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Teleportation of n-Particle State via n Pairs of EPR Channels

CAO Min, ZHU Shi-Qun,* and FANG Jian-Xing

School of Physical Science and Technology, Suzhou University, Suzhou 215006, China

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Abstract *The teleportation of an arbitrary n-particle state ($n \geq 1$) is proposed if n pairs of identical EPR states are utilized as quantum channels. Independent Bell state measurements are performed for joint measurement. By using a special Latin square of order $2^n (n \geq 1)$, explicit expressions of outcomes after the Bell state measurements by Alice (sender) and the corresponding unitary transformations by Bob (receiver) can be derived. It is shown that the teleportation of n-particle state can be implemented by a series of single-qubit teleportation.*

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Recently, the theoretical and experimental investigations of quantum teleportation have been paid much attention due to its important applications in computation and communication.[1−5] In the original idea of quantum teleportation, the nonlocal correction between an Einstein–Podolsky–Rosen (EPR) pair of particles was utilized to transfer the quantum information of an unknown quantum state to a remote place. An arbitrary two-level single particle state is transmitted from a sender (Alice) to a receiver (Bob) via an EPR channel with the help of some classical information.[1] Experimental demonstrations of quantum teleportation were achieved.[2−5] Some schemes of quantum teleportation of two-particle and three-particle states via different channels were presented.[6−10]

In this paper, the quantum teleportation of an arbitrary two-level n-particle state is proposed. Identical EPR pairs are utilized for the quantum channels and independent Bell state measurements are performed on particles owned by Alice. Explicit expressions of outcomes after the measurements of Alice and the corresponding unitary transformations of Bob are derived if a special Latin square of order 2^n ($n \geq 1$) is employed. These unitary transformations can be divided into n local single-qubit operations.[11]

An arbitrary n-particle state with unknown coefficients x_i ($i = 0, 1, \ldots, 2^n - 1$) can be written as

$$|\psi\rangle_{12\cdots n} = x_0|\underbrace{0\cdots 0}_{n}\rangle + x_1|0\cdots 1\rangle + \cdots + x_a|\cdots 1_i \cdots 0_j \cdots\rangle + \cdots + x_{2^n-1}|1\cdots 1\rangle\,, \quad (1)$$

where $a \in \{i\}$, coefficients x_i are complex numbers and satisfy normalization condition with

$$\sum_{i=0}^{2^n-1} |x_i|^2 = 1\,,$$

$|\cdots 1_i \cdots 0_j \cdots\rangle$ denotes that there is a "1" at the position "i" and a "0" at the position "j", and

$$\{|0\cdots 0\rangle, |0\cdots 1\rangle, \cdots, |\cdots 1_i \cdots 0_j \cdots\rangle, \cdots, |1\cdots 1\rangle\}$$

is the basis of a 2^n-dimensional Hilbert space.

To teleport the state $|\psi\rangle_{12\cdots n}$, Alice needs to set up n pairs of identical EPR states as quantum channels between her (sender) and Bob (receiver),

$$|\phi^+\rangle_{(n+1)(n+2)} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)\,, \quad (2)$$

$$|\phi^+\rangle_{(n+3)(n+4)} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)\,, \quad (3)$$

$$\vdots$$

$$|\phi^+\rangle_{(3n-1)(3n)} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)\,, \quad (4)$$

where particles $(n+1), (n+3), \ldots, (3n-1)$ belong to the sender (Alice) and particles $(n+2), (n+4), \ldots, (3n)$ belong to the receiver (Bob).

The state of the system is

$$|\Psi\rangle = |\psi\rangle_{12\cdots n}|\phi^+\rangle_{(n+1)(n+2)} \times |\phi^+\rangle_{(n+3)(n+4)} \cdots |\phi^+\rangle_{(3n-1)(3n)}$$

at this time. The four Bell states of particle kl ($l = n + 2k - 1$, $k = 1, 2, \ldots, n$) can be expressed as

$$|\phi^\pm\rangle_{kl} = \frac{1}{\sqrt{2}}(|00\rangle_{kl} \pm |11\rangle_{kl})\,, \quad (5)$$

$$|\psi^\pm\rangle_{kl} = \frac{1}{\sqrt{2}}(|01\rangle_{kl} \pm |10\rangle_{kl})\,. \quad (6)$$

In order to realize the teleportation, Alice operates in turn a series of Bell state measurements on particles k and l from $k = 1$ to $k = n$. There are 2^n classes of different measurements that Alice can perform and accordingly there are 4^n kinds of outcomes in all. The first 2^n kinds of measurements can be listed in the form

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*Correspondence author, e-mail: szhu@suda.edu.cn

$$\begin{aligned}\langle\phi^{\pm}|_{n(3n-1)}\cdots\langle\phi^{\pm}|_{1(n+1)}\Psi\rangle &= \frac{1}{2^n}\{x_0|0\cdots0\rangle+\cdots\pm x_1|0\cdots1\rangle+\cdots\\ &\quad+(\cdots\pm_i\cdots+_j\cdots x_a|\cdots1_i\cdots0_j\cdots\rangle)+\cdots+(\pm\cdots\pm x_{2^n-1}|1\cdots1\rangle)\}_{(n+2)(n+4)\cdots3n}\,,\end{aligned}\tag{7}$$

where coefficients x_0 of basis vector $|0\cdots0\rangle$ and others are the same as the original state in Eq. (1), while the sign of each corresponding coefficient is altered. The sign "±" or "+" from left to right corresponds to the Bell state measurement of particle $k\,l$ from $k=1$ to $k=n$. It is noticed that whenever there is a "0" in position "j", there should be a "+" sign in the same position. When there is a "1" in position "i", there should be corresponding "±" sign at the same position. For example, if $n=4$, one has

$$\begin{aligned}\langle\phi^{\pm}|_{411}\langle\phi^{\pm}|_{39}\langle\phi^{\pm}|_{27}\langle\phi^{\pm}|_{15}\Psi\rangle = \frac{1}{2^4}\{&x_0|0000\rangle+++\pm x_1|0001\rangle++\pm+x_2|0010\rangle++\pm\pm x_3|0011\rangle+\pm++\\ &x_4|0100\rangle+\pm+\pm x_5|0101\rangle+\pm\pm+x_6|0110\rangle+\pm\pm\pm x_7|0111\rangle\pm+++\\ &x_8|1000\rangle\pm++\pm x_9|1001\rangle\pm+\pm+x_{10}|1010\rangle\pm+\pm\pm x_{11}|1011\rangle\pm\pm++\\ &x_{12}|1100\rangle\pm\pm+\pm x_{13}|1101\rangle\pm\pm\pm+x_{14}|1110\rangle\pm\pm\pm\pm x_{15}|1111\rangle\}_{681012}\,.\end{aligned}\tag{8}$$

The outcomes of other (2^n-1)-class Bell-state measurements belong to the 2^n-dimensional Hilbert space. It is necessary to determine the corresponding coefficient and its sign of each basic vector when a unitary transformation is performed.

In order to educe the (2^n-1) classes of $2^n(2^n-1)$ kinds of outcomes of the measurements, a Latin square of order $2^n (n\geq1)$ can be introduced, where it is a $2^n\times2^n$ array of integers chosen from the set $L=\{l_0,l_1,\ldots,l_{2^n-1}\}$. In the Latin square, each integer occurs exactly once in each row and column.

The special Latin square of order 2^n can be constructed by using the Latin square of order 2. The recursion formula can be written as

$$L_{2^{n+1}}=\begin{bmatrix}L_{2^n} & L'_{2^n}\\ L'_{2^n} & L_{2^n}\end{bmatrix},\tag{9}$$

where $L'_{2^n}=L_{2^n}+2^nO$, O is a special matrix of order 2^n with the value of each element being one. The Latin square of order 2 can be written as

$$L_2=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix},\tag{10}$$

where n pairs of $|\phi^+\rangle$ are used as quantum channels.

If the Bell state measurement $|\phi^{\pm}\rangle$ is denoted by binary bit of "0", and $|\psi^{\pm}\rangle$ by binary bit of "1", each joint measurement $(|\phi^{\pm}\rangle_{1(n+1)}\cdots|\phi^{\pm}\rangle_{n(3n-1)})$, $(|\phi^{\pm}\rangle_{1(n+1)}\cdots(|\psi^{\pm}\rangle_{n(3n-1)}),\cdots,(|\psi^{\pm}\rangle_{1(n+1)}),\cdots,|\psi^{\pm}\rangle_{n(3n-1)})$ will correspond to a binary number $(0\cdots0)$, $(0\cdots1),\ldots$, $(1\cdots1)$. Any binary number has its decimal correspondence. So $|\phi^{\pm}\rangle_{1(n+1)}\cdots|\phi^{\pm}\rangle_{n(3n-1)}$ can be interpreted as the 1st class of Bell-state measurements, and $|\phi^{\pm}\rangle_{1(n+1)}\cdots|\psi^{\pm}\rangle_{n(3n-1)}$ can be interpreted as the 2nd class of Bell-state measurements. Consequently, $|\psi^{\pm}\rangle_{1(n+1)},\ldots,|\psi^{\pm}\rangle_{n(3n-1)}$ can be interpreted as the 2^n-th class of Bell-state measurements.

It is noticed that the outcomes of the 1st class measurements satisfy the relation when the basis vectors of the array are from $|0\cdots0\rangle$ to $|1\cdots1\rangle$. The corresponding coefficients x_i of the array are from $i=0$ to $i=2^n-1$, which is the first row of the $2^n\times2^n$ Latin square in Eq. (9). If the position of the basis vectors is kept, the coefficients of the 2nd class of Bell-state measurements in the array can be determined by the second row of the Latin square. Analogically, the coefficient of the 2^n-th class of outcome states can be determined by the 2^n-th row of the Latin square. While the sign of the coefficient will be the same as that in Eq. (7). If one takes $n=4$ as an example, one has

$$\begin{aligned}\langle\psi^{\pm}|_{411}\langle\phi^{\pm}|_{39}\langle\psi^{\pm}|_{27}\langle\phi^{\pm}|_{15}\Psi\rangle = \frac{1}{2^4}\{&+\pm+\pm x_5|0000\rangle+\pm++x_4|0001\rangle+\pm\pm\pm x_7|0010\rangle+\pm\pm+x_6|0011\rangle\\ &+++\pm x_1|0100\rangle++++x_0|0101\rangle++\pm\pm x_3|0110\rangle++\pm+x_2|0111\rangle\\ &\pm\pm+\pm x_{13}|1000\rangle\pm\pm++x_{12}|1001\rangle\pm\pm\pm\pm x_{15}|1010\rangle\pm\pm\pm+x_{14}|1011\rangle\\ &\pm++\pm x_9|1100\rangle\pm+++x_8|1101\rangle\pm+\pm\pm x_{11}|1110\rangle\pm+\pm+x_{10}|1111\rangle\}_{681012}\,.\end{aligned}\tag{11}$$

Thus the formulations of outcome states after 4^n kinds of Bell state measurements performed by Alice can be determined. After the measurements, Alice informs Bob which measurement she chooses to her particles via a classical channel. According to the message of Alice, Bob can reconstruct the original state with his particles $(n+2),(n+4),\ldots,(3n)$. It is evident that there are 4^n kinds of relevant unitary operations that can be performed on his particles against the different measurements by Alice. Table 1 shows all 4^n kinds of corresponding unitary transformations by Bob accompanied by the measurements of Alice. It will be mentioned that the symbol "±" of the particles $(n+2), (n+4), \ldots, (3n)$ in the right column is relied on the Bell state measurements of particles $1(n+1), 2(n+3), \ldots, n(3n-1)$, respectively. The symbol "±" is "+" if the Bell state is "+", and the sign is "−" in other cases. Whenever there is a Bell state measurement $|\psi^{\pm}\rangle_{i(n+2i-1)}$ on particles "i" and "$n+2i-1$", there is a Pauli transformation $(|0\rangle\langle 1| \pm |1\rangle\langle 0|)_{n+2i}$ on particle $(n+2i)$.

Table 1 Measurements of Alice and the unitary transformations of Bob when n pairs of $|\phi^{+}\rangle$ are used as quantum channels.

Alice's measurements	Bob's unitary transformations
$\lvert\phi^{\pm}\rangle_{1(n+1)}, \ldots, \lvert\phi^{\pm}\rangle_{n(3n-1)}$	$(\lvert 0\rangle\langle 0\rvert \pm \lvert 1\rangle\langle 1\rvert)_{n+2} \otimes \cdots \otimes (\lvert 0\rangle\langle 0\rvert \pm \lvert 1\rangle\langle 1\rvert)_{3n}$
$\lvert\phi^{\pm}\rangle_{1(n+1)}, \ldots, \lvert\psi^{\pm}\rangle_{n(3n-1)}$	$(\lvert 0\rangle\langle 0\rvert \pm \lvert 1\rangle\langle 1\rvert)_{n+2} \otimes \cdots \otimes (\lvert 0\rangle\langle 1\rvert \pm \lvert 1\rangle\langle 0\rvert)_{3n}$
$\vdots$	$\vdots$
$\lvert\psi^{\pm}\rangle_{1(n+1)}, \ldots, \lvert\psi^{\pm}\rangle_{n(3n-1)}$	$(\lvert 0\rangle\langle 1\rvert \pm \lvert 1\rangle\langle 0\rvert)_{n+2} \otimes \cdots \otimes (\lvert 0\rangle\langle 1\rvert \pm \lvert 1\rangle\langle 0\rvert)_{3n}$

Whenever there is a Bell state measurement $|\phi^{\pm}\rangle_{j(n+2j-1)}$ on particles "j" and "$n+2j-1$", there is a Pauli transformation $(|0\rangle\langle 0| \pm |1\rangle\langle 1|)_{n+2j}$ on particle $(n+2j)$. For example, when $n=4$, the Bell state measurements of Alice are $|\phi^{\pm}\rangle_{15}|\psi^{\pm}\rangle_{27}|\phi^{\pm}\rangle_{39}|\psi^{\pm}\rangle_{411}$, the corresponding unitary transformations of Bob can immediately be written as

$$(|0\rangle\langle 0| \pm |1\rangle\langle 1|)_6 \otimes (|0\rangle\langle 1| \pm |1\rangle\langle 0|)_8 \otimes (|0\rangle\langle 0| \pm |1\rangle\langle 1|)_{10} \otimes (|0\rangle\langle 1| \pm |1\rangle\langle 0|)_{12}\,. \tag{12}$$

When n pairs of $|\phi^{-}\rangle$, $|\psi^{+}\rangle$, and $|\psi^{-}\rangle$ are utilized as quantum channels, the measurements of Alice and the corresponding unitary transformations of Bob are listed in Tables 2, 3, and 4, respectively.

Table 2 Measurements of Alice and the unitary transformations of Bob when n pairs of $|\phi^{-}\rangle$ are used as quantum channels.

Alice's measurements	Bob's unitary transformations
$\lvert\phi^{\pm}\rangle_{1(n+1)}, \ldots, \lvert\phi^{\pm}\rangle_{n(3n-1)}$	$(\lvert 0\rangle\langle 0\rvert \mp \lvert 1\rangle\langle 1\rvert)_{n+2} \otimes \cdots \otimes (\lvert 0\rangle\langle 0\rvert \mp \lvert 1\rangle\langle 1\rvert)_{3n}$
$\lvert\phi^{\pm}\rangle_{1(n+1)}, \ldots, \lvert\psi^{\pm}\rangle_{n(3n-1)}$	$(\lvert 0\rangle\langle 0\rvert \mp \lvert 1\rangle\langle 1\rvert)_{n+2} \otimes \cdots \otimes (\lvert 0\rangle\langle 1\rvert \mp \lvert 1\rangle\langle 0\rvert)_{3n}$
$\vdots$	$\vdots$
$\lvert\psi^{\pm}\rangle_{1(n+1)}, \ldots, \lvert\psi^{\pm}\rangle_{n(3n-1)}$	$(\lvert 0\rangle\langle 1\rvert \mp \lvert 1\rangle\langle 0\rvert)_{n+2} \otimes \cdots \otimes (\lvert 0\rangle\langle 1\rvert \mp \lvert 1\rangle\langle 0\rvert)_{3n}$

Table 3 Measurements of Alice and the unitary transformations of Bob when n pairs of $|\psi^{+}\rangle$ are used as quantum channels.

Alice's measurements	Bob's unitary transformations
$\lvert\phi^{\pm}\rangle_{1(n+1)}, \ldots, \lvert\phi^{\pm}\rangle_{n(3n-1)}$	$(\lvert 0\rangle\langle 1\rvert \pm \lvert 1\rangle\langle 0\rvert)_{n+2} \otimes \cdots \otimes (\lvert 0\rangle\langle 1\rvert \pm \lvert 1\rangle\langle 0\rvert)_{3n}$
$\lvert\phi^{\pm}\rangle_{1(n+1)}, \ldots, \lvert\psi^{\pm}\rangle_{n(3n-1)}$	$(\lvert 0\rangle\langle 1\rvert \pm \lvert 1\rangle\langle 0\rvert)_{n+2} \otimes \cdots \otimes (\lvert 0\rangle\langle 0\rvert \pm \lvert 1\rangle\langle 1\rvert)_{3n}$
$\vdots$	$\vdots$
$\lvert\psi^{\pm}\rangle_{1(n+1)}, \ldots, \lvert\psi^{\pm}\rangle_{n(3n-1)}$	$(\lvert 0\rangle\langle 0\rvert \pm \lvert 1\rangle\langle 1\rvert)_{n+2} \otimes \cdots \otimes (\lvert 0\rangle\langle 0\rvert \pm \lvert 1\rangle\langle 1\rvert)_{3n}$

Table 4 Measurements of Alice and the unitary transformations of Bob when n pairs of $|\psi^{-}\rangle$ are used as quantum channels.

Alice's measurements	Bob's unitary transformations
$\lvert\phi^{\pm}\rangle_{1(n+1)}, \ldots, \lvert\phi^{\pm}\rangle_{n(3n-1)}$	$(\lvert 0\rangle\langle 1\rvert \mp \lvert 1\rangle\langle 0\rvert)_{n+2} \otimes \cdots \otimes (\lvert 0\rangle\langle 1\rvert \mp \lvert 1\rangle\langle 0\rvert)_{3n}$
$\lvert\phi^{\pm}\rangle_{1(n+1)}, \ldots, \lvert\psi^{\pm}\rangle_{n(3n-1)}$	$(\lvert 0\rangle\langle 1\rvert \mp \lvert 1\rangle\langle 0\rvert)_{n+2} \otimes \cdots \otimes (\lvert 0\rangle\langle 0\rvert \mp \lvert 1\rangle\langle 1\rvert)_{3n}$
$\vdots$	$\vdots$
$\lvert\psi^{\pm}\rangle_{1(n+1)}, \ldots, \lvert\psi^{\pm}\rangle_{n(3n-1)}$	$(\lvert 0\rangle\langle 0\rvert \mp \lvert 1\rangle\langle 1\rvert)_{n+2} \otimes \cdots \otimes (\lvert 0\rangle\langle 0\rvert \mp \lvert 1\rangle\langle 1\rvert)_{3n}$

In conclusion, a scheme for teleportation of a completely unknown n-particle state via n pairs of identical EPR channels is reported. The formulation of the states after the Bell state measurements by Alice and the corresponding unitary transformations by Bob is derived through a Latin square. It is shown that when separate EPR pairs are utilized for the quantum channels and joint measurement is decomposable into independent Bell state measurements, unitary transformation can be divided into n local single-qubit operations. This implies that teleportation of n-particle state can be implemented by a series of single-qubit teleportation.

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