Anisotropic Bianchi Type I Cosmological Models with Generalized Chaplygin Gas and Dynamical Gravitational and Cosmological Constants

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Abstract *This paper deals with study of generalized Chaplygin gas model with dynamical gravitational and cosmological constants. In this paper a new set of exact solutions of Einstein field equations for spatially homogeneous and anisotropic Bianchi type I space-time have been obtained. The solutions of the Einstein's field equations are obtained by considering (i) the power law relation between Hubble parameter H and scale factor R and (ii) scale factor of the form $R = -1/t + t^2$, $t > 1$. The assumptions lead to constant and variable deceleration parameter respectively. The physical and dynamical behaviors of the models have been discussed with the help of graphical representations. Also we have discussed the stability and physical acceptability of solutions for solution type-I and solution type-II.*

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1 Introduction

On the basis of recent cosmological observations obtained by distant type Ia Supernovae,[1] WMAP,[2] SDSS,[3] and X-Ray[4] it has been established that currently the Universe is undergoing the accelerated phase of expansion. Physical mechanism and driving force of the accelerated expansion of the Universe is yet to achieve. To understand the accelerated behavior of the Universe the cosmological constant has played a very important role. Many researchers have proposed cosmological models with time varying cosmological constant in order to solve the discrepancy between the cosmological constant inferred from observations and the vacuum energy density resulting from quantum field theories. Number of researchers[5−9] have investigated cosmological constant problem and consequences on cosmology with a time varying cosmological constant.

A variation of G has many interesting consequences in geology and astrophysics. Ever Since Dirac[10] first considered the possibility of variable G, then has been numerous modifications of general relativity to allow a variable G. Canuto and Narlikar[11] have shown that G varying cosmology is consistent with cosmological observations. Singh and Kotambkar[12] have discussed cosmological models with G and Λ in higher dimensional space-time. Vishwakarma[13] has investigated a Bianchi type I model with variable G and Λ. Singh *et al.*[14] have studied a new class of cosmological models with variable G and Λ. Bali and Yadav[15] have investigated Bianchi type IX viscoud fluid cosmological model in general relativity. Bali and Tinkar[16] have suggested Bianchi type V bulk viscous barotropic fluid cosmological model with variable G and Λ. Singh and Baghel[17] have investigated Bianchi type V Universe with bulk viscous matter and time varying gravitational and cosmological constants.

It is believed that during early stages of the Universe, it was not having isotropic behavior. Astronomical and astrophysical observations suggest that observed Universe is homogeneous and isotropic, hence space-time is usually described by Friedman–Lemaitra–Robertson–Walkar (FLRW) cosmology. It is widely believed that FLRW model does not give correct matter description in the early stage of the Universe. Anisotropic model plays a significant role in description of evolution of the early phase of Universe. Bianchi models I to IX present a middle way between FRW model and inhomogeneous and anisotropic Universe and thus important in modern cosmology.[18−26]

According to recent observational evidence the expansion of the Universe is accelerated, which is dominated by smooth component with negative pressure so called dark energy. Among the different theories put forward to understand the nature of dark energy, single component fluid known as Chaplygin gas (CG) with equation of state (EoS) $p = -B/\rho$,[27] where ρ and p are energy density and pressure respectively and B is a constant has attracted large interest in cosmology. Chaplygin gas is also an interest-

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ing subject of holography[28] and string theory.[29] Also it is the only kind of fluid that accepts a supper symmetric generation.[30] Chaplygin gas describes a transition from a decelerated cosmological expansion to the present cosmic acceleration and perhaps submit a deformation of Λ-CDM model. The inhomogeneous Chaplygin gas can combine dark energy and dark matter and play the unification role of them.[31−32] As CG model does not pass the tests connected with structure formation and observed strong oscillations of matter power spectrum,[33] the generalized Chaplygin gas has been proposed with the equation of state (EoS) $p = -B/\rho^{\alpha}$ with $0 \le \alpha \le 1$. The GCG is also interesting from holographic point of view. Since the inferences from GCG models are almost similar to the CDM models, this GCG model is modified as Modified Generalized Chaplygin gas model with EoS as $p = \mu\rho - B/\rho^{\alpha}$, where μ is a positive constant.[34] The evolution of generalized Chaplygin gas has been discussed by Wu *et al.*[35] Kamenshchik,[36] and Saha[37] have studied Bianchi type I cosmological model in the presence of perfect fluid and dark energy given by cosmological constant.

Inspired by the above research, it is worthwhile to study the anisotropic Bianchi type I cosmological models with dynamical gravitational and Cosmological constants in presence of GCG.

2 Field Equations

In this paper we have considered the gravitational field of matter distribution represented by a Bianchi type I space-time as

$$\mathrm{d}s^2 = \mathrm{d}t^2 - (R_1^2(t)\mathrm{d}x^2 + R_2^2(t)\mathrm{d}y^2 + R_3^2(t)\mathrm{d}z^2)\,. \quad (1)$$

Einstein's field equations with gravitational and cosmological "constant" for perfect fluid distribution are given as

$$R_{ij} - \frac{1}{2}Rg_{ij} = -8\pi G T_{ij} + \Lambda g_{ij}\,, \quad (2)$$

where T_{ij} is the energy momentum tensor, which is given as

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij}\,. \quad (3)$$

Here ρ is the energy density, p represents perfect fluid pressure, and u^i is the four velocity vector.

The Einstein's field equation (2) for the space-time metric (1) yields following equations

$$\frac{\ddot{R}_2}{R_2} + \frac{\ddot{R}_3}{R_3} + \frac{\dot{R}_2}{R_2}\frac{\dot{R}_3}{R_3} = -8\pi G p + \Lambda\,, \quad (4)$$

$$\frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_3}{R_3} + \frac{\dot{R}_1}{R_1}\frac{\dot{R}_3}{R_3} = -8\pi G p + \Lambda\,, \quad (5)$$

$$\frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_2}{R_2} + \frac{\dot{R}_1}{R_1}\frac{\dot{R}_2}{R_2} = -8\pi G p + \Lambda\,, \quad (6)$$

$$\frac{\dot{R}_1}{R_1}\frac{\dot{R}_2}{R_2} + \frac{\dot{R}_2}{R_2}\frac{\dot{R}_3}{R_3} + \frac{\dot{R}_3}{R_3}\frac{\dot{R}_1}{R_1} = 8\pi G \rho + \Lambda\,. \quad (7)$$

By a combination of equations (4)–(7) one can easily obtain

$$\dot{\rho} + (\rho + p)\Big(\frac{\dot{R}_1}{R_1} + \frac{\dot{R}_2}{R_2} + \frac{\dot{R}_3}{R_3}\Big) + \rho\frac{\dot{G}}{G} + \frac{\dot{\Lambda}}{8\pi G} = 0\,. \quad (8)$$

The energy momentum conservation equation ($T^{ij}_{;j}$) suggests

$$\dot{\rho} + 3(\rho + p)H = 0\,, \quad (9)$$

where H is the Hubble parameter, which is defined as

$$H = \frac{1}{3}\Big(\frac{\dot{R}_1}{R_1} + \frac{\dot{R}_2}{R_2} + \frac{\dot{R}_3}{R_3}\Big)\,.$$

From Eqs. (8) and (9) we have

$$\rho\frac{\dot{G}}{G} + \frac{\dot{\Lambda}}{8\pi G} = 0\,. \quad (10)$$

Further, Eqs. (4)–(6) yield the solutions

$$R_1 = c_1 R \exp\Big[\frac{k_1}{3}\int\Big(\frac{1}{R^3}\Big)\Big]\mathrm{d}t\,, \quad (11)$$

$$R_2 = c_2 R \exp\Big[\frac{k_2}{3}\int\Big(\frac{1}{R^3}\Big)\Big]\mathrm{d}t\,, \quad (12)$$

$$R_3 = c_3 R \exp\Big[\frac{k_3}{3}\int\Big(\frac{1}{R^3}\Big)\Big]\mathrm{d}t\,. \quad (13)$$

Here k_i and c_i ($i = 1, 2, 3$) are constants obeying the relation

$$\sum_{i=1}^{3} k_i = 0\,, \qquad \prod_{i=1}^{3} c_i = 1\,.$$

The above results suggest that the metric potential can be explicitly expressed in terms of scale factor represented by Bianchi type I space-time.

3 Cosmological Solutions

It can be easily seen that we have four equations (4)–(7) with six unknowns R_1, R_2, R_3, ρ, G and Λ. Hence to solve the system of equations completely we need two additional physically plausible relations among these variables.

3.1 *Solution Type I*

Consider the power law relation between Hubble parameter H and scale factor R as

$$H = \frac{\beta}{R^m}\,. \quad (14)$$

In order to solve Eq. (14), we consider two cases as $m \neq 0$ and $m = 0$.

(i) ***Case 1 ($m \neq 0$)*** In this case, on integration of Eq. (14) gives us

$$R = (m\beta t + D_1)^{1/m}\,, \quad (15)$$

where $D_1 = mc$ and c is an integration constant. With the help of Eqs. (11)–(13), and (15) take the form

$$R_1 = c_1(m\beta t + D_1)^{1/m} \times \exp\Big[\frac{k_1}{3\beta(m-3)}(m\beta t + D_1)^{1-3/m}\Big]\,, \quad (16)$$

$$R_2 = c_2(m\beta t + D_1)^{1/m} \times \exp\Big[\frac{k_2}{3\beta(m-3)}(m\beta t + D_1)^{1-3/m}\Big]\,, \quad (17)$$

$$R_3 = c_3(m\beta t + D_1)^{1/m}$$

$$\times \exp\Big[\frac{k_3}{3\beta(m-3)}(m\beta t+D_1)^{1-3/m}\Big]. \tag{18}$$

It is known that ordinary matter fields available from standard model of particle physics in general relativity, fails to account the present observations. Therefore modifications of the matter sector of the Einstein–Hilbert action with exotic matter are considered in the literature. Chaplygin gas (CG) is considered to be one such candidate for dark energy so in this case we have considered the equation of state for an exotic background fluid, the Chaplygin gas, described by equation of state

$$p=-\frac{A}{\rho^\alpha}\,,\quad A>0 \quad \text{and} \quad 0\le\alpha\le 1\,. \tag{19}$$

From Eqs. (9), (14) and (19) one can easily get the expression for energy density as

$$\rho=\big[A+D_2^{1+\alpha}(m\beta t+D_1)^{-3(1+\alpha)/m}\big]^{1/(1+\alpha)}\,, \tag{20}$$

here D_2 is a constant of integration. From Eq. (20) and Fig. 1, one can see that energy density is a decreasing function of time. Also it is noticed that, $\rho\nrightarrow 0$ as $t\to\infty$ which means that energy density maintain small constant value ($\lim_{t\to\infty}\rho^{1+\alpha}=A$) with evolution of time. From Eqs. (5), (7), (16)–(18), and (19)–(20), we can obtain the expression for Gravitational constant

$$G=\frac{1}{8\pi D_2^{1+\alpha}}\big[A+D_2^{1+\alpha}(m\beta t+D_1)^{-3(1+\alpha)/m}\big]^{\alpha/(1+\alpha)}\Big[\frac{2m\beta^2}{(m\beta t+D_1)^{-3(1+\alpha)/m+2}}-\frac{k_1^2+k_2^2+k_3^2}{9(m\beta t+D_1)^{3(1-\alpha)/m}}\Big]\,. \tag{21}$$

By substituting the values from Eqs. (16)–(18), (20), and (21) in Eq. (7), one can easily obtain

$$\Lambda=\frac{3\beta^2}{(m\beta t+D_1)^2}+\frac{k}{9(m\beta t+D_1)^{6/m}}-\frac{1}{D_2^{1+\alpha}}\big[A+D_2^{1+\alpha}(m\beta t+D_1)^{-3(1+\alpha)/m}\big]$$
$$\times\Big[\frac{2m\beta^2}{(m\beta t+D_1)^{-3(1+\alpha)/m+2}}-\frac{k_1^2+k_2^2+k_3^2}{9(m\beta t+D_1)^{3(1-\alpha)/m}}\Big]\,, \tag{22}$$

here $k=k_1k_2+k_1k_3+k_2k_3$. We have observed from Eq. (21) that G is positive or negative, it depends up on the term

$$\Big[\frac{2m\beta^2}{(m\beta t+D_1)^{-3(1+\alpha)/m+2}}-\frac{k_1^2+k_2^2+k_3^2}{9(m\beta t+D_1)^{3(1-\alpha)/m}}\Big]=T_1(\text{say})\,.$$

G is positive for $T_1>0$ and G is negative for $T_1<0$. After simple calculations it is found that $G>0$ for $18m\beta^2(m\beta t+D_1)^{6/m-2}>k_1^2+k_2^2+k_3^2$ and $G<0$ for $18m\beta^2(m\beta t+D_1)^{6/m-2}<k_1^2+k_2^2+k_3^2$. As a representative case we have considered two different values of $k_1^2+k_2^2+k_3^2(0.007, 531\,442)$, which is shown in Figs. 3 and 4 for different values of α. Also G is an increasing function of time. Cosmological constant Λ is plotted with the same considered values as in G. Cosmological constant takes values from positive to negative or positive values with the evolution of time, which can be observed from Figs. 5 and 6. It is interesting to note that when $G>0$ or $G<0$, Λ takes values from positive to negative or positive respectively. From the above observations one can conclude that G and Λ are opposite in nature.

The physical quantities of observational interest viz. the expansion scalar Θ, shear scalar σ^2, the anisotropic parameter A_m and deceleration parameter q are defined as follows:

$$\Theta=3H\,, \tag{23}$$

where H is the mean Hubble parameter and which is defined as $H=(1/3)(H_1+H_2+H_3)$. $H_1=\dot{R}_1/R_1$, $H_2=\dot{R}_2/R_2$, and $H_3=\dot{R}_3/R_3$ stand for the directional Hubble parameters along the coordinate axes.

$$\sigma^2=\frac{1}{2}\Big[\Big(\frac{\dot{R}_1}{R_1}\Big)^2+\Big(\frac{\dot{R}_2}{R_2}\Big)^2+\Big(\frac{\dot{R}_3}{R_3}\Big)^2\Big]-\frac{\Theta^2}{6}\,, \tag{24}$$

$$A_m=\frac{1}{3}\sum_{i=1}^{3}\Big(\frac{H_i-H}{H}\Big)^2\,, \tag{25}$$

$$q=\frac{\mathrm{d}}{\mathrm{d}t}\Big(\frac{1}{H}\Big)-1\,. \tag{26}$$

For present model the physical quantities are obtained as

$$\Theta=\frac{3\beta}{m\beta t+D_1}\,, \tag{27}$$

$$\sigma^2=\frac{k_1^2+k_2^2+k_3^2}{18(m\beta t+D_1)^{6/m}}\,, \tag{28}$$

$$A_m=\frac{k_1^2+k_2^2+k_3^2}{18(m\beta t+D_1)^{6/m-2}}\,, \tag{29}$$

$$q=m-1\,. \tag{30}$$

For an accelerating expansion of the Universe the deceleration parameter $q<0$, for $m<1$. Considering present day limit for deceleration parameter $q=-0.53^{+0.17}_{-0.13}$[38] suggests $0.3\le m\le 0.64$. From Eqs. (27)–(29), it is observed that Θ, σ^2, $A_m\to 0$ when $t\to\infty$. This model is shear free as shear is die out with the evolution of time. The state finder parameters are defined as

$$r=\frac{\dddot{R}}{RH^2} \quad \text{and} \quad s=\frac{r-1}{3(q-0.5)}\,. \tag{31}$$

For the present model, state finder pair $\{r,s\}$ is expressed as

$$r=\frac{\beta(m-1)(2m-1)}{m\beta t+D_1}\,, \tag{32}$$

$$s=\frac{2\beta(m-1)(2m-1)-2(m\beta t+D_1)}{3(2m-3)(m\beta t+D_1)}\,. \tag{33}$$

The relationship between r and s is obtained as

$$s = \frac{2(r-1)}{3(2m-3)}\,. \tag{34}$$

From the literature, it is known that state finder pair $\{r, s\}$ is useful to discriminate the different dark energy models. The pairs

$$\{r, s\} = \{1, 1\} \quad \{r, s\} = \{1, 0\}$$

represent the SCDM and ΛCDM model respectively. In this case of study we have observed that our model approaches towards ΛCDM model and nearer to the SCDM model with the evolution of time, which can be seen from Fig. 2.

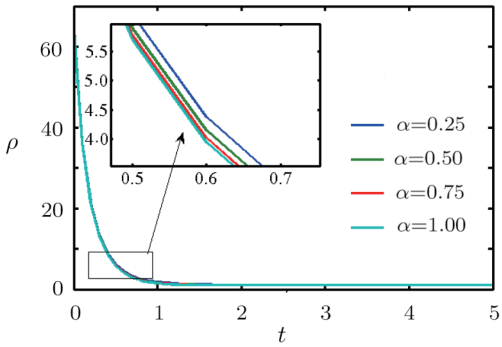

Fig. 1 Variation of energy density ρ versus time t for $A = 1$, $D_2 = 1$, $m = 0.5$, $\beta = 1$, $c = 1$, and different α.

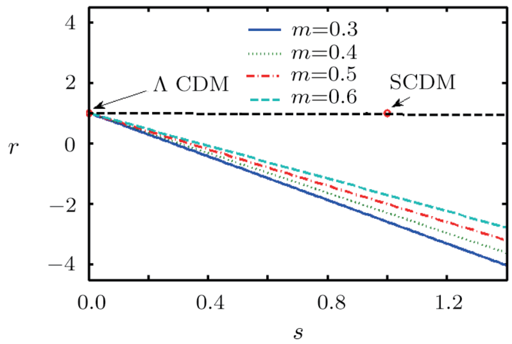

Fig. 2 Variation of r–s plane for different m.

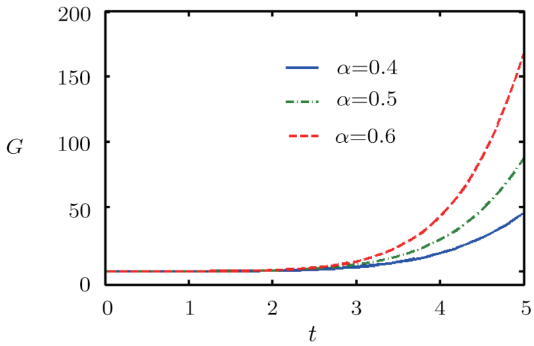

Fig. 3 Variation of gravitational constant G versus time t for $A = 1$, $D_2 = 1$, $m = 0.5$, $\beta = 1$, $c = 1$, $k_1^2 + k_2^2 + k_3^2 = 0.007$, and different α.

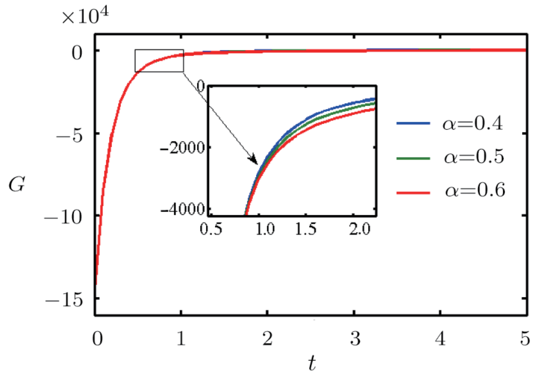

Fig. 4 Variation of gravitational constant G versus time t for $A = 1$, $D_2 = 1$, $m = 0.5$, $\beta = 1$, $c = 1$, $k_1^2 + k_2^2 + k_3^2 = 531\,442$, and different α.

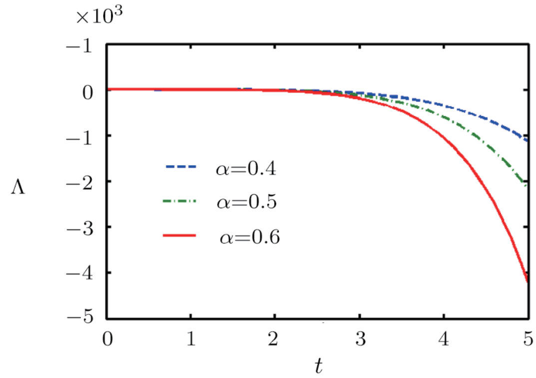

Fig. 5 Variation of Cosmological constant Λ versus time t for $A = 1$, $D_2 = 1$, $m = 0.5$, $\beta = 1$, $c = 1$, $k_1^2 + k_2^2 + k_3^2 = 0.007$, and different α.

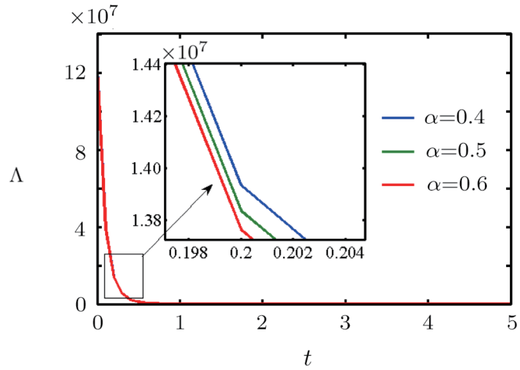

Fig. 6 Variation of Cosmological constant Λ versus time t for $A = 1$, $D_2 = 1$, $m = 0.5$, $\beta = 1$, $c = 1$, $k_1^2 + k_2^2 + k_3^2 = 531\,442$, and different α.

(ii) ***Case 2 (m=0)*** In this case, Eq. (14) takes the form

$$H = \beta\,. \tag{35}$$

On integrating Eq. (35), we get

$$R = D_3\,\mathrm{e}^{\beta t}\,. \tag{36}$$

With the help of Eqs. (11)–(13) and (36), take the form

$$R_1 = c_1 D_3 \exp\Big[\beta t - \frac{k_1\,\mathrm{e}^{-3\beta t}}{9\beta D_3^3}\Big]\,, \tag{37}$$

$$R_2 = c_2 D_3 \exp\Big[\beta t - \frac{k_2\,\mathrm{e}^{-3\beta t}}{9\beta D_3^3}\Big]\,, \tag{38}$$

$$R_3 = c_3 D_3 \exp\left[\beta t - \frac{k_3\, \mathrm{e}^{-3\beta t}}{9\beta D_3^3}\right]. \tag{39}$$

Using Eqs. (9), (19), and (34), we get

$$\rho = [A + D_4^{1+\alpha}\, \mathrm{e}^{-3\beta(1+\alpha)t}]^{1/(1+\alpha)}, \tag{40}$$

where D_4 is a constant of integration. From Eq. (40) and Fig. 7 one can see that energy density is a decreasing function of time and after some time maintains a constant value ($\lim_{t\to\infty} \rho^{1+\alpha} = A$) with evolution of time. Using Eqs. (5), (7), (19), (37)–(39), and (40) we have obtained

$$G = -\frac{k_1^2 + k_2^2 + k_3^2}{72\pi D_3^6 D_4^{1+\alpha}} + [A + D_4^{1+\alpha}\, \mathrm{e}^{-3\beta(1+\alpha)t}]^{\alpha/(1+\alpha)}\, \mathrm{e}^{3\beta t(\alpha-1)}. \tag{41}$$

Here the gravitational constant is negative and increasing function of time. In this case $G \to 0$ as $t \to \infty$, which can be observed from Fig. 9 and Eq. (41). Substituting the values from Eqs. (37)–(39) and (40)–(41) in Eq. (7), one can easily get the equation for cosmological constant as

$$\Lambda = 3\beta^2 + \frac{k}{9D_3^6}\, \mathrm{e}^{-6\beta t} + \frac{[A + D_4^{1+\alpha}\, \mathrm{e}^{-3\beta(1+\alpha)t}](k_1^2 + k_2^2 + k_3^2)\, \mathrm{e}^{3\beta t(\alpha-1)}}{9D_3^6 D_4^{1+\alpha}}. \tag{42}$$

Again with the same considered values as in G, we have plotted the cosmological constant in Fig. 10. From Eq. (42) and Fig. 10, one can observe that cosmological constant is positive and decreasing with evolution of time. Also it approaches to a constant value with the evolution of time, i.e. $\Lambda \nrightarrow 0$ as $t \to \infty$.

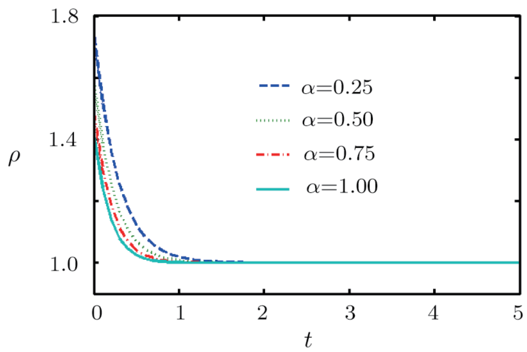

Fig. 7 Variation of energy density ρ versus time t for $A = 1$, $D_4 = 1$, $\beta = 1$, and different α.

For present model the physical quantities are obtained as

$$\Theta = 3\beta, \tag{43}$$

$$\sigma^2 = \frac{k_1^2 + k_2^2 + k_3^2}{18 D_3^6\, \mathrm{e}^{6\beta t}}, \tag{44}$$

$$A_m = \frac{k_1^2 + k_2^2 + k_3^2}{27 D_3^6 \beta^2\, \mathrm{e}^{6\beta t}}, \tag{45}$$

$$q = -1. \tag{46}$$

Equations (44) and (45) show that σ^2, $A_m \to 0$ when $t \to \infty$. This model is also shear free as shear is die out with the evolution of time. The state finder pair $\{r, s\}$ for this case is expressed as

$$r = \beta, \quad s = \frac{2}{9}(1-\beta). \tag{47}$$

The relationship between r and s is obtained as

$$s = \frac{2}{9}(1-r). \tag{48}$$

The variation of r–s plane is presented in Fig. 8. In this case also our model approaches towards ΛCDM model and nearer to the SCDM model.

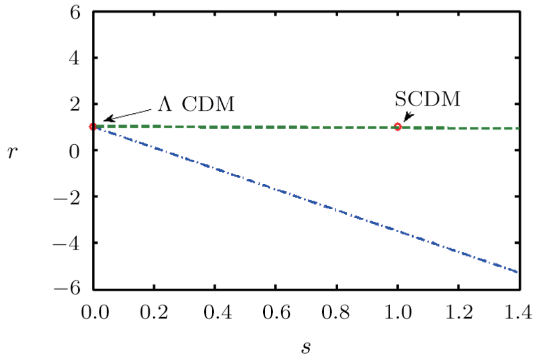

Fig. 8 Variation of r–s plane.

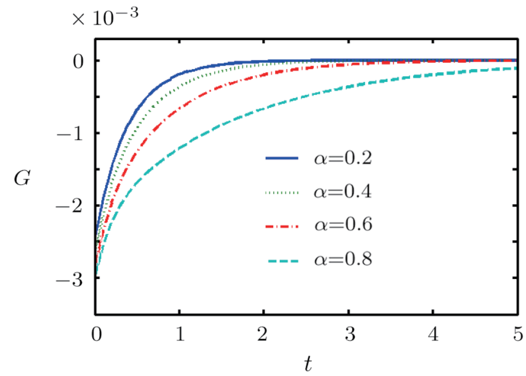

Fig. 9 Variation of gravitational constant G versus time t for $A = 1$, $D_3 = 1$, $D_4 = 1$, $\beta = 1$, $k_1^2 + k_2^2 + k_3^2 = 0.5$ and different α.

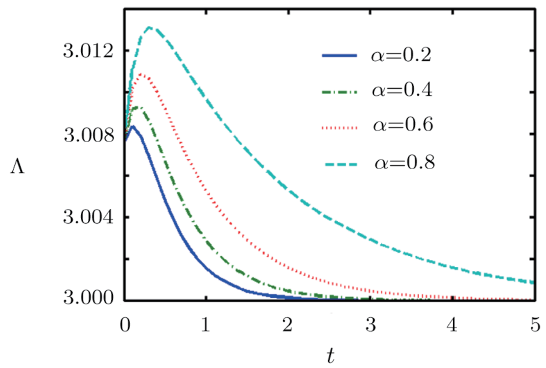

Fig. 10 Variation of Cosmological constant Λ versus time t for $A = 1$, $D_3 = 1$, $D_4 = 1$, $\beta = 1$, $k_1^2 + k_2^2 + k_3^2 = 0.5$ and different α.

3.2 *Solution Type II*

In the previous section solution type I, the solutions of the field equations are obtained by considering the power law relation between Hubble parameter and scale factor. This assumptions leads to a constant deceleration parameter. Also we are interested in the time varying deceleration parameter. Thus in this section solution type II, we consider the scale factor of the form

$$R = -\frac{1}{t} + t^2\,, \quad t > 1\,. \tag{49}$$

With the help of Eq. (49) and Eqs. (11)–(13) take the form

$$R_1 = c_1\Big(-\frac{1}{t} + t^2\Big)\,\mathrm{e}^{k_1\chi(t)/3}\,, \tag{50}$$

$$R_2 = c_2\Big(-\frac{1}{t} + t^2\Big)\,\mathrm{e}^{k_2\chi(t)/3}\,, \tag{51}$$

$$R_3 = c_3\Big(-\frac{1}{t} + t^2\Big)\,\mathrm{e}^{k_3\chi(t)/3}\,, \tag{52}$$

where

$$\chi(t) = \frac{1}{54}\ln\Big(\frac{t^2+t+1}{(t-1)^2}\Big) + \frac{\sqrt{3}}{27}\arctan\Big(\frac{1+2t}{\sqrt{3}}\Big) - \frac{t(t^3+2)}{18(t^2+t+1)^2(t-1)^2}\,.$$

From Eqs. (9), (19), and (49) one can easily get the energy density as

$$\rho = (A + D_5^{1+\alpha}t^{3(1+\alpha)}(t^3-1)^{-3(1+\alpha)})^{1/(1+\alpha)}\,, \tag{53}$$

where D_5 is a positive constant of integrations. From Eqs. (5), (7), (19), and (50)–(53), one can obtain the gravitational constant as

$$G = (A + D_5^{1+\alpha}t^{3(1+\alpha)}(t^3-1)^{-3(1+\alpha)})^{\alpha/(1+\alpha)} \times \frac{(36t^{18} - 378t^{12} + 792t^9 - (k_1^2+k_2^2+k_3^2)t^8 - 648t^6 + 216t^3 - 18)(t^3-1)^{3(1+\alpha)}}{72t^2(t-1)^6(t^2+t+1)^6\pi D_5^{1+\alpha}t^{3(1+\alpha)}}\,. \tag{54}$$

Using Eqs. (50)–(54) in Eq. (7), one can easily obtain the expression for Cosmological constant as

$$\Lambda = \frac{108t^{18} - 324t^{15} + 243t^{12} + 108t^9 + (k_3k_1 + k_2k_1 + k_2k_3)t^8 - 162t^6 + 27}{9t^2(t-1)^6(t^2+t+1)^6} \times (A + D_5^{1+\alpha}t^{3(1+\alpha)}(t^3-1)^{-3(1+\alpha)})(t^3-1)^{3(1+\alpha)} - \frac{(36t^{18} - 378t^{12} + 792t^9 - (k_1^2+k_2^2+k_3^2)t^8 - 648t^6 + 216t^3 - 18)}{9t^2(t-1)^6(t^2+t+1)^6D_5^{1+\alpha}t^{3+3\alpha}}\,. \tag{55}$$

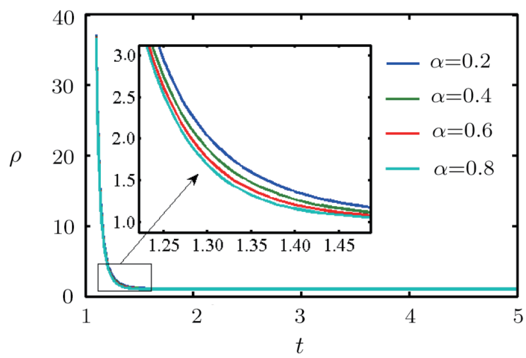

Fig. 11 Variation of energy density ρ versus time t for $A = 1$, $D_5 = 1$ and different α.

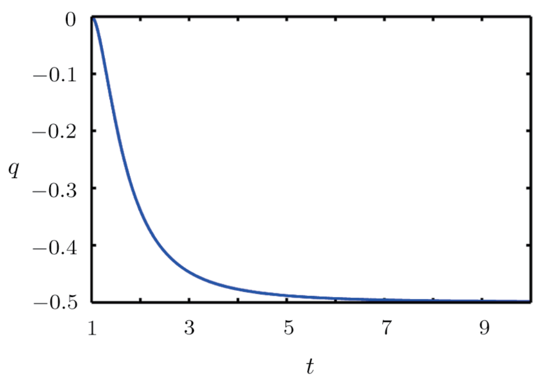

Fig. 12 Variation of deceleration parameter q versus time t.

In this case the energy density also possesses the same qualitative behaviour as that of energy density in case-1 and case-2, which can be seen from Fig. 11. The gravitational constant is an increasing function of time where as cosmological constant is a decreasing function of time. Also one can notice from Figs. 13 and 14 that, $G > 0$ and $\Lambda < 0$.

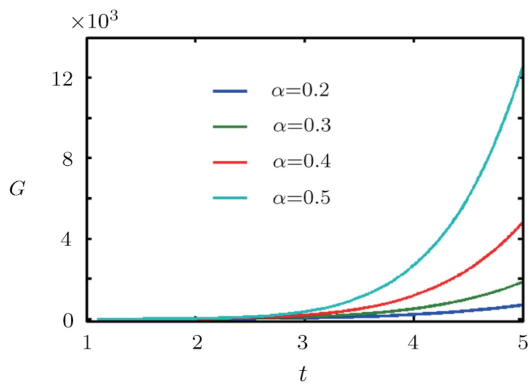

Fig. 13 Variation of gravitational constant G versus time t for $A = 1$, $D_5 = 1$, $k_1^2 + k_2^2 + k_3^2 = 0.5$ and different α.

The physical quantities like Hubble parameter (H), expansion scalar (Θ), Anisotropic parameter (A_m), and Shear scalar (σ^2) are expressed as

$$H = \frac{1}{3}(H_1 + H_2 + H_3) = \frac{2t^3+1}{(t^3-1)t}\,, \tag{56}$$

where

$$H_1 = \frac{\dot{R}_1}{R_1} = \frac{6t^9 - 9t^6 + k_1 t^4 + 3}{3t(t^3-1)^3}\,,$$
$$H_2 = \frac{\dot{R}_2}{R_2} = \frac{6t^9 - 9t^6 + k_2 t^4 + 3}{3t(t^3-1)^3}\,,$$
$$H_3 = \frac{\dot{R}_3}{R_3} = \frac{6t^9 - 9t^6 + k_3 t^4 + 3}{3t(t^3-1)^3}$$

are directional Hubble parameter along x, y and z direction respectively.

$$\Theta = \frac{3(2t^3+1)}{(t^3-1)t}\,, \tag{57}$$
$$A_m = \frac{(k_1^2+k_2^2+k_3^2)t^8}{27(2t^3+1)^2(t^3-1)^4}\,, \tag{58}$$
$$\sigma^2 = \frac{(k_1^2+k_2^2+k_3^2)t^6}{18(t^3-1)^6}\,. \tag{59}$$

The deceleration parameter q is expressed as

$$q = -\frac{2(t^3-1)^2}{(2t^3+1)^2}\,. \tag{60}$$

From Eqs. (56)–(59), one can find that H, Θ, A_m, $\sigma^2 \to 0$ as $t \to \infty$. This model is also shear free. Here $q \to -0.5$ as $t \to \infty$, which is in the fare agreement with the observational data (see Fig. 12).

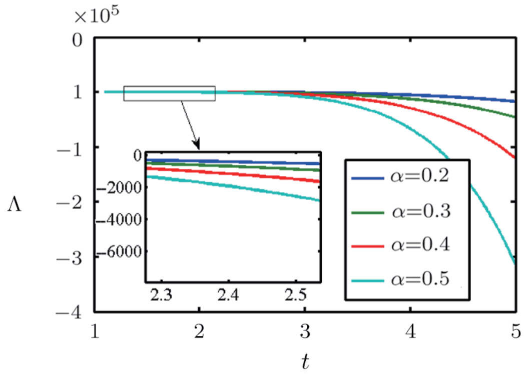

Fig. 14 Variation of Cosmological constant Λ versus time t for $A = 1$, $D_5 = 1$, $k_1^2 + k_2^2 + k_3^2 = 0.5$ and different α.

4 Stability and Physical Acceptability of Solutions

For solution type-I and solution type-II, we have discussed the stability of our solutions by considering the squared sound velocity. The condition to have stable theory is

$$C_s^2 = \frac{\partial p}{\partial \rho} \geq 0\,. \tag{61}$$

In our present work of study the condition of stability in presence of generalized Chaplygin gas is expressed as

$$C_s^2 = \frac{\partial p}{\partial \rho} = \frac{A\alpha}{\rho^{1+\alpha}} = \begin{cases} \dfrac{A\alpha}{A + D_2^{1+\alpha}(m\beta t + D_1)^{-3(1+\alpha)/m}}\,, & \text{Solution type I (Case-1)} \\ \dfrac{A\alpha}{A + D_4^{1+\alpha}\,\mathrm{e}^{-3\beta(1+\alpha)t}}\,, & \text{Solution type I (Case-2)} \\ \dfrac{A\alpha}{A + D_5^{1+\alpha} t^{3(1+\alpha)}(t^3-1)^{-3(1+\alpha)}}\,, & \text{Solution type II}\,. \end{cases} \tag{62}$$

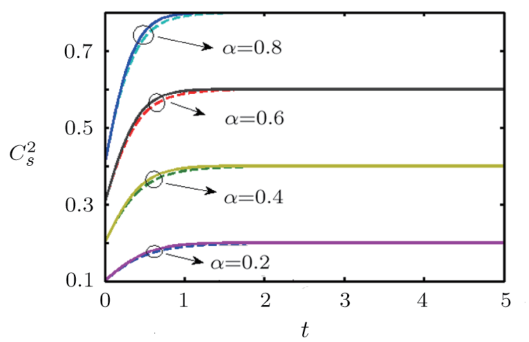

Fig. 15 Variation of squared of sound velocity C_s^2 versus time t for solution type-I (Case-1 (Dotted line) and Case-2 (simple line)).

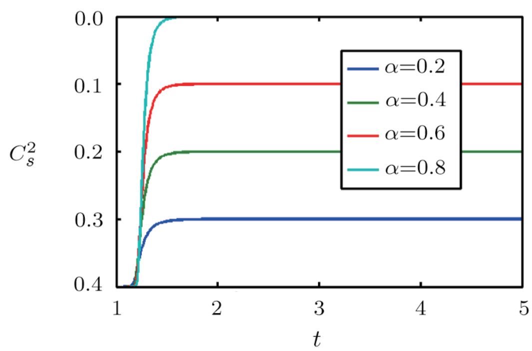

Fig. 16 Variation of squared of sound velocity C_s^2 versus time t for solution type-II.

From Figs. 15 and 16, it is noticed that squared of velocity is positive ($C_s^2 > 0$) and $C_s^2 < 1$ in both solution type-I and solution type-II. Also it is concluded that our solutions are stable. Next we have discussed about the weak energy conditions (WEC), dominant energy conditions (DEC) and strong energy conditions (SEC) for solution type-I and solution type-II. The conditions are given as follows: (i) $\rho > 0$, $\rho - p \geq 0$ (WEC); (ii) $\rho + p \geq 0$ (DEC); (iii) $\rho + 3p \geq 0$ (SEC).

Here we observe that for both solution type-I and solution type-II, WEC and DEC conditions are satisfied where as SEC is violated (see Figs. 17, 18, and 19). This violation may lead us to accelerated expansion of our Universe.

Thus on account of the above discussions, our solutions are physically acceptable.

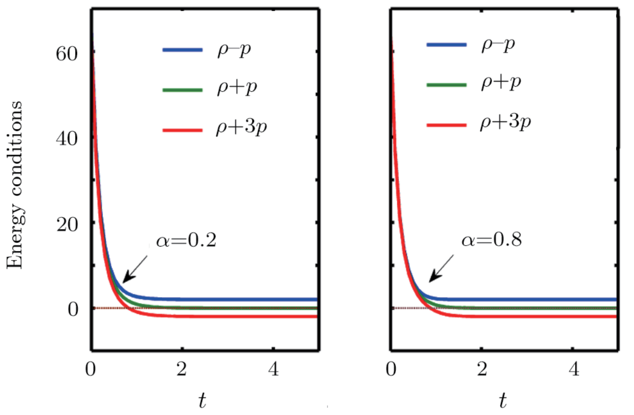

Fig. 17 Variation of energy conditions versus time (Solution type-I, case-1).

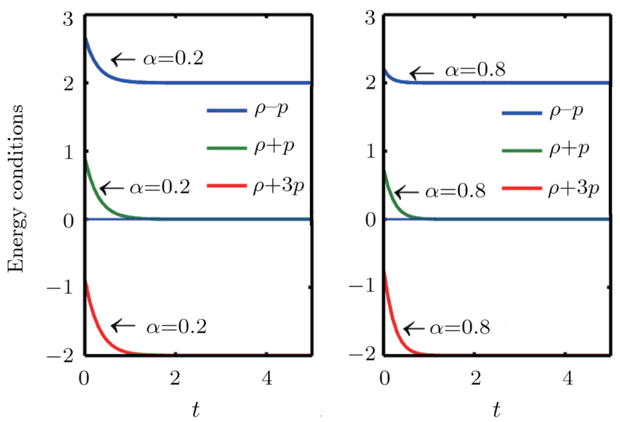

Fig. 18 Variation of energy conditions versus time (Solution type-I, case-2).

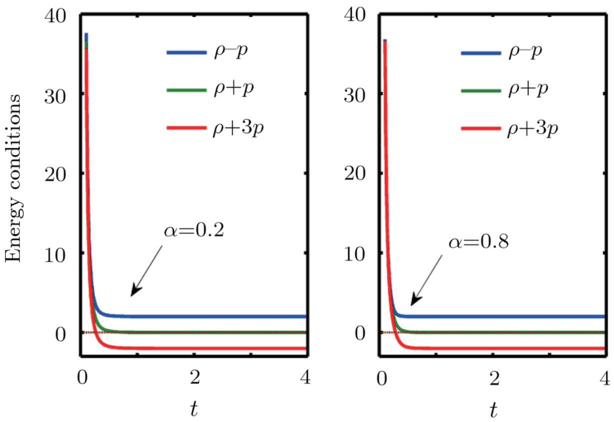

Fig. 19 Variation of energy conditions versus time (Solution type-II).

5 Concluding Remarks

In this paper we have investigated the cosmological constant Λ and gravitational constant G in presence of generalized chaplygin gas for Bianchi type-I space time. According to the assumption of the physically plausible relation, we have discussed two type of solution namely solution type-I and solution type-II, which consist of three models (solution type-I (case-1), solution type-I (case-2) and solution type-II). The concluding remarks of the article are as follows:

- For solution type-I (i) In both cases, energy density is decreasing with the evolution of time and approaches to a constant value ($\lim_{t\to\infty} \rho^{1+\alpha} = A$). (ii) In case-I, we have found that $G > 0$ and $G < 0$ depending on the value of $k_1^2 + k_2^2 + k_3^2$. Also with the same considered values, $\Lambda > 0$ for $G < 0$ and $\Lambda < 0$ for $G > 0$. (iii) In case-II, we have noticed that $G < 0$, $\Lambda > 0$, $G \to 0$ as $t \to \infty$ and $\Lambda \to 3\beta^2$ as $t \to \infty$.
- For solution type-II (i) energy density has the same qualitative behavior as that of energy density of solution type-I. (ii) Gravitaional constant is positive where as Cosmological constant is negative.
- All the models discussed here are shear free for late time.
- The assumption of the power law relation between Hubble parameter H and scale factor R and scale factor of the form $R = -1/t + t^2$, $t > 1$, leads us to constant and time dependent deceleration parameter respectively, which is well accepted in literature.[39−41]
- The solutions discussed in the article are stable and physically acceptable.
- WEC and DEC conditions are satisfied for all three models where as SEC is violated. This violation may be responsible for the accelerated expansion of the Universe.
- The ratio of square of shear scalar and square of expansion scalar is given as

$$\frac{\sigma^2}{\theta^2} = \begin{cases} \dfrac{k_1^2+k_2^2+k_3^2}{162\beta^2}\,\dfrac{(m\beta t + D_1)^2}{(m\beta t + D_1)^{6/m}}\,, & \text{Solution type I (Case-1)}\,,\\ \dfrac{k_1^2+k_2^2+k_3^2}{162\beta^2 D_3^6}\,\dfrac{1}{\mathrm{e}^{6\beta t}}\,, & \text{Solution type I (Case-2)}\,,\\ \dfrac{k_1^2+k_2^2+k_3^2}{162}\,\dfrac{t^8}{(t^3-1)^4(2t^3+1)^2}\,, & \text{Solution type II}\,. \end{cases}$$

Here we observe that, in case of solution type I (Case-1) $\sigma^2/\theta^2 \to 0$ when $t \to \infty$ for $m < 3$ and $\sigma^2/\theta^2 \nrightarrow 0$ when $t \to \infty$ for $m > 3$. In the present study we have $0.3 \leq m \leq 0.64$. Thus model with this case approaches to isotropy for late time. In case of solution type I (Case-2), $\sigma^2/\theta^2 \to 0$ when $t \to \infty$ for $\beta > 0$. Thus in this case the model approaches to isotropy for late time. Also for solution type II, the model approaches to isotropy as $\sigma^2/\theta^2 \to 0$ when $t \to \infty$.

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