Determination of Deuteron Dipole Moment in Nuclear Quark-Like Model

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Abstract *Using the quark-like model, we have improved the existing deviation between theoretical and experimental values of magnetic dipole moment of deuteron. Based upon Pauli Exclusion Principle, the constituent quarks form a ground state for $l = 0$. The expectation value of the deuteron magnetic dipole moment operator is determined to be equal to $0.861\,597\,8\mu_N$ in better agreement with the measured value of $0.857\,437\,6\mu_N$ as compared to the shell model calculations.*

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1 Introduction

According to present quark model, hadrons are made of quarks with three generation and six flavors. Each quark has its own spin, charge, bare mass and different effective masses in baryon and mesons.[1] Deuteron is made of one proton and one neutron. In shell model, the magnetic dipole moment of deuteron μ_D is calculated in ground state ($l = 0$) by using the proton and neutron spin g-factors namely, $g_{\rm sp} = 5.585\,691\,2$ and $g_{\rm sn} = -3.826\,083\,7$.[2] In shell model the obtained value of μ_D is given as $\mu_D = 0.879\,804\mu_N$ where μ_N stands for nuclear magenton.[3] The experimental measured value of μ_D is given as $\mu_D^{\rm exp} = 0.857\,437\,6\mu_N$.[3–5] In order to reduce the difference between the measured and calculated values, it was assumed that the deuteron ground state is a combination of $l = 0$ and $l = 2$ states.[3] By using the same spin g-factor for proton and neutron, the magnetic dipole moment of other nuclei were also determined resulting in larger difference between the measured and calculated values.

In order to achieve a better match it was assumed that the spin g-factors of free protons and neutrons are different from bound protons and neutrons and their approximation relation is, $g_s^{\rm bound} \cong 0.6 g_s^{\rm free}$.[6]

In this work our approach is quite different and based upon the constituent quarks, the magnetic dipole moment of deuteron is determined in quark-like model. Based upon this model new binding energy formula is presented and all the magic numbers are obtained and new magic number namely 184 is predicted.[7–10] Using this model, the deuteron wave function is given in terms of its quark structure. Therefore deuteron is made of three up and three down quarks with different colors.

Considering the Pauli Exclusion Principle all six quarks can form ground state for $l = 0$ case. Then this ground state deuteron wave function is used to calculate the expectation value of the magnetic dipole moment operator of the deuteron.

2 Determination of Deuteron Wave Function

The deuteron wave function is written in terms of four different parts namely, space, spin, flavor and color parts as:[1,8]

$$\psi = \psi(\text{space})\psi(\text{spin})\psi(\text{flavor})\psi(\text{color})\,. \tag{1}$$

The total wave function is anti-symmetric (under exchange of two nucleons in nuclei or two quarks in nucleon). Although the functional dependency of the ground state of the space part is not known but it is symmetric due to the even parity of the deuteron. Assuming that the strong force acting among the quarks have a radical nature, one can conclude that the states $l = 0, 1, 2$, and ... do not mix and the ground state involve only $l = 0$ state. The three colors are generators of SU(2) color symmetry and three colors together make one decuplet and two octet and one color singlet namely,

$$3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1\,. \tag{2}$$

Naturally made particles are colorless and color singlet. In SU(3) color singlet is anti-symmetric.[1] Therefore, the deuteron wave function is written in terms of proton and neutron wave functions as follow:

$$\psi = \psi(\text{space})(|p\rangle|n\rangle - |n\rangle|p\rangle)/\sqrt{2}\,. \tag{3}$$

The total wave function of a system of fermions is anti-symmetric. Nucleons are made of 3 quarks. The spin part is obtained from SU(2) group[8] as follow:

$$2 \otimes 2 \otimes 2 = 4 \oplus 2 \oplus 2\,, \tag{4}$$

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$$\left.\begin{aligned}&\left|\frac{3}{2},\frac{3}{2}\right\rangle=(\uparrow\uparrow\uparrow)\\&\left|\frac{3}{2},\frac{1}{2}\right\rangle=\frac{(\uparrow\uparrow\downarrow+\uparrow\downarrow\uparrow+\downarrow\uparrow\uparrow)}{\sqrt{3}}\\&\left|\frac{3}{2},\frac{-1}{2}\right\rangle=\frac{(\uparrow\downarrow\downarrow+\downarrow\uparrow\downarrow+\downarrow\downarrow\uparrow)}{\sqrt{3}}\\&\left|\frac{3}{2},\frac{-3}{2}\right\rangle=(\downarrow\downarrow\downarrow)\end{aligned}\right\}\quad\text{for spin}\frac{3}{2},\ (\psi_s)\,,\qquad(5)$$

$$\left.\begin{aligned}&\left|\frac{1}{2},\frac{1}{2}\right\rangle_{12}=\frac{(\uparrow\downarrow-\downarrow\uparrow)\uparrow}{\sqrt{2}}\\&\left|\frac{1}{2},\frac{-1}{2}\right\rangle_{12}=\frac{(\uparrow\downarrow-\downarrow\uparrow)\downarrow}{\sqrt{2}}\end{aligned}\right\}\quad\text{for spin}\frac{1}{2},\ (\psi_{12}\ )\,,\qquad(6)$$

$$\left.\begin{aligned}&\left|\frac{1}{2},\frac{1}{2}\right\rangle_{23}=\frac{\uparrow(\uparrow\downarrow-\downarrow\uparrow)}{\sqrt{2}}\\&\left|\frac{1}{2},\frac{-1}{2}\right\rangle_{23}=\downarrow(\uparrow\downarrow-\downarrow\uparrow)\sqrt{2}\end{aligned}\right\}\quad\text{for spin}\frac{1}{2},\ (\psi_{23})\,.\qquad(7)$$

All members of spin 3/2 are symmetric and for 1/2 are partially anti-symmetric and change sign under exchange of two particles. We can have also the following combination

$$\left.\begin{aligned}&\left|\frac{1}{2},\frac{1}{2}\right\rangle_{13}=\frac{(\uparrow\uparrow\downarrow-\downarrow\uparrow\uparrow)}{\sqrt{2}}\\&\left|\frac{1}{2},\frac{-1}{2}\right\rangle_{13}=(\uparrow\downarrow\downarrow-\downarrow\downarrow\uparrow)\sqrt{2}\end{aligned}\right\}\quad\text{for spin}\frac{1}{2},\ (\psi_{13})\,,\qquad(8)$$

which is not independent namely,

$$|\,\rangle_{13}=|\,\rangle_{12}+|\,\rangle_{23}\,.\qquad(9)$$

The flavor part of nucleons, ψ(flavor) is similar to the spin part due to the fact that are made of 2 quarks u and d and belong to the SU(2) group. In fact, by substituting $\uparrow$ spin for u and $\downarrow$ spins for d, the flavor part is studied. As stated before, the color function is anti-symmetric for proton and neutron and the space function is symmetric, therefore, ψ(flavor)ψ(spin) must be symmetric. Now since nucleons have spin 1/2 therefore, they have mixed symmetry and so is the case for ψ(flavor). Then we may write:

$$\psi(\text{nucleon})=\frac{\sqrt{2}}{3}[\psi_{12}(\text{flavor})\psi_{12}(\text{spin})+\psi_{23}(\text{flavor})\psi_{23}(\text{spin})+\psi_{13}(\text{flavor})\psi_{13}(\text{spin})]\,,\qquad(10)$$

$$\begin{aligned}\left|p:\frac{1}{2},\frac{1}{2}\right\rangle&=\Big\{\frac{1}{2}(\uparrow\downarrow\uparrow-\downarrow\uparrow\uparrow)(\text{udu - duu})+\frac{1}{2}(\uparrow\uparrow\downarrow-\uparrow\downarrow\uparrow)(\text{uud - udu})+\frac{1}{2}(\uparrow\uparrow\downarrow-\downarrow\uparrow\uparrow)(\text{uud - duu})\Big\}\frac{\sqrt{3}}{2}\\&=\frac{2}{3\sqrt{2}}(u(\uparrow)u(\uparrow)d(\downarrow)-\frac{1}{3\sqrt{2}}(u(\uparrow)u(\downarrow)d(\uparrow)-\frac{1}{3\sqrt{2}}(u(\downarrow)u(\uparrow)d(\uparrow)+\cdots,\end{aligned}\qquad(11)$$

including nine terms. Similarly neutron wave function is obtained by exchanging $u\leftrightarrow d$ in proton wave function. Now let us write the deuteron wave function, namely,

$$|D\rangle=\frac{1}{\sqrt{2}}[|p\rangle|n\rangle-|n\rangle|p\rangle]\,.\qquad(12)$$

Considering the spin components, the deuteron wave function is

$$\begin{aligned}|D:1,+1\rangle&=\frac{1}{\sqrt{2}}\Big\{\left|p:\frac{1}{2},\frac{1}{2}\right\rangle\left|n:\frac{1}{2},\frac{1}{2}\right\rangle-\left|n:\frac{1}{2},\frac{1}{2}\right\rangle\left|p:\frac{1}{2},\frac{1}{2}\right\rangle\Big\}\,,\\|D:1,0\rangle&=\frac{1}{2}\Big\{\Big(\left|p:\frac{1}{2},\frac{1}{2}\right\rangle\left|n:\frac{1}{2},\frac{-1}{2}\right\rangle+\left|p:\frac{1}{2},\frac{-1}{2}\right\rangle\left|n:\frac{1}{2},\frac{1}{2}\right\rangle\Big)-\Big(\left|n:\frac{1}{2},\frac{1}{2}\right\rangle\left|p:\frac{1}{2},\frac{-1}{2}\right\rangle+\left|n:\frac{1}{2},\frac{-1}{2}\right\rangle\left|p:\frac{1}{2},\frac{1}{2}\right\rangle\Big)\Big\},\\|D:1,-1\rangle&=\frac{1}{\sqrt{2}}\Big\{\left|p:\frac{1}{2},\frac{-1}{2}\right\rangle\left|n:\frac{1}{2},\frac{-1}{2}\right\rangle-\left|n:\frac{1}{2},\frac{-1}{2}\right\rangle\left|p:\frac{1}{2},\frac{-1}{2}\right\rangle\Big\}\end{aligned}\qquad(13)$$

Finally we obtain

$$\left|P:\frac{1}{2},\frac{1}{2}\right\rangle\left|n:\frac{1}{2},\frac{1}{2}\right\rangle=\frac{1}{18}\{4u(\uparrow)u(\uparrow)d(\downarrow)d(\uparrow)d(\uparrow)u(\downarrow)-2u(\uparrow)u(\uparrow)d(\downarrow)d(\uparrow)d(\downarrow)u(\uparrow)+\cdots\}\,,\qquad(14)$$

involving $9\times 9=81$ terms, which is given in appendix in details. The $|n\rangle|p\rangle$ part also has 81 terms and is obtained by $u\leftrightarrow d$ exchange in the first term.

3 Determination of Deuteron Magnetic Dipole Moment

In the absence of orbital motion, the magnetic dipole moment of deuteron is simply the vector sum of the six constituent quarks namely,

$$\mu=\mu_1+\mu_2+\mu_3+\mu_4+\mu_5+\mu_6\,.\qquad(15)$$

The terms of this equation depend upon the flavor of quarks (since up and down quarks have different magnetic dipole moments) and also depend upon the spin direction of six quarks.

The magnetic dipole moment of spin 1/2 particle with mass m and charge q is given[1,3] as

$$\mu=\frac{q}{mc}S\,.\qquad(16)$$

Substituting for spin 1/2, we have

$$\mu=\frac{q\hbar}{2mc}\,.\qquad(17)$$

Now for up and down quarks we obtain μ_u and μ_d as:

$$\mu_u = \frac{2}{3}\frac{e\hbar}{2m_u c}\,, \quad \mu_d = \frac{-1}{3}\frac{e\hbar}{2m_d c}\,. \tag{18}$$

Therefore the deuteron magnetic dipole moment is given as:

$$\mu_D = \langle D:1,1|\mu_1+\mu_2+\mu_3+\mu_4+\mu_5+\mu_6|D:1,1\rangle$$
$$-\frac{2}{\hbar}\sum_{i=1}^{6}\langle D:1,1|\mu_i s_{iz}|D:1,1\rangle\,. \tag{19}$$

The detailed calculations of the deuteron wave function are given in appendix. The first term of this wave function is $(4/18)[uudddu(\uparrow\uparrow\downarrow\uparrow\uparrow\downarrow)]$ therefore,

$$\frac{2}{\hbar}\Big(\sum_{i=1}^{6}\mu_i s_{iz}\Big)[uudddu(\uparrow\uparrow\downarrow\uparrow\uparrow\downarrow)]$$
$$= (\mu_u+\mu_u-\mu_d+\mu_d+\mu_d-\mu_u)[uudddu(\uparrow\uparrow\downarrow\uparrow\uparrow\downarrow)]$$
$$= (\mu_u+\mu_d)[uudddu(\uparrow\uparrow\downarrow\uparrow\uparrow\downarrow)]\,. \tag{20}$$

The expectation value for magnetic dipole moment of this first term is

$$\Big(\frac{4}{18}\Big)^2\frac{2}{\hbar}\sum_{i=1}^{6}\langle uudddu(\uparrow\uparrow\downarrow\uparrow\uparrow\downarrow)|\mu_i\mu_{iz}|uudddu(\uparrow\uparrow\downarrow\uparrow\uparrow\downarrow)\rangle = \frac{16}{324}(\mu_u+\mu_d)\,. \tag{21}$$

Similar calculations are carried out for each remaining terms of deuteron wave function and we obtained, the following results,

$$\frac{1}{2}\Big\langle n:\frac{1}{2},\frac{1}{2}\Big|\Big\langle p:\frac{1}{2},\frac{1}{2}\Big|\Big(\frac{2}{\hbar}\sum_{i=1}^{6}\mu_i\mu_{iz}\Big)\Big|P:\frac{1}{2,}\frac{1}{2}\Big\rangle\Big|n:\frac{1}{2},\frac{1}{2}\Big\rangle = \frac{1}{2}(\mu_u+\mu_d)\,. \tag{22}$$

Similarly for the second term, we get,

$$\frac{1}{2}\Big\langle p:\frac{1}{2},\frac{1}{2}\Big|\Big\langle n:\frac{1}{2},\frac{1}{2}\Big|\Big(\frac{2}{\hbar}\sum_{i=1}^{6}\mu_i\mu_{iz}\Big)\Big|n:\frac{1}{2,}\frac{1}{2}\Big\rangle\Big|p:\frac{1}{2},\frac{1}{2}\Big\rangle = \frac{1}{2}(\mu_u+\mu_d)\,. \tag{23}$$

Finally the magnetic dipole moment of deuteron is

$$\mu_D = \langle D:1,1|\mu|D:1,1\rangle = \mu_u+\mu_d = 0.861\,597\,8\mu_N\,, \tag{24}$$

where

$$\mu_u = \frac{2}{3}\frac{e\hbar}{2m_u c} = \frac{2}{3}\frac{m_p}{m_u}\frac{e\hbar}{2m_p c} = \frac{2m_p}{3m_u}\mu_N$$
$$= 1.723\,195\,6\mu_N\,, \tag{25}$$

$$\mu_d = \frac{-1}{3}\frac{e\hbar}{2m_d c} = \frac{-m_p}{3m_d}\mu_N = -0.861\,597\,8\mu_N\,, \tag{26}$$

where μ_N stands for nuclear magenton and

$$m_p = 938.280\ \mathrm{Mev/c^2}$$

and $m_u = m_d = 363\ \mathrm{Mev/c^2}$.[1]

4 Conclusion

We have improved the deviation of the previously found theoretical and experimental values of magnetic dipole moment of deuteron. The experimental value for μ_D is $0.857\,437\,6\mu_N$. In the shell model for $l=0$ the values $\mu_D = 0.874\,804\,6\mu_N$ where as our finding for μ_D is $0.861\,597\,8\mu_N$ which is in better agreement with experimental measurement. Having chosen a tensor component for the strong force, then we could find the higher state mixture by using the experimental data of magnetic dipole moment of deuteron, similar to shell model calculations for two particles. Our investigation indicates that the amount of higher state mixture in the ground state is less than 3 percent.

Appendix

$$\Big|p:\frac{1}{2},\frac{1}{2}\Big\rangle\Big|n:\frac{1}{2},\frac{1}{2}\Big\rangle = \frac{1}{18}\{4uudddu(\uparrow\uparrow\downarrow\uparrow\uparrow\downarrow) - 2uudddu(\uparrow\uparrow\downarrow\uparrow\downarrow\uparrow) - 2uudddu(\uparrow\uparrow\downarrow\downarrow\uparrow\uparrow)$$
$$- 2uuddud(\uparrow\uparrow\downarrow\uparrow\uparrow\downarrow) + 4uuddud(\uparrow\uparrow\downarrow\uparrow\downarrow\uparrow) - 2uuddud(\uparrow\uparrow\downarrow\downarrow\uparrow\uparrow)$$
$$- 2uududd(\uparrow\uparrow\downarrow\uparrow\uparrow\downarrow) - 2uududd(\uparrow\uparrow\downarrow\uparrow\downarrow\uparrow) + 4uududd(\uparrow\uparrow\downarrow\downarrow\uparrow\uparrow)$$
$$- 2uudddu(\uparrow\downarrow\uparrow\uparrow\uparrow\downarrow) + uudddu(\uparrow\downarrow\uparrow\uparrow\downarrow\uparrow) + uudddu(\uparrow\downarrow\uparrow\downarrow\uparrow\uparrow)$$
$$+ uuddud(\uparrow\downarrow\uparrow\uparrow\uparrow\downarrow) - 2uuddud(\uparrow\downarrow\uparrow\uparrow\downarrow\uparrow) + uuddud(\uparrow\downarrow\uparrow\downarrow\uparrow\uparrow)$$
$$+ uududd(\uparrow\downarrow\uparrow\uparrow\uparrow\downarrow) + uududd(\uparrow\downarrow\uparrow\uparrow\downarrow\uparrow) - 2uududd(\uparrow\downarrow\uparrow\downarrow\uparrow\uparrow)$$
$$- 2uudddu(\downarrow\uparrow\uparrow\uparrow\uparrow\downarrow) + uudddu(\downarrow\uparrow\uparrow\uparrow\downarrow\uparrow) + uudddu(\downarrow\uparrow\uparrow\downarrow\uparrow\uparrow)$$
$$+ uuddud(\downarrow\uparrow\uparrow\uparrow\uparrow\downarrow) - 2uuddud(\downarrow\uparrow\uparrow\uparrow\downarrow\uparrow) + uuddud(\downarrow\uparrow\uparrow\downarrow\uparrow\uparrow)$$

$$
\begin{aligned}
&+ uududd(\downarrow\uparrow\uparrow\uparrow\uparrow\downarrow) + uududd(\downarrow\uparrow\uparrow\uparrow\downarrow\uparrow) - 2uududd(\downarrow\uparrow\uparrow\downarrow\uparrow\uparrow)\\
&- 2ududdu(\uparrow\uparrow\downarrow\uparrow\uparrow\downarrow) + ududdu(\uparrow\uparrow\downarrow\uparrow\downarrow\uparrow) + ududdu(\uparrow\uparrow\downarrow\downarrow\uparrow\uparrow)\\
&+ ududud(\uparrow\uparrow\downarrow\uparrow\uparrow\downarrow) - 2ududud(\uparrow\uparrow\downarrow\uparrow\downarrow\uparrow) + ududud(\uparrow\uparrow\downarrow\downarrow\uparrow\uparrow)\\
&+ uduudd(\uparrow\uparrow\downarrow\uparrow\uparrow\downarrow) + uduudd(\uparrow\uparrow\downarrow\uparrow\downarrow\uparrow) - 2uduudd(\uparrow\uparrow\downarrow\downarrow\uparrow\uparrow)\\
&+ 4ududdu(\uparrow\downarrow\uparrow\uparrow\uparrow\downarrow) - 2ududdu(\uparrow\downarrow\uparrow\uparrow\downarrow\uparrow) - 2ududdu(\uparrow\downarrow\uparrow\downarrow\uparrow\uparrow)\\
&- 2ududud(\uparrow\downarrow\uparrow\uparrow\uparrow\downarrow) + 4ududud(\uparrow\downarrow\uparrow\uparrow\downarrow\uparrow) - 2ududud(\uparrow\downarrow\uparrow\downarrow\uparrow\uparrow)\\
&- 2uduudd(\uparrow\downarrow\uparrow\uparrow\uparrow\downarrow) - 2uduudd(\uparrow\downarrow\uparrow\uparrow\downarrow\uparrow) + 4uduudd(\uparrow\downarrow\uparrow\downarrow\uparrow\uparrow)\\
&- 2ududdu(\downarrow\uparrow\uparrow\uparrow\uparrow\downarrow) + ududdu(\downarrow\uparrow\uparrow\uparrow\downarrow\uparrow) + ududdu(\downarrow\uparrow\uparrow\downarrow\uparrow\uparrow)\\
&+ ududud(\downarrow\uparrow\uparrow\uparrow\uparrow\downarrow) - 2ududud(\downarrow\uparrow\uparrow\uparrow\downarrow\uparrow) + ududud(\downarrow\uparrow\uparrow\downarrow\uparrow\uparrow)\\
&+ uduudd(\downarrow\uparrow\uparrow\uparrow\uparrow\downarrow) + uduudd(\downarrow\uparrow\uparrow\uparrow\downarrow\uparrow) - 2uduudd(\downarrow\uparrow\uparrow\downarrow\uparrow\uparrow)\\
&- 2duuddu(\uparrow\uparrow\downarrow\uparrow\uparrow\downarrow) + duuddu(\uparrow\uparrow\downarrow\uparrow\downarrow\uparrow) + duuddu(\uparrow\uparrow\downarrow\downarrow\uparrow\uparrow)\\
&+ duudud(\uparrow\uparrow\downarrow\uparrow\uparrow\downarrow) - 2duudud(\uparrow\uparrow\downarrow\uparrow\downarrow\uparrow) + duudud(\uparrow\uparrow\downarrow\downarrow\uparrow\uparrow)\\
&+ duuudd(\uparrow\uparrow\downarrow\uparrow\uparrow\downarrow) + duuudd(\uparrow\uparrow\downarrow\uparrow\downarrow\uparrow) - 2duuudd(\uparrow\uparrow\downarrow\downarrow\uparrow\uparrow)\\
&- 2duuddu(\uparrow\downarrow\uparrow\uparrow\uparrow\downarrow) + duuddu(\uparrow\downarrow\uparrow\uparrow\downarrow\uparrow) + duuddu(\uparrow\downarrow\uparrow\downarrow\uparrow\uparrow)\\
&+ duudud(\uparrow\downarrow\uparrow\uparrow\uparrow\downarrow) - 2duudud(\uparrow\downarrow\uparrow\uparrow\downarrow\uparrow) + duudud(\uparrow\downarrow\uparrow\downarrow\uparrow\uparrow)\\
&+ duuudd(\uparrow\downarrow\uparrow\uparrow\uparrow\downarrow) + duuudd(\uparrow\downarrow\uparrow\uparrow\downarrow\uparrow) - 2duuudd(\uparrow\downarrow\uparrow\downarrow\uparrow\uparrow)\\
&+ 4duuddu(\downarrow\uparrow\uparrow\uparrow\uparrow\downarrow) - 2duuddu(\downarrow\uparrow\uparrow\uparrow\downarrow\uparrow) - 2duuddu(\downarrow\uparrow\uparrow\downarrow\uparrow\uparrow)\\
&- 2duudud(\downarrow\uparrow\uparrow\uparrow\uparrow\downarrow) + 4duudud(\downarrow\uparrow\uparrow\uparrow\downarrow\uparrow) - 2duudud(\downarrow\uparrow\uparrow\downarrow\uparrow\uparrow)\\
&- 2duuudd(\downarrow\uparrow\uparrow\uparrow\uparrow\downarrow) - 2duuudd(\downarrow\uparrow\uparrow\uparrow\downarrow\uparrow) + 4duuudd(\downarrow\uparrow\uparrow\downarrow\uparrow\uparrow)\} \,.
\end{aligned}
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