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Dynamic Behavior of Lambda-Type Three-Level Atoms and Two-Mode Cavity Field*

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Abstract *A system comprising of Lambda-type three-level atoms and the two-mode cavity field is considered in this paper. Under the adiabatical approximation and the large detuning condition, the effective Hamiltonian of the system in the interaction picture can be given out. If the two identical three-level atoms pass through the cavity in turn, the entangled state atoms can be generated. When the interaction time is taken to an appropriate value, the maximally entangled states are created. At the same time, the dynamic behaviors of the system are studied in detail.*

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Key words: entangled atoms, dynamic behavior

1 Introduction

As a potential resource for communication and information processing, quantum entanglement has been the subject of many studies in recent years. It is the quantum mechanical property that Schrödinger pointed out many years ago as "the characteristic trait of quantum mechanics".[1] A pure state of a pair of quantum systems is called entangled if it is unfactorizable, for example, for the singlet state of two spin-1/2 particles, $(1/\sqrt{2})(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$. A mixed state is entangled if it cannot be represented as a mixture of factorizable pure states. One can realize its richness with many different interesting phenomena, such as collapse-revival phenomenon,[2] squeezing,[3] antibunching,[4] chaos,[5] trapping states,[6] etc.

The nonlinear and the quantum effects of the interactions involving one atom with a few energy levels and one or more near-resonant electromagnetic (em) fields are generally studied either by quantizing the interacting field or by allowing the atom to undergo quantized motion around its c.m. by placing it in a harmonic trap. The effects arising due to the first method have been investigated extensively theoretically as well as experimentally, whereas not enough studies are made on the effects due to the quantization of the atomic motion. The dynamics of the atomic inversion of a two-level trapped atom have been studied for initial coherent vibration of the atomic c.m.[7] We should point out that research of the interactions of three-level atoms with uncorrelated fields have been carried out previously.[8−11] In particular, Li and Peng[11] considered a three-level atom in the λ configuration interacting with a quantized field prepared either in the two-mode coherent state or the two-mode thermal state.

In this paper, we let a Lambda-type three-level atom pass through a two-mode cavity field. After the first atom leaves the cavity, another identical atom passes through the cavity, too. Via the procedure, the entangled atoms can be created. When the interaction time is taken to be an appropriate value, the maximally entangled atoms can be created. Here, the ground hyperfine levels of the Lambda-type three-level atoms are used as qubit, which is more stable than the qubit composed of the excited state and ground state of a two-level state because the former is not related to the spontaneous emission of the atom. In the same time, it is convenient to let the atoms pass through the cavity, which is more operable in experiments than the situation that the atom moves in the cavity. There is no need to measure the states of the atoms in the cavity. This paper also attempts to throw some light on the dynamical and statistical properties of the system.

The remainder of the paper is organized as follows. A system comprises a Lambda-type three-level atom and two-mode cavity is introduced in detail in Sec. 2. The effective Hamilton can be achieved under the limit of large detuning. In Sec. 3, we draw our attention to the situation that one atom passes through the two-mode cavity. In Sec. 4, another identical atom passes through the cavity, thus, the entangled atoms can be attained. With the wave functions of the system in hand, we analyze the dynamic behaviors of the multipartite of the system in Sec. 5. Meantime, we can get the EPR state. In the last section, we summarize our results.

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2 The Model

Let us consider a three-level atomic configuration, which is shown in Fig. 1. It interacts with a two-mode cavity field.

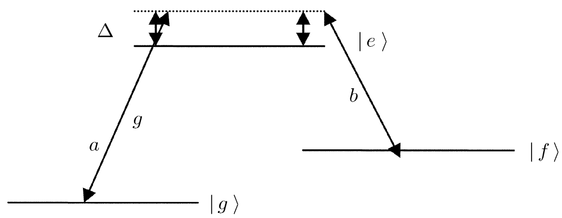

Fig. 1 Three-level atomic configuration with levels $|g\rangle$, $|f\rangle$, and $|e\rangle$ interacting with two orthogonal modes of the cavity, described by annihilation operators a and b. Here, $|g\rangle$, $|f\rangle$ correspond to the ground hyperfine levels and $|e\rangle$ represents the excited level of the atom. g stands for the atom-cavity coupling of the modes with the corresponding transitions, Δ is the detuning of the modes from the corresponding atomic transition.

The Hamiltonian under rotating wave approximation can be written as

$$H = \hbar[\omega_{eg}|e\rangle\langle e| + \omega_{fg}|f\rangle\langle f| + \omega_1 a^\dagger a + \omega_2 b^\dagger b + (g_1|e\rangle\langle g|a + g_2|e\rangle\langle f|b + g_1 a^\dagger|g\rangle\langle e| + g_2 b^\dagger|f\rangle\langle e|)]\,, \tag{1}$$

where a and b are the annihilation operators of the two modes interacting with $|e\rangle \leftrightarrow |g\rangle$ and $|e\rangle \leftrightarrow |f\rangle$ transitions, respectively. They are corresponding to the different polarization directions. ω_{lg} $(l \in e, f)$ is the atomic transition frequency, ω_i $(i \in 1, 2)$ is the frequency of the cavity modes, and g_i is the atom-cavity coupling constant, which is assumed real. The interaction Hamiltonian in the interaction picture can be written as

$$H' = \hbar(g_1|e\rangle\langle g|a\,\mathrm{e}^{\mathrm{i}\Delta_1 t} + g_2|e\rangle\langle f|b\,\mathrm{e}^{\mathrm{i}\Lambda_2 t} + g_1 a^\dagger|g\rangle\langle e|\,\mathrm{e}^{-\mathrm{i}\Delta_1 t} + g_2 b^\dagger|f\rangle\langle e|\,\mathrm{e}^{-\mathrm{i}\Delta_2 t})\,, \tag{2}$$

where $\Delta_i = \omega_{eg,f} - \omega_i$ $(i \in 1, 2)$ is the detuning of the cavity modes from the corresponding atomic transition.

Now, it is supposed that $g_1 = g_2 = g$, $\Delta_1 = \Delta_2 = \Delta$, and $\hbar = 1$. Under the adiabatical approximation and the large detuning approximation, the effective Hamiltonian can be given as

$$H_{\rm eff} = \frac{g^2}{\Delta}(|g\rangle\langle g|a^+ a + |g\rangle\langle f|a^\dagger b + |f\rangle\langle g|b^\dagger a + |f\rangle\langle f|b^\dagger b)\,. \tag{3}$$

Here, the first term and the last term represent the Stark shifts and the rest two terms demonstrate the interaction leading to a transition from the initial state to the final state. From the effective Hamiltonian, it is apparent that the Stark shift terms are of the same order of the magnitude as the coupling term. So they cannot be ignored from the effective Hamiltonian.

Thus, we can find the general solution of the Schrödinger equation. The state vector of the system at an arbitrary moment t can be written as

$$|\psi(t)\rangle = \sum_{n,m} a_{n,m}(t)|f, n, m\rangle + b_{n+1,m-1}(t)|g, n+1, m-1\rangle\,. \tag{4}$$

Substituting Eq. (4) into the Schrödinger equation of interaction picture, we get the following set of differential equations:

$$\mathrm{i}\dot{a}_{n,m}(t) = \frac{g^2}{\Delta}[m a_{n,m}(t) + \sqrt{m(n+1)}\,b_{n+1,m-1}(t)]\,,$$
$$\mathrm{i}\dot{b}_{n+1,m-1}(t) = \frac{g^2}{\Delta}[\sqrt{m(n+1)}\,a_{n,m}(t) + (n+1)b_{n+1,m-1}(t)]\,. \tag{5}$$

Solving the above set of differential equations, we get the following results:

$$a_{n,m}(t) = \frac{1}{\sqrt{m(n+1)}}\Big\{C_2\frac{\mathrm{i}\Delta}{g^2}\frac{m}{m+n+1}\,\mathrm{e}^{-\mathrm{i}(g^2/\Delta)(m+n+1)t} - C_1(n+1)\Big\}\,,$$
$$b_{n+1,m-1}(t) = \frac{\mathrm{i}\Delta}{g^2(m+n+1)}C_2\,\mathrm{e}^{-\mathrm{i}(g^2/\Delta)(m+n+1)t} + C_1\,. \tag{6}$$

So the general solutions of the Schrödinger equation can be given as

$$|\psi(t)\rangle = \sum_{n,m}\frac{1}{\sqrt{m(n+1)}}\Big\{C_2\frac{\mathrm{i}\Delta}{g^2}\frac{m}{m+n+1}\,\mathrm{e}^{-\mathrm{i}(g^2/\Delta)(m+n+1)t} - C_1(n+1)\Big\}|f, n, m\rangle \times\Big[\frac{\mathrm{i}\Delta}{g^2(m+n+1)}C_2\,\mathrm{e}^{-\mathrm{i}(g^2/\Delta)(m+n+1)t} + C_1\Big]|\,g, n+1, m-1\rangle\,. \tag{7}$$

3 Interaction of One Atom with Cavity

Now, we let a Lambda-type atom, which is shown in Fig. 1, passes though the two-mode cavity. It is assumed that the two-mode cavity is in the state $|10\rangle$ initially and the atom arrives at $t = 0$ in the state $|g\rangle$ and leaves it at $t = t_1$. Under the interaction of the effective Hamiltonian, one can obtain the following equations, which display the evolution of the state,

$$|g\rangle|10\rangle \to \frac{1}{2}\Big[\Big(1+\cos\frac{2g^2}{\Delta}t_1 - \mathrm{i}\sin\frac{2g^2}{\Delta}t_1\Big)|g\rangle|10\rangle + \Big(-1+\cos\frac{2g^2}{\Delta}t_1 - \mathrm{i}\sin\frac{2g^2}{\Delta}t_1\Big)|f\rangle|01\rangle\Big]\,. \tag{8}$$

The probability that the three-level atom is found in the state $|g\rangle$ at $t=t_1$ can be obtained from Eq. (8),

$$w_g^{(1)}(t_1) = \frac{1}{2}\Big(1+\cos\frac{2g^2}{\Delta}t_1\Big)\,, \tag{9}$$

where the superscript "(1)" and the subscript "1" refer to the first atom.

It is known from the above equation that the probability of finding the atom in the state $|g\rangle$ depends on the coupled constant g, interaction time t_1 and the detuning of the cavity-mode from the atom transition Δ. When $t_1=(2k+1)\pi\Delta/2g^2$ (k is an arbitrary integral), the incident atom is transferred from the state $|g\rangle$ to the state $|f\rangle$ under the effect of the quantified cavity modes. The procedure is just like the Raman procedure.

Next, if the atom arrives at $t=0$ in the state $|f\rangle$ and the two-mode cavity is in the Fock state $|01\rangle$ initially, the wave function after the evolution can be given as

$$|f\rangle|01\rangle \to \frac{1}{2}\Big[\Big(\cos\frac{2g^2}{\Delta}t_1' - \mathrm{i}\sin\frac{2g^2}{\Delta}t_1' - 1\Big)|g\rangle|10\rangle + \Big(\cos\frac{2g^2}{\Delta}t_1' - \mathrm{i}\sin\frac{2g^2}{\Delta}t_1' + 1\Big)|f\rangle|01\rangle\Big]\,. \tag{10}$$

From the above equations, it is easy to calculate the probability that the atom is found in the state $|g\rangle$ at $t=t_1'$,

$$w{\prime}_g^{(1)}(t_1') = \frac{1}{2}\Big(1-\cos\frac{2g^2}{\Delta}t_1'\Big)\,. \tag{11}$$

In the same way, if the interaction time $t_1' = (2k'+1)\pi\Delta/2g^2$ (k' is an arbitrary integer), the probability of finding the three-level atom in the state $|g\rangle$ when it leaves the cavity is unity. Under the above condition, the atom is transferred from the state $|f\rangle$ to the state $|g\rangle$ with the interaction of the two-mode cavity. It is similar to the situation of the first initial condition.

4 Interaction of the Second Atom with Cavity

Now, let us talk about the first initial condition of the system.

The First Initial Condition

When the first atom leaves the cavity, we let the second identical atom pass though the two-mode cavity. Via the quantum field inside the cavity, the two atoms become entangled. Equation (3) can also be applied to the second atom when it enters the cavity and interacts with the field. If $|g\rangle_1|g\rangle_2|10\rangle$ is chosen as the initial state, the wave function in the interaction picture of the two atoms and the two field modes now has the form as

$$\begin{aligned}|\psi(t_1,t_2)\rangle = {}& \mathrm{e}^{-\mathrm{i}(g^2/\Delta)(t_1+t_2)}\cos\frac{g^2}{\Delta}t_1\cos\frac{g^2}{\Delta}t_2|g\rangle_1|g\rangle_2|10\rangle \\ & - \mathrm{i}\,\mathrm{e}^{-\mathrm{i}(g^2/\Delta)(t_1+t_2)}\cos\frac{g^2}{\Delta}t_1 \\ & \times\sin\frac{g^2}{\Delta}t_2|g\rangle_1|f\rangle_2|01\rangle \\ & - \mathrm{i}\,\mathrm{e}^{-\mathrm{i}(g^2/\Delta)t_1}\sin\frac{g^2}{\Delta}t_1|f\rangle_1|g\rangle_2|01\rangle\,. \end{aligned} \tag{12}$$

The subscript "j" ($j=1,2$) refers to the j-th atom. It is obvious that when $t_1=(8k+1)\pi\Delta/4g^2$ and $t_2=(4m+1)\pi\Delta/2g^2$ (k, m are arbitrary integers), equation (12) can be transformed as

$$|\psi(t_1,t_2)\rangle = \frac{\sqrt{2}}{2}\big(|g\rangle_1|f\rangle_2 + |f\rangle_1|g\rangle_2\big)|01\rangle\,. \tag{13}$$

Via the procedure, the maximally entangled atoms have been created. The phenomenon can be explained as follows. When the first three-level atom passes through the cavity, it exchanges the information with the two-mode cavity field. Similarly, when the second atom passes through the cavity, it stores some of the information in the cavity field, too. So the two atoms become entangled with each other via the cavity field. The entangled ensemble behaves as a collective entity, rather than as two individual particles. This atom-atom correlation displays nonclassical features although the two atoms follow two spatially separated paths.

5 Dynamic Behavior of Atom-Cavity System

It is well known that the degree of atomic population inversion is perhaps the simplest nontrivial physical quantity in the atom-field interaction problem. Its dynamical behavior in a photon number state has been found to be purely sinusoidal. In this section we shall discuss the atomic population inversion. This can be attained from the wave function given by Eq. (12). The time evolution of the average population inversion in the state $|f\rangle$ of the first atom is governed by[12]

$$\langle\sigma_z^{(1)}(t)\rangle = \langle(|f\rangle_1\langle f| - |g\rangle_1\langle g|)\rangle = -\cos 2\frac{g^2}{\Delta}t_1\,, \tag{14}$$

where the subscript "1" and superscript "1" refer to the first atom.

From the above equation one can easily find the time for the k-th transition process,

$$t_1(k) = \frac{(2k+1)\Delta\pi}{2g^2}\,, \tag{15}$$

where k is an arbitrary integer.

On the other hand, the average population in the state $|f\rangle$ of the second atom is expressed as

$$\langle\sigma_z^{(2)}(t)\rangle = \langle(|f\rangle_2\langle f| - |g\rangle_2\langle g|)\rangle = -\frac{1}{2}\Big(\cos 2\frac{g^2}{\Delta}t_2 - \cos 2\frac{g^2}{\Delta}t_1 + \cos 2\frac{g^2}{\Delta}t_2 \cos 2\frac{g^2}{\Delta}t_1 + 1\Big)\,. \quad (16)$$

If

$$\cos 2\frac{g^2}{\Delta}t_2 = \frac{-3 + (\cos 2g^2 t_1/\Delta)}{1 + (\cos 2g^2 t_1/\Delta)}\,, \quad \langle\sigma_z^{(2)}(t)\rangle = 1\,.$$

From the above equations, one can get

$$t_2 = \frac{\Delta}{2g^2}\arccos\Big[\frac{-3 + (\cos 2g^2 t_1/\Delta)}{1 + (\cos 2g^2 t_1/\Delta)}\Big]\,. \quad (17)$$

It is obvious that the time of the second atom is dependent on the interaction time of the first atom with the field. This phenomenon shows the correlation between the first atom and the second atom.

In Fig. 2 we have plotted the atomic population inversion of the second atom given in Eq. (16) as a function of scaled time t_1 and t_2.

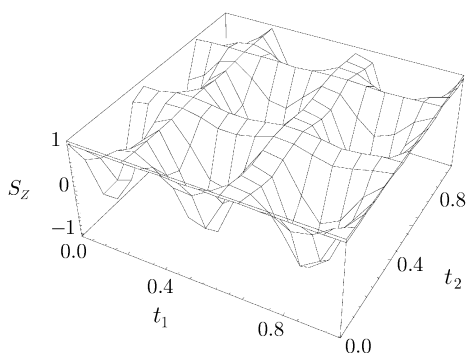

Fig. 2 The atomic inversion evolution for the second atom in the state $|g\rangle_1|g\rangle_2$ and the field in the state $|10\rangle$ initially with $\Delta = 10g$, $g/2\pi \approx 25$ MHz.[15]

Next, we turn our attention to study the behavior of the photon number against time related to the present system. In fact there is a photon number distribution, i.e., the probability of finding n photons in the mode 1 or mode 2 at an arbitrary time. According to Eq. (12), the time evolution of the average photon number for the first mode of the field is governed by

$$\langle N_1(t_1,t_2)\rangle = \langle a_1^\dagger(t)a_1(t)\rangle = \cos^2\frac{g^2}{\Delta}t_1\cos^2\frac{g^2}{\Delta}t_2\,, \quad (18)$$

while the average excitation number of the second mode $\langle N_2\rangle$ can be given as

$$\langle N_2(t_1,t_2)\rangle = \langle a_2^\dagger(t)a_2(t)\rangle = \cos^2\frac{g^2}{\Delta}t_1\sin^2\frac{g^2}{\Delta}t_2 + \sin^2\frac{g^2}{\Delta}t_1\,. \quad (19)$$

It is apparent that the average excitation numbers $\langle N_1\rangle$ and $\langle N_2\rangle$ are both functions of interaction time t_1 and t_2 from the above equation. The time evolutions of the average photon number of the two modes of the cavity field have been plotted in Fig. 3. Comparing Fig. 3(a) and Fig. 3(b), one can easily note that the time evolutions of the photon number of the two modes have the same period. When the photon number of the first mode come to the maximum value, that of the second mode reduces to the minimum value.

The dipole squeezing properties of the atoms are central to the topic of quantum optics. For this reason we discuss the squeezing phenomenon of the quantum fluctuations in atomic dipole variables related to the present system. In order to investigate the squeezing properties of the atomic dipole variables, we follow the standard procedure to define the slowly varying operators.[13]

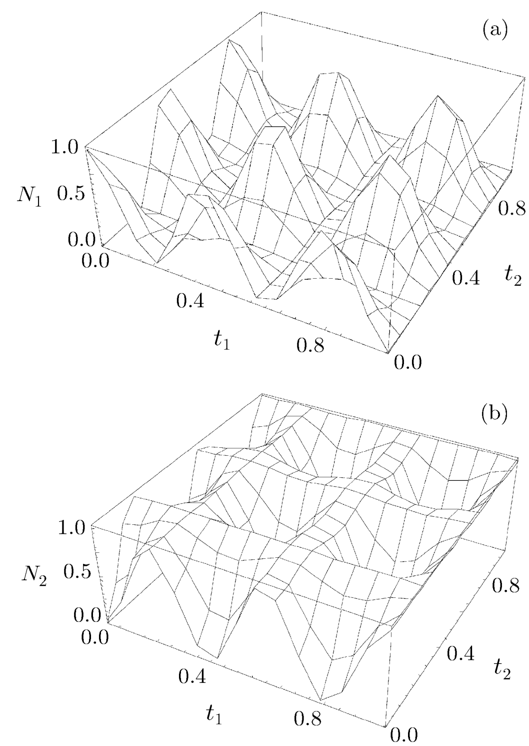

Fig. 3 (a) The average photon number evolution for the first mode of the cavity field in the state $|g\rangle_1|g\rangle_2$ and the field in the state $|10\rangle$ initially with $\Delta = 10g$, $g/2\pi \approx 25$ MHz;[15] (b) The average photon number for the second mode of the cavity field under the same initial condition as above.

One can give a pair of conjugate dipole operators of the first atom and the second atom,

$$\hat{S}_1 = \frac{1}{2}(|f\rangle_1\langle g| + |f\rangle_2\langle g| + |g\rangle_1\langle f| + |g\rangle_2\langle f|)\,,$$

$$\hat{S}_2 = \frac{1}{2\mathrm{i}}(|f\rangle_1\langle g| + |f\rangle_2\langle g| - |g\rangle_1\langle f| - |g\rangle_2\langle f|)\,. \quad (20)$$

And they satisfy $\langle(\Delta S_1)^2\rangle\langle(\Delta S_2)^2\rangle \geq \frac{1}{16}\langle S_Z\rangle^2$, which is resulted from the Heisenberg uncertainty relation, where $(\Delta S_j)^2 = \langle S_j^2\rangle - \langle S_j\rangle^2$ is the quantum variance in S_j ($j =$

1, 2). Following the above equations, the variance of the S_j ($j = 1, 2$) can be given as

$$\langle(\Delta\hat{S}_1)^2\rangle = \langle(\Delta\hat{S}_2)^2\rangle = \sin\frac{g^2}{\Delta}t_1\cos\frac{g^2}{\Delta}t_1\sin\frac{g^2}{\Delta}t_2\cos\frac{g^2}{\Delta}t_2 + \frac{1}{4}\Big(\cos^2\frac{g^2}{\Delta}t_1\cos^2\frac{g^2}{\Delta}t_2+1\Big)\,. \qquad (21)$$

It can be easily attained that $\langle(\Delta\hat{S}_1)^2\rangle\langle(\Delta\hat{S}_2)^2\rangle \geq \frac{1}{16}(\sigma_z^{(1)}+\sigma_z^{(2)})^2$, where $\sigma_z^{(1)}$ and $\sigma_z^{(2)}$ have been presented in Eqs. (14) and (16). The superscript "1" and "2" represent the first atom and the second atom, respectively. It shows that the atomic dipole squeezing has been generated.

According to the above equations, the evolution of the variance of the conjugate dipole operators can be obtained by the technique of numerical analysis.

The variance of the S_1 or S_2 have been shown in Fig. 4.

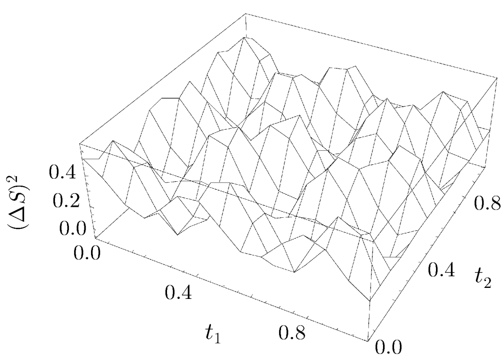

Fig. 4 The variance evolution for the S_1 or S_2 for the atom ensemble in the state $|g\rangle_1|g\rangle_2$ and the field in the state $|10\rangle$ initially with $\Delta = 10g$, $g/2\pi \approx 25$ MHz.[15]

Next, let us turn to the topic about the sub-Poissonian statistics. This problem is one of the nonclassical phenomena of the quantized electromagnetic radiation field. A state (of a single mode for convenience) which displays sub-Poisson statistics is characterized by the fact that the variance of the photon number $\langle(\Delta\hat{n}_i(t))^2\rangle$ is less than the average photon number $\langle\hat{a}_i^\dagger(t)\hat{a}_i(t)\rangle = \langle\hat{n}_i(t)\rangle$. This can be expressed by means of the normalized second-order correlation function as[14]

$$g_i^{(2)}(t) = \frac{\langle\hat{a}_i^{\dagger 2}(t)\hat{a}_i^2(t)\rangle}{\langle\hat{a}_i^\dagger(t)\hat{a}_i(t)\rangle^2} = 1 + \frac{\langle(\Delta\hat{n}_i(t))^2\rangle - \langle\hat{a}_i^\dagger(t)\hat{a}_i(t)\rangle}{\langle\hat{a}_i^\dagger(t)\hat{a}_i(t)\rangle^2}\,, \qquad (22)$$

where the subscript i is related to the i-th mode. Then it holds that $g_i^{(2)}(t) < 1$ for sub-Poissonian distribution, $g_i^{(2)} > 1$ for super-Poissonian distribution, $g_i^{(2)}(t) = 2$ for the thermal light when $g_i^{(2)} = 1$ Poisson distribution of photons occurs. Now we investigate the statistics of the correlation function for the above initial condition. In this case, it becomes $\langle(\Delta\hat{n}_i(t))^2\rangle = \langle\hat{a}_i^\dagger(t)\hat{a}_i(t)\rangle$. So one can find $g_i^{(2)} = 1$, which represents the occurring of the Poisson distribution of photons.

The Second Initial Condition

Now, let us consider the situation that the first atom enters the cavity in the state $|f\rangle$ and the second atom enters the cavity in the state $|g\rangle$ initially. The system is in the state $|f\rangle_1|g\rangle_2|01\rangle$ at this time. Taking the advantage of Eq. (7) and the initial condition, the wave function of the two atoms and the two-mode field system can be reached as

$$|\psi'(t_1,t_2)\rangle = -\,\mathrm{i}\sin\frac{g^2}{\Delta}t_1'\cos\frac{g^2}{\Delta}t_2'\exp\Big[-\mathrm{i}\frac{g^2}{\Delta}(t_1'+t_2')\Big]|g\rangle_1|g\rangle_2|10\rangle - \sin\frac{g^2}{\Delta}t_1'\sin\frac{g^2}{\Delta}t_2' \times\exp\Big[-\mathrm{i}\frac{g^2}{\Delta}(t_1'+t_2')\Big]|g\rangle_1|f\rangle_2|01\rangle + \cos\frac{g^2}{\Delta}t_1'\exp\Big(-\mathrm{i}\frac{g^2}{\Delta}t_1'\Big)|f\rangle_1|g\rangle_2|01\rangle\,. \qquad (23)$$

Similarly, following the above wave function, it can be seen that when $t_1' = (8k+1)\pi\Delta/4g^2$ and $t_2' = (4m+1)\pi\Delta/2g^2$ (k, m are arbitrary integers), the wave function of the system reduces to the state

$$|\psi'(t_1,t_2)\rangle = \frac{\sqrt{2}}{2}(|g\rangle_1|f\rangle_2 + |f\rangle_1|g\rangle_2)|01\rangle\,. \qquad (24)$$

Thus, the max entangled state has been generated.

In this case, the atomic population inversion of the first atom can be achieved from Eq. (23). It can be given as

$$\langle\sigma_z'^{(1)}\rangle = \langle(|f\rangle_1\langle f| - |g\rangle_1\langle g|)\rangle = \cos 2\frac{g^2}{\Delta}t_1'\,. \qquad (25)$$

It is apparent that when $t_1'(k') = k'\pi\Delta/g^2$, $\langle\sigma_z'^{(1)}\rangle = 1$. ($k'$ is an arbitrary integer.)

The population inversion of the second atom is given as

$$\langle\sigma_z'^{(2)}\rangle = -\sin^2\frac{g^2}{\Delta}t_1'\cos 2\frac{g^2}{\Delta}t_2' - \cos^2\frac{g^2}{\Delta}t_1'\,. \qquad (26)$$

And when

$$\cos 2\frac{g^2}{\Delta}t_2' = -\frac{1+\cos^2 g^2/\Delta t_1'}{\sin^2 g^2/\Delta t_1'}\,, \quad \langle\sigma_z'^{(2)}\rangle = 1\,.$$

The time of the second atom depends on the interaction time of the first atom and the two-mode cavity field.

In Fig. 5, we plot the time evolution of the atomic inversion for the second atom under this initial condition.

Now, the excited occupation of the first mode and the second mode of the field is easily acquired,

$$\langle N_1'\rangle = \sin^2\frac{g^2}{\Delta}t_1'\cos^2\frac{g^2}{\Delta}t_2'\,, \qquad (27)$$

$$\langle N_2'\rangle = \sin^2\frac{g^2}{\Delta}t_1'\sin^2\frac{g^2}{\Delta}t_2' + \cos^2\frac{g^2}{\Delta}t_1'\,. \qquad (28)$$

The time evolution of the excited occupation of the two modes can be shown in Fig. 6. Comparing Fig. 6(a) and Fig. 6(b), we can get the conclusion that the two time evolution functions have the same period. Like the situation of the initial condition $|g\rangle_1|g\rangle_2|10\rangle$, when one of the values of the functions comes to the maximum value, the other comes down to the minimum value.

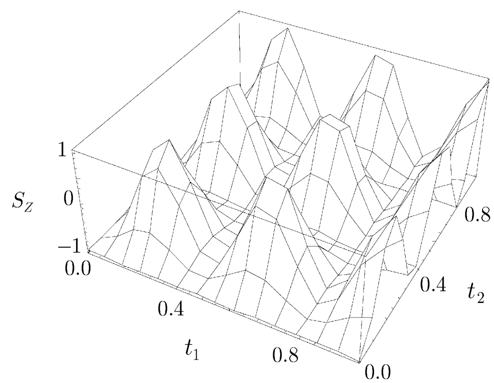

Fig. 5 The atomic inversion evolution for the second atom in the state $|f\rangle_1|g\rangle_2$ and the field in the state $|01\rangle$ initially with $\Delta = 10g$, $g/2\pi \approx 25$ MHz.[15]

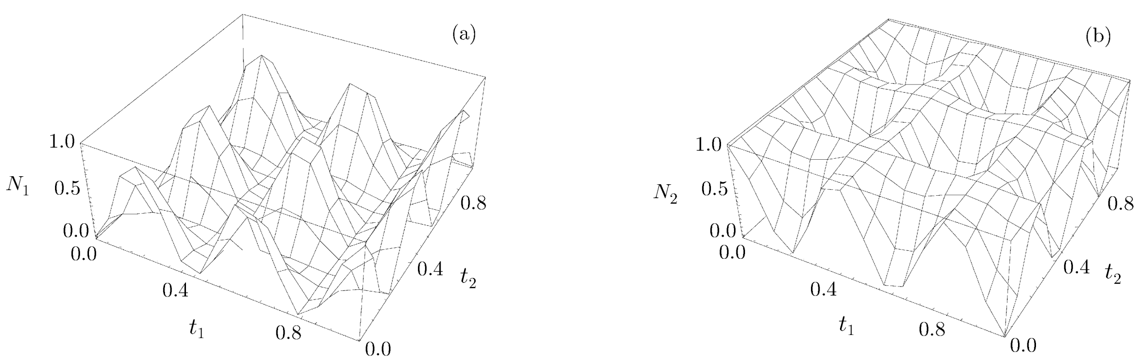

Fig. 6 (a) The photon number evolution for the first mode in the state $|f\rangle_1|g\rangle_2$ and the field in the state $|01\rangle$ initially with $\Delta = 10g$, $g/2\pi \approx 25$ MHz;[15] (b) The time evolution of the photon number for the second mode under the same initial condition.

Next, the variance of the conjugate dipole operators S_1 and S_2 can be given as

$$\langle(\Delta S_1')^2\rangle = \langle(\Delta S_2')^2\rangle = \frac{1}{2} - \sin\frac{g^2}{\Delta}t_1' \cos\frac{g^2}{\Delta}t_1' \sin\frac{g^2}{\Delta}t_2' \cos\frac{g^2}{\Delta}t_2' \,. \tag{29}$$

They can be displayed in Fig. 7.

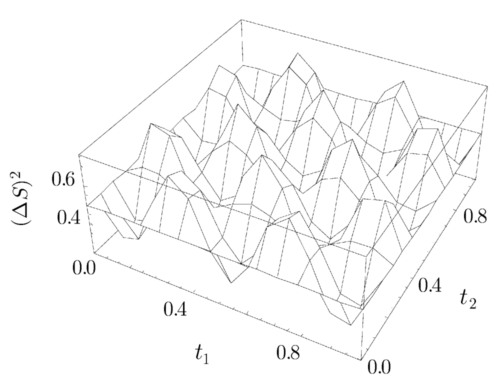

Fig. 7 The variance evolution of the S_1 or S_2 for the atom ensemble in the state $|f\rangle_1|g\rangle_2$ and the field in the state $|01\rangle$ initially with $\Delta = 10g$, $g/2\pi \approx 25$ MHz.[15]

Now, let us talk about the phenomenon of the sub-Poissonian distribution. Being similar to the situation of the first initial condition, $g_i^{(2)}$ still has the value unity under the initial condition $|f\rangle_1|g\rangle_2|01\rangle$. That is to say, the Poissonian distribution appears in this case.

The Third Initial Condition

In this condition, it is supposed that the first atom goes into the cavity in the state $|f\rangle$ and the second atom goes into the cavity in the state $|f\rangle$ initially. The two modes of the cavity field are in the coherent state $|\alpha\beta\rangle$ at the time $t = 0$.

Following Eq. (7) and using this initial condition, the time evolution of the state of the system can be achieved as

$$
\begin{aligned}
|\psi''(t_1'', t_2'')\rangle = {} & \sum_{n,m} \mathrm{e}^{-(|\alpha|^2+|\beta|^2)/2} \frac{\alpha^n\beta^m}{(m+n+1)^2(n!m!)^{1/2}} \Big\{ \big(m\,\mathrm{e}^{-\mathrm{i}(g^2/\Delta)(m+n+1)t_1''} + n + 1 \big) \\
& \times \big(m\,\mathrm{e}^{-\mathrm{i}(g^2/\Delta)(m+n+1)t_2''} + n + 1 \big) |f\rangle_1 |f\rangle_2 |n, m\rangle \\
& + \sqrt{m(n+1)} \big(m\,\mathrm{e}^{-\mathrm{i}(g^2/\Delta)(m+n+1)t_1''} + n + 1 \big) \big(\mathrm{e}^{-\mathrm{i}(g^2/\Delta)(m+n+1)t_2''} - 1 \big) |f\rangle_1 |g\rangle_2 |n+1, m-1\rangle \\
& + \sqrt{m(n+1)} \big(\mathrm{e}^{-\mathrm{i}(g^2/\Delta)(m+n+1)t_1''} - 1 \big) \big[(m-1)\,\mathrm{e}^{-\mathrm{i}(g^2/\Delta)(m+n+1)t_2''} + n + 2 \big] |g\rangle_1 |f\rangle_2 |n+1, m-1\rangle \\
& + \sqrt{m(n+1)(m-1)(n+2)} \big(\mathrm{e}^{-\mathrm{i}(g^2/\Delta)(m+n+1)t_1''} - 1 \big) \big(\mathrm{e}^{-\mathrm{i}(g^2/\Delta)(m+n+1)t_2''} - 1 \big) \\
& \times |g\rangle_1 |g\rangle_2 |n+2, m-2\rangle \Big\}\,, \qquad (30)
\end{aligned}
$$

where t_1'' and t_2'' represent the interaction time of the two atoms with the cavity field, respectively.

According to the above equation, we can attain the atomic population inversion of the first atom as

$$
\langle \sigma_z''^{(1)}(t_1'') \rangle = \sum_{n,m} \frac{|\alpha|^{2n}|\beta|^{2m}}{n!m!(m+n+1)^2} \mathrm{e}^{-(|\alpha|^2+|\beta|^2)} \Big[(m-n-1)^2 + 4m(n+1)\cos(m+n+1)\mathrm{i}\frac{g^2}{\Delta}t_1'' \Big]\,. \qquad (31)
$$

Having obtained the exact expression of the atomic inversion, it is apparent that if

$$
\cos(m+n+1)\mathrm{i}\frac{g^2}{\Delta}t''
$$

is taken as the unity, $\langle \sigma_z''^{(1)}(t_1'') \rangle = 1$. That is to say,

$$
t_1'' = \frac{2k''\Delta\pi}{\mathrm{i}g^2(m+n+1)} \quad (k'' \text{ is an arbitrary integer}). \qquad (32)
$$

Thus in the large-detuning limit, the times for the first atomic inversion to achieve the maximum value under this initial condition are independent of α, β. Strictly speaking, only the time t_1'' in (above) Eq. (32) for which k'' is an even number corresponds to the Rabi oscillations,

$$
\begin{aligned}
\langle \sigma_z''^{(2)} \rangle = {} & \sum_{n,m} \frac{|\alpha|^{2n}|\beta|^{2m}}{n!m!(m+n+1)^4} \mathrm{e}^{-(|\alpha|^2+|\beta|^2)} \Big\{ \Big[(m-n-3)^2 + 4(m-1)(n+2)\cos\mathrm{i}\frac{g^2}{\Delta}t_2'' \Big] m(n+1) \\
& \times 4\sin^2\mathrm{i}\frac{g^2}{2\Delta}(m+n+1)t_1'' + \Big[\Big(m^2 + (n+1)^2 + 2m(n+1)\cos\mathrm{i}\frac{g^2}{\Delta}(m+n+1)t_1'' \Big) \Big] \\
& \times \Big[(m-n-1)^2 + 4m(n+1)\cos\mathrm{i}\frac{g^2}{\Delta}(m+n+1)t_2'' \Big] \Big\}\,. \qquad (33)
\end{aligned}
$$

The above equation displays the time evolution of the atomic population inversion of the second atom. In the examination, we found that different values of α, β and the detuning parameter Δ will bring different results.

Similarly, the average photon number of the two modes can be given as

$$
\begin{aligned}
\langle N_1''(t_1'', t_2'') \rangle = {} & \sum_{n,m} \frac{|\alpha|^{2n}|\beta|^{2m}}{n!m!(m+n+1)^4} \mathrm{e}^{-(|\alpha|^2+|\beta|^2)} \Big\{ \Big[m^2 + (n+1)^2 + 2m(n+1)\cos(m+n+1)\mathrm{i}\frac{g^2}{\Delta}t_1'' \Big] \\
& \times \Big[nm^2 + (2m+n)(n+1)^2 - 2m(n+1)\cos(m+n+1)\mathrm{i}\frac{g^2}{\Delta}t_2'' \Big] + 4\sin^2\mathrm{i}\frac{g^2}{2\Delta}(m+n+1)t_1'' \\
& \times \Big\{ m(n+1)^2[(n+2)^2 + (m-1)^2] + 2m(m-1)(n+1)(n+2) \\
& \times \Big[n + 2 - \cos(m+n+1)\mathrm{i}\frac{g^2}{\Delta}t_2'' \Big] \Big\} \Big\}\,, \qquad (34)
\end{aligned}
$$

$$
\langle N_2''(t_1'', t_2'') \rangle = \sum_{n,m} \frac{|\alpha|^{2n}|\beta|^{2m}}{n!m!(m+n+1)^4} \mathrm{e}^{-(|\alpha|^2+|\beta|^2)} \Big\{ \Big[m^2 + (n+1)^2 + 2m(n+1)\cos(m+n+1)\mathrm{i}\frac{g^2}{\Delta}t_1'' \Big]
$$

$$\times \Big[m^3 + m(n+1)^2 + 2m(m-1)(n+1) + 2m(n+1)\cos(m+n+1)\,\mathrm{i}\,\frac{g^2}{\Delta}t_2''\Big]$$
$$+ 4m(m-1)(n+1)\sin^2(m+n+1)\,\mathrm{i}\,\frac{g^2}{2\Delta}t_1''\Big[(m-1)^2 + (n+2)^2 + 2(m-1)(n+2)$$
$$\times \cos(m+n+1)\,\mathrm{i}\,\frac{g^2}{\Delta}t_2'' + (m-2)(n+2)4\sin^2\mathrm{i}\,\frac{g^2}{2\Delta}(m+n+1)t_2''\Big]\Big\}\,. \tag{35}$$

Thus, the photon distribution statistics of the atom-cavity system in the coherent state at $t = 0$ have been achieved. From the above two equations, it can be easily found that the average photon number of the two modes are the functions of α, β, the coupled constant g and the detuning parameter Δ.

Subsequently, we investigate the time evolution of the variance of the conjugate dipole operators $\hat{S}_1$, $\hat{S}_2$. They can be delivered as

$$\begin{aligned}\langle(\Delta\hat{S}_1'')^2\rangle\langle(\Delta\hat{S}_2'')^2\rangle = \sum_{n,m}\frac{|\alpha|^{2n}|\beta|^{2m}}{n!m!(m+n+1)^4}\,\mathrm{e}^{-(|\alpha|^2+|\beta|^2)}\frac{1}{(m+n+1)^4}\frac{1}{2}\big\{\big(1-\,\mathrm{e}^{-(m+n+1)\mathrm{i}(g^2/\Delta)t_1''}\big)\\
\times\big(1-\mathrm{e}^{(m+n+1)\mathrm{i}(g^2/\Delta)t_2''}\big)\big(m\,\mathrm{e}^{(m+n+1)\mathrm{i}(g^2/\Delta)t_1''}+n+1\big)\\
\times\big[(m-1)\,\mathrm{e}^{-(m+n+1)\mathrm{i}(g^2/\Delta)t_2''}+n+2\big]m(n+1)\\
+\big(1-\mathrm{e}^{(m+n+1)\mathrm{i}(g^2/\Delta)t_1''}\big)\big(1-\,\mathrm{e}^{(m+n+1)\mathrm{i}(g^2/\Delta)t_2''}\big)\big(m\,\mathrm{e}^{-(m+n+1)\mathrm{i}(g^2/\Delta)t_1''}+n+1\big)\\
\times\big[(m-1)\,\mathrm{e}^{(m+n+1)\mathrm{i}(g^2/\Delta)t_2''}+n+2\big]m(n+1)+1\big\}\,.\end{aligned} \tag{36}$$

As the same as the former two initial conditions, we get

$$\langle(\Delta\hat{S}_1'')\rangle\langle(\Delta\hat{S}_1'')\rangle \geq \frac{1}{4}(\sigma_z''^{(1)} + \sigma_z''^{(2)})\,,$$

where $\sigma_z''^{(1)}$ and $\sigma_z''^{(2)}$ have been demonstrated in Eqs. (31) and Eq. (33), respectively.

On the other hand, the second-order correlation function of the system under the initial condition can be attained,

$$g_1^{(2)}(t_1'', t_2'') = \frac{\langle N_1''^2\rangle - \langle N_1''\rangle}{\langle N_1''\rangle^2}\,, \tag{37}$$

where the subscript "1" represent the first mode and

$$\begin{aligned}\langle N_1''^{(2)}\rangle = \sum_{n,m}\frac{|\alpha|^{2n}|\beta|^{2m}}{n!m!(m+n+1)^4}\,\mathrm{e}^{-(|\alpha|^2+|\beta|^2)}\Big\{\Big[m^2+(n+1)^2+2m(n+1)\cos(m+n+1)\,\mathrm{i}\,\frac{g^2}{\Delta}t_1''\Big]\\
\times\Big[m^2n^2+n^2(n+1)^2+2m(n+1)^3-2m(n+1)(2n+1)\cos(m+n+1)\,\mathrm{i}\,\frac{g^2}{\Delta}t''\Big]\\
+m(n+1)4\sin^2(m+n+1)\,\mathrm{i}\,\frac{g^2}{2\Delta}t_1''\big[(m-1)^2(n+1)^2+(n+1)^2(n+2)^2\\
+(m-1)(n+2)^3-2(m-1)(n+2)(2n+3)\cos(m+n+1)\,\mathrm{i}\,\frac{g^2}{\Delta}t_2''\Big]\Big\}\,,\end{aligned} \tag{38}$$

$\langle N_1''\rangle$ has been given in Eq. (34).

The second-order correlation functions of the mode two can be delivered as

$$g_2^{(2)}(t_1'', t_2'') = \frac{\langle N_2''^2\rangle - \langle N_2''\rangle}{\langle N_2''\rangle^2}\,, \tag{39}$$

where

$$\begin{aligned}\langle N_2''^2\rangle = \sum_{n,m}\frac{|\alpha|^{2n}|\beta|^{2m}}{n!m!(m+n+1)^4}\,\mathrm{e}^{-(|\alpha|^2+|\beta|^2)}\Big\{\Big[m^2+(n+1)^2+2m(n+1)\cos(m+n+1)\,\mathrm{i}\,\frac{g^2}{\Delta}t_1''\Big]\\
\times\Big[m^4+m^2(n+1)^2+2m(n+1)(m-1)^2+2m(2m-1)(n+1)\cos(m+n+1)\,\mathrm{i}\,\frac{g^2}{\Delta}t_2''\Big]\\
+4m(m-1)(n+1)\sin^2(m+n+1)\,\mathrm{i}\,\frac{g^2}{2\Delta}t_1''\Big[(m-1)^3+2(n+2)(m-2)^2+(m-1)(n+2)^2\\
+2(n+2)(2m-3)\cos(m+n+1)\,\mathrm{i}\,\frac{g^2}{\Delta}t''\Big]\Big\}\,.\end{aligned} \tag{40}$$

We have achieved the expression of $\langle N_2''\rangle$ in Eq. (35).

Having the expression of the second-order correlation functions of the two modes, we can find that the phenomenon of the sub-Poissonian is in relation to α, β, etc.

6 Conclusion

In this paper, we consider a system consisting of λ-type three-level atoms and two-mode cavity field. Under the adiabatical approximation and the large detuning condition, the effective Hamiltonian of the system can be acquired. Following the effective Hamiltonian, the wave functions of the system under the different initial conditions can be obtained. When we let another atom pass through the cavity, the entangled atoms can be generated. When the interaction time is choose to the right time, the maximally entangled atoms can be generated, which is an essential resource for quantum computation. The highly entangled state plays a key role in an efficient realization of quantum information processing including quantum teleportation, cryptography, dense coding, and computation. In these protocols, the maximally entangled states are required. On the other hand, we concentrate our attention to the dynamic behavior of the atom-cavity system under the three different initial conditions. Following the wave functions under the three kinds of initial conditions, the population inversion of the atoms, the photon number distribution of the cavity modes, the atomic dipole squeezing behavior and the second-order correlation function have been demonstrated in detail. They all built up on the coupling parameter g and the detuning parameter Δ.

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