

Plane Symmetric Solutions in $f(R, T)$ Gravity

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Abstract The main purpose of this paper is to investigate the exact solutions of plane symmetric spacetime in the context of $f(R, T)$ gravity [Phys. Rev. D **84** (2011) 024020], where $f(R, T)$ is an arbitrary function of Ricci scalar R and trace of the energy momentum tensor T . We explore the exact solutions for two different classes of $f(R, T)$ models. The first class $f(R, T) = R + 2f(T)$ yields a solution which corresponds to Taub's metric while the second class $f(R, T) = f_1(R) + f_2(T)$ provides two additional solutions which include the well known anti-deSitter spacetime. The energy densities and corresponding functions for $f(R, T)$ models are evaluated in each case.

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1 Introduction

The most popular issue in the modern day cosmology is the current expansion of universe. Observational and theoretical facts indicate that our universe is in the phase of accelerated expansion.^[1] The existence of dark matter and dark energy is another interesting topic of discussion.^[2] Almost a century ago, Einstein introduced the concept of dark energy by adding a small positive cosmological constant in the field equations. But he rejected this idea later on. However, it is now thought that the cosmological constant may become a suitable candidate for dark energy. Modified theories of gravity seem attractive to explain late time acceleration of the universe. An interesting modified theory of gravity is the $f(R)$ theory which involves a generic function of Ricci scalar in standard Einstein–Hilbert Lagrangian.

In recent years, $f(R)$ gravity has been investigated by many authors in different contexts.^[3–17] Some interesting review articles^[18] can be helpful to understand the theory. Bamba *et al.*^[19] discussed curvature singularity appearing in the collapse process of a star in this theory. They proved that curvature singularity could be avoided by adding R^α term in the viable $f(R)$ gravity models. Thermodynamics of the apparent horizon in the Palatini $f(R)$ gravity has been discussed by Bamba and Geng.^[20] Capozziello *et al.*^[21] found spherically symmetric solutions in $f(R)$ gravity via Noether symmetries. Cylindrically symmetric vacuum and non-vacuum solutions have also been explored in this theory.^[22] Sharif and Shamir^[23] investigated plane symmetric solutions in metric $f(R)$ gravity. The same authors^[24] obtained the solutions of Bianchi types I and V cosmologies for vacuum and non-vacuum cases. Kucukakca and Camci^[25] investigated Palatini $f(R)$ gravity

using Noether gauge symmetry approach. For this purpose, they considered a flat Friedmann–Robertson–Walker (FRW) universe and it was shown that the resulting form of $f(R)$ gravity model yielded a power law expansion for the scale factor of the universe. Conserved quantities in metric $f(R)$ gravity using Noether symmetry approach have been calculated.^[26]

Another modified theory named as $f(R, T)$ gravity has been developed by Harko *et al.*^[27] It is the generalization of $f(R)$ gravity and based upon the coupling of matter and geometry. In this theory, gravitational Lagrangian involves an arbitrary function of the scalar curvature R and the trace of the energy momentum tensor T . The equations of motions after the addition of an appropriate function $f(T)$ indicate the presence of an extra force acting on test particles. The investigation of perihelion shift of Mercury using $f(R, T)$ gravity provide an upper limit on the magnitude of the extra acceleration in the solar system which indicates the presence of dark energy.^[27] Thus the study of $f(R, T)$ gravity models may also provide better results as compared to the predictions of standard theory of general relativity (GR). The action for $f(R, T)$ theory of gravity is given by^[27]

$$S = \int \sqrt{-g} \left(\frac{1}{2\kappa} f(R, T) + L_m \right) d^4x, \quad (1)$$

where g is the determinant of the metric tensor $g_{\mu\nu}$ and L_m is the usual matter Lagrangian. It would be worthwhile to mention that if we replace $f(R, T)$ with $f(R)$, we obtain the action for $f(R)$ gravity and replacement of $f(R, T)$ with R leads to the action of GR. The energy momentum tensor $T_{\mu\nu}$ is defined as^[28]

$$T_{\mu\nu} = - \frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}L_m)}{\delta g^{\mu\nu}}. \quad (2)$$

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When we assume that the dependance of matter Lagrangian is merely on the metric tensor $g_{\mu\nu}$ rather than its derivatives, we get

$$T_{\mu\nu} = L_m g_{\mu\nu} - 2 \frac{\delta L_m}{\delta g^{\mu\nu}}. \quad (3)$$

Many authors have investigated this theory in recent years and a reasonable amount of work has been done so far.

Adhav^[29] explored the exact solutions of $f(R, T)$ field equations for locally rotationally symmetric Bianchi type I spacetime. Bianchi Type V cosmology with cosmological constant has been studied in this theory by Ahmed and Pradhan.^[30] Jamil *et al.*^[31] reconstructed cosmological models in $f(R, T)$ gravity and it was concluded that the dust fluid reproduced Λ CDM, phantom-non-phantom era and the phantom cosmology. Gödel type universe was studied in the framework of $f(R, T)$ gravity by Santos.^[32] Sharif and Zubair^[33] discussed the reconstruction and stability of $f(R, T)$ gravity with Ricci and modified Ricci dark energy. The same authors^[34] analyzed the laws of thermodynamics in this theory. However, it has been proved that the first law of black hole thermodynamics is violated for $f(R, T)$ gravity.^[35] Houndjo^[36] reconstructed $f(R, T)$ gravity by taking $f(R, T) = f_1(R) + f_2(T)$ where it was shown that $f(R, T)$ gravity allowed transition of matter from dominated phase to an acceleration phase. Harko and Lake^[37] found cylindrically symmetric interior string like solutions in $f(R, L_m)$ theory of gravity. In a recent paper,^[38] we investigated the exact solutions of cylindrically symmetric spacetime in $f(R, T)$ gravity and recovered two solutions which corresponded to an exterior metric of cosmic string and a non-null electromagnetic field. We explored the exact solutions of locally rotationally symmetric Bianchi Type I universe in the context of $f(R, T)$ gravity.^[39] In another work,^[40] we investigated Bianchi type I cosmology in $f(R, T)$ gravity with some interesting results. It was concluded that equation of state parameter $w \rightarrow -1$ as $t \rightarrow \infty$ which suggested an accelerated expansion of the universe. Thus it is hoped that $f(R, T)$ gravity may explain the resent phase of cosmic acceleration of our universe. This theory can be used to explore many issues and may provide some satisfactory results.

In this paper, we focus ourselves to investigate the exact solutions of plane symmetric spacetime in the framework of $f(R, T)$ gravity. The plan of paper is as follows: In Sec. 2, we give some basics of $f(R, T)$ gravity. Section 3 provides the exact solutions for plane symmetric spacetime using two different classes of $f(R, T)$ models. Summary and concluding remarks are given in the last section.

2 Some Basics of $f(R, T)$ Gravity

The $f(R, T)$ gravity field equations are obtained by varying the action S in Eq. (1) with respect to the metric

tensor $g_{\mu\nu}$

$$\begin{aligned} f_R(R, T)R_{\mu\nu} - \frac{1}{2}f(R, T)g_{\mu\nu} - (\nabla_\mu \nabla_\nu - g_{\mu\nu}\square)f_R(R, T) \\ = \kappa T_{\mu\nu} - f_T(R, T)(T_{\mu\nu} + \Theta_{\mu\nu}), \end{aligned} \quad (4)$$

where ∇_μ denotes the covariant derivative and

$$\begin{aligned} \square \equiv \nabla^\mu \nabla_\mu, \quad f_R(R, T) = \frac{\partial f_R(R, T)}{\partial R}, \\ f_T(R, T) = \frac{\partial f_R(R, T)}{\partial T}, \quad \Theta_{\mu\nu} = g^{\alpha\beta} \frac{\delta T_{\alpha\beta}}{\delta g^{\mu\nu}}. \end{aligned}$$

Contraction of Eq. (4) yields

$$\begin{aligned} f_R(R, T)R + 3\square f_R(R, T) - 2f(R, T) \\ = \kappa T - f_T(R, T)(T + \Theta), \end{aligned} \quad (5)$$

where $\Theta = \Theta_\mu{}^\mu$. This is an important equation because it provides a relationship between Ricci scalar R and the trace T of energy momentum tensor. Using matter Lagrangian L_m , $\Theta_{\mu\nu}$ and the standard matter energy-momentum tensor are derived respectively as

$$\Theta_{\mu\nu} = -2T_{\mu\nu} + g_{\mu\nu}L_m - 2g^{\alpha\beta} \frac{\partial^2 L_m}{\partial g^{\mu\nu} \partial g^{\alpha\beta}}. \quad (6)$$

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu}, \quad (7)$$

satisfying the equation of state

$$p = w\rho, \quad (8)$$

where $u_\mu = \sqrt{g_{00}}(1, 0, 0, 0)$ is the four-velocity in comoving coordinates and ρ and p denote energy density and pressure of the fluid respectively. Perfect fluids problems involving energy density and pressure are not any easy task to deal with. Moreover, there does not exist any unique definition for matter Lagrangian. Thus we can assume the matter Lagrangian as

$$L_m = -p. \quad (9)$$

Using this value in Eq. (3) and Eq. (7), we get a constraint equation

$$2 \frac{\delta p}{\delta g^{\mu\nu}} = (p + \rho)u_\mu u_\nu. \quad (10)$$

Similarly, using Eq. (6) and Eq. (9), we get

$$\Theta_{\mu\nu} = -pg_{\mu\nu} - 2T_{\mu\nu}, \quad (11)$$

which gives an extra condition on p ,

$$g^{\alpha\beta} \frac{\partial^2 p}{\partial g^{\mu\nu} \partial g^{\alpha\beta}} = 0. \quad (12)$$

Using Eq. (11), the field equations (4) take the form

$$\begin{aligned} f_R(R, T)R_{\mu\nu} - \frac{1}{2}f(R, T)g_{\mu\nu} - (\nabla_\mu \nabla_\nu - g_{\mu\nu}\square)f_R(R, T) \\ = \kappa T_{\mu\nu} + f_T(R, T)(T_{\mu\nu} + pg_{\mu\nu}). \end{aligned} \quad (13)$$

It is mentioned here that these field equations depend on the physical nature of matter field. Many theoretical models corresponding to different matter contributions for $f(R, T)$ gravity are possible. However, Harko *et al.*^[27]

gave three classes of these models

$$f(R, T) = \begin{cases} R + 2f(T), \\ f_1(R) + f_2(T), \\ f_1(R) + f_2(R)f_3(T). \end{cases}$$

In this paper, we consider the first and second class only to investigate the exact plane symmetric solutions.

3 Exact Plane Symmetric Solutions

Here we shall find plane symmetric static solutions of the field equations in $f(R, T)$ gravity.

3.1 Plane Symmetric Spacetimes

We consider the general static plane symmetric spacetime given by

$$ds^2 = A(x)dt^2 - C(x)dx^2 - B(x)(dy^2 + dz^2). \quad (14)$$

For the sake of simplicity, we choose $C(x) = 1$ so that the above spacetime becomes

$$ds^2 = A(x)dt^2 - dx^2 - B(x)(dy^2 + dz^2). \quad (15)$$

The corresponding Ricci scalar takes the form

$$R = \frac{1}{2} \left[\frac{2A''}{A} - \left(\frac{A'}{A} \right)^2 + \frac{2A'B'}{AB} + \frac{4B''}{B} - \left(\frac{B'}{B} \right)^2 \right], \quad (16)$$

where prime represents derivative with respect to x . Now, we explore the solutions of the field equations for two classes of $f(R, T)$ models.

3.2 $f(R, T) = R + 2f(T)$

For the model $f(R, T) = R + 2f(T)$, the field equations become

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \kappa T_{\mu\nu} + 2f_T(T)T_{\mu\nu} + [f(T) + 2pf_T(T)]g_{\mu\nu}. \quad (17)$$

Here we find the most basic possible solution of this theory due to the complicated nature of field equations. However, in the next subsection we will explore the solutions with more general case. For the sake of simplicity, we use natural system of units ($G = c = 1$) and $f(T) = \lambda T$, where λ is a dynamical parameter. In this case, the gravitational field equations take the form similar to GR

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \lambda(T + 2p)g_{\mu\nu} = (8\pi + 2\lambda)T_{\mu\nu}, \quad (18)$$

where the term $\lambda(T + 2p)$ may play the role of cosmological constant Λ of the GR field equations. It would be worthwhile to mention here that the dependence of the cosmological constant Λ on the trace of the energy momentum tensor T has already been proposed by Poplawski^[41] where the cosmological constant in the gravitational Lagrangian is a function of T , and consequently the model was named as “ $\Lambda(T)$ gravity”. It was argued that recent cosmological data favour a variable cosmological constant which was consistent with $\Lambda(T)$ gravity. $\Lambda(T)$ gravity has

been shown to be more general than the Palatini $f(R)$ gravity.^[42]

Now using Eq. (18), we obtain a set of differential equations for plane symmetric spacetime,

$$\frac{B'^2}{4B^2} - \frac{B''}{B} = (8\pi + 3\lambda)\rho - p\lambda, \quad (19)$$

$$\frac{A'B'}{2AB} + \frac{B'^2}{4B^2} = (8\pi + 3\lambda)p - \lambda\rho, \quad (20)$$

$$\frac{A'B'}{4AB} + \frac{B''}{2B} - \frac{A'^2}{4A^2} + \frac{A''}{2A} - \frac{B'^2}{4B^2} = (8\pi + 3\lambda)p - \lambda\rho. \quad (21)$$

Thus we get three independent field equations in four unknowns namely A , B , ρ and p . Subtracting Eq. (20) and Eq. (21), it follows that

$$\left[\left(\frac{A'}{A} \right)^2 + 2 \left(\frac{B'}{B} \right)^2 - 2 \left(\frac{A''}{A} + \frac{B''}{B} \right) + \frac{A'B'}{AB} \right] = 0. \quad (22)$$

We solve this equation as follows:

Case I

As a first case, we solve Eq. (22) by the power law assumption, i.e., $A \propto x^r$ and $B \propto x^l$, where r and l are any real numbers. Thus we use $A = k_1x^r$ and $B = k_2x^l$, where k_1 and k_2 are constants of proportionality. It follows that

$$r = -\frac{2}{3}, \quad l = \frac{4}{3}, \quad (23)$$

and hence the solution metric becomes

$$ds^2 = k_1x^{-2/3}dt^2 - dx^2 - k_2x^{4/3}(dy^2 + dz^2). \quad (24)$$

It can be shown that these values of r and l lead to $R = 0$. By substituting the solution $A = k_1x^{-2/3}$ and $B = k_2x^{4/3}$ back in Eqs. (19)–(21), we get either $\lambda = -2\pi$ or $\lambda = -4\pi$ which shows that λ is not an arbitrary constant. If we re-define the parameters, i.e., $\sqrt{k_1}t \rightarrow \tilde{t}$, $\sqrt{k_2}y \rightarrow \tilde{y}$ and $\sqrt{k_2}z \rightarrow \tilde{z}$, the above metric takes the form

$$ds^2 = x^{-2/3}d\tilde{t}^2 - dx^2 - x^{4/3}(d\tilde{y}^2 + d\tilde{z}^2), \quad (25)$$

which is exactly the same as the well-known Taub's metric.^[43]

Case II

Now we assume $B = A^n$ so that Eq. (22) gives

$$(3n + 1)A'^2 - 2(n + 1)AA'' = 0. \quad (26)$$

This equation yields a solution given by

$$A = k_3[(n - 1)x + 2k_4(n + 1)]^{2(n+1)/(1-n)}, \quad n \neq 1, \quad (27)$$

where k_3 and k_4 are integration constants. Without loss of generality, we can choose $k_3 = 1$ and $k_4 = 0$ so that Eq. (27) takes the form

$$A = [(n - 1)x]^{2(n+1)/(1-n)}, \quad (28)$$

and thus B turns out to be

$$B = [(n - 1)x]^{2n(n+1)/(1-n)}, \quad (29)$$

and the solution metric takes the form

$$ds^2 = [(n-1)x]^{2(n+1)/(1-n)} dt^2 - dx^2 - [(n-1)x]^{2n(n+1)/(1-n)} (dy^2 + dz^2). \quad (30)$$

In this case, the energy density becomes

$$\rho = \frac{8n[2\pi(2-3n-8n^2-3n^3) - \lambda(2-n-5n^2-2n^3)]}{(n-1)^2 x^2}, \quad (31)$$

while the pressure of universe turns out to be

$$p = \frac{8n[2\pi(2+5n+4n^2+n^3) + \lambda(2+3n+n^2)]}{(n-1)^2 x^2}. \quad (32)$$

It would be worthwhile to mention here that we can recover Taub's solution when $n = -2$.

3.3 $f(R, T) = f_1(R) + f_2(T)$

Now we explore the solutions with more general class. Here the field equations for the model $f(R, T) = f_1(R) + f_2(T)$ become

$$f_{1R}(R)R_{\mu\nu} - \frac{1}{2}f_1(R)g_{\mu\nu} - (\nabla_\mu \nabla_\nu - g_{\mu\nu} \square) f_{1R}(R) = [\kappa + f_{2T}(T)]T_{\mu\nu} + [pf_{2T}(T) + \frac{1}{2}f_2(T)]g_{\mu\nu}. \quad (33)$$

Contracting the field equations (33), we obtain

$$Rf_{1R}(R) - 2f_1(R) + 3\square f_{1R}(R) = \kappa T + 2f_2(T) + [T + 4p]f_{2T}(T). \quad (34)$$

Using this, we can write

$$f_1(R) = \frac{3\square f_{1R}(R) + Rf_{1R}(R) - \kappa T - 2f_2(T) - [T + 4p]f_{2T}(T)}{2}. \quad (35)$$

Inserting this in Eq. (33), we get

$$\frac{f_{1R}(R)R_{\mu\nu} - \nabla_\mu \nabla_\nu f_{1R}(R) - (\kappa + f_{2T}(T))T_{\mu\nu}}{g_{\mu\nu}} = \frac{Rf_{1R}(R) - \square f_{1R}(R) - \kappa T - Tf_{2T}(T)}{4}. \quad (36)$$

Since the metric (15) depends only on x , one can view Eq. (36) as the set of differential equations for $f_{1R}(x)$, $f_{2T}(x)$, A and B . It follows from Eq. (36) that the combination

$$A_\mu \equiv \frac{f_{1R}(R)R_{\mu\mu} - \nabla_\mu \nabla_\mu f_{1R}(R) - (\kappa + f_{2T}(T))T_{\mu\mu}}{g_{\mu\mu}}, \quad (37)$$

is independent of the index μ and hence $A_\mu - A_\nu = 0$ for all μ and ν . Thus $A_0 - A_1 = 0$ yields

$$\frac{A'B'}{AB} + \left(\frac{B'}{B}\right)^2 - \frac{2B''}{B} - 2\frac{f_{1R}''(R)}{f_{1R}(R)} + \frac{A'f_{1R}'(R)}{Af_{1R}(R)} - 2\left[\frac{\kappa + f_{2T}(T)}{f_{1R}(R)}\right](\rho + p) = 0. \quad (38)$$

Also, $A_0 - A_2 = 0$ provides

$$\frac{A''}{A} - \frac{1}{2}\left(\frac{A'}{A}\right)^2 + \frac{A'B'}{2AB} - \frac{B''}{B} + \left[\frac{A'}{A} - \frac{B'}{B}\right]\frac{f_{1R}'(R)}{f_{1R}(R)} - 2\left[\frac{\kappa + f_{2T}(T)}{f_{1R}(R)}\right](\rho + p) = 0. \quad (39)$$

Thus we get two non-linear differential equations with five unknowns namely A , B , f_{1R} , f_{2T} , ρ and p . The solution of these equations could not be found straightforwardly. Subtracting Eq. (38) and Eq. (39), we obtain

$$\left[\frac{A'B'}{AB} + \left(\frac{A'}{A}\right)^2 + 2\left(\frac{B'}{B}\right)^2 - 2\left(\frac{A''}{A} + \frac{B''}{B}\right)\right]f_{1R}(R) + 2\frac{B'}{B}f_{1R}'(R) - 4f_{1R}''(R) = 0. \quad (40)$$

Due to highly non-linear nature of Eq. (40), here we also use the assumption $B = A^n$. Thus Eq. (40) reduces to

$$(3n+1)\left(\frac{A'}{A}\right)^2 - 2(n+1)\left(\frac{A''}{A}\right) + 2n\frac{A'f_{1R}'(R)}{Af_{1R}(R)} - 4\frac{f_{1R}''(R)}{f_{1R}(R)} = 0, \quad (41)$$

and the Ricci scalar turns out to be

$$R = \frac{1}{2}\left[(3n^2 - 2n - 1)\left(\frac{A'}{A}\right)^2 + (4n + 2)\frac{A''}{A}\right]. \quad (42)$$

We follow the approach of Nojiri and Odintsov^[44] and take the assumption $f_{1R}(R) \propto f_0 R^m$, where f_0 is an arbitrary constant. So using Eqs. (41), (42) and after some tedious calculations we obtain

$$\begin{aligned} & [8m(1+4n-2n^2-12n^3+9n^4) + 16m^2(1+4n-2n^2-12n^3+9n^4) \\ & - 1 - 3n + 6n^2 + 10n^3 - 21n^4 + 9n^5]A'^6 + [-8m(5+20n+2n^2-36n^3+9n^4) \\ & - 32m^2(2+8n-n^2-18n^3+9n^4) + 2(3+11n-4n^2-22n^3+21n^4-9n^5)] \\ & \times A'^4 A'' A + [-8m(1+4n+4n^2-9n^4) + 32m^2(1+4n+n^2-6n^3)] \end{aligned}$$

$$\begin{aligned}
& + 4n(1 + 4n + n^2 - 6n^3)]A'^3 A'''' A^2 + [8m(8 + 32n + 17n^2 - 30n^3 - 9n^4) \\
& + 16m^2(4 + 16n + 4n^2 - 24n^3 + 9n^4) - 4(3 + 13n + 10n^2 - 8n^3)]A'^2 A''^2 A^2 \\
& + [16m(1 + 4n + n^2 - 6n^3) - 32m^2(2 + 8n + 5n^2 - 6n^3) - 8n(1 + 4n + 4n^2)] \\
& \times A' A'' A'''' A^3 + 16m(1 + 4n + 4n^2)A'''' A'' A^4 - 8m(1 + 4n + n^2 - 6n^3)A'''' A^2 A^3 \\
& + [16m^2(1 + 4n + 4n^2) - 16m(1 + 4n + 4n^2)]A''''^2 A^4 \\
& + [-16m(2 + 8n + 5n^2 - 6n^3) + 8(1 + 5n + 8n^2 + 4n^3)]A''^3 A^3 = 0. \tag{43}
\end{aligned}$$

Many solutions can be reconstructed using this equation. However we discuss only three cases here.

Case III

In this case we try to recover the Taub's solution. For this purpose, we substitute $A = x^{-2/3}$ in Eq. (43). After some lengthy calculations, we obtain a constraint equation

$$18m^2 + 9m - 1 + n = 0. \tag{44}$$

We can obtain $B = x^{4/3}$ for $n = -2$. Thus Eq. (44) reduces to

$$6m^2 + 3m - 1 = 0. \tag{45}$$

The roots of Eq. (45) turn out to be $m = (-3 \pm \sqrt{33})/12$. Thus, we have

$$f_{1R}(R) = f_0 R^{(-3 \pm \sqrt{33})/12}. \tag{46}$$

After integration, we obtain

$$\begin{aligned}
f_1(R) &= \hat{f}_0(R)^{(9+\sqrt{33})/12} + k_5, \\
f_1(R) &= \check{f}_0(R)^{(9-\sqrt{33})/12} + k_6, \tag{47}
\end{aligned}$$

where $\hat{f}_0 = 12f_0/(9 + \sqrt{33})$, $\check{f}_0 = 12f_0/(9 - \sqrt{33})$ and k_5 , k_6 are integration constants. It has been shown that the terms with positive powers of the curvature support the inflationary epoch.^[45] The corresponding Ricci scalar is also zero in this case. Using first root $m = (-3 + \sqrt{33})/12$, EoS parameter ω and Eqs. (38), (39), the energy density of the universe turns out to be

$$\begin{aligned}
\rho &= \frac{f_0}{36(\kappa + f_{2T}(T))(1 + \omega)} \\
&\times \left[\frac{6^{\frac{15-\sqrt{33}}{12}}}{x^{\frac{1+\sqrt{33}}{6}}} + \frac{5(-3+\sqrt{33})6^{\frac{15-\sqrt{33}}{12}}}{x^{\frac{9+\sqrt{33}}{6}}} - \frac{1}{6x^6} \right], \\
\omega &\neq -1. \tag{48}
\end{aligned}$$

We can choose the sign of f_0 depending upon the values of ω to get the positive energy density. Similarly, we can find expression for energy density in the case of other root $m = (-3 - \sqrt{33})/12$.

Case IV

Here we take $A = 1/x$ in Eq. (43) to obtain a constraint equation

$$16m^2 + 8m - 3n + 3 = 0. \tag{49}$$

Using this equation, it follows that

$$B = x^{-(16m^2+8m+3)/3}. \tag{50}$$

The solution metric takes the form

$$ds^2 = \frac{1}{x} dt^2 - dx^2 - x^{-(16m^2+8m+3)/3} (dy^2 + dz^2). \tag{51}$$

The corresponding Ricci scalar becomes

$$R = \frac{3(n^2 + 2n + 1)}{2x^2}. \tag{52}$$

We can construct different $f_1(R)$ models for different values of m satisfying Eq. (49). However, an interesting logarithmic form of $f_1(R)$ models is obtained for $m = -1$

$$f_1(R) = f_0 \ln R + k_7, \tag{53}$$

where k_7 is an integration constant. In this case Ricci scalar becomes $R = 98/3x^2$ and the solution metric takes the form

$$ds^2 = \frac{1}{x} dt^2 - dx^2 - \frac{1}{x^{11/3}} (dy^2 + dz^2). \tag{54}$$

The matter density turns out to be

$$\rho = \frac{-f_0}{441(\kappa + f_{2T}(T))(1 + \omega)} \left[93 + \frac{470596}{x^6} \right]. \tag{55}$$

Similarly for $m = -2$, we obtain

$$f_1(R) = -f_0 R^{-1} + k_8, \tag{56}$$

where k_8 is an integration constant. This model is also cosmologically important as it has been proved that negative power of curvature serves as an effective dark energy supporting the current cosmic acceleration.^[45] Obviously one can work out the Ricci scalar, energy density and the solution metric in this case.

Case V

Here we consider $A = e^{\alpha x}$, where α is a non-zero real number. In this case, we obtain a constraint equation using Eq. (43)

$$(n-1)(3n^2 + 2n + 1)^2 = 0, \tag{57}$$

which does not involve parameter m . So this choice will yield a solution for any $f_1(R)$ model in power law or logarithmic form. The roots of Eq. (57) turn out to be

$$n = 1, \quad \frac{-3 \pm i\sqrt{3}}{6}. \tag{58}$$

We discard the imaginary roots and consider the real value of n to get a physical solution. In this case the Ricci scalar turns out to be non-zero constant, i.e., $R = 3$. The energy density becomes

$$\rho = \frac{-3f_0}{16(\kappa + f_{2T}(T))(1 + \omega)}, \tag{59}$$

and the solution metric becomes

$$ds^2 = e^{\alpha x} (dt^2 - dy^2 - dz^2) - dx^2. \tag{60}$$

This corresponds to the well-known anti-deSitter spacetime in GR.^[46]

3.4 Physical Importance of the Solutions

The spacetime admitting three parameter group of motions of the Euclidean plane is said to possess plane symmetry and is known as a plane symmetric spacetime. Such spacetime possesses many properties equivalent to those of spherical symmetry. The plane symmetric spacetime has been extensively investigated by many researchers from various standpoints. Taub,^[47] Bondi,^[48] Bondi and Pirani–Robinson^[49] defined and studied plane wave solutions. They considered the concept of group of motions of spacetime which played a fundamental role in plane gravitational waves. It has been established that the spacetime Eq. (15) admits the plane wave solutions of GR field equations.^[50]

In this study, we have explored plane symmetric solutions in the context of $f(R, T)$ gravity. The non-vacuum plane symmetric solutions provide Taub's universe with a singularity at $x = 0$, which suggests the presence of black hole. Another solution (51) suggests that an object falling into a black hole approaches the singularity at $x = 0$. However, non-singular solution is obtained in the shape of an anti-deSitter spacetime. An anti-deSitter space is a GR like spacetime, where in the absence of matter or energy, the curvature of spacetime is naturally hyperbolic. From geometrical point of view, an anti-de Sitter space has a curvature analogous to a flat cloth sitting on a saddle, with a very slight curvature because it is so large. Thus it would correspond to a negative cosmological constant. Anti-deSitter space can also be thought as empty space having negative energy, which causes this spacetime to collapse at a greater rate. The existence of quantum-corrected deSitter space has been predicted as an outcome of a nontrivial solution for constant curvature R_0 in $f(R)$ gravity.^[51] One may play with the parameters of the theory under consideration in such a way that the deSitter space can provide a solution to the cosmological constant problem. Thus the physical relevance of the solutions is obvious.

4 Concluding Remarks

This paper is devoted to explore the exact static plane symmetric solutions in $f(R, T)$ gravity. To our knowl-

edge, this is the first attempt to investigate plane symmetric solutions in $f(R, T)$ gravity. In this work, we consider two classes of $f(R, T)$ models. First we take $f(R, T) = R + 2f(T)$. For this class, we investigate two cases and recover a solution which corresponds to the well known Taub's spacetime.^[43] The expression for matter density ρ and pressure p have been calculated for this solution.

The second class with $f(R, T) = f_1(R) + f_2(T)$ is the more general choice to explore the solutions. We assume $f_{1R}(R) \propto f_0 R^m$, where f_0 and m are arbitrary real constants. The corresponding field equations are solved for three cases, which provide one new solution and two important solutions namely Taub's and anti-desitter spacetimes are recovered. So the physical relevance of these solutions is obvious. The function of Ricci scalar $f_1(R)$ contains positive power of curvature in the first case while the second case corresponds to negative power of curvature. It is worth mentioning here that the terms with positive power of curvature support the inflationary epoch while the term with negative power of curvature serve as an effective dark energy which supports the current cosmic acceleration.^[45] We also provide a choice for solution with logarithmic form of $f(R)$ model.

We have discussed five cases in this paper. However, many other cases can also be explored and different cosmologically important $f(R, T)$ models can be reconstructed. It would be worthwhile to mention here that such solutions may provide a gateway towards the solution of dark energy and dark matter problems. It also seems interesting to explore the exact solutions in other modified theories of gravity. The investigation of plane symmetric solutions in $f(G)$ gravity is under process.

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