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The Relativistic Covariance of the Fermion Green Function and Minimal Quantization of Electrodynamics*

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Abstract *This paper is devoted to the one-loop calculation of the fermion Green function in QED within the framework of the minimal quantization method, based on an explicit solution of the constraint equations and the gauge-invariance principle. The relativistic invariant expression for the fermion Green function with correct analytical properties is obtained.*

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1 Introduction

Quantum electrodynamics (QED) is a relativistic quantum field theory, which is characterized by the U(1) gauge invariance and the related vanishing photon mass. QED has many different formulations, which are obtained by choosing different gauge conditions, all leading to identical physical predictions. The success of QED in the explanation of a wide range of physical phenomena (in particular, the anomalous moment of the electron and the Lamb shift) has made it the most striking achievement of relativistic quantum theory. QED has been established earlier than other field theories and was the prototype for them.[1−5] In spite of its success, there still remains in the different formulations of QED the problem of determining the fermion wavefunction renormalization (or the residue of the one-fermion Green's function $R = \lim_{\hat{p}\to m_R}(\hat{p} - m_R)G_R(p)$, where $G_R(p)$ is the renormalized Green's function). It was shown by the author in Ref. [6] that the residue R of the one-fermion Green's function has not be solved, after analyzing all standard proofs of gauge invariance. For instance, in the usual relativistic covariant gauge one supplies the Gauss equation with the gauge condition and all components are quantized on an equal footing (Hereafter we call such approach a covariant quantization method where the problem of the gauge choice arises). The introduction of the superfluous longitudinal variables[3] changes the singularity of the electron Green function,[2] $G(p) \sim (p^2 - m^2)^b$, $b = -1 + (\alpha/2\pi)(3 - d)$, where $\alpha = e^2/4\pi$. In particular, for the Landau gauge ($d = 0$) and the gauge corresponding to ($d = 1$) instead of the usual pole, the branch appears so that the residue of the Green function R is equal to zero. Therefore, to reconstruct physical analytical properties, it is necessary to choose a nonsingular asymptotical interaction involving longitudinal components. In relativistic and nonrelativistic cases, one cannot treat all components on an equal footing. In this sense, the dependence of the Green function on the choice of gauge is an inevitable defect of quantization. In the nonrelativistic Coulomb gauge, the residue of the Green function is given by $R = 1$ in the rest frame $p^\mu = (p_0, \vec{p})$ and $\vec{p} = 0$, whereas in a uniformly moving reference frame the quantity R becomes velocity-dependent and in general loses its meaning because of infrared divergences.[7−10] Many attempts have been made to solve this old problem, but the same kind difficulty exists in both nonrelativistic[9] and relativistic gauges,[3] where the Green function exhibits a cut in place of a pole, and the quantity R can be equal to zero or to infinity, depending on the gauges. There existed some opinion that fermion Green functions to a certain extent are nonphysical quantities because physical quantities must be gauge-independent and their analytical properties do not reflect the gauge-invariant content of a gauge theory.

This problem is of appreciable interest for investigating the soluble models, and is also necessary for logical completeness of quantum electrodynamics.[6,8] Furthermore, the problem of recovering the relativistic invariance of Coulomb gauge becomes of practical importance for QCD, where the "Coulomb" version of confinement is used as a basis for the violation of chiral symmetry.[11,12]

In the present paper we attempt to solve the previously mentioned problem in the framework of the "minimal" canonical quantization method of gauge theories developed systematically by the author and collaborators in

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Ref. [13]. The approach is based on a quantization of physical variables obtained by means of an explicit solution of the constraint equations at the classical level and of the gauge invariance principle.

The paper is organized as follows. In Sec. 2 we briefly describe the minimal quantization method[13] for QED. It is shown that this quantization scheme, based on the explicit solution of the Gauss equation and on the gauge-invariant Belinfante tensor, does not need a gauge condition as an initial supposition, and that the Lorentz transformations of classical and quantum fields coincide at the operator level. These transformations contain an additional gauge rotation as first remarked by Pauli and Heisenberg.[14] In Sec. 3, at the level of Feynman diagrams, this additional gauge transformation leads to an extra set of diagrams in perturbation theory which provide the correct relativistic transformation properties of the observables such as the residues of the Green function. Section 4 is devoted to our conclusions. We use here the conventions $g_{\mu\nu} = (1, -1, -1, -1)$ and $\hbar = c = 1$.

2 Minimal Quantization Method of Electrodynamics

Following the quantization method for gauge theories given in Ref. [13], we consider the interaction between the electromagnetic field and electron-positron field.

The Lagrangian and Belinfante energy momentum tensor of the system can be chosen in according with the gauge invariance principle in the following forms,

$$\begin{aligned} L(x) &= -\frac{1}{4}F^2_{\mu\nu} + \bar{\Psi}[\gamma_\mu(\mathrm{i}\partial_\mu - eA_\mu) - m]\Psi\,, \\ S &= \int \mathrm{d}x L(x)\,, \end{aligned} \tag{1}$$

$$\begin{aligned} T^B_{\mu\nu} &= F_{\mu\lambda}F_{\lambda\nu} + \bar{\Psi}(\mathrm{i}\partial_\mu - eA_\mu)\Psi \\ &\quad - g_{\mu\nu}L + \frac{\mathrm{i}}{4}\partial_\nu(\bar{\Psi}\Gamma_{\lambda\mu\nu}\Psi)\,, \\ \Gamma_{\lambda\mu\nu} &= \frac{1}{2}[\gamma_\lambda, \gamma_\mu]\gamma_\nu + g_{\nu\mu}\gamma_\lambda - g_{\lambda\nu}\gamma_\mu\,, \end{aligned} \tag{2}$$

where the spinor field Ψ describes the fermion, A_μ the electromagnetic field. The Lagrangian (1) and Belinfante tensor (2) are invariant under the gauge transformations

$$\begin{aligned} \hat{A}^g_\mu &= g(\hat{A}_\mu + \partial_\mu)g^{-1}\,, \quad \hat{A}_\mu = \mathrm{i}eA_\mu\,, \\ \Psi^g &= g\Psi\,, \qquad\qquad g = \exp(\mathrm{i}e\lambda(\vec{x}, t)) \end{aligned} \tag{3}$$

for arbitrary function $\lambda(\vec{x}, t)$.

To construct the Hamiltonian of the theory, one should explicitly separate out the true dynamical physical variables. Lagrangian (1) is degenerate, namely, it does not contain the time derivative of the field A_0. As a result, the corresponding canonical momentum of A_0 is identically equal to zero. Here the variable A_0 is not a true dynamical physical variable and variation of the action with respect to A_0 leads to a constraint equation (the Gauss equation). Therefore, for quantization of the Lagrangian (1) two possibilities exist: either to use a modified Dirac canonical formalism[14–20] or to eliminate the nonphysical variable A_0 prior to the quantization by explicit solution of the constraint equation. We shall adhere to the second possibility, i.e., use the Gauss equation

$$\frac{\partial S}{\partial A_0} = 0 \Rightarrow \partial_i^2 A_0 = \partial_i\partial_0 A_i + j_0 \tag{4}$$

as constraint equation to express A_0 in terms of the physical dynamical variables

$$A_0 = \frac{1}{\vec{\partial}^2}(\partial_i\partial_0 A_i + j_0)\,, \tag{5}$$

where $1/\vec{\partial}^2$ is an integral operator represented through the corresponding Green function. The term $(1/\vec{\partial}^2)j_0$ in Eq. (5) can be written as

$$\frac{1}{\vec{\partial}^2}j_0(\vec{x}, t) = \frac{1}{4\pi}\int \mathrm{d}^3y \frac{1}{|\vec{y} - \vec{x}|}j_0(\vec{y}, t)\,, \tag{6}$$

and describes the Coulomb field of the instantaneous charge distribution $j_0(\vec{y}, t) = e\Psi^+(\vec{y}, t)\cdot\Psi(\vec{y}, t)$.

The substitution of Eq. (5) into Eq. (1) gives the following expressions for the Lagrangian,

$$\begin{aligned} L(x) &= \frac{1}{2}F^2_{0i}[A^T] - \frac{1}{4}F^2_{ij} - j^T_i A^T_i \\ &\quad + j^T_0 \frac{1}{\vec{\partial}^2} j^T_0 + \bar{\Psi}^T[\mathrm{i}\gamma_\mu\partial_\mu - m]\Psi^T\,, \end{aligned} \tag{7}$$

$$F_{0i}[A^T] = \dot{A}^T_i - \partial_i A^T_0\,, \quad A^T_0 = \frac{1}{\vec{\partial}^2}j^T_0\,, \tag{8}$$

where the following notations are introduced,

$$\begin{aligned} \hat{A}^T_i[A] &= \Big(\delta_{ij} - \partial_i\frac{1}{\vec{\partial}^2}\partial_i\Big)A_j \\ &= \delta^T_{ij}A_j = v^{-1}[A](\hat{A}_i + \partial_i)v[A]\,, \end{aligned} \tag{9}$$

$$\Psi^T[A, \Psi] = v[A]\Psi\,, \tag{10}$$

$$\delta^T_{ij} = \Big(\delta_{ij} - \partial_i\frac{1}{\vec{\partial}^2}\partial_j\Big),$$

$$v[A] = \exp\Big\{\int^t \mathrm{d}t'\frac{1}{\vec{\partial}^2}\partial_i\partial_{0'}\hat{A}_i\Big\} = \exp\Big\{\frac{1}{\vec{\partial}^2}\partial_i\hat{A}_i\Big\}. \tag{11}$$

Using the Belinfante tensor (2) expressed in terms of Eqs (9) $\sim$ (11), we obtain the following expressions for the Hamiltonian, momentum and Lorentz boots,

$$\begin{aligned} H &= \int \mathrm{d}^3x T^B_{00} \\ &= \int \mathrm{d}^3x\Big[\frac{1}{2}F^2_{0i} + \frac{1}{4}F^2_{ij} + \bar{\Psi}^T[\mathrm{i}\gamma_\mu\partial_\mu - m]\Psi^T\Big], \end{aligned} \tag{12}$$

$$\begin{aligned} P_k &= \int \mathrm{d}^3x T^B_{0k} \\ &= \int \mathrm{d}^3x\Big[F_{0i}F_{ki} + \Psi^{+T}_i + \frac{\mathrm{i}}{4}\partial_i(\Psi^{+T}[\gamma_k, \gamma_i]\Psi^T)\Big], \end{aligned} \tag{13}$$

$$M_{0k} = x_k H - tP_k + \int \mathrm{d}^3y(y_k - x_k)T_{00}\,. \tag{14}$$

It can be seen that the Lagrangian L and the Belinfante energy-momentum tensor $T^B_{\mu\nu}$ are now expressed only in terms of A_i^T and Ψ^T connected with the initial fields (8) $\sim$ (10) in nonlocal way.

According to Eqs (3), (4), (9) and (10) the gauge factor $v[A]$ transforms as

$$v[A^g] = v[A]g^{-1}\,. \qquad (15)$$

It is easy to check that nonlocal variables A_i^T and Ψ^T are invariant under gauge transformation (3) of the initial fields

$$A_i^T[A^g] = A_i^T[A]\,, \qquad (16)$$

$$\Psi^T[A^g, \Psi^g] = \Psi^T[A, \Psi]\,. \qquad (17)$$

This means that the variables $A_i^T[A]$ and $\Psi^T[A,\Psi]$ contain only physical degrees of freedom and are independent of pure gauge function $g(\vec{x},t)$. They satisfy the transversality condition

$$\partial_i A_i^T[A] = 0\,, \qquad (18)$$

which is not an initial assumption in the minimal quantization method and it changes under Lorentz transformations.

Thus, the substitution of the explicit solution of the constraint equation (5) into the gauge-invariant Lagrangian (1) and Belinfante tensor (2) also eliminates all nonphysical variables as obvious in Eqs (7), (8) and (12) $\sim$ (14). As a consequence, the gauge-invariant expressions (7), (8) and (12) $\sim$ (14) depend only on two nonlocal transverse variables $A_i^T[A]$, $\Psi^T[A,\Psi]$ which are themselves gauge-invariant functionals of the initial fields.

Let us consider now the Lorentz boot transformation

$$x'_k = x_k + \varepsilon_k t\,,$$
$$t' = t + \varepsilon_k x_k\,, \qquad |\varepsilon_k| \ll 1\,. \qquad (19)$$

Using the solution of the constraint equation (5) and the relations

$$\delta_L^0 \partial_k = \varepsilon_k \partial_0\,, \quad \delta_L^0(1/\vec{\partial}^2) = -2\varepsilon_k(1/\vec{\partial}^2)\partial_k\partial_0(1/\vec{\partial}^2)$$

for nonlocal physical transverse variables A_i^T and Ψ^T, we find the following expressions,

$$\delta_L^0 A_i^T[A] = \delta_L^0 A_k^T(x') + \varepsilon_k \Lambda(x')\,,$$
$$(\delta_L^0 A_k^T(x') = \varepsilon_i(x'_i\partial_{0'} - t'\partial_i^{x'})A_k^T(x) + \varepsilon_k A_0^T(x'))\,, \qquad (20)$$

$$\delta_L^0 \Psi^T[A,\Psi] = \delta_L^0 \Psi^T(x') + \mathrm{i}e\Lambda(x')\Psi^T(x')\,,$$
$$\Big(\delta_L^0 \Psi^T(x') = \varepsilon_i(x'_i\partial_{0'} - t'\partial_i^{x'})\Psi^T(x') + \frac{1}{4}\varepsilon_k[\gamma_0,\gamma_k]\Psi^T(x')\Big), \qquad (21)$$

where δ_L^0 is the ordinary Lorentz transformation and

$$\Lambda = \varepsilon_k \frac{1}{\vec{\partial}^2}(\partial_0 A_k^T + \partial_k A_0^T) \qquad (22)$$

is the additional gauge transformation which transforms A_i^T into the transverse field $A_\mu^{T\ell}$ one in the new coordinate system $\ell_\mu = \ell_\mu^0 + \delta_L^0 \ell_\mu^0$,

$$\partial_\mu^\ell A_\mu^{T\ell} = 0\,,$$
$$\big(\partial_\mu^\ell = \partial_\mu - \ell_\mu(\partial\ell)\,, \quad A_\mu^{T\ell} = A_\mu^T - \ell_\mu(A^T\cdot\ell)\big). \qquad (23)$$

The dynamical system of quantized fields A_i^T and Ψ^T follows a rotation of the time axis in relativistic transformations.

Thus, at the classical level we have three results which differ from the usual covariant method.

i) An explicit solution of the constraint equation and a transition to nonlocal invariant variables;
ii) The choice of a gauge-invariant energy-momentum Belinfante tensor (The gauge invariance principle here is extended to the variables themselves and dynamical observables (12) $\sim$ (14));
iii) The Lorentz transformations of nonlocal physical fields (9) $\sim$ (11).

In the usual covariant method the physical fields form a subspace in the space of initial 4-components A_μ by constraint conditions. These conditions contain an extra gauge choice $f(A) = 0$. In the minimal method the physical subspace of transverse fields is formed by the nonlocal projection of the space of initial fields which arises automatically for an explicit solution of the Gauss equation (4). It is necessary to notice that the approach considered here cannot be described by the general scheme of choosing gauge conditions which is applied to relativistic gauge.[16] The explicit solution of the Gauss equation and the gauge invariance principle allow one to remove just two nonphysical variables from the gauge-invariant expressions L and $T_{\mu\nu}^B$. In constructing variables (9) $\sim$ (11) we have only the arbitrariness in the choice of time axis or of the reference frame $\ell_\mu^0 = (1,0,0,0)$, $A_0 = (\ell_\mu^0 A_\mu)$. We can choose instead any vector ℓ_μ connected to ℓ_μ^0 by the Lorentz transformation $\ell_\mu = \ell_\mu^0 + \delta_L^0\ell_\mu^0$.

To discuss the quantum theory, we determine the canonical momenta and write the equal-time ($x_0 = y_0 = t$) commutation relations

$$\mathrm{i}[F_{0i}(\vec{x},t), A_j^T(\vec{y},t)] = \delta_{ij}^T\delta^3(\vec{x}-\vec{y})\,, \qquad (24)$$

$$\{\Psi_\alpha^T(\vec{x},t), \Psi_\beta^{+T}(\vec{y},t)\} = \delta_{\alpha\beta}\delta^3(\vec{x}-\vec{y})\,, \qquad (25)$$

where $F_{0i}(\vec{x},t) = F_{0i}[A^T]$, $\delta_{ij}^T = (\delta_{ij} - \partial_i(1/\vec{\partial}^2)\partial_i)$. Note that the temporal component A_0^T is not an independent field, and is determined via Ψ^T in Eq. (8).

Therefore, A_0^T satisfies the equal-time ($x_0 = y_0 = t$) commutation relations

$$[A_0^T(\vec{x},t), \Psi_\alpha^T(\vec{y},t)] = \frac{\alpha}{4\pi|\vec{x}-\vec{y}|}\Psi_\alpha^T(y)\,. \qquad (26)$$

All other commutators are equal to zero. Thus in the minimal quantization method operators of the quantum fields A_i^T, Ψ^T expressed in the forms of nonlocal functionals (9) $\sim$ (11) satisfy the nonlocal commutation relations. In the following we shall show that all of them have the same Lorentz transformations. Using commutation relations (24) $\sim$ (26) it is easy to show that the operators H,

P_k, M_{ij}, M_{0k} satisfy the algebra of commutators of the Poincaré group in the physical sector of gauge fields.[13]‡

In the present theory the Heisenberg relations for fields $A^T_\mu = \Big(A^T_0 = (1/\vec{\partial}^2)j^T_0, A^T_i\Big)$, Ψ^T,

$$\mathrm{i}[P_\mu, A^T_\nu(x)] = \partial_\mu A^T_\nu(x)\,, \quad \mathrm{i}\big[P_\mu, \Psi^T(x)] = \partial_\mu \Psi^T(x)\,, \tag{27}$$

and the Schwinger criterion of Lorentz invariance

$$\mathrm{i}[T_{00}(x), T_{00}(y)] = -(T_{0k}(x) + T_{0k}(y))\partial_k \delta^3(\vec{x} - \vec{y}) \tag{28}$$

are fulfilled. These relations can be proved by direct calculations.

Making an infinitesimal Lorentz rotation (produced by the boost M_{0k}) one can see that the operators A^T_μ and Ψ^T acquire additional gauge-dependent terms

$$\delta A^T_\mu(x) = \mathrm{i}\varepsilon_k\big[M_{0k}, A^T_\mu(x)\big] = \delta^0_L A^T_\mu(x) + \partial_\mu \Lambda\,, \tag{29}$$

$$\delta \Psi^T(x) = \mathrm{i}\varepsilon_k[M_{0k}, \Psi^T(x)] = \delta^0_L \Psi^T(x) + \mathrm{i}e\Lambda\Psi^T\,, \tag{30}$$

where δ^0_L is the ordinary Lorentz transformation and

$$\Lambda = \varepsilon_k \frac{1}{\vec{\partial}^2}(\partial_0 A^T_k + \partial_k A^T_0) \tag{31}$$

is the gauge operator function.[22] Note that the result (31) is exactly the same as Eq. (22) that we obtained in classical theory. The physical meaning of the transformation is that the very decomposition into Coulomb and transverse parts in this scheme has really a covariant structure. In other words, the Lorentz transformation simultaneously changes the gauge.

Note that the transformations (29) and (30) were discussed at first by Heisenberg and Pauli[14] with a reference to an unpublished remark by J. von Neuman. Also, we stress that the transformations (29) and (30) of quantum fields are exactly the same as the transformations of classical fields, Eqs (20) and (21).

The formulation of the Coulomb gauge in the frame work of the usual covariant quantization method leads to the usual canonical Hamiltonian that differs from the Belinfante one, Eq. (2), by a total derivative which contributes to the first terms of the boost operators (29) and (30). Strictly speaking, the Coulomb gauge leads to another gauge functional Λ in Eq. (31) in addition to those in Eq. (9). This gauge breaks the usual relativistic covariance of matrix elements of the type of Green functions. According to the interpretation of the usual covariant method the term Λ in Eqs (29) and (30) is treated as the gauge transformation which does not affect the physical results. But we know from Refs [6]–[13] that, this interpretation cannot be applied to off-mass shell amplitudes, bound states and one-fermion Green function. In our minimal quantization method, the new type of diagrams (39) with the gauge functional $\Lambda(x)$ defined by Eq. (31) restores the conventional relativistic properties of the Green functions in each order of radiative corrections.[13]

The diagram technique in the minimal quantization method, as was shown in Ref. [13], differs from the usual Feynman rule only by the form of the photon propagator of

$$D^T_{\mu\nu}(q) = \frac{1}{q^2 + \mathrm{i}\varepsilon}\Big[-g_{\mu\nu} - \frac{q_\mu q_\nu}{(q\ell^0)^2 - q^2} + \frac{(q\ell^0)(q_\mu \ell^0_\nu + \ell^0_\mu q_\nu)}{(q\ell^0)^2 - q^2}\Big]\,. \tag{32}$$

In the last expression the vector $\ell^0_\mu = (1,0,0,0)$ is determined in that Lorentz frame of reference where the quantization carried out. Other Feynman rules remain in fact.

The QED constructed by us satisfies all standard requirements of relativistic quantum theory. Such a quantization scheme is most close to the method of Schwinger[21] who has assumed the set of postulates: 1) transversality of physical variables; 2) the Belinfante tensor; 3) nonlocal commutation relations. The difference between the scheme proposed here and the Schwinger quantization method consists in that the physical variables are not postulated, but rather they are constructed explicitly by projecting the Lagrangian and Belinfante tensor onto the solution of the Gauss equation. In Ref. [13] it has been shown that the nonlocal physical variables obtained by the solution of the Gauss equation contain new physical information about the specific character of strong interaction theory§ that is absent in the covariant or Schwinger quantization methods.

3 The Relativistic Covariance of the Fermion Green Function in QED

The transverse variables which appear naturally in solving the constraint equations in the minimal quantization method are convenient in calculating some tangible physical effects. For example, the Lamb shift corrections $O(\alpha^6)$ are calculated only by the use of these variables.[7,23] On the other hand, just for the transverse variables the wavefunction renormalization is momentum-dependent because of the absence of a manifestly relativistic covariant expression for the electron Green function. Let us calculate the Green function from the formula

$$(2\pi)^4\delta^4(p-q)G(p) = \int \mathrm{d}^4x\,\mathrm{d}^4y\,\mathrm{e}^{(px-qx)}\langle 0[T(\Psi^T(x)\bar{\Psi}^T(y))|0\rangle\,, \tag{33}$$

where Ψ^T and $\bar{\Psi}^T$ are operators in the Heisenberg representation. In the one-loop approximation, $G(p)$ has the form

$$G(p) = G_0(p) + G_0(p)\Sigma(p)G_0(p) + 0(\alpha^2)\,, \tag{34}$$

where $\Sigma(p)$ is the electron self-energy at order α which contains the contributions from transverse fields and the

‡It is important to notice that the Belinfante tensor is a unique tensor which allow one to prove closed algebra of Poincaré group in the physical sector of gauge theories.

§In QCD this quantization method also leads to a new picture of colour confinement.[13] The latter is based on the destructive interference of the phase factors which appear in theories with topological degeneracies $[\Pi_3(\mathrm{SU}(N)) = Z]$.

Coulomb interaction

$$\Sigma(p)=\int\frac{(\mathrm{d}q)}{q_\mu^2}\Big[\Big(\delta_{i,j}-\frac{q_iq_j}{\vec{q}^{\,2}}\Big)\gamma_iG_0'\gamma_j+\gamma_0G_0'\gamma_0\frac{q_\mu^2}{\vec{q}^{\,2}}\Big],\quad(35)$$

where

$$(\mathrm{d}q)=\frac{e^2}{(2\pi)^4}\mathrm{i}\,\mathrm{d}^4q\,,\quad q_\mu^2=q_0-\vec{q}^{\,2}=q^2\,,\quad G_0'=G_0(p-q)\,.$$

Let us prove the invariance of the Green function (34) under the Lorentz transformation of the operators Ψ^T and $\bar{\Psi}^T$. By "invariance" we mean the equality[4]

$$G_0(p')=S_{p'p}G_0(p)S_{p'p}^{-1}\,.\quad(36)$$

That is, we shall take into account the Lorentz transformation of the γ-matrices. In this case $\delta_L^0G_0(p)=0$. It is known[4] that equation (35) can be represented by a sum of the invariant $\Sigma_F(p)$ and $\Delta\Sigma(p)$ terms,

$$\Sigma_F(p)=-\int\frac{(\mathrm{d}q)}{q^2}\gamma_\mu G_0'\gamma_\mu\,,\quad \delta_L^0\Sigma_F(p)=0\,,$$

$$\Delta\Sigma(p)=\int\frac{(\mathrm{d}q)}{q_\mu^2\vec{q}^{\,2}}[\hat{q}G_0'\hat{q}+\underline{q}G_0'\hat{q}+\hat{q}G_0'\underline{q}\,]\,,$$

$$\hat{q}=\gamma_\mu q_\mu\,,\qquad \underline{q}=\vec{\gamma}\vec{q}.\quad(37)$$

The response of $\Delta\Sigma(p)$ to the Lorentz transformation (36) can be obtained by changing the integration variables in Eq. (37),

$$\delta_L^0q_0=\varepsilon_kq_k\,,\qquad \delta_L^0q_k=\varepsilon_kq_0\,,$$

$$\delta_L^0\Delta\Sigma(p)=\varepsilon_k\int\frac{(\mathrm{d}q)}{q_\mu^2\vec{q}^{\,2}}[B_kG_0'\hat{q}+\hat{q}G_0'B_k]\,,$$

where

$$B_k=q_k\gamma_0+\gamma_kq_0-\frac{2q_0q_k}{\vec{q}^{\,2}}q_i\gamma_i-\frac{q_0q_k}{\vec{q}^{\,2}}\hat{q}\,.\quad(38)$$

The total Lorentz transformation for the Green function contains also the additional gauge transformations (29) ∼ (31),

$$\delta_L[(2\pi)^4\delta^4(p-q)\mathrm{i}G(p)]=\mathrm{i}e\varepsilon_k\int\mathrm{d}^4x\,\mathrm{d}^4y\exp(\mathrm{i}px-\mathrm{i}qy)$$
$$\times[\langle0|T(\Psi^T(x)\bar{\Psi}^T(y)\Lambda_k(y))|0\rangle$$
$$-\langle0|T(\Lambda_k(x)\Psi^T(x)\bar{\Psi}^T(y))|0\rangle]\,.\quad(39)$$

Using the explicit form for $\Lambda_k(x)=\Lambda_k^T(x)+\Lambda_k^c(x)$ (see Fig. 1),

$$\Lambda_k^T(\vec{x},t)=-\frac{1}{4\pi}\int\mathrm{d}^3y\frac{\partial_0A_k^T(\vec{y},t)}{|\vec{y}-\vec{x}\,|}\,,$$

$$\Lambda_k^c(\vec{x},t)=-\frac{1}{4\pi}\int\mathrm{d}^3y\frac{\partial_kA_0^c(\vec{y},t)}{|\vec{y}-\vec{x}\,|}\,,$$

we obtain the following expression,

$$\delta_\Lambda\Sigma=-\varepsilon_k\int\frac{(\mathrm{d}q)}{q_\mu^2\vec{q}^{\,2}}[B_kG_0'(\hat{p}-m)+(\hat{p}-m)G_0'B_k]\,,\quad(40)$$

where B_k is given by formula (38). Since

$$G_0(p-q)(\hat{p}-m)=1+G_0(p-q)\hat{q}\,,\qquad\int(\mathrm{d}q)\frac{B_k}{q_\mu^2\vec{q}^{\,2}}=0\,,\,(41)$$

the total response of Eqs (39) and (40) to the Lorentz transformations (29) and (30) is equal to zero,

$$\delta_{L,\mathrm{total}}\Sigma(p)=(\delta_L^0+\delta_\Lambda)\Sigma(p)=0\,.\quad(42)$$

Fig. 1 Diagrams responding to contribution from the gauge part of the Lorentz transformation: (a) Coulomb contribution; (b) Transverse contributions. Here H is defined by Eq. (12).

Therefore, it is sufficient to calculate expression (33) in the rest frame of the electron $p_\mu=(p_0,0,0,0)$ for the choice $\ell_\mu^0=(1,0,0,0)$,

$$\Sigma(p)=\int\frac{(\mathrm{d}q)}{q_\mu^2}\frac{2}{\hat{p}-\hat{q}+m}-\int\frac{(\mathrm{d}q)}{\vec{q}^{\,2}}\gamma_0\frac{1}{\hat{q}+m}\gamma_0\,.\quad(43)$$

Using the dimensional regularization, the integral (43) is equal to

$$\Sigma(p_\mu)=\frac{\alpha}{4\pi}\big[m(3D+4)-D(\hat{p}-m)\big]+\Sigma_R(p_\mu)\,,\quad(44)$$

where $D=1/\varepsilon-\gamma_E+\ln(4\pi)$ and

$$\Sigma_R(p_\mu)=\frac{\alpha}{2\pi}\Big[-\frac{\hat{p}}{4}+\int_0^1\mathrm{d}x[x\hat{p}-m]\ln\Big(1-\frac{p^2}{m^2}x\Big)\Big]$$
$$=\frac{\alpha}{2\pi}(\hat{p}-m)\Big\{\frac{\hat{p}+m}{p^2}\Big[\ln\Big(\frac{m^2-p^2}{m^2}\Big)\Big]$$
$$\times\Big[1+\frac{\hat{p}(\hat{p}-m)}{2p^2}\Big]-\frac{\hat{p}}{2p^2}\Big\},\quad(45)$$

To pass to a uniformly moving reference frame $p_\mu'=(p_0',\vec{p}\,')$ we should take into account Eq. (39) which leads to the change of the gauge,

$$q_iA_i^T(q)=0\Rightarrow[q_\mu-\ell_\mu(\ell q)]A_\mu^T(q)=0\,,\quad(46)$$

$$\ell_\mu=p_\mu'/\sqrt{p'^2}\,.\quad(47)$$

We must also consider the new diagrams (39) dictated by the "minimal" quantization method. This leads to the motion of the Coulomb field

$$K=\gamma_0V(\vec{q})\gamma_0\Rightarrow\gamma_{\mu\|}V(q^\perp)\gamma_{\mu\|}\,,\quad(48)$$

$$\gamma_{\mu\|}=\ell_\mu(\ell\cdot\gamma)\,,\qquad q_\mu^\perp=q_\mu-\ell_\mu(q\cdot\ell)\,.\quad(49)$$

The use of these diagrams is a principal difference between the "minimal" quantization method and standard Coulomb gauge used in many papers.[4,7−9] We stress that the electron self-energy $\Sigma(p)$ in Eq. (45) has no infrared divergences and allows the renormalization with subtraction at physical values of the momentum $\hat{p}=m$,

$$\Sigma(\hat{p}=m)=\delta m=\frac{m\alpha}{4\pi}(3D+4)\,,$$

$$\Sigma'(\hat{p}=m) = Z-1\,, \quad Z = 1-\frac{\alpha}{4\pi}D\,. \qquad (50)$$

The probability of finding an electron with the mass $m_R = m+\delta m$ calculated from formula ($R(p) = \lim_{\hat{p}\to m_R}(\hat{p}-m_R)G_R(p) = |\Psi|^2$) is equal to unity ($|\Psi|^2=1$). These results cannot be obtained in any relativistic gauge. These results represent a solution to the renormalization problem on mass shell for transverse variables.

A mistake in Refs [4] and [8] consists not only in ignoring correct transformation properties (29) and (31) to the construction of $\Sigma(p)$ but also in a nonphysical choice of the initial vector (the time axis) that fixes the component of the Coulomb field. For example, in expression (33) where $p_\mu = (p_0, \vec{p}\neq 0)$ the vector $\ell^0_\mu = (1,0,0,0)$ is chosen so that the electron has a velocity different from that of the Coulomb field. As a result, they lead to difficulties with manifest Lorentz invariance and infrared divergences. On the other hand, the correct transition (33) to the electron rest frame $p_\mu = (p_0, \vec{p}=0)$ does not remove these difficulties as we simultaneously rotate the initial gauge $\ell^0_\mu = (1,0,0,0)$, thus leaving velocities of the electron and its proper field being different. So, a choice of ℓ^0_μ must be defined in a physical formulation of the problem, in this case ℓ^0_μ is the unit vector along the momentum $\ell_\mu \sim p_\mu$. We note that nonphysical infrared divergences in the calculation of R arise if we use the Lorentz transformation corresponding to the canonical energy momentum tensor $T^c_{\mu\nu}$,[24] or local commutation relations $\mathrm{i}[E_i(\vec{x},t), A_j(\vec{y},t)] = \delta_{ij}\delta^3(\vec{x}-\vec{y})$.

One has taken into account the additional diagrams which are induced by the Λ when passing to another Lorentz frame. Thus, the proof of manifestly relativistic covariance of the fermion Green function in the one-loop approximation, based on the quantization only of physical transverse variables, can be made at the level of Feynman diagrams. The results of this paper solve the problem of renormalization of physical quantities on mass shell for the transverse variables.

4 Conclusion

In the framework of the minimal quantization methods of QED the electron's relativistically covariant Green function with correct (from a physical point of view) analytical properties has been obtained. We have shown that the physical residues of the one-particle Green function $\lim_{\hat{p}\to m_R}(\hat{p}-m_R)G_R(p) = 1$. The main difference between the "conventional Coulomb gauge" and "our minimal quantization method" consists in the proof and interpretation of the additional gauge transformations (29) $\sim$ (31). Our result can be explained by using additional gauge transformation in the calculation scheme for the physical residues of the one-particle Green function, as described here. Moreover, the approach considered in this paper can be used for investigation of the interaction and spectrum of bound states in QED and QCD.

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