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A Simple Approach to Study the Isospin Effect in Mass Splitting of Three-Nucleon Systems by Using Hyperspherical Functions

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Abstract *In this work, the binding energy and wavefunctions of three-nucleon systems are obtained by using hyperspherical harmonic approach. We have used a mathematical modification method to obtain the eigenvalues and eigenfunctions of Schrödinger equation for three-nucleon systems in calculation. Next, we have used a simple approach to obtain the difference between binding energy of* ^3H *and* ^3He *where gives us mass splitting of three-nucleon systems. We have compared our results with the other works and experimental values.*

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Key words: three-nucleon systems, binding energy, H.H method, isospin

1 Introduction

The nature of the strong force among nucleons that binds nuclei together, which is still not fully understood, coupled to the many-body characteristics of these systems, has given rise to a rich and complex field of scientific inquiry, bringing forth a very creative research area. The fact that nuclei contain many particles but not nearly enough to treat them statistically, explains why they can be alternatively described both as a collection of individual nucleons and as a single object akin to a charged, dense liquid drop.[1] Investigation of the details of the nucleon-nucleon (NN) interaction is one of the most extensively explored fields in nuclear physics. An obvious step in testing NN force models is to check whether their application leads to a full description of observable in systems where more than two nucleons interact. The three-nucleon (3N) system is the simplest testing ground in which the quality of modern NN interactions can be probed in a non-trivial environment.[2] During the past years, several methods have been developed to solve the nonrelativistic Schrödinger equation accurately for few-nucleon bound states, by using realistic nuclear potentials. These methods are the CRCGV,[3] the SV,[4] the H.H,[5] the GFMC,[6] the NCSM,[7] the EIHH[8] and the Faddeev. Among this methods, the H.H method is one of the best framework to study the few-nucleon systems because the many-body forces can be introduced and treated more easily within H.H space. Because of the near equality of the masses and of the interactions between nucleons, the hamiltonian of the nucleus is (approximately) invariant with respect to transformations between neutron and proton states. It can be expressed as $[H, I] = 0$. The formal implication is thus that H becomes invariant with respect to a rotation in isospin space.[1] In this paper, by using the Jacobian coordinates and with this assumption that nuclear hamiltonian H is isospin invariant, we have tried to investigate the binding energy of three-nucleon systems. Advantage of this model is that changes the corresponding equations to the single variable and solving these equations is more easier than coupled equations.

2 Jacobi Coordinates and Hyperspherical Functions

In the theory of many-particle systems, Jacobi coordinates often are used to simplify the mathematical formulation. For a system of N identical mass particles, the schrödinger equation can be solved by using the Jacobin coordinates transformations as follows[9–11]

$$\vec{\xi}_i = \sqrt{\frac{i}{i+1}}\Big(\frac{1}{i}\sum_{j=1}^{i}\vec{r}_j - \vec{r}_{i+1}\Big), \quad i = 1, 2, \ldots, N-1\,, \tag{1}$$

and the center of mass coordinate can be shown as

$$\vec{R} = \vec{\xi}_N = \frac{\vec{r}_1 + \vec{r}_2 + \cdots + \vec{r}_N}{N}\,. \tag{2}$$

Each Jacobian coordinate $\vec{\xi}_i$ is defined as absolute vector that connects the centre of mass of a subsystem with $i-1$ particle to the remaining particle or to the centre of mass of another subsystem. The hyperradius function can be written as

$$X = \Big[\sum_{i=1}^{N-1}\xi_i^2\Big]^{1/2} = \Big[\sum_{i=1}^{N-1}\frac{i+1}{i}(\vec{r}_{i+1} - \vec{R})^2\Big]^{1/2}, \tag{3}$$

and volume element takes the form

$$\mathrm{d}X = \prod_{i=1}^{N}\mathrm{d}\vec{r}_i = N^{3/2}\mathrm{d}\vec{R}\prod_{i=1}^{N-1}\mathrm{d}\vec{\xi}_i\,. \tag{4}$$

The many-body forces are more easily introduced and treated within the hyperspherical harmonics. In H.H. space, if the interaction potential between the particles

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is dependent on relative distances of particles from each other only, it can be written in terms of hyperradius function. In this case, this potentials are called hypercentral potentials. For hypercentral potentials, the Schrödinger equation simply reduced to a single hyperradial equation in the hyperspherical coordinates, while the angular and hyperangular parts of the A-particles are the known hyperspherical harmonics.[10−11]

3 Schrödinger Equation for Three-Nucleon Systems

From a variety of scattering experiments, We know that the nucleons in a nucleus are in motion with kinetic energies of the order 10 MeV[12] so we can with confidence use nonrelativistic quantum mechanics. For three-nucleon systems, the Hamiltonian is given by

$$H=\sum_{i=1}^{3}\Bigl(\frac{-\hbar^2}{2m_i}\frac{\partial^2}{\partial\vec{r}_i^{\,2}}\Bigr)+\sum_{\substack{i,j=1\\ j>i}}^{3}V_{ij}(\vec{r}_i\,,\vec{r}_j)\,. \qquad (5)$$

where V_{ij} is interaction potential between i and j nucleon. The suitable interaction potential between nucleons is Woods–Saxon potential[13−14] but the Schrödinger equation for this potential cannot be solved exactly at the value $l\neq 0$.[15] Therefore we chose harmonic oscillator potential which is closest form to the Woods–Saxon potential and also satisfies the boundary conditions.[16] Normal form of this potential can not be adopted because the nucleon separation energy becomes infinite. Also from the scattering experiments, we know that there is strong evidence for a nucleon-nucleon spin-orbit interaction, therefore we add the spin-orbit interaction for each nucleon.[14,16] The spin-orbit interaction can be written as $(V_{\rm so}(r)/\hbar^2)\,l.s$ where the form of $V_{\rm so}(r)$ is as $(1/r)({\rm d}/{\rm d}r)V(r)$ and $V(r)$ is the radial part of the interacting potential. Without the spin-orbit term, the energy does not depend on whether the nucleon spin is aligned or anti-aligned with the orbital angular momentum and the binding energy can not be obtained correctly. We adopt the nucleon-nucleon interaction potential as follow

$$V_{ij}=V'_{ij}+\Bigl[\frac{R^2(\vec{S}_i\cdot\vec{L}_i)}{\hbar^2(\vec{r}_i-\vec{r}_j)}\frac{\partial}{\partial(\vec{r}_i-\vec{r}_j)}\Bigr]V'_{ij}\,, \qquad (6)$$

where

$$V'_{ij}=V_0\Bigl[\Bigl(\frac{\vec{r}_i-\vec{r}_j}{R}\Bigr)^2-1\Bigr],\quad |\vec{r}_i-\vec{r}_j|\leq R\,, \qquad (7)$$

since nucleon separation energy must not be infinite, if $|\vec{r}_i-\vec{r}_j|>R$ then $V_0=0$. For three-nucleon systems, the Schrödinger equation with this potential is written as

$$\sum_{i=1}^{3}\Bigl(\frac{-\hbar^2}{2m_i}\frac{\partial^2}{\partial\vec{r}_i^{\,2}}\Bigr)\psi(\vec{r}_1,\vec{r}_2,\vec{r}_3)+V_0\sum_{\substack{i,j=1\\ j>i}}^{3}\Bigl[1+\frac{R^2(\vec{S}_i\cdot\vec{L}_i)}{\hbar^2(\vec{r}_i\cdot\vec{r}_j)}\frac{\partial}{\partial(\vec{r}_i\cdot\vec{r}_j)}\Bigr]\Bigl[\Bigl(\frac{\vec{r}_i\cdot\vec{r}_j}{R}\Bigr)^2-1\Bigr]\psi(\vec{r}_1,\vec{r}_2,\vec{r}_3)=E\psi(\vec{r}_1,\vec{r}_2,\vec{r}_3)\,. \qquad (8)$$

In Eq. (8), $\vec{L}_i$ is angular momentum of i-th nucleon relative to the center of mass and $\vec{S}_i$ is its spin. If $m_1=m_2=m_3$, Jacobian coordinates Eqs. (1) and (2) for three-nucleon system can be introduced as

$$\vec{\rho}=\frac{\vec{r}_1-\vec{r}_2}{\sqrt{2}},\quad \vec{\lambda}=\frac{\vec{r}_1+\vec{r}_2-2\vec{r}_3}{\sqrt{6}}\,,\quad \vec{R}=\frac{\vec{r}_1+\vec{r}_2+\vec{r}_3}{3}\,. \qquad (9)$$

The center of mass $\vec{R}$ can be eliminated, so the hyperradius and hyperangle are found as

$$X=\sqrt{\rho^2+\lambda^2},\quad \Omega=\arctan\Bigl(\frac{\rho}{\lambda}\Bigr)\,. \qquad (10)$$

In this way we can use the hyperspherical harmonic formalism.[9−11,17] In H.H approach, the wavefunction can be written as

$$\psi(\vec{r}_1,\vec{r}_2,\vec{r}_3)=\psi(X)\Lambda_{K,n}(\Omega)\,, \qquad (11)$$

where $\psi(X)$ and $\Lambda_{K,n}(\Omega)$ are called hyperradius wavefunction and hyperangular wavefunction respectively. Substitution wavefunction Eq. (11) and jacobian coordinates Eq. (9) into Eq. (8), we can obtain

$$\Bigl[\frac{-\hbar^2}{2\mu}\nabla^2+3V_0\Bigl(\frac{2(\vec{S}\cdot\vec{L})}{\hbar^2}+\frac{X^2}{R^2}-1\Bigr)-E_{K,n}\Bigr]\psi_{K,n}(X)=0\,. \qquad (12)$$

The Laplace operator for the three-nucleon systems becomes

$$\nabla^2=\frac{{\rm d}^2}{{\rm d}X^2}+\frac{5}{X}\frac{\rm d}{{\rm d}X}-\frac{L^2}{X^2}\,, \qquad (13)$$

where L^2 is the angular part of the Laplace operator and characterizes the standard centrifugal barrier term, which involves the angular coordinates. The eigenvalue of L^2 are given as follow

$$\text{Ł}^2\Lambda_{K,n}(\Omega)=K(K+4)\Lambda_{K,n}(\Omega)\,. \qquad (14)$$

where K is the grand angular momentum quantum number. By introducing a new function

$$U_{K,n}(X)=X^{5/2}\,\psi_{K,n}(X)\,, \qquad (15)$$

and by introducing the notions

$$D=\Bigl(\frac{6\mu V_0}{R^2\hbar^2}\Bigr)^{1/2},\quad \xi=\sqrt{D}X\,, \qquad (16)$$

we can obtain

$$\frac{{\rm d}^2U_{K,n}(\xi)}{{\rm d}\xi^2}-\Bigl[\xi^2+\frac{(2K+3)(2K+5)}{4\xi^2}-\frac{2\mu[E+3V_0(2K+1)]}{D\hbar^2}\Bigr]U_{K,n}(\xi)=0\,. \qquad (17)$$

In Eq. (17), we use the following relations

$$S^2\equiv\frac{3}{4}\hbar^2,\quad L^2\equiv K(K+4)\hbar^2, \qquad (18)$$

$$\frac{\vec{S}\cdot\vec{L}}{\hbar^2}=\begin{cases}-K, & J=K+\frac{1}{2}\,,\\ -2K-\frac{1}{2}, & J=K-\frac{1}{2}\,,\end{cases}\tag{19}$$

where J is total angular momentum quantum number and $J>0$. Let us assume

$$\frac{(2K+3)(2K+5)}{4}=t(t+1)\ \Rightarrow\ t=K+\frac{3}{2},\ t=-K-\frac{5}{2}\,,\tag{20}$$

where t is a new parameter. It is now easy to find that in the limits $\xi\to 0$ and $\xi\to\infty$, $U_{K,n}(\xi)$ becomes proportional to ξ^{t+1} and $\exp(-\xi^2/2)$ respectively. Now we make an ansatz for the wavefunction

$$U_{K,n}(\xi)=f(\xi)\exp\Big(-\frac{\xi^2}{2}\Big)\xi^{t+1}.\tag{21}$$

With substitution Eq. (21) into Eq. (17) we obtain for $f(\xi)$

$$\xi f''+2(t+1-\xi^2)f'+\xi\Big(\frac{2\mu[E+3V_0(2K+1)]}{D\hbar^2}-2t-3\Big)f=0\,.\tag{22}$$

Finally the substitution $S=\xi^2$ leads to

$$Sf''+\Big(t+\frac{3}{2}-S\Big)f'+\frac{1}{2}\Big(\frac{\mu[E+3V_0(2K+1)]}{D\hbar^2}-t-\frac{3}{2}\Big)f=0\,.\tag{23}$$

The latter equation is recognizable as the differential equation for the confluent hypergeometric function or Kummer function, $F(a,c;z)$ which fulfills the Kummer differential equation[16,19−21]

$$zF''+(c-z)F'-aF=0\,.\tag{24}$$

The eigenvectors of the Kummer differential equation are given in standard handbooks but are also easy to derive. With comparison of Eq. (23) and Eq. (24) we obtain

$$U_{K,n}(S)=N_0\,F\Big(a,t+\frac{3}{2};S\Big)\exp\Big(-\frac{S}{2}\Big)S^{(t+1)/2},\tag{25}$$

where F is kummer function and

$$a=-\frac{1}{2}\Big(\frac{\mu(E+3V_0(2K+1))}{D\hbar^2}-t-\frac{3}{2}\Big)\,.\tag{26}$$

Assume a power series expansion

$$F=a_0+a_1S+a_2S^2+\cdots\,,\tag{27}$$

and substitute into Eq. (23) leads to a simple recursion relation

$$\frac{a_{k+1}}{a_k}=\frac{k+a}{(k+1)(k+c)}\,.\tag{28}$$

Thus for large k, $a_{k+1}/a_k\to 1/k$. This corresponds to a divergence at large distances, which cannot be allowed. Therefore, it must be required that the series terminates at some finite $k=n$, i.e. $a_n\neq 0$ but $a_{n+1}=0$. This condition, which according to the recursion relation corresponds to $a=-n$, leads to an energy quantization in the original wave function

$$\frac{1}{2}\Big(\frac{\mu[E+3V_0(2K+1)]}{D\hbar^2}-t-\frac{3}{2}\Big)=n\,,\qquad n=0,1,2,\ldots\,,\tag{29}$$

and the energy levels will be found as

$$E_{K,n}=\Big(2n+t+\frac{3}{2}\Big)\frac{D\hbar^2}{\mu}-3V_0(2K+1)\,.\tag{30}$$

With substitution $V_0=25$ MeV, $K=1$, $\mu=469.41\ \mathrm{MeV/c^2}$ and $R=2.06$ fm, the binding energy of three-nucleon systems are found as

$$E_B=-7.94\ \mathrm{MeV}\,.$$

The eigenfunction of ground state in terms of ξ will be obtained as

$$U_{K,0}(\xi)=N_0\,F\Big(a_0,t+\frac{3}{2};\xi^2\Big)\exp\Big(-\frac{\xi^2}{2}\Big)\xi^{t+1}.\tag{31}$$

With the obtained wavefunctions Eq. (31), we can calculate the root-mean-squared radii of three nucleon systems. The rms radii of three nucleon systems is defined as[16]

$$R_{\mathrm{rms}}=\Big(\frac{5}{3}\langle x^2\rangle\Big)^{1/2},\tag{32}$$

where

$$\langle x^2\rangle=\int_0^\infty\psi_{K,n}(X)\,X^2\,\psi_{K,n}(X)\,\mathrm{d}^3(X)\,.\tag{33}$$

The expression for the Kummer function follows directly from the relation

$$F(a,c;z)\equiv 1+\frac{a}{c}\frac{z}{1!}+\frac{a(a+1)}{c(c+1)}\frac{z^2}{2!}+\cdots\tag{34}$$

By using the Eqs. (15), (31), and (34) and via numerical integration, we can obtain the rms radii of three nucleon systems as follow

$$R_{\mathrm{rms}}=1.82\ \mathrm{fm}\,.\tag{35}$$

The experimental values of ms radii of ^{3}H and ^{3}He are 1.70 ± 0.05: fm and 1.88 ± 0.05 fm respectively[22−23] so the rms radii obtained here is agree to experimental values. Since we have assumed hamiltonian H is isospin invariant, we cant derive any differences between the binding energy and radii of ^{3}H and ^{3}He.

4 Isospin Formalism

In previous section, we assumed the hamiltonian of the nucleus is invariant with respect to transformations between neutron and proton states. It means that, the Coulomb interaction between the protons is neglected and furthermore it is assumed that the strong interaction does not distinguish between neutrons and protons. Explicitly, invariance under the isospin algebra $\mathrm{SU}(2)=[I_3,I_\pm]$ follows from

$$[H,I_3]=[H,I_\pm]=0\,.$$

As a consequence of these commutation relations, the many-particle eigenstates of H have good isospin symmetry. The Coulomb interaction between the protons destroys the equivalence between the nucleons and hence breaks isospin symmetry. The main effect of the Coulomb interaction is a dynamical breaking of isospin symmetry. This can be shown by writing the Coulomb interaction[1,13]

$$V=\sum_{k<l}\Big(\frac{1}{2}+I_3(k)\Big)\Big(\frac{1}{2}+I_3(l)\Big)\frac{e^2}{|\vec{r}_k-\vec{r}_l|}\,.$$

The electromagnetic interaction produces Two effects. (i) Mass difference between protons and neutrons. (ii) A repulsive force between protons in nuclei. Various methods are used to obtain the isospin effect in the few-nucleon systems. The simplest method is determination of density function in terms of isospin and calculation its effect on the energy levels.[13,16] In this case, the density function where we will choose is fermi function

$$\rho=\rho_0\Big[1+\exp\Big(\frac{r-R_0}{a}\Big)\Big]^{-1}. \tag{37}$$

Density integral over all space is equal to the total charge of nuclei, so we have

$$\int_{r=0}^{\infty}\rho\,\mathrm{d}v=\sum_{i=1}^{A}q_i=e\Big(I_3+\frac{1}{2}A\Big),\quad A=3\,. \tag{38}$$

By numerical integration, it can be obtained

$$\int_{r=0}^{\infty}\Big[1+\exp\Big(\frac{r-1.82}{0.52}\Big)\Big]^{-1}4\pi\rho_0 r^2\,\mathrm{d}r=44.66\rho_0\,. \tag{39}$$

The coulomb energy $\triangle E_c$ is given by[26]

$$\triangle E_c=\frac{\hbar c\alpha}{\sqrt{3}}\Big\langle\frac{1}{r}\Big\rangle=\frac{\hbar c\alpha}{\sqrt{3}}\int\mathrm{d}^3(r)\frac{\rho(r)}{r}\,, \tag{40}$$

where α is the fine structure constant. By using the Eqs. (38), (39), and (40) and by numerical integration, we can obtain the Coulomb energy as follow

$$\rho_0(^3\mathrm{He})\approx 7.16\times10^{-21}\ \mathrm{C}\cdot\mathrm{fm}^{-3},$$
$$\triangle\, E_I\approx 0.86\ \mathrm{MeV}\,, \tag{41}$$
$$\rho_0(^3\mathrm{H})\approx 3.58\times10^{-21}\ \mathrm{C}\cdot\mathrm{fm}^{-3}\,,$$
$$\triangle\, E_I\approx 0.28\ \mathrm{MeV}\,. \tag{42}$$

Results obtained here and in other works are given in Table 1. The energy difference of mirror nuclei ^{3}He and ^{3}H addition of isospin, is also due to other effects[13,18,26]

$$\triangle E_B=\triangle E_I+\text{Other effects}\,.$$

The effects of other interaction that are not considered in this paper are given in Table 2. As you can see, a good value of system's binding energy is obtained. The difference between binding energy of ^{3}H and ^{3}He obtained here is comparable with experimental value in Table 2. It is highly important to say although some methods gives us more accurate binding energy but these methods are very complicated (these methods such as AV18 have free parameters about 45 in each model) and advantage of this model is that its equations are very simple and this method can be extended to more three-nucleon systems easily and calculate the properties of these nucleuses.

Table 1 Binding energies of ^{3}H and ^{3}He within different methods.[18]

Hamiltonian	^{3}H	^{3}He
AV18	7.624 MeV	6.925 MeV
N3LO-Idaho	7.618 MeV	6.917 MeV
AV18+URIX	8.479 MeV	7.750 MeV
N3LO-Idaho+N2LOL	8.474 MeV	7.742 MeV
Our model	7.66 MeV	7.08 MeV
Experimental value	8.482 MeV	7.718 MeV

Table 2 Different contribution to the (^{3}H-^{3}He) mass difference.[18,26]

Interaction term	$B(^3\mathrm{H})$-$B(^3\mathrm{He})$
Nuclear CSB	65 KeV
Point Coulomb*	677 KeV
Magnetic moment	17 KeV
Orbit-orbit force	7 KeV
n-*p* mass difference	14 KeV
Total	751 KeV
Coulomb obtained here*	580 KeV
Experimental value	764 KeV

5 Conclusions

In this paper, the schrödinger equation for three-nucleon systems has been solved by using the Jacobian coordinates and H.H approach. We have applied a mathematical modification method to obtain the binding energy and wavefunction of three-nucleon systems in isospin invariant space then we have calculated the isospin effect on the binding energy of ^{3}H and ^{3}He where leads to mass splitting of three-nucleon systems. A good value of system's binding energy is obtained and the wavefunction satisfies the boundary conditions. This results are agree well with the experimental values. We stress that even though the problem has been attacked by different methods but our elegant methodology is powerful because it is simple and it can be extended to more three-nucleon systems easily.

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