

Interaction of a Surface Acoustic Wave with a Two-dimensional Electron Gas*

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Abstract When a surface acoustic wave (SAW) propagates on the surface of a GaAs semiconductor, coupling between electrons in the two-dimensional electron gas beneath the interface and the elastic host crystal through piezoelectric interaction will attenuate the SAW. The coupling coefficient is calculated for the SAW propagating along an arbitrary direction. It is found that the coupling strength is strongly dependent on the propagating direction. When the SAW propagates along the [011] direction, the coupling becomes quite weak.

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Key words: surface acoustic wave, piezoelectric coupling

1 Introduction

Experiments which include coupling of surface acoustic waves (SAW's) with electrons in a two-dimensional electron gas (2DEG) have intrigued great interest in these years. SAW experiments that have been carried out in the fractional quantum Hall regime near a filling factor $\nu = 1/2$, strongly supporting the composite fermion approach to the compressible state.^[1] In quantum Hall systems the formation of unidirectionally modulated electron structures can occur at half-filled high Landau levels. These new phases were predicted theoretically and confirmed experimentally by observation of a strong anisotropy of the conductivity. The piezoelectric coupling is considered as the origin of the orientation of stripes or electron crystal.^[2,3]

On the other hand, SAW has also been demonstrated as another method of creating quantum wires in 2DEG.^[4,5] This method involves an intense SAW in a hybrid structure containing a semiconductor quantum-well and a strongly piezoelectric coupling host crystal. In the experiments a homogeneous 2DEG in the hybrid structure turns into moving wires of electrons due to a very strong piezoelectric potential induced by an SAW. This effect has been demonstrated in the above-mentioned experiments for the classical regime of acoustic-electric interaction at room-temperature. For low temperatures and a sufficiently strong piezoelectric potential of an SAW, the electron spectrum of wires can be quantized in the direction of the SAW momentum.^[6,7]

It is well known that the propagation of acoustic wave on the surface of piezoelectric crystals can be effected by nearby conductors.^[8] In a heterostructure where a 2-dimensional electron gas forms beneath the interface, the interaction of the electron gas with the SAW will attenuate

the propagation with a velocity shift δv and an attenuation coefficient κ , which satisfy the relation,^[9]

$$\frac{\delta v}{v} - \frac{i\kappa}{q} = \frac{\alpha^2/2}{1 + i\sigma_{xx}(q, \omega)/\sigma_m}, \quad (1)$$

where $\sigma_{xx}(q, \omega)$ is the longitudinal conductivity of the adjoining medium at wave vector q and frequency $\omega = vq$. The coefficients and $\alpha^2/2$ depend on material parameters, which will be discussed in this work.

Unlike conductivity measurements in which a current is driven through the 2DEG and the voltage is measured, where only the conductivity at zero wave vector can be probed, SAW measurements allow one to probe the finite wave vector conductivity.^[10] In earlier SAW experiments, the wavelength of the SAW was much larger than the distance d of the 2DEG from the surface. In this case, the depth can be neglected, and the coefficients σ_m and $\alpha^2/2$ can be assumed to be constant. When the wavelength is comparable to the distance d , one should be ware of the wave-vector dependence of the coefficients.^[10,11] Roughly one might expect that the coupling $\alpha^2/2$ should decay approximately as e^{-2qd} .

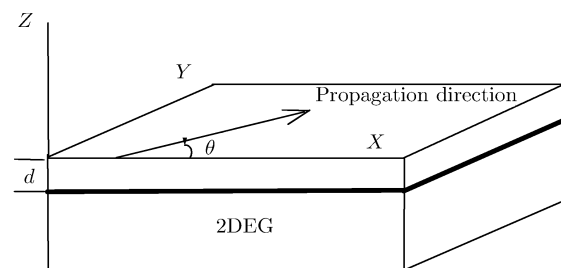


Fig. 1 A Schematic description of the sample on which the SAW transports. The 2DEG is located a distance d beneath the surface.

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In this work, we use a model which takes into account the anisotropy of the elastic constants of the crystalline matrix. The physical properties of the GaAs system are described by three elastic constants c_{11} , c_{12} , and c_{44} , one piezoelectric modulus e_{14} , and the dielectric constant ϵ . We study the propagation of SAW along an arbitrary direction with and without piezoelectric coupling where the 2DEG is located at a distance d below the surface, as shown in Fig. 1. In Sec. 2 we describe the transport of the SAW and in Sec. 3 we introduce the energy shift by piezoelectric coupling. The coupling coefficient of the SAW with 2DEG is calculated in Sec. 4. A brief discussion is given in Sec. 5.

2 Transport of SAW

Consider an infinite piezoelectric medium. We study the elastic wave movement by defining a displacement vector \mathbf{u}_k . The elastic stress tensor satisfies $T_{ij} = c_{ijkl}u_{kl} - e_{kij}E_k$, where $u_{kl} = \frac{1}{2}(\partial_k u_l + \partial_l u_k)$ and \mathbf{E} is electric field vector.^[12] c_{ijkl} is the elastic modulus which in cubic symmetric crystals has only three independent parameters c_{11} , c_{12} , and c_{44} . For GaAs at low temperatures, these elastic constants are measured as 12.26×10^{10} , 5.71×10^{10} , and 6.00×10^{10} N/m², respectively. e_{kij} is the piezoelectric stress tensor, which has only one nonzero component e_{14} in GaAs and AlAs crystals. It has an accepted value of approximately 0.15 C/m² for GaAs. The electric displacement vector $D_j = e_{jkl}u_{kl} + \epsilon E_j$ by taking account of the piezoelectric coupling. Here ϵ is the dielectric constant of the medium which is assumed to be isotropic. The elastic wave equation is given by

$$\begin{aligned} c_{ijkl}\partial_l\partial_j u_k + e_{kij}\partial_j\partial_k\phi &= \rho\partial_t^2 u_i, \\ e_{jkl}\partial_l\partial_j u_k - \epsilon\nabla^2\phi &= 0, \end{aligned} \quad (2)$$

where ρ is the mass density. For GaAs crystal, $\rho = 5300$ kg/m³.

We first consider an SAW propagating on a free surface by disregarding of the piezoelectric coupling, i.e., the crystal is non-piezoelectric. The boundary condition in the \hat{z} direction is

$$T_{zk} = c_{zklm}\partial_l u_m|_{z=0} = 0. \quad (3)$$

A surface acoustic wave is propagating along an arbitrary direction with an azimuth angle θ to the \hat{x} axis. The plane wave solution has the following form,

$$u_i = u_{0i} \exp[-i\omega t + i\mathbf{q} \cdot \mathbf{r} + iq_z z], \quad (4)$$

where $\mathbf{q} = (q_x, q_y)$ is the wave vector in the X-Y plane and q_z is the wave vector in the \hat{z} direction. Substitute formula (4) into the first line of Eqs. (2) by omitting the piezoelectric coupling term, one obtains

$$[c_{ijkl}q_l q_j - \rho\omega^2\delta_{ik}]u_{0k} = 0. \quad (5)$$

For surface acoustic waves, q_z should be a complex quantity which implies the wave decay exponentially into the bulk. The eigenvalues of the wave vectors $q_z^{(n)}$ and the respective eigen solutions $u_{0i}^{(n)}$ ($n = 1, 2, 3$) are determined by Eq. (5). To satisfy the boundary condition (3), a linear combination of the eigen solutions should be taken,

$$u_i = \sum_{n=1}^3 C(n)u_{0i}^{(n)} \exp[-i\omega t + i\mathbf{q} \cdot \mathbf{r} + iq_z^{(n)}z]. \quad (6)$$

From the above equation, the velocity of the SAW and relevant coefficients of $C(n)$ are computed. Figure 2 displays a non-monotonous relation of v_s with the propagation direction in a GaAs crystal. The velocity v_s reaches a peak when the SAW transports along the direction of $\theta \sim 25^\circ$ while at $\theta = 45^\circ$ it falls to a minimum.

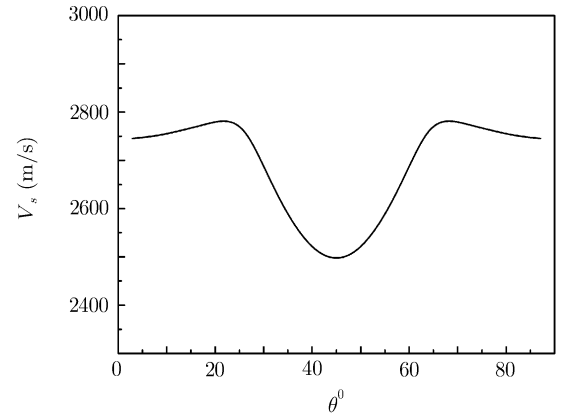


Fig. 2 The velocity of the SAW versus the transport direction without piezoelectric coupling.

3 Interaction-Induced Energy Shift

Due to the piezoelectric coupling, an external scalar potential $\phi^{\text{ext}} = Ae_{14}F(qd)/\epsilon$ is induced in the 2DEG, where A is the amplitude of the SAW and F is a dimensionless function of qd that represents the fact that the SAW decays into the bulk of the crystal. The induced energy density per unit area due to this external potential is given by^[13]

$$\delta U = \frac{\epsilon_{\text{eff}} q}{4\pi[1 - i\sigma_m/\sigma_{xx}(q, \omega)]} |\phi^{\text{ext}}|^2, \quad (7)$$

where ϵ_{eff} is the effective background dielectric constant which is wave-vector-dependent with the form

$$\frac{\epsilon_{\text{eff}}}{\epsilon} = \frac{1}{2} \left[\frac{(\epsilon + 1) \exp(qd)}{\epsilon \cosh(qd) + \sinh(qd)} \right]. \quad (8)$$

We want to measure this energy shift with respect to the shift for $\sigma_{xx} \rightarrow \infty$,

$$\Delta U \equiv \delta U - \delta U(\sigma_{xx} \rightarrow \infty). \quad (9)$$

It is found below that the surface acoustic wave has an energy density proportional to $A^2 q^2$. Furthermore, the

wave decays exponentially into the bulk with a decay constant proportional to q . Thus, when integrated in the \hat{z} direction, the energy U per unit surface area is given by

$$U = qA^2H, \quad (10)$$

where H is a factor that depends on material parameters and is determined by calculating the energy density induced by SAW.

Combined the above results, the fractional energy shift is then given by

$$\frac{\Delta U}{U} = \frac{-\alpha^2/2}{1 + i\sigma_{xx}(q, \omega)/\sigma_m}, \quad (11)$$

where the coefficient $\alpha^2/2$ in Eq. (1) is derived as

$$\frac{\alpha^2}{2} = \frac{\epsilon_{\text{eff}}}{\epsilon} \frac{e_{14}^2}{\epsilon_0 \epsilon H} |F(qd)|^2. \quad (12)$$

4 Piezoelectric Coupling with 2DEG

Since the piezoelectric coupling e_{14} is small, it is seen from the second equation of Eqs. (2) that ϕ is of order e_{14} smaller than u . Thus the first equation will be solved by the non-piezoelectric solution discussed above with corrections only at order e_{14}^2 . The mechanical boundary conditions in the piezoelectric case are

$$c_{i\hat{z}k\hat{l}}\partial_{\hat{l}}u_k + e_{ik\hat{z}}\partial_k\phi = 0. \quad (13)$$

The electrical boundary condition for the second equation of Eqs. (2) is determined by setting the potential to be continuous at $z = 0$,

$$e_{z\hat{k}\hat{l}}\partial_{\hat{l}}u_k - \epsilon\partial_z\phi|_{z=0^-} = -\partial_z\phi|_{z=0^+}. \quad (14)$$

The piezoelectric case for ϕ is solved by use of the non-piezoelectric solution for displacement u in Sec. 2. The supposed form of solution is

$$\begin{aligned} \phi = \frac{e_{14}}{\epsilon} e^{i(\mathbf{q}\cdot\mathbf{r}-\omega t)} [A(1)e^{iq_z^{(1)}z} + A(2)e^{iq_z^{(2)}z} \\ + A(3)e^{iq_z^{(3)}z} + A(4)e^{qz}]. \end{aligned} \quad (15)$$

Substitute the above solution and formula (6) into Eq. (2), one gets

$$\begin{aligned} A(n) = \frac{1}{q^2 + (q_z^{(n)})^2} [C(n)u_{01}^{(n)}q_yq_z^{(n)} \\ + C(n)u_{02}^{(n)}q_xq_z^{(n)} + C(n)u_{03}^{(n)}q_xq_y], \\ (n = 1, 2, 3), \end{aligned} \quad (16)$$

and

$$\begin{aligned} A(4) = \frac{1}{q(\epsilon + 1)} \sum_{n=1}^3 [i\epsilon A(n)q_z^{(n)} - qA(n) \\ - \frac{i\epsilon}{2}(q_xC(n)u_{02}^{(n)} + q_yC(n)u_{01}^{(n)})], \end{aligned} \quad (17)$$

where $A(1) = -A(2)^*$ and $A(3), A(4)$ are pure imaginaries. The explicit form of the potential solution is then

$$\begin{aligned} \phi = \frac{iAe_{14}}{\epsilon} [2\text{Im}(A(1)e^{iq_z^{(1)}z}) + \text{Im}A(3)e^{iq_z^{(3)}z} \\ + \text{Im}A(4)e^{qz}] e^{i\mathbf{q}\cdot\mathbf{r}-i\omega t}. \end{aligned} \quad (18)$$

Thus we obtain the dimensionless function F in formula (12) as

$$\begin{aligned} F(q) = 2|A(1)|e^{-\beta d} \sin(\xi - \alpha d) + \text{Im}(A(3))e^{-\gamma d} \\ + \text{Im}(A(4))e^{-qd}, \end{aligned} \quad (19)$$

where α, β , and ξ are the real part, imaginary part and phase of $A(1)$, respectively.

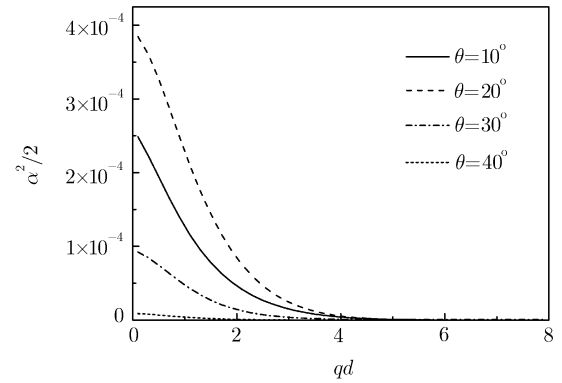


Fig. 3 Piezoelectric coupling coefficient of the SAW with 2DEG under various transport directions. As the transport angle increases, the coupling constant experiences first a rise and then a fall process. At $\theta = 45^\circ$, the coupling becomes quite weak.

Figure 3 shows the dependence of the coupling coefficient $\alpha^2/2$ on the orientation angle that the SAW propagates in a GaAs crystal. They exponentially decay into the bulk of the sample, as we have expected. However, For different transport angles, the coefficients differ largely. They also exhibit a non-monotonous relation with the angle. For example, near the region of $\theta \sim 25^\circ$, the coefficient reaches the maximum while when the SAW transports along the direction of $\theta = 45^\circ$, the coupling between the 2DEG and the SAW becomes quite weak.

5 Discussions

We have studied the interaction of SAW with the 2DEG by introducing the piezoelectric coupling. It is found that the coupling is most strong when the SAW transports along the direction of $\theta \sim 25^\circ$ while it reaches a minimum when transports along the $\theta = 45^\circ$ direction. This result is very helpful for experimentalist when designing devices for different purposes of SAW transport on piezoelectric crystals.

References

- [1] B.I. Halperin, P.A. Lee, and N. Read, *Phys. Rev. B* **47** (1993) 7312.
- [2] E.Ya. Sherman, *Phys. Rev. B* **52** (1995) 1512.
- [3] D.V. Fil, *Low Temp. Phys.* **26** (2000) 581.
- [4] A.O. Govorov, A.V. Kalameitsev, V.M. Kovalev, H.J. Kutschera, and A. Wixforth, *Phys. Rev. Lett.* **87** (2001) 226803.
- [5] F. Alsina, P.V. Santos, H.P. Schröter, W. Seidel, and K.H. Ploog, *Phys. Rev. B* **66** (2002) 165330.
- [6] A. Aharony and O. Entin-Wohlman, *Phys. Rev. B* **65** (2002) 241401.
- [7] V.I. Talyanskii, *et al.*, *Phys. Rev. B* **56** (1997) 15180.
- [8] K.A. Ingebrigtsen, *J. Appl. Phys.* **49** (1969) 2681.
- [9] A.L. Efros and Y.M. Galperin, *Phys. Rev. Lett.* **64** (1990) 1959.
- [10] R.L. Willett, K.W. West, and L.N. Pfeiffer, *Phys. Rev. Lett.* **75** (1995) 2988.
- [11] R.L. Willett, K.W. West, and L.N. Pfeiffer, *Phys. Rev. Lett.* **71** (1993) 3846.
- [12] L.D. Landau and E.M. Lifshitz, *Theory of Elasticity*, 3rd ed., Pergamon, Oxford (1986).
- [13] S.H. Simon, *Phys. Rev. B* **54** (1996) 13878.