged at each vertex and is given a number, which increases from 1 to N orderly. Each player is a pure strategist, adopting either a cooperative or a defecting strategy, and is connected, on average, to other k neighbors. The edges connecting two players enable the interaction between them.

Following common practice,[10−11] we begin with by rescaling the games such that each depends on a single parameter. For the PD, we make $T = b > 1$, $R = 1$, and $P = S = 0$, where b represents the advantage of defectors over cooperators, being taken $1 < b \le 2$. For the SG, we take $T = 1 + r$, $R = 1$, $S = 1 - r$, and $P = 0$, where r is the cost-to-benefit ratio of mutual cooperation, being taken $0 \le r \le 1$.

A round of play consists of the confrontation of every player with all its neighbors directly connected. P_i is the accumulated payoff earned by a player with the number i in a time step, and it is not accumulated from round to round. Due to being difficulty to inspect the profit and the strategies of its neighbors, the players know very limited information about its far neighbors. Thus, it is assumed that the focal player i chooses one j of its neighbors to be imitated with probability ρ_{ij}, which is declined in the form of the exponential with the geographical distance between the players i and j, namely

$$\rho_{ij} = \frac{1}{z} e^{-L_{ij}/V}\,, \tag{1}$$

$$z = \sum_{j \in \Omega_i} e^{-L_{ij}/V}\,, \tag{2}$$

where Ω_i is the set of neighbors of element i. z is set in order to satisfy that the total option probability is equal to 1. L_{ij} is the geographical distance between the players i and j, which is the shorter arc length between them. If $|i-j| \le N/2$, $L_{ij} = |i-j|$, otherwise $L_{ij} = N - |i-j|$. According to Eq. (1), if the player j is far away from the focal player i, the probability being imitated by the player i is very little. V is defined as the inspect-ability to reflect the ability that the player i inspects its neighbors and knows the information. $V \to 0$ implies that the focal player is very myopic, thus can only know the information of its nearest neighbor and imitate the behavior. $V \to \infty$ implies that the player has very keen vision and can inspect the information about the far neighbors, thus can choose one of their neighbors to be imitated uniformly, similar to most of the existing replica dynamics.[10−11,22−24] V may depend on the players. However, for simplicity, V is a number independent of the players in the model. Once the being imitated neighbor j is taken, whenever $P_j > P_i$, the focal player i adopts the strategy of the player j for the next round play with probability given by $(P_j - P_i)/(Dk_m)$, where k_m is the larger one between k_i and k_j and $D = T - S$ for the PD and $D = T - P$ for the SG. Otherwise, the player i keeps its own strategy. Simulations were carried out for a population of $N = 1000$ individuals. The initial strategies are assigned at random with equal probability. Equilibrium frequencies of cooperators result from an average over 10 independent realizations, running 1000 generations after a transient time of 5000 generations, in which it is confirmed that averaging over 2000 periods or using a transient time of 10000 generations or over 20 independent realizations did not change the results qualitatively.

3 Results and Discussion

The key quantity for characterizing the cooperative behavior is frequency of cooperators, which is defined as the fraction of cooperation in the whole population for the system reaches a steady state. Figure 1 shows the results of simulations carried out for the PD and the SG respectively on the small-world network with the rewiring probability $p = 0.9$ for the different inspect-abilities V when the average connectivity $k = 6$. Deviations from the well-mixed population limits (the diagonal line $1 - r$ for the SG and the zero baseline for the PD) are pronounced. Under the very hospitable conditions, namely $r \to 0$ for the SG and $b \to 1$ for the PD, frequencies of cooperators are not all equal to 1, and frequency of cooperators decreases as the inspect-ability V varies small for the both game. It is logical that due to the learning information being not very lucid, the players are fear the circumambient environment, leading to distrust of their far neighbors and do not adopt their strategies. As a result, large cooperative clusters cannot emerge, which inhibits cooperation. When the inspect-ability V becomes smaller, the focal player knows much less information about the payoff and the strategies of the far neighbors, leading to the bigger cooperative clique being inhibited easily. Under the hostile condition, namely $r \to 1$ for the SG and $b \to 2$ for the PD, frequencies of cooperators are not equal to 0, and the frequency of the cooperators increases as V decreases for both of the games. It is considered that when V is small, i.e. the player is myopic, the imitated neighbors are near and the extent about the learning is limited. Ultimately, the players form some small compact cooperative clusters to minimize the exploitation by defectors to keep cooperation some level. It is more advantageous to survival of the cooperators as the player is more myopic, leading to the higher cooperative level. It is a very interesting result. Therefore, I investigate dynamic evolution in the hostile condition in detail below.

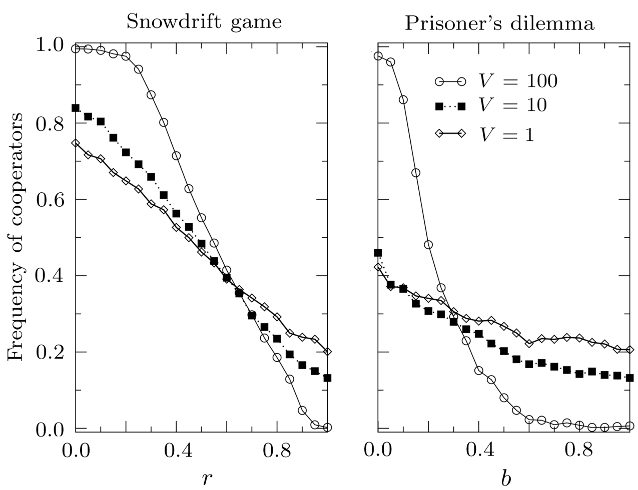

Fig. 1 Frequency of cooperators on the small-world network with rewiring probability $p = 0.9$ and the average connectivity $k = 6$ for the different inspect-ability V. Results shown as functions of the cost-to-benefit ratio r for the SG (left panel) and the temptation to defect b for PD (right panel). The results correspond to ten independent realizations of 1000 elements, run for 1000 rounds after a transient of 5000 rounds.

Figure 2 shows the results of simulations carried out on the small-world network with the rewiring probability $p = 0.9$ for different average connectivity k, which reflects that the frequency of cooperators depends on the inspect-ability V in the hostile condition in which $r = 0.9$ for the SG game and $b = 1.9$ for the PD respectively. It shows that the frequencies of cooperators reach about 0.38 for both the SG and PD when the focal player is very myopic, namely $V \to 0$, in which the player only knows the information of the nearest neighbor and learns its strategy to form very compact cooperative cluster to resist the aggression of the defectors. The frequencies vary small as the average sizes of the group k vary big. It implies that large size of the group inhibits the resistive ability. In addition, frequencies of cooperators keep almost invariability when $V > 50$ for the SG and $V > 100$ for the PD respectively, which indicates that when the compact intensity is less than some magnitude, the cluster effect of cooperators disappears in the sea of defectors for both the PD and SG games in the hostile condition.

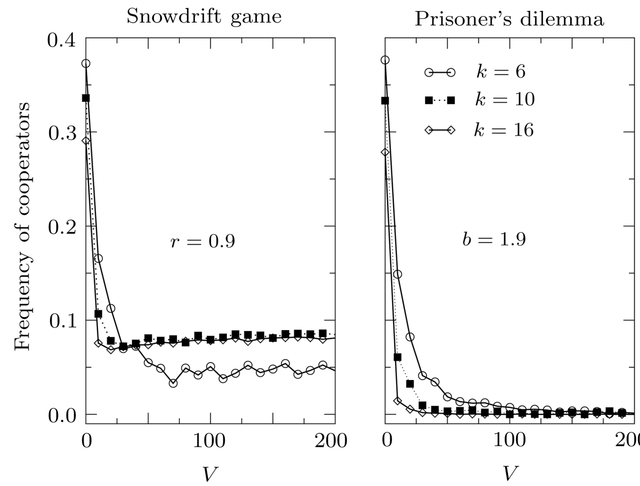

Fig. 2 Frequency of cooperators on the small-world network with rewiring probability $p = 0.9$ and the different average connectivity $k = 6$, 10, 16. Results shown as functions of the inspect-ability V in the hostile conditions for the SG $r = 0.9$ (left panel) and for PD $b = 1.9$ (right panel). The results correspond to ten independent realizations of 1000 elements, run for 1000 rounds after a transient of 5000 rounds.

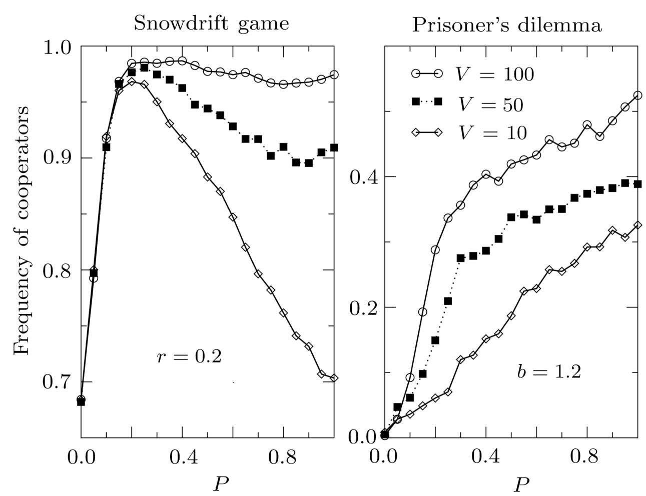

Fig. 3 Frequency of cooperators as a function of the rewiring probability p about the network with the average connectivity $k = 6$. The three lines correspond to different inspect-ability V. The cost-to-benefit ratio $r = 0.2$ for the SG (left panel) and the temptation to defect $b = 1.2$ for PD (right panel). The results correspond to ten independent realizations of 1000 elements, run for 1000 rounds after a transient of 5000 rounds.

In Fig. 3, we show a plot of frequency of cooperators depending on the rewiring probability p with the average connectivity $k = 6$, to emphasize the changes about the behaviors as the structure of the network varies. The three curves correspond to the different inspect-abilities V in the benign cooperative environments in which the value $r = 0.2$ for the SG and the value $b = 1.2$ for the PD. We can find three features under the benign condition: (i) frequency of cooperators is almost independent on the inspect-ability V for the two games when the network is regular, i.e., the rewiring probability $p = 0$; (ii) on the small-world network with the same rewiring probability p, frequency of cooperators increases with the inspect-ability V, which is accordant with Fig. 1. (iii) For the SG, frequency of cooperators gains the maximal magnitude when p is about 0.2. It is because that the random network favors cooperation[10] for the SG, but the limited learning information inhibits cooperation under the benign condition, which results in that the frequency of cooperators reaches balance at some place. We find also that when the inspect-ability varies bigger, i.e., the learning information becomes more, the place vanishes gradually. For the PD, frequency of cooperators increases monotonously with the rewiring probability p and reaches the maximal magnitude at random network. We consider that when p is bigger, the network is more out-of-order, which makes the far away players form the same incompact group, in which cooperators can outweigh their losses against defectors by gains from interactions within the cluster under the benign condition along the boundary so as to make cooperators resist easily the defectors to invade. Dynamic evolutions have also been investigated under the hostile condition for the two games, which is shown in Fig. 4. It indicates that the cooperation increases all monotonously with randomness.

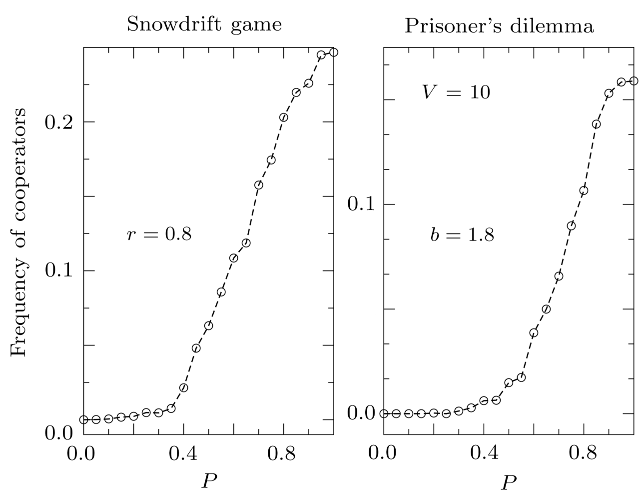

Fig. 4 Frequency of cooperators as a function of the rewiring probability p about the network with the average connectivity $k = 6$. The inspect-ability V is 10. The cost-to-benefit ratio $r = 0.8$ for the SG (left panel) and the temptation to defect $b = 1.8$ for PD (right panel). The results correspond to ten independent realizations of 1000 elements, run for 1000 rounds after a transient of 5000 rounds.

We also draw the similar conclusions with the different inspect-abilities V. The reason is explained in the following. When the randomness p increases, the number of the far neighbors increases. According to the limited learning information model, the probability imitated about the far neighbors is very small. However, the total option probability is the same, thus the probability imitated about the near neighbors increases. It results in some impact small cooperative clusters come into being, which keeps cooperation some levels. The more random the networks are, the nearer the imitated neighbors are, the cooperative clusters are more compact, which is more effective to resist aggression by the defectors, keeping the bigger cooperative level.

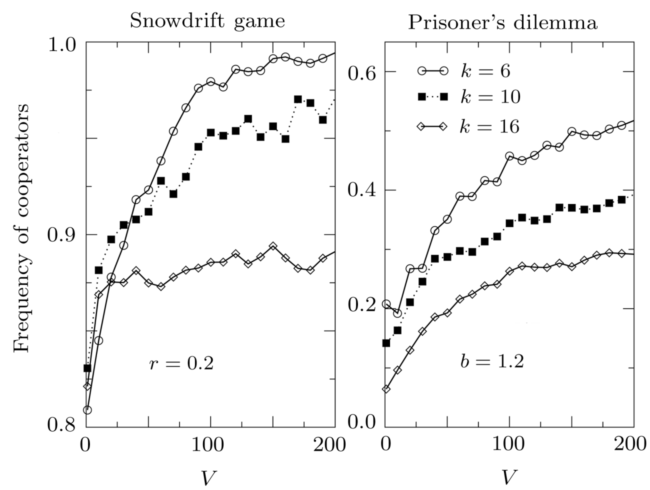

Fig. 5 Frequency of cooperators on the small-world network with rewiring probability $p = 0.6$ and three different average connectivity k. Results shown as functions of the inspect-ability V for the SG (left panel) with cost-to-benefit ratio $r = 0.2$ and for PD (right panel) with the temptation to defect $b = 1.2$. The results correspond to ten independent realizations of 1000 elements, run for 1000 rounds after a transient of 5000 rounds.

Figure 5 shows the effects of the inspect-ability V on frequency of cooperators on the small-world network with the rewiring probability $p = 0.6$. The three curves correspond to the different average connectivity k for the benign cooperative environment in which the value $r = 0.2$ for the SG and the value $b = 1.2$ for the PD. It is found that frequencies of cooperators increase monotonously with the inspect-ability V for the both games, which is accordant with that in Figs. 1 and 3. It indicates that too much information favors cooperation for both of the games in the benign condition. For the PD, frequency of cooperators decreases as the average connectivity k increases, which is consistent with the theory that spatial structure promotes the evolution of cooperation.[11] For the SG, when the inspect-ability V is smaller than 40, frequency of cooperators is chaotic with the average connectivity k; when the inspect-ability V is bigger than 40, frequency of cooperators decreases as the average connectivity k increases. It results from the two common interactions that spatial structure often inhibits the evolution of cooperation in the snowdrift game;[10] however, the keen eyesight of the players is advantageous to cooperation for the benign condition. The detailed reason will be explored in the future.

4 Conclusions

Different from classical game theory, in our model the learning information is limited, and players are not allowed to inspect the profit collected by its neighbors, which results in that the focal player cannot choose randomly a neighbor or the wealthiest one and compare its payoff to copy its strategy. Here, the player copies only an adjacent neighbor's strategy with a probability, which is declined in the form of the exponential with the geographical distance between the players. When the model is played on the small-world network, we discovered some complicated dynamic behaviors. According to classical game theory, it is widely accepted that spatial structure often inhibits the evolution of cooperation for the SG[10] and promotes the evolution of cooperation for the PD.[11] However, in our model, for the SG, the small-world network with the moderate rewiring probability is advantageous to cooperation under the benign condition, whereas for the hostile one, the randomness favors cooperation, and the small average connectivity is helpful to cooperation as the eyes are keen. In contrast, for the PD, cooperation increases with randomness under any environment and is inhibited by the average connectivity under the benign one.

In summary, the inspect-ability V plays a very important role in the limited learning information model. Whether the players play the SG or the PD, under the benign condition, a significant enhancement of cooperation is shown when the inspect-ability is powerful; under the hostile condition, the level of the cooperation becomes low when the inspect-ability increases, namely the learning information is more abundant. In other words, too much learning information favors cooperation under the benign condition, whereas limited learning information tends to cooperation under the hostile condition. We infer that the features observed here come from the competition between the stability of clusters of cooperators and their exploitation by treacherous neighbors at the borders. Under the benign condition, treacherous neighbors at the border of a cooperative group cannot penetrate deep, which results in that cooperators may survive in large groups. Here, when the inspect-ability V grows, cooperators belonging to faraway neighbors may be connected to form large clusters able to survive to increase cooperative intensity. Under the hostile condition, the defectors at the border invade easily the cooperative clusters, in which the focal player imitates the strategy of the nearest neighbor with the small inspect-ability V to form very compact cooperative cluster to resist the aggression of the defectors, which leads to keep some cooperative level.

In real world, the information among the interaction players is limited. Moreover, the extent of the information depends on the inspect-ability of the player. Therefore, the present numerical simulation and analysis may help understand the cooperative behaviors in human societies and insect world.

References

[1] J. Mayanrd Smith, *Evolution and the Theory of Games*, Cambridge University Press, Cambridge (1982).

[2] H. Ginitis, *Game Theory Evolving*, Princeton University, Princeton (2000).

[3] R. Axelrod and W.D. Hamilton, Science **211** (1981) 1390.

[4] W.D. Hamilton, J. Theor. Biol. **7** (1964) 1.

[5] R. Axelrod, *The Evolution of Cooperation*, Basic Books, New York (1984).

[6] M.A. Nowak and R.M. May, Nature (London) **437** (2005) 1291.

[7] H. Ohtsuki, C. Hauert, E. Lieberman, and M.A. Nowak, Nature (London) **441** (2006) 502.

[8] A. Traulsen and M.A. Nowak, Proc. Natl. Acad. Sci. U.S.A. **103** (2006) 10952.

[9] R. Sugden, *The Economics of Rights, Co-operation and Welfare*, Blackwell, Oxford, UK (1986).

[10] C. Hauert and M. Doebeli, Nature (London) **428** (2004) 643.

[11] M.A. Nowak and R.M. May, Nature (London) **359** (1992) 826.

[12] S. Milgram, Psychol. Today **2** (1967) 60.

[13] D.J. Watts and S.H. Strogatz, Nature (London) **393** (1998) 440.

[14] G. Abramson and M. Kuperman, Phys. Rev. E **63** (2001) 030901 (R).

[15] Z. Xi, X.J. Xu, Z.G. Huang, S.J. Wang, and Y.H. Wang, Phys. Rev. E **74** (2006) 021107.

[16] J.Y. Guan, Z.X. Xu, Z.G. Huang, X.J. Xu, and Y.H. Wang, Europhys. Lett. **76** (2006) 1214.

[17] S. Lozano, A. Arenas, and A. Sanchez, PLOS ONE **3** (2008) e1892.

[18] G. Szabo and G. Fanth, Phys. Rep. **446** (2007) 97.

[19] J.M. Sinkula, Journal of Marketing **58** (1994) 35.

[20] M.A. Meyer, Review of Economic Studies **58** (1991) 15.

[21] A. Marcet, Journal of Political Economy **97** (1989) 1306.

[22] S. Assenza, J. Gómez-Gardenes, and V. Latora, Phys. Rev. E **78** (2008) 017101.

[23] F.C. Santos and J.M. Pacheco, Phys. Rev. Lett. **95** (2005) 098104.

[24] J. Gómez-Gardenes, M. Campillo, L.M. Floría, and Y. Moreno, Phys. Rev. Lett. **98** (2007) 108103.