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Comment on "Computer Algebra and Solutions to the Karamoto–Sivashinsky Equation" [Commun. Theor. Phys. (Beijing, China) 43 (2005) 39][*]

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Abstract *In a recent article [Commun. Theor. Phys. (Beijing, China)* ***43*** *(2005) 39], Xie et al. improved the extended tanh function method by introducing a generalized Riccati equation and its new solutions. Then they choose the Karamoto–Sivashinsky (KS) equation to illustrate their approach and obtain many exact solutions of the KS equation. So they claim that, by using their method, one not only can successfully recover the previously known formal solutions but also construct new and more general formal solutions for some nonlinear evolution equations. In this comment, we will show that the claim is incorrect.*

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Recently, to seek for new exact solutions of nonlinear evolution equations, Xie *et al.* further improved the extended tanh function method[1−3] by generalizing the Riccati equation and picking up its new solutions.[4] They firstly extended the Riccati equation to the general form,

$$\phi' = r + p\phi + q\phi^2\,, \tag{1}$$

where r, p, and q are all real constants, and found the following solutions to Eq. (1) with some restrained conditions.

(i) When $p^2-4rq>0$ and $p\,q\neq 0$ (or $qr\neq 0$),

$$\phi_1 = -\frac{1}{2q}\Big[p+\sqrt{p^2-4rq}\,\tanh\Big(\frac{\sqrt{p^2-4rq}}{2}\,\xi\Big)\Big]\,, \tag{2}$$

$$\phi_2 = -\frac{1}{2q}\Big[p+\sqrt{p^2-4rq}\,\coth\Big(\frac{\sqrt{p^2-4rq}}{2}\,\xi\Big)\Big]\,; \tag{3}$$

(ii) When $p^2-4rq>0$ and $qr\neq 0$,

$$\phi_5 = \frac{2r\cosh(\sqrt{p^2-4rq}\,\xi/2)}{\sqrt{p^2-4rq}\,\sinh(\sqrt{p^2-4rq}\,\xi/2)-p\cosh(\sqrt{p^2-4rq}\,\xi/2)}\,, \tag{4}$$

$$\phi_6 = \frac{-2r\sinh(\sqrt{p^2-4rq}\,\xi/2)}{p\sinh(\sqrt{p^2-4rq}\,\xi/2)-\sqrt{p^2-4rq}\,\cosh(\sqrt{p^2-4rq}\,\xi/2)}\,; \tag{5}$$

(iii) When $r=0$ and $p\,q\neq 0$,

$$\phi_9 = \frac{-p\,w}{q\,(w+\cosh(p\,\xi)-\sinh(p\,\xi))}\,,\qquad \phi_{10} - \frac{p\,(\cosh(p\,\xi)+\sinh(p\,\xi))}{q\,(w+\cosh(p\,\xi)+\sinh(p\,\xi))}\,, \tag{6}$$

where w is an arbitrary constant.

Remark In Ref. [4], when $p^2-4rq<0$ and $p\,q\neq 0$ (or $qr\neq 0$), ϕ_3, ϕ_4, ϕ_7, and ϕ_8 are triangular function solutions of Eq. (1). They can be constructed directly from ϕ_1, ϕ_2, ϕ_5, and ϕ_6 by use of the identical formula $\tanh(\mathrm{i}\,\alpha)=\mathrm{i}\tan\alpha$, $\mathrm{sech}(\mathrm{i}\,\alpha)=\sec\alpha$, $\coth(\mathrm{i}\,\alpha)=-\mathrm{i}\cot\alpha$, $\mathrm{csch}(\mathrm{i}\,\alpha)=-\mathrm{i}\csc\alpha$, here we omit them.

Then, Xie *et al.* studied the Karamoto–Sivashinsky (KS) equation

$$u_t + uu_x + \alpha u_{xx} + \beta u_{xxxx} = 0\,. \tag{7}$$

Supposing that equation (7) has the formal solution

$$u = a_0 + a_1\phi + a_2\phi^2 + a_3\phi^3\,, \tag{8}$$

and by using direct algebra method,[5−8] they obtained the algebraic system and its five solutions ***Cases 1 ∼ 5***. Finally, substituting ***Cases*** 1 ∼ 5 into Eq. (8) along with Eqs. (2) ∼ (6), they obtained many exact solutions of Eq. (7) and claimed that new solutions to KS equation had been obtained.

Unfortunately, we find that the solutions (2) ∼ (6) ***of Eq. (1) are related, which can be easily shown by the following simple calculations.***

For the sake of brevity, we denote $\theta=(\sqrt{p^2-4rq}\,\xi)/2$. Firstly, we can prove the solutions ϕ_5 and ϕ_1 of Eq. (1), the solutions ϕ_6 and ϕ_2 of Eq. (1) are the same in waveform and only different in phase. So the solutions ϕ_5 and ϕ_6 are not new solutions of Eq. (1).

In fact, set $p/\sqrt{4rq}=\cosh\alpha_1$, thus $\sqrt{p^2-4rq}/\sqrt{4rq}=\sinh\alpha_1$. Notice the identical formula

$$\sinh\theta\cosh\alpha - \cosh\theta\sinh\alpha = \sinh(\theta-\alpha)\,, \tag{9}$$

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$$\cosh\theta\cosh\alpha - \sinh\theta\sinh\alpha = \cosh(\theta-\alpha)\,, \tag{10}$$

we have

$$\frac{p\sinh\theta - \sqrt{p^2-4rq}\cosh\theta}{\sqrt{p^2-4rq}\sinh\theta - p\cosh\theta} = \frac{\sinh\theta\cosh\alpha_1 - \cosh\theta\sinh\alpha_1}{\sinh\theta\sinh\alpha_1 - \cosh\theta\cosh\alpha_1} = -\tanh(\theta-\alpha_1)\,, \tag{11}$$

so

$$\begin{aligned}\phi_5 &= -\frac{p}{2q} + \frac{p}{2q} + \frac{2r\cosh\theta}{\sqrt{p^2-4rq}\,\sinh\theta - p\cosh\theta} = -\frac{p}{2q} + \frac{p\sqrt{p^2-4rq}\,\sinh\theta - p^2\cosh\theta + 4rq\cosh\theta}{2q[\sqrt{p^2-4rq}\sinh\theta - p\cosh\theta\,]}\\ &= -\frac{p}{2q} + \frac{\sqrt{p^2-4rq}}{2q}\Big(\frac{p\sinh\theta - \sqrt{p^2-4rq}\cosh\theta}{\sqrt{p^2-4rq}\sinh\theta - p\cosh\theta}\Big) = -\frac{p}{2q} - \frac{\sqrt{p^2-4rq}}{2q}\tanh(\theta-\alpha_1)\,.\end{aligned} \tag{12}$$

Similar to the above proof, we have

$$\frac{p\cosh\theta - \sqrt{p^2-4rq}\sinh\theta}{p\sinh\theta - \sqrt{p^2-4rq}\cosh\theta} = \coth(\theta-\alpha_1)\,, \tag{13}$$

so

$$\begin{aligned}\phi_6 &= -\frac{p}{2q} + \frac{p}{2q} + \frac{-2r\sinh\theta}{p\sinh\theta - \sqrt{p^2-4rq}\,\cosh\theta} = -\frac{p}{2q} + \frac{-p^2\sinh\theta + p\sqrt{p^2-4rq}\cosh\theta + 4qr\sinh\theta}{2q[-p\sinh\theta + \sqrt{p^2-4rq}\cosh\theta]}\\ &= -\frac{p}{2q} - \frac{\sqrt{p^2-4rq}}{2q}\Big(\frac{p\cosh\theta - \sqrt{p^2-4rq}\sinh\theta}{p\sinh\theta - \sqrt{p^2-4rq}\cosh\theta}\Big) = -\frac{p}{2q} - \frac{\sqrt{p^2-4rq}}{2q}\coth(\theta-\alpha_1)\,.\end{aligned} \tag{14}$$

Secondly, we can prove the solutions ϕ_9, ϕ_{10} of Eq. (1) and the solutions ϕ_1, ϕ_2 are the same in waveform and only different in phase, so the solutions ϕ_9 and ϕ_{10} are not new solutions of Eq. (1).

In fact,

$$\begin{aligned}\phi_9 &= \frac{-p\,w}{q[w + \cosh(p\,\xi) - \sinh(p\,\xi)]} + \frac{p}{2q} - \frac{p}{2q} = -\frac{p}{2q} - \frac{p\,(w - \mathrm{e}^{-p\,\xi})}{2q(w + \mathrm{e}^{-p\,\xi})} = -\frac{p}{2q} - \frac{p}{2q}\Big(\frac{w\,\mathrm{e}^{p\,\xi} - 1}{w\,\mathrm{e}^{p\,\xi} + 1}\Big)\\ &= \begin{cases} -\dfrac{p}{2q} - \dfrac{p}{2q}\tanh\Big(\dfrac{p\,\xi + \ln w}{2}\Big), & \text{if} \quad w > 0\,,\\ 0, & \text{if} \quad w = 0\,,\\ -\dfrac{p}{2q} - \dfrac{p}{2q}\coth\Big(\dfrac{p\,\xi + \ln(-w)}{2}\Big), & \text{if} \quad w < 0\,.\end{cases}\end{aligned} \tag{15}$$

$$\begin{aligned}\phi_{10} &= \frac{-p\,[\cosh(p\,\xi) + \sinh(p\,\xi)]}{q[w + \cosh(p\,\xi) + \sinh(p\,\xi)]} + \frac{p}{2q} - \frac{p}{2q} = -\frac{p}{2q} - \frac{p\,(\mathrm{e}^{p\,\xi} - w)}{2q(\mathrm{e}^{p\,\xi} + w)} = -\frac{p}{2q} - \frac{p}{2q}\Big(\frac{\mathrm{e}^{p\,\xi}/w - 1}{\mathrm{e}^{p\,\xi}/w + 1}\Big)\\ &= \begin{cases} -\dfrac{p}{2q} - \dfrac{p}{2q}\tanh\Big(\dfrac{p\,\xi - \ln w}{2}\Big), & \text{if} \quad w > 0\,,\\ -\dfrac{p}{q}, & \text{if} \quad w = 0\,,\\ -\dfrac{p}{2q} - \dfrac{p}{2q}\coth\Big(\dfrac{p\,\xi - \ln(-w)}{2}\Big), & \text{if} \quad w < 0\,.\end{cases}\end{aligned} \tag{16}$$

In summary, we have shown that the solutions ϕ_j ($j = 5, 6, 9, 10$) and ϕ_k ($k = 1, 2$) are the same in waveform and only different in phase. **In addition, we note that the solutions *Cases* $2 \sim 5$ of the algebraic system in Ref. [4] are the special circumstances of *Case 1*, and so are redundant.**

To conclude, due to no new solutions of Eq. (1) and the algebraic system in Ref. [4] were found, using the proposed method by Xie *et al.*, no new solutions to the KS equation were obtained.

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