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Critical Behavior of Ising Model with Long Range Correlated Quenched Impurities*

CHEN Yuan

Department of Physics, Guangzhou University, Guangzhou 510405, China

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Abstract *The theoretic renormalization-group approach is applied to the study of the critical behavior of the d-dimensional Ising model with long-range correlated quenched impurities, which has a power-like correlations* $r^{-(d-\rho)}$. *The asymptotic scaling law is studied in the framework of the expansion in* $\epsilon = 4 - d$. *In* $d < 4$, *the dynamic exponent* z *is calculated up to the second order in* ρ *with* $\rho = O(\epsilon^{1/2})$. *The shape function is obtained in one-loop calculation. When* $d = 4$, *the logarithmic corrections to the critical behavior are found. The finite size effect on the order parameter relaxation rate is also studied.*

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It has been known that the static and dynamic critical properties would be modified by the short-range correlated quenched impurities (SCQI).[1−7] Those works are consistent with the Harris criterion,[8] which states that the critical behavior of the system is unaffected by the SCQI if $d\nu_p > 2$, where ν_p is the correlation-length exponent of the pure system. When the long-range correlated quenched impurities (LCQI) whose correlations behave as $r^{-(d-\rho)}$ for large distances r (where d is the spatial dimension, and ρ characterizes the decay rate of the long-range correlations)[9−15] are taken into account, the critical properties can be changed further, and the Harris criterion is extended.

The case of the LCQI was first considered by Weinrib and Halperin.[9] They found that the modified Harris criterion for the LCQI to be irrelevant is $(d - \rho)\nu - 2 > 0$ for $\rho > 0$, but $d\nu - 2 > 0$ for $\rho \leq 0$. When the LCQI is relevant, it leads to a new critical behavior with the exponent $\nu = 2/(d - \rho)$. However, their results are obtained in one-loop approximation. As is known, in the case of Ising model, there is an accidental degeneracy in the renormalization group (RG) recursion relations when only the SCQI is concluded, making the SCQI fixed point and its eigenvalues have the order of $\epsilon^{1/2}$ rather than $\epsilon = 4-d$.[2,3] Including the LCQI, the degeneracy is broken. However, working only to the $\epsilon^{1/2}$ term, Weinrib and Halperin[9] have not found the stable LCQI fixed point, and have not proved that the crossover between fixed points is in accord with the extended Harris criterion.

In the present paper, I will study the critical behavior of Ising model with LCQI in two-loop approximation. The stability of LCQI fixed point is calculated to the order of ϵ. The shape function, which describes the frequency and wave vector spectrum of critical order-parameter fluctuations of the systems with nonconvered parameter, is calculated. It may be used to determine the cross section for the inelastic scattering of neutrons as a function of the energy loss and momentum transfer.[5] In the presence of the LCQI, the logarithmic correction to the critical behavior in $d = 4$ as well as the finite size effect on the order parameter relaxation rate, is also studied.

In this paper how the critical behavior of Ising systems is affected by the LCQI will be studied within the model A dynamics.[16] This kind of systems can be described by the dynamical Ginzburg–Landau model with LCQI, which is defined by the Langevin equation

$$\partial_t s(x,t) = -\lambda \frac{\delta H[s]}{\delta s(x,t)} + \xi(x,t)\,, \tag{1}$$

$$H[s] = \int \mathrm{d}^d x \Big\{ \frac{1}{2}(\bigtriangledown s)^2 + \frac{\tau}{2}s^2 + \frac{g}{4!}s^4 + \frac{1}{2}\phi s^2 \Big\}\,, \tag{2}$$

where s is one-component order parameter field, and λ is the kinetic coefficient. $\xi(x,t)$ and $\phi(x)$ are the Gaussian random force and static Gaussian random-impurity noise with zero mean and the correlations as, respectively,

$$\langle \xi(x,t)\xi(x',t')\rangle_\xi = 2\lambda\delta(x-x')\delta(t-t')\,, \tag{3}$$

$$\langle \phi(x)\phi(x')\rangle_\phi = [\,g_1 + g_2(-\bigtriangledown^2)^{-\rho/2}]\delta(x-x')\,. \tag{4}$$

The Fourier transform of $\langle \phi(x)\phi(x')\rangle_\phi$ for small k is $g_1 + g_2 k^{-\rho}$. The non-interacting case, i.e. $g = 0$, has been investigated in Ref. [15], which can be extended to study the statistics of self-avoiding linear polymers.

By introducing a response field $\tilde{s}(x,t)$,[17] the dynamic process can be described by a path-integral description,[18] which is equivalent to the formulation (1). After averaging over the static random degrees of freedom, one has an effective generating functional for all the connected correlation functions and response functions,

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$$W[h,\tilde{h}] = \ln\int \mathcal{D}(\mathrm{i}\tilde{s},s)\exp\Big(-\mathcal{L}[\tilde{s},s] + \int_0^\infty \mathrm{d}t\int \mathrm{d}^d x(hs+\tilde{h}\tilde{s})\Big)\,, \tag{5}$$

where the effective action functional is

$$\mathcal{L}[\tilde{s},s] = \int_0^\infty \mathrm{d}t\int \mathrm{d}^d x\Big\{\tilde{s}\Big[\dot{s}+\lambda(\tau-\nabla^2)s+\frac{\lambda g}{6}ss^2\Big]-\lambda\tilde{s}^2\Big\}-\frac{\lambda^2}{2}\int \mathrm{d}^d x(g_1+g_2(-\nabla^2)^{-\rho/2})\Big(\int_0^\infty \mathrm{d}t\,\tilde{s}(x,t)s(x,t)\Big)^2\,. \tag{6}$$

Here we have used a prepoint discretization with respect to time so that the step function $\Theta(t=0)=0$. Then the contribution ($\propto\Theta(0)$) to $\mathcal{L}[\tilde{s},s]$ arising from the functional determinant $\det[\delta\xi(x,t)/\delta s(x,t)]$ vanishes.[19,20]

Since the criticality occurs at a macroscopic scale, one should in principle average over the microscopic fast degrees of freedom and obtain an effective theory for the slow variables. In such average, divergence appears if the theory is defined in the microscopic scale. To obtain a meaningful theory the divergence must be absorbed into the renormalizations of the model parameter and the fields. We will adopt the dimensional regularization with minimal subtraction scheme,[21] and introduce renormalized quantities through some multiplicative factors,

$$s_b = Z_s^{1/2}s\,,\quad \tilde{s}_b = Z_{\tilde{s}}^{1/2}\tilde{s}\,,\quad \lambda_b = (Z_s/Z_{\tilde{s}})^{1/2}\lambda\,,\quad \tau_b = Z_s^{-1}Z_\tau\tau\,,$$
$$g_b = \mu^\epsilon K_d^{-1}Z_s^{-2}Z_u u\,,\quad g_{1b} = \mu^\epsilon K_d^{-1}Z_s^{-2}Z_{u_1}u_1\,,\quad g_{2b} = \mu^{\epsilon+\rho}K_d^{-1}Z_s^{-2}Z_{u_2}u_2\,. \tag{7}$$

Here the subscript b denotes the bare quantity, $K_d = 2^{1-d}\pi^{-d/2}[\Gamma(d/2)]^{-1}$. In the following, the double expansion in $\epsilon = 4-d$ and ρ is used to calculate the multiplicative factors.

As usual, the unrenormalized Green functions, which are defined by $G^{(b)M}_{N\tilde{N}} = \langle s_b^N\tilde{s}_b^{\tilde{N}}\rangle$, are independent of the external momentum scale μ. This leads to the renormalization group equation,

$$\Big[\mu\partial_\mu + \zeta\lambda\partial_\lambda + \kappa\tau\partial_\tau + \beta_u\partial_u + \beta_{u_1}\partial_{u_1} + \beta_{u_2}\partial_{u_2} + \frac{1}{2}(N\gamma+\tilde{N}\tilde{\gamma})\Big]G_{N\tilde{N}} = 0\,, \tag{8}$$

for the renormalized Green functions $G_{N\tilde{N}} = \langle s^N\tilde{s}^{\tilde{N}}\rangle$. Here $\beta_w = \mu\partial_\mu w|_0$ (for $w = u, u_1, u_2$) and $X = \mu\partial_\mu \ln Y|_0$ (for $X = \gamma_s, \gamma_{\tilde{s}}, \kappa, \zeta$, and $Y = Z_s, Z_{\tilde{s}}, \tau,\ \lambda$, respectively) are Wilson functions. The symbol $|_0$ means that μ-derivatives are calculated at fixed bare parameters. Since the momentum dependence of the LCQI term in Eq. (6) is not analytic, it is not renormalized and the condition $Z_\tau = Z_{u_2}^{1/2}$ is satisfied. By use of Eq. (7), we arrive at the set of beta functions

$$\beta_u = u[-\epsilon + 2\gamma_s - \mu\partial_\mu \ln Z_u|_0]\,, \tag{9}$$

$$\beta_{u_1} = u_1[-\epsilon + 2\gamma_s - \mu\partial_\mu \ln Z_{u_1}|_0]\,, \tag{10}$$

$$\beta_{u_2} = u_2[-\epsilon - \rho + 2\kappa]\,. \tag{11}$$

Here the relations $\kappa = \gamma_s - \mu\partial_\mu \ln Z_\tau|_0$ and $Z_\tau = Z_{u_2}^{1/2}$ have been used. Making $u, u_1, u_2 \to \bar{u}(l), \bar{u}_1(l), \bar{u}_2(l)$ and $\mu\to\mu l$, we obtain the flow equations

$$l\partial_l \bar{w}(l) = \beta_w(\bar{u}(l),\bar{u}_1(l),\bar{u}_2(l))\,,$$
$$\bar{w}(l) = \bar{u}(l),\bar{u}_1(l),\bar{u}_2(l)\,, \tag{12}$$

$$l\partial_l \ln\bar{\tau}(\bar{u}(l),\bar{u}_1(l),\bar{u}_2(l)) = \kappa(\bar{u}(l),\bar{u}_1(l),\bar{u}_2(l))\,, \tag{13}$$

$$l\partial_l \ln\bar{\lambda}(\bar{u}(l),\bar{u}_1(l),\bar{u}_2(l)) = \zeta(\bar{u}(l),\bar{u}_1(l),\bar{u}_2(l)) \tag{14}$$

with $\bar{A}|_{l=1} = A$, $A = u, u_1, u_2, \tau, \lambda$.

Then the renormalization group equation has a scaling solution for the Green functions

$$G_{N\tilde{N}}(\{x,t\},\tau,\lambda,u,u_1,u_2,\mu) = l^{(d-2)N/2+(d+2)\tilde{N}/2}\exp\Big\{\int_1^l [N\gamma_s(l')+\tilde{N}\gamma_{\tilde{s}}(l')]\frac{\mathrm{d}l'}{2l'}\Big\}$$
$$\times G_{N\tilde{N}}(\{lx,\bar{\lambda}(l)l^2t\},\bar{\tau}(l)l^{-2},1,\bar{u}(l),\bar{u}_1(l),\bar{u}_2(l),\mu)\,, \tag{15}$$

Equation (15) allows us to study the scaling behavior of Green functions on large length and time scales. In the limit $l\to 0$, the scalings are governed by the fixed points $w^* = (u^*, u_1^*, u_2^*)$ of the systems, i.e., the Gaussian, pure, unphysical and SCQI and LCQI fixed points. At the fixed point $w^* = (u^*, u_1^*, u_2^*)$, we derive the scaling laws

$$G_{N\tilde{N}}(\{x,t\},\tau,\tau_0^{-1},\lambda,w^*,\mu) = l^{(d-2+\eta_s)N/2+(d+2+\eta_{\tilde{s}})\tilde{N}/2}G_{N\tilde{N}}(\{lx,l^z t\},\tau l^{-1/\nu},\lambda,w^*,\mu)\,, \tag{16}$$

where $\eta_s = \gamma(w^*)$ and $\eta_{\tilde{s}} = \gamma_{\tilde{s}}(w^*)$ are the anomalous dimensions. The critical exponents is determinated by $\eta = \eta_s$, $z = 2 + (\eta_{\tilde{s}} - \eta)/2$, $1/\nu = 2 - \kappa(w^*)$. At the LCQI fixed point $w^*\neq 0$, we obtain the correlation length exponent characteristic of the LCQI system

$$\nu = 2/(4-\epsilon-\rho)\,, \tag{17}$$

which is exact to all orders in perturbation theory. The derivatives of Eqs. (9) $\sim$ (11) give the eigenvalues λ_u, λ_{u_1}, and λ_{u_2}. When $u_2^* = 0$, it is easy to obtain

$$\lambda_{u_2} = 4-\epsilon-\rho-2/\nu_0\,. \tag{18}$$

Here the exponent ν_0 is the correlation length exponent characteristic of the system without LCQI, such as the Gaussian ν_g, the pure ν_p, the SCQI ν_s, etc. $\lambda_{u_2} > 0$ shows that the sable regime of the SCQI fixed point yields $(d-\rho)\nu_s > 2$ for all ρ. The pure fixed point is unstable unless $d\nu_p > 2$ for $\rho \le 0$ or $(d-\rho)\nu_p > 2$ for $\rho > 0$. Those are in accordance with Refs. [9] and [11]. However, only $\lambda_{u_2} > 0$ for $u_2^* = 0$ is not sufficient to obtain the stability of all fixed points. We shall need the values of λ_u and λ_{u_1}. For this purpose, we have to calculate the Wilson functions. At the two-loop level, they are given by

$$\beta_u = -\epsilon u + \frac{3}{2}u^2 - 6(u_1+u_2)u - \frac{17}{12}u^3 + \frac{23}{2}(u_1+u_2)u^2 - \frac{41}{2}(u_1+u_2)^2 u + \frac{\rho}{2}u_2 u\,, \tag{19}$$

$$\begin{aligned}\beta_{u_1} = &-\epsilon u_1 + uu_1 - 2(u_1+u_2)u_1 - 2(u_1+u_2)^2 - 8(u_1+u_2)^3\\ &- \frac{5}{2}(u_1+u_2)^2u_1 - \frac{5}{12}u^2u_1 + 3u(u_1+u_2)^2 + \frac{5}{2}u(u_1+u_2)u_1 + \frac{\rho}{2}u_2u_1\,,\end{aligned} \tag{20}$$

$$\beta_{u_2} = -(\epsilon+\rho)u_2 + uu_2 - 2(u_1+u_2)u_2 - \frac{5}{12}u^2u_2 - \frac{5}{2}(u_1+u_2)^2u_2 + \frac{5}{2}u(u_1+u_2)u_2 + \frac{\rho}{2}u_2^2\,, \tag{21}$$

$$\gamma_s = \frac{1}{24}u^2 - \frac{1}{4}u(u_1+u_2) + \frac{1}{4}(u_1+u_2)^2 + \frac{\rho}{4}u_2\,, \tag{22}$$

$$\gamma_{\tilde{s}} = 2(u_1+u_2) + \frac{1}{24}\Big(12\ln\frac{4}{3} - 1\Big)u^2 - \frac{3}{4}u(u_1+u_2) + \frac{11}{4}(u_1+u_2)^2 - \frac{\rho}{4}u_2\,, \tag{23}$$

$$\kappa = \frac{1}{2}u - u_1 - u_2 - \frac{5}{24}u^2 - \frac{5}{4}(u_1+u_2)^2 + \frac{5}{4}u(u_1+u_2) + \frac{\rho}{4}u_2\,, \tag{24}$$

and $\zeta = (\gamma_{\tilde{s}} - \gamma_s)/2$.

In the following, we only list the physical meaningfully fixed points and their corresponding eigenvalues in Tables 1 and 2, respectively. The case with $u^* = 0$ and $u_1^* < 0$ or $u_1^* + u_2^* < 0$ has been discussed in Refs. [9] and [15]. Notice that the eigenvalues of LCQI fixed point are derived in the u, u_1+u_2, u_2 subspace in Table 2 in order to avoid the complicated expressions in the u, u_1, u_2 subspace.

Table 1 Physically fixed points.

Fixed point	u^*	u_1^*	u_2^*
Gaussian	0	0	0
Pure	$2\epsilon/3 + 34\epsilon^2/81$	0	0
SCQI	$4\sqrt{6\epsilon/53}$	$\sqrt{6\epsilon/53} - 6\epsilon/53$	0
LCQI	$2(2\epsilon+3\rho)/3 + \rho^2/18$	$(2\epsilon+3\rho)/6 - 17\rho^2/72$	$-\epsilon/6 + 53\rho^2/144$

Table 2 Eigenvalues of the fixed points in Table 1.

Eigenvalues	λ_u	λ_{u_1}	λ_{u_2}
Gaussian	$-\epsilon$	$-\epsilon$	$-\epsilon-\rho$
Pure	$\epsilon - 17\epsilon^2/27$	$-\epsilon/3 + 19\epsilon^2/81$	$-\rho - \epsilon/3 + 19\epsilon^2/81$
SCQI	2ϵ	$2\sqrt{6\epsilon/53}$	$-\rho + 2\sqrt{6\epsilon/53} - 36\epsilon/53$
LCQI	$\rho - 107\rho^2/36 + 2\epsilon/3$	$265\rho^2/144 + \epsilon/6 \pm \mathrm{i}(\sqrt{53/24}\,\rho^{3/2} - \sqrt{6/53\rho}\,\epsilon)$	

For $d < 4$, the Gaussian fixed point is unstable. It is stable if $d > 4$ and $\epsilon+\rho < 0$. The pure fixed point is stable against u_2 perturbations under condition $0 < \rho < \rho_p = \epsilon/3 + (19/81)\epsilon^2$ and $27/19 < \epsilon < 27/17$ if $d < 4$. When $\rho \le 0$, the pure fixed point is stable for $27/19 < \epsilon < 27/17$. In this case the randomness is irrelevant and pure criticality is expected to occur. Similarly, the SCQI fixed point is stable for $\rho < \rho_s = 2\sqrt{6\epsilon/53} - 36\epsilon/53$ with $\epsilon > 0$. For $\rho = O(\epsilon^{1/2})$, the LCQI fixed point goes into the physical region with the order of $\epsilon^{1/2}$, and is stable since the real parts of its eigenvalues are positive. The crossover between the LCQI and SCQI fixed points is at $\rho = \rho_s$. However, in the region $\rho = O(\epsilon)$, i.e. $\rho_p < \rho < \rho_s$, the LCQI fixed point of order ϵ is unstable. Those are in agreement with the extended Harris criterion.

At the LCQI fixed point, to second order in ρ, the static and dynamic exponents are given by

$$\eta = -\rho^2/48\,, \tag{25}$$

$$\nu \equiv \frac{2}{d-\rho} = \frac{1}{2} + \frac{1}{8}(\epsilon+\rho) + \frac{\rho^2}{32}\,, \tag{26}$$

$$z = 2 + \frac{1}{6}(\epsilon+3\rho) + \Big(\frac{1}{36} + \ln\frac{4}{3}\Big)\rho^2 \tag{27}$$

for $\rho > \rho_s$. The values of the exponents η and ν agrees with Ref. [10]. The values of z in the first order in ρ, have already been obtained in Refs. [13] and [14]. At $\rho = \rho_s$, the LCQI exponents cross over the corresponding SCQI ones.[3−5] For example, the exponents η, ν, and z for $d = 3$ are listed in Table 3, where the pure and SCQI ones are taken from Refs. [16], and [3] ∼ [5], respectively.

In the following, we discuss the effect of the LCQI on the shape functions $f(q,y)$, which describes the frequency ω and wave vector spectrum of critical order-parameter fluctuations of systems with nonconvered parameter. $f(q,y)$ is defined by the equation

$$f(q,y) = \omega_c(k)C(k,\omega)(\lim_{\omega\to 0} G(k,\omega))^{-1} \tag{28}$$

with $q = k\tau^{-\nu}$ and $y = \omega/\omega_c(k)$. Here $G(k,\omega)$ and $C(k,\omega)$ are the Fourier transforms of the response function $G_{11}(x-x';t-t')$ and the correlation function $G_{20}(x-x';t-t')$, respectively. The characteristic frequency $\omega_c(k)$ can be given by[4,19]

$$\omega_c(k) = 2\lambda G(k,0)/C(k,0)\,. \tag{29}$$

One-loop calculation shows that the shape function $f(q,y)$ is given by

$$f(q,y) = \frac{2}{y}\,\mathrm{Im}\Big\{\frac{F(q,y)}{F(q,y)-\mathrm{i}y}\Big\}\,, \tag{30}$$

where

$$F(q,y) = \Big[\frac{\mathrm{i}y(1-\tilde{y})}{\tilde{y}}\Big]^{\rho/4}\Big[\frac{1-\rho\ln(1-\tilde{y})}{4\tilde{y}}\Big] \tag{31}$$

with $\tilde{y} = \mathrm{i}y(1+q^2)$. The result displays that impurities lead to an enhancement of $f(q,y)$ near $y = 0$. For one-loop, equation (31) crosses over to the SCQI one if $\rho = 2\sqrt{(6/53)\epsilon}$.[4,5]

We now concentrate on the critical behavior of the LCQI model at $d = 4$. In this case, all the fixed points become degenerate with the Gaussian fixed points, i.e., $u^* = u_1^* = u_2^* = 0$. The pure and SCQI cases have been studied in Refs. [14], [20], and [22] ∼ [24]. In the following we are interested in how the LCQI affects the scaling. In the presence of u_2, the solutions for the flow equations are given by

$$\bar{u}(l) = 4\Big(-\frac{53}{3}\ln\Big(\frac{l}{l_0}\Big)\Big)^{-1/2}\,, \tag{32}$$

$$\bar{u}_1(l) + \bar{u}_2(l) = \Big(-\frac{53}{3}\ln\Big(\frac{l}{l_0}\Big)\Big)^{-1/2} - \Big(-\frac{53}{3}\ln\Big(\frac{l}{l_0}\Big)\Big)^{-1}\,, \tag{33}$$

$$u_2(l) = u_2\exp\Big\{-4\sqrt{\frac{3}{53}}\Big[\Big(-\ln\frac{l}{l_0}\Big)^{1/2} - (\ln l_0)^{1/2}\Big]\Big\}\Big[-\frac{\ln(l/l_0)}{\ln l_0}\Big]^{-17/106}\,, \tag{34}$$

$$\bar{\lambda}(l) = \lambda\Big(\frac{\bar{u}_2(l)}{u_2}\Big)^{1/2}\Big[-\frac{\ln(l/l_0)}{\ln l_0}\Big]^{17/106-12\ln(4/3)/53}\,, \tag{35}$$

$$\bar{\tau}(l) = \tau\Big(\frac{\bar{u}_2(l)}{u_2}\Big)^{1/2} \tag{36}$$

with $u(1) = 4[(53/3)\ln l_0]^{-1/2}$.

Choosing $\bar{u}_2(l) = 1$, at the critical point, the correlation $G_{20}(x-x';t-t') = \langle s(x,t)\tilde{s}(x',t')\rangle$ and the susceptibility $\chi = \lambda\int \mathrm{d}^d(x-x')\mathrm{d}(t-t')G_{11}(x-x';t-t')$ have the asymptotic scaling laws as follows:

$$G_{20}(x-x';t-t') \sim u_2^{1/68}(t-t')^{-1}\exp\Big\{-\frac{1}{34}\sqrt{\frac{6}{53}}\Big(\ln\frac{t-t'}{t_0-t_0'}\Big)^{1/2}\Big\}\,, \tag{37}$$

$$\chi \sim \tau^{-1}u_2^{35/68}\exp\Big\{-\frac{1}{34}\sqrt{\frac{6}{53}}\,\Big|\ln\Big(\frac{\tau}{\tau_0}\Big)\Big|^{1/2}\Big\}\,. \tag{38}$$

Here $\tau_0 u^{-1/2} l_0^{-2} = 1$, and t_0 is a microscopic time scale for the LCQI case. The condition $t-t' > t_0 - t_0'$ is satisfied.

If the finite size effect is taken into account, the critical scaling is dependent on the linear size of the system L.[21] Dynamical properties of a system are determined by relaxation times. In contrast to the pure Ising model, the disordered systems are characterized by the continuous relaxation time spectra.[25] Here we are particularly interested in the order parameter relaxation $\omega_s = \omega_c(k)|_{k=0}$[20] affected by the LCQI. The inverse of ω_s is the familiar linear relaxation time. From Eq. (15), one gets

$$\omega_s(L,\tau,u,u_1,u_2) = l^2\bar{\lambda}\omega_s(lL,\bar{\tau}l^{-2},\bar{u},\bar{u}_1,\bar{u}_2)\,, \tag{39}$$

where L is the linear size of the system. At the critical point, we have

$$\omega_s \sim \begin{cases} \rho\lambda L^{-z}\,, & (d<4)\,,\\ \lambda u^{-1/2+a} L^{-2}\exp\Big(-4a\sqrt{\dfrac{3}{53}}\,(\ln L)^{1/2}\Big)\,, & (d=4)\\ \lambda(u_2 L^{\rho-d})^{1/2}\,, & (d>4)\,, \end{cases}$$

where $a = 1-(24/17)\ln(4/3)$.

Summarizing, the critical behavior of the Ising model with LCQI is studied by the theoretic renormalization-group approach to two-loop order. In dimensions $d<4$, the critical exponent z and the shape function are attained. For $\rho>\rho_s$, the LCQI scaling regime is stable. At $\rho=\rho_s$, the LCRI exponents cross over the corresponding SCQI ones. The logarithmic corrections to critical behavior affected by the LCQI in $d=4$ are also studied. The factor $1/34\sqrt{6/53}$ in Eqs. (37) and (38) is a universal constant. Above four dimensions, dynamic finite size scaling is found to be broken if $\omega_s \sim L^{(\rho-d)/4}$ as $L\to\infty$. Our results may be checked by some experiments of the magnetic systems (such as Ho and Tb)[12] or by the dynamical Monte Carlo measurement.[26]

Table 3 η, ν, and z for $d=3$. The SCQI and pure values are taken from Refs. [3] ∼ [5] and [16], respectively.

	LCQI ($d=3,\rho=1$)	SCQI ($d=3$)	pure ($d=3$)
η	−0.021	−0.009	0.019
ν	1	0.638	0.627
z	2.982	2.306	2.013

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