

# Teleportation of Squeezed Entangled State\*

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**Abstract** Based on the coherent entangled state  $|\alpha, x\rangle$  we introduce the squeezed entangled state (SES). Then we propose a teleportation protocol for the SES by using Einstein–Podolsky–Rosen entangled state  $|\eta\rangle$  as a quantum channel. The calculation is greatly simplified by virtue of the Schmidt decompositions of both  $|\alpha, x\rangle$  and  $|\eta\rangle$ . Any bipartite states that can be expanded in terms of  $|\alpha, x\rangle$  may be teleported in this way due to the completeness of  $|\alpha, x\rangle$ .

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## 1 Introduction

Quantum information is very fascinating due to the involvement of quantum entanglement. Quantum teleportation (QT), one of the most striking features in quantum information, can be realized based on entangled states. QT has drawn much attention due to its close relation to quantum computing,<sup>[1]</sup> quantum cryptography,<sup>[2–6]</sup> and dense coding.<sup>[7,8]</sup> The first theoretical proposal of QT in the pioneer article of Bennett *et al.*<sup>[3]</sup> has stimulated experiments of QT using different types of Einstein–Podolsky–Rosen (EPR) pairs.<sup>[5,6,9,10]</sup> In the scheme of QT, Alice sends an unknown quantum state to Bob, using EPR pairs as a quantum channel for the faithful transmission and a classical channel to deliver her measuring results to Bob. The experimental teleportation of discrete variable quantum states has been successfully carried out by Zeilinger’s group.<sup>[5]</sup> The theoretical analysis of the teleportation of continuous quantum states was first made by Vaidman<sup>[11]</sup> followed by subsequent works.<sup>[4,12–14]</sup>

In an entangled state with continuous variables, the measurement performed on one part of the system provided information on the remaining part,<sup>[15]</sup> as firstly pointed out by EPR in their famous paper arguing the incompleteness of quantum mechanics.<sup>[16]</sup> It has been shown that the EPR entangled state representation  $|\eta\rangle$  can be directly and effectively used in discussing the QT of continuous states.<sup>[17]</sup> By using the Schmidt decomposition of  $|\eta\rangle$  state,<sup>[18–20]</sup> the discussion of teleportation can be conveniently converted into the coordinate-momentum representation or into the particle-number representation as one’s wish, so one can concisely recapitulate teleportation scheme in a simple manner.<sup>[21]</sup> Moreover, the teleported states can be calculated more explicitly, so one can reach the point straightforwardly. On the other hand,  $|\eta\rangle$  can be either viewed as an ideal quantum channel or measure-

ment basis of quadrature phase, thus it plays an essential role in QT theory.

The expression of state  $|\eta\rangle$  in two-mode Fock space<sup>[18–20]</sup> is

$$|\eta\rangle = \exp\left[-\frac{1}{2}|\eta|^2 + \eta\hat{a}_1^\dagger - \eta^*\hat{a}_2^\dagger + \hat{a}_2^\dagger\hat{a}_1^\dagger\right]|00\rangle_{12},$$

$$\eta = \frac{1}{\sqrt{2}}(\eta_1 + i\eta_2), \quad (1)$$

which is the simultaneous eigenstate of commutative operators  $(\hat{X}_1 - \hat{X}_2, \hat{P}_1 + \hat{P}_2)$ , i.e.,

$$(\hat{X}_1 - \hat{X}_2)|\eta\rangle_{1,2} = \eta_1|\eta\rangle_{1,2},$$

$$(\hat{P}_1 + \hat{P}_2)|\eta\rangle_{1,2} = \eta_2|\eta\rangle_{1,2}, \quad (2)$$

where  $|00\rangle_{1,2}$  is the two-mode vacuum state,  $(\hat{a}_i, \hat{a}_i^\dagger)$ ,  $i = 1, 2$ , are the two-mode Bose annihilation and creation operators in Fock space, related to  $\hat{X}_i$  and  $\hat{P}_i$  by  $\hat{X}_i = (1/\sqrt{2})(\hat{a}_i + \hat{a}_i^\dagger)$ ,  $\hat{P}_i = (1/\sqrt{2}i)(\hat{a}_i - \hat{a}_i^\dagger)$ . The state  $|\eta\rangle$  not only satisfies the completeness relation,

$$\int \frac{d^2\eta}{\pi} |\eta\rangle\langle\eta| = 1, \quad d^2\eta = \frac{1}{2}d\eta_1 d\eta_2, \quad (3)$$

but also possesses the orthogonal property,

$$\langle\eta'|\eta\rangle = \pi \delta(\eta - \eta')\delta(\eta^* - \eta'^*). \quad (4)$$

So  $|\eta\rangle$  makes up a new quantum mechanical representation and can be considered as a continuous Bell basis, we name it EPR eigenstate with continuous variables. To teleport a single particle quantum state, target state  $|\psi_{in}\rangle_3$ , an entangled EPR pair  $|\eta\rangle_{12}$  is used as the quantum channel shared by Alice and Bob. (see Fig. 1). So the total initial state of the whole system is given by

$$|\phi\rangle_{in} = |\psi_{in}\rangle_3 \otimes |\eta\rangle_{12}. \quad (5)$$

A joint Bell measurement (quadrature phase measurement) of  $\hat{X}_1 - \hat{X}_3$  and  $\hat{P}_1 + \hat{P}_3$  performed by Alice on particles 1 and 3 projects  $|\phi\rangle_{in}$  onto the EPR entangled

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tion, the completeness relation can be proved

$$\int_{-\infty}^{\infty} \frac{dx}{\sqrt{\pi}} \int \frac{d^2\alpha}{2\pi} |\alpha, x\rangle \langle \alpha, x| = 1, \quad (11)$$

$|\alpha, x\rangle$  is of importance since it makes up a quantum mechanical representation. The Schmidt decomposition of  $|\alpha, x\rangle$  can be obtained by firstly making the Fourier transform

$$\int_{-\infty}^{\infty} \frac{d\alpha_2}{2\pi} |\alpha = \alpha_1 + i\alpha_2, x\rangle e^{-iu\alpha_2} = e^{-(\alpha_1^2/8) - (u^2/2) + (u\alpha_1/2)} \left| \frac{x}{\sqrt{2}} + \tau \right\rangle_1 \otimes \left| \frac{x}{\sqrt{2}} - \tau \right\rangle_2, \quad (12)$$

and then taking its inverse Fourier transform. In so doing we have

$$|\alpha = \alpha_1 + i\alpha_2, x\rangle = e^{-\alpha_1^2/8} \int_{-\infty}^{\infty} du e^{u(\alpha_1 + 2i\alpha_2 - u)/2} \left| \frac{x}{\sqrt{2}} + \tau \right\rangle_1 \otimes \left| \frac{x}{\sqrt{2}} - \tau \right\rangle_2, \quad (13)$$

where  $\tau = (u + \alpha_1/2)/\sqrt{2}$  and

$$|x\rangle_i = \pi^{-1/4} \exp\left(-\frac{1}{2}x^2 + \sqrt{2}x\hat{a}_i^\dagger - \frac{1}{2}\hat{a}_i^{\dagger 2}\right)|0\rangle_i \quad (14)$$

is the coordination eigenstate. Equation (13) shows manifestly that  $|\alpha, x\rangle$  is an entangled state. An ideal coherent entangled state  $|\alpha, x\rangle$  can be implemented by operating two local oscillator displacements,

$$D_1\left[\frac{1}{2}(x + \alpha)\right] = \exp\left[\frac{1}{2}(x + \alpha)\hat{a}_1^\dagger - \frac{1}{2}(x + \alpha^*)\hat{a}_1\right], \quad D_2\left[\frac{1}{2}(x - \alpha)\right] = \exp\left[\frac{1}{2}(x - \alpha)\hat{a}_2^\dagger - \frac{1}{2}(x - \alpha^*)\hat{a}_2\right], \quad (15)$$

on the state  $\exp[-(\hat{a}_1^\dagger + \hat{a}_2^\dagger)^2/4]|00\rangle$ , i.e.,

$$D_1 D_2 \exp\left[-\frac{1}{4}(\hat{a}_1^\dagger + \hat{a}_2^\dagger)^2\right]|00\rangle = \exp\left\{-\frac{1}{4}\left[\hat{a}_1^\dagger - \frac{1}{2}(x + \alpha^*) + \hat{a}_2^\dagger - \frac{1}{2}(x - \alpha^*)\right]^2\right\} D_1|0\rangle_1 \otimes D_2|0\rangle_2 = |\alpha, x\rangle, \quad (16)$$

where the displacement  $D_i$  can be realized experimentally by reflecting the light field from an almost perfect reflecting mirror and adding through the mirror a field with phase and amplitude modulated according to the values  $(x \pm \alpha)/2$ .

### 3 Quadrature-Amplitude Measure for Squeezed Entangled State and Its Properties

Based on Eq. (11) we can introduce a new squeezing operator constructed<sup>[24]</sup> as

$$U(r, s, \mu) \equiv \sqrt{\mu s} \int_{-\infty}^{\infty} \frac{dx}{\sqrt{\pi}} \int \frac{d^2\alpha}{2\pi} |s\alpha - r\alpha^*, \mu x\rangle \langle \alpha, x|, \quad (17)$$

where  $r$  and  $s$  are complex and satisfy the unimodularity condition  $ss^* - rr^* = 1$ , so  $U(r, s, \mu)$  is a unitary operator with four real parameters. Using the normal ordering of the two-mode vacuum projector  $|00\rangle\langle 00| = : \exp[-\hat{a}_1^\dagger \hat{a}_1 - \hat{a}_2^\dagger \hat{a}_2] :$  and the IWOP method, we can derive the normal ordering form of  $U(r, s, \mu)$ <sup>[24]</sup>

$$U(r, s, \mu) = \frac{\text{sech}^{1/2}\lambda}{\sqrt{s^*}} \exp\left[\frac{\tanh\lambda}{2}\left(\frac{\hat{a}_1^\dagger + \hat{a}_2^\dagger}{\sqrt{2}}\right)^2 - \frac{r}{2s^*}\left(\frac{\hat{a}_1^\dagger - \hat{a}_2^\dagger}{\sqrt{2}}\right)^2\right] \\ \times V(s, \mu) \exp\left[-\frac{\tanh\lambda}{2}\left(\frac{\hat{a}_1 + \hat{a}_2}{\sqrt{2}}\right)^2 + \frac{r^*}{2s^*}\left(\frac{\hat{a}_1 - \hat{a}_2}{\sqrt{2}}\right)^2\right], \quad (18)$$

where we have set  $e^\lambda = \mu$ ,  $\text{sech}\lambda = 2\mu/(\mu^2 + 1)$ ,  $\tanh\lambda = (\mu^2 - 1)/(\mu^2 + 1)$ , and

$$V(s, \mu) = : \exp\left[(\hat{a}_1^\dagger, \hat{a}_2^\dagger)[\Lambda - I]\begin{pmatrix} \hat{a}_1 \\ \hat{a}_2 \end{pmatrix}\right] :, \quad (19)$$

$$\Lambda \equiv \begin{pmatrix} \frac{1}{2}(\text{sech}\lambda + \frac{1}{s^*}) & \frac{1}{2}(\text{sech}\lambda - \frac{1}{s^*}) \\ \frac{1}{2}(\text{sech}\lambda - \frac{1}{s^*}) & \frac{1}{2}(\text{sech}\lambda + \frac{1}{s^*}) \end{pmatrix}. \quad (20)$$

Operating the squeezing operator  $U(r, s, \mu)$  on  $|00\rangle$ , a generalized squeezed vacuum state (named squeezed entangled state) is obtained,

$$U(r, s, \mu)|00\rangle = \frac{\text{sech}^{1/2}\lambda}{\sqrt{s^*}} \exp\left[\frac{\tanh\lambda}{2}\left(\frac{\hat{a}_1^\dagger + \hat{a}_2^\dagger}{\sqrt{2}}\right)^2 - \frac{r}{2s^*}\left(\frac{\hat{a}_1^\dagger - \hat{a}_2^\dagger}{\sqrt{2}}\right)^2\right]|00\rangle. \quad (21)$$

In this state the quantum fluctuation of the two pairs of quadrature phase amplitudes

$$\hat{Q}_1 = \frac{1}{2}(\hat{X}_1 + \hat{X}_2), \quad \hat{Q}_2 = \frac{1}{2}(\hat{P}_1 + \hat{P}_2), \quad [\hat{Q}_1, \hat{Q}_2] = \frac{i}{2}, \quad \hat{Q}'_1 = \frac{1}{2}(\hat{X}_1 - \hat{X}_2), \quad \hat{Q}'_2 = \frac{1}{2}(\hat{P}_1 - \hat{P}_2), \quad [\hat{Q}'_1, \hat{Q}'_2] = \frac{i}{2}, \quad (22)$$

are as follows:

$$(\Delta\hat{Q}_1)^2 = \frac{1}{8}e^{-2\lambda}, \quad (\Delta\hat{Q}_2)^2 = \frac{1}{8}e^{2\lambda}, \quad \Delta\hat{Q}_1\Delta\hat{Q}_2 = \frac{1}{8}, \quad (23)$$

and

$$(\Delta\hat{Q}'_1)^2 = \frac{1}{8}|s + r|^2, \quad (\Delta\hat{Q}'_2)^2 = \frac{1}{8}|s - r|^2, \quad \Delta\hat{Q}'_1\Delta\hat{Q}'_2 = \frac{1}{8}|s^2 - r^2|. \quad (24)$$

While equation (23) (just relating to  $\lambda$ ) seems the same as the squeezing effect caused by the ordinary two-mode squeezing operator, equation (24) is related to  $s$ ,  $r$  and is quite different from that of the ordinary one. The squeezing transform generated by  $U(r, s, \mu)$  is

$$\hat{a}_1^{\dagger} \equiv U(r, s, \mu) \hat{a}_1^{\dagger} U^{-1}(r, s, \mu) = \frac{1}{2}(\hat{a}_1^{\dagger} + \hat{a}_2^{\dagger}) \cosh \lambda - \frac{1}{2}(\hat{a}_1 + \hat{a}_2) \sinh \lambda + \frac{s}{2}(\hat{a}_1 + \hat{a}_2) + \frac{r^*}{2}(\hat{a}_1 - \hat{a}_2), \quad (25)$$

from which we see that  $U(r, s, \mu)$  generates squeezing for both the rotated modes  $(\hat{a}_1^{\dagger} + \hat{a}_2^{\dagger})/\sqrt{2}$  and  $(\hat{a}_1^{\dagger} - \hat{a}_2^{\dagger})/\sqrt{2}$ , corresponding to the squeezing parameters  $\mu = e^{\lambda}$  and  $(s, r)$ , respectively. Thus if a 50:50 symmetric beam splitter transforms the input modes  $\hat{a}_1^{\dagger}$  and  $\hat{a}_2^{\dagger}$  to  $(\hat{a}_1^{\dagger} + \hat{a}_2^{\dagger})/\sqrt{2}$  and  $(\hat{a}_1^{\dagger} - \hat{a}_2^{\dagger})/\sqrt{2}$  as the output, then the squeezing operator  $U(r, s, \mu)$  will play a role of squeezing for them separately with squeezing parameters  $\mu = e^{\lambda}$  and  $(s, r)$ , respectively. These can be implemented in experiments.

When we make a single-mode quadrature-amplitude measure, say  $|x\rangle_{11}\langle x|$ ,  $|x\rangle_1$  is the eigenvector of the quadrature operator  $\hat{X}_1 = (\hat{a}_1 + \hat{a}_1^{\dagger})/\sqrt{2}$ , on the squeezed entangled state Eq. (21), then using the completeness relation  $\int \frac{d^2z}{\pi} |z\rangle_{11}\langle z| = 1$  of coherent state  $|z\rangle_1$  [25] and the following formula:

$$\int \frac{d^2z}{\pi} \exp[\zeta|z|^2 + \xi z + \eta z^* + f z^2 + g z^{*2}] = \frac{1}{\sqrt{\zeta^2 - 4fg}} \exp\left[\frac{-\zeta\xi\eta + \xi^2g + \eta^2f}{\zeta^2 - 4fg}\right], \quad (26)$$

whose convergent condition is either

$$\text{Re}(\zeta + f + g) < 0, \quad \text{Re}\left(\frac{\zeta^2 - 4fg}{\zeta + f + g}\right) < 0, \quad (27)$$

or

$$\text{Re}(\zeta - f - g) < 0, \quad \text{Re}\left(\frac{\zeta^2 - 4fg}{\zeta - f - g}\right) < 0, \quad (28)$$

we have

$$\begin{aligned} {}_1\langle x|U(r, s, \mu)|00\rangle &= \frac{\text{sech}^{1/2}\lambda}{\sqrt{s^*}} \int \frac{d^2z}{\pi} {}_1\langle x|z\rangle_{11}\langle z| \exp\left[\frac{\tanh\lambda}{4}(z^* + \hat{a}_2^{\dagger})^2 - \frac{r}{4s^*}(z^* - \hat{a}_2^{\dagger})^2\right] |0\rangle_1|0\rangle_2 \\ &= \frac{\text{sech}^{1/2}\lambda}{\sqrt{s^*\pi^{1/4}}} \int \frac{d^2z}{\pi} \exp\left[-|z|^2 - \frac{x^2}{2} + \sqrt{2}xz - \frac{z^2}{2}\right] \exp\left[\frac{\tanh\lambda}{4}(z^* + \hat{a}_2^{\dagger})^2 - \frac{r}{4s^*}(z^* - \hat{a}_2^{\dagger})^2\right] |0\rangle_2 \\ &= \frac{\pi^{-1/4}\text{sech}^{1/2}\lambda}{\sqrt{(1+\epsilon/2)s^*}} \exp\left\{\frac{1}{1+\epsilon/2}\left[-(1-\epsilon)\frac{x^2}{2} + \frac{x\varepsilon}{\sqrt{2}}\hat{a}_2^{\dagger} + \left(\frac{1}{4}\epsilon - \frac{1}{2}\frac{r}{s^*}\tanh\lambda\right)\hat{a}_2^{\dagger 2}\right]\right\} |0\rangle_2, \end{aligned} \quad (29)$$

where  $\epsilon = \tanh\lambda - r/s^*$ ,  $\varepsilon = \tanh\lambda + r/s^*$ . Equation (29) shows that after the measurement the second-mode collapses to a single-mode squeezed state, which is related to three parameters  $\mu$ ,  $r$ , and  $s$ . In particular, when  $\tanh\lambda = r/s^*$ , so  $|s|^2 = \text{sech}^{-2}\lambda$ , then setting  $\sqrt{s^*} = \text{sech}^{-1/2}\lambda$ , equation (29) becomes

$${}_1\langle x|U(r, s, \mu)|00\rangle = \frac{\text{sech}\lambda}{\pi^{1/4}} \exp\left[-\frac{x^2}{2} + \sqrt{2}x \tanh(\lambda)\hat{a}_2^{\dagger} - \frac{\hat{a}_2^{\dagger 2}}{2} \tanh^2\lambda\right] |0\rangle_2, \quad (30)$$

which is just a single-mode squeezed state. [26] In this sense the state  $U(r, s, \mu)|00\rangle$  is worth of paying much attention. A measurement by  $|x\rangle_{22}\langle x|$  on it will yield a similar result.

#### 4 Teleportation of Squeezed Entangled State

Recently, the two-mode squeezed state has been applied to continuous variables QT. [27] Here we discuss how to teleport the squeezed entangled state  $U(r, s, \mu)|00\rangle$  from Alice to Bob by using the EPR state  $|\eta\rangle$  as quantum channel. Instead of using Eq. (21) we appeal to the Schmidt decomposition of  $|\alpha, x\rangle$  in Eq. (13) to decompose

$$\begin{aligned} U(r, s, \mu)|00\rangle &= \sqrt{\mu s} \int_{-\infty}^{\infty} \frac{dx}{\sqrt{\pi}} \int \frac{d^2\alpha}{2\pi} |s\alpha - r\alpha^*, \mu x\rangle \exp\left(-\frac{1}{4}|\alpha|^2 - \frac{1}{2}x^2\right) \\ &= \sqrt{\mu s} \int_{-\infty}^{\infty} \frac{dx}{\sqrt{\pi}} \int \frac{d^2\alpha}{2\pi} \int_{-\infty}^{\infty} du F(x, \alpha, u) \left|\frac{\mu x}{\sqrt{2}} + \tau'\right\rangle_5 \otimes \left|\frac{\mu x}{\sqrt{2}} - \tau'\right\rangle_6, \end{aligned} \quad (31)$$

where  $\tau' = (u + \alpha'_1/2)/\sqrt{2}$ ,  $\alpha'_1 = \text{Re}(s\alpha - r\alpha^*)$ ,  $\alpha'_2 = \text{Im}(s\alpha - r\alpha^*)$ , and

$$F(x, \alpha, u) \equiv \exp\left[\frac{1}{2}u(\alpha'_1 + 2i\alpha'_2 - u) - \frac{1}{8}\alpha'^2_1 - \frac{1}{4}|\alpha|^2 - \frac{1}{2}x^2\right]. \quad (32)$$

It is necessary to provide two quantum channels, i.e.,  $|\eta\rangle_{13}\otimes|\eta'\rangle_{24}$  for teleporting the 2-mode quantum state (see Fig. 2). Let particles 1 and 3 be prepared in an EPR entangled state  $|\eta\rangle_{13}$ , and particles 2 and 4 in  $|\eta'\rangle_{24}$ . The unknown SES  $U(r, s, \mu)|00\rangle_{56}$  is in modes 5 and 6. In addition, the quadrature phase measurements (joint Bell measurements) are

performed on two pairs of particles, 3 and 5, 4 and 6, respectively, i.e, the projection state is  $|\eta''\rangle_{53} \otimes |\eta'''\rangle_{64}$ , which can be viewed as continuous Bell basis. So the total initial state in this scheme is given by

$$U(r, s, \mu)|00\rangle_{56} \otimes |\eta\rangle_{13} \otimes |\eta'\rangle_{24}. \quad (33)$$

According to Refs. [18] ~ [20] the Schmidt decomposition of  $|\eta\rangle_{ij}$  in coordinate space is

$$|\eta\rangle_{ij} = e^{-i\eta_1\eta_2/2} \int_{-\infty}^{\infty} dx |x\rangle_1 \otimes |x - \eta_1\rangle_3 e^{i\eta_2 x}, \quad \eta = \frac{1}{\sqrt{2}}(\eta_1 + i\eta_2), \quad (34)$$

while in momentum space is

$$|\eta\rangle_{ij} = e^{-i\eta_1\eta_2/2} \int_{-\infty}^{\infty} dp |p + \eta_2\rangle_1 \otimes |-p\rangle_3 e^{-ip\eta_1}, \quad (35)$$

where  $|p\rangle_i$  is the eigenstate of  $\hat{P}_i$ ,

$$|p\rangle_i = \pi^{-1/4} \exp\left(-\frac{1}{2}p^2 + i\sqrt{2}p\hat{a}_i^\dagger + \frac{1}{2}\hat{a}_i^{\dagger 2}\right)|0\rangle_i. \quad (36)$$

Then noticing  ${}_i\langle x|p\rangle_i = (1/\sqrt{2\pi})e^{ipx}$  we can calculate

$$\begin{aligned} & {}_{64}\langle \eta''' | \otimes {}_{53}\langle \eta'' | \eta\rangle_{13} \otimes |\eta'\rangle_{24} \otimes U(r, s, \mu)|00\rangle_{56} \\ &= \frac{A}{2\pi} \int_{-\infty}^{\infty} dp'' e^{ip''\eta_1'''} \int_{-\infty}^{\infty} dp' e^{ip'\eta_1''} \int_{-\infty}^{\infty} dx e^{i(x-\eta_1)p'} \int_{-\infty}^{\infty} dx' e^{i(x'-\eta_1')p''} e^{ix\eta_2} e^{ix'\eta_2'} \\ & \quad \times {}_6\langle p'' + \eta_2''' | \otimes {}_5\langle p' + \eta_2'' | U(r, s, \mu)|00\rangle_{56} \otimes |x\rangle_1 \otimes |x'\rangle_2 \\ &= A \int_{-\infty}^{\infty} dp'' e^{ip''(\eta_1''' - \eta_1')} \int_{-\infty}^{\infty} dp' e^{ip'(\eta_1'' - \eta_1)} {}_6\langle p'' + \eta_2''' | \otimes {}_5\langle p' + \eta_2'' | U(r, s, \mu)|00\rangle_{56} \\ & \quad \times |p' + \eta_2\rangle_1 \otimes |p'' + \eta_2'\rangle_2 \equiv |\Phi\rangle, \end{aligned} \quad (37)$$

where

$$A \equiv \exp\left[\frac{1}{2}i(\eta_1''' \eta_2''' + \eta_1'' \eta_2'' - \eta_1' \eta_2' - \eta_1 \eta_2)\right]. \quad (38)$$

Using the relation

$$e^{ik\hat{X}_1}|p\rangle_1 = |p+k\rangle_1, \quad e^{ik\hat{P}_1}|x\rangle_1 = |x-k\rangle_1, \quad (39)$$

$$|x\rangle_1 = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dp e^{-ixp}|p\rangle_1, \quad (40)$$

we can rewrite Eq. (37) as (comparing with Eq. (31))

$$\begin{aligned} |\Phi\rangle &= A\sqrt{\mu s} \int_{-\infty}^{\infty} \frac{dx}{\sqrt{\pi}} \int \frac{d^2\alpha}{2\pi} \int_{-\infty}^{\infty} du F(x, \alpha, u) \int_{-\infty}^{\infty} dp' e^{ip'(\eta_1'' - \eta_1)} {}_5\langle p' + \eta_2'' | \frac{\mu x}{\sqrt{2}} + \tau' \rangle_5 |p' + \eta_2\rangle_1 \\ & \quad \otimes \int_{-\infty}^{\infty} dp'' e^{ip''(\eta_1''' - \eta_1')} {}_6\langle p'' + \eta_2''' | \frac{\mu x}{\sqrt{2}} - \tau' \rangle_6 |p'' + \eta_2'\rangle_2 \\ &= BY\sqrt{\mu s} \int_{-\infty}^{\infty} \frac{dx}{\sqrt{\pi}} \int \frac{d^2\alpha}{2\pi} \int_{-\infty}^{\infty} du F(x, \alpha, u) \left| \frac{\mu x}{\sqrt{2}} + \tau' \right\rangle_1 \otimes \left| \frac{\mu x}{\sqrt{2}} - \tau' \right\rangle_2 \\ &= BY \cdot U(r, s, \mu)|00\rangle_{12}, \end{aligned} \quad (41)$$

where

$$B = A e^{i\eta_2''(\eta_1 - \eta_1')} e^{i\eta_2'''(\eta_1' - \eta_1''')}, \quad (42)$$

$$\begin{aligned} Y &= e^{i[(\eta_2 - \eta_2')\hat{X}_1 + (\eta_2' - \eta_2'')\hat{X}_2]} \\ & \quad \times e^{i[\hat{P}_1(\eta_1'' - \eta_1) + \hat{P}_2(\eta_1''' - \eta_1')]} \end{aligned} \quad (43)$$

Comparing Eq. (41) with Eq. (31), we see that up to the inessential factor and a unitary transform, the outcome state in modes 1 and 2 is the same as the incoming state  $U(r, s, \mu)|00\rangle_{56}$ . After Alice informs Bob of the data of  $\eta, \eta'',$  and  $\eta'''$  through a classical channel, Bob then makes the unitary transformation  $Y^{-1}$  to obtain the unknown squeezed entangled state. In this way the teleportation

process is carried out successfully. Due to the completeness property of  $|\alpha, x\rangle$ , any bipartite state  $|\rangle_{56}$  can be written as

$$|\rangle_{56} = \int_{-\infty}^{\infty} \frac{dx}{\sqrt{\pi}} \int \frac{d^2\alpha}{2\pi} \mathcal{F}|\alpha, x\rangle_{5,6}, \quad (44)$$

where  $\mathcal{F} = {}_{56}\langle \alpha, x | \rangle_{56}$ , thus any bipartite state with definite  $\mathcal{F}$  can be teleported in this way.

## 5 Conclusion

We have proposed a quantum protocol to teleport the 2-mode squeezed entangled state, which possesses remarkable squeezing property, by employing the quantum chan-

nels composed of two EPR eigenstates. Our calculations have been greatly simplified by virtue of the Schmidt decomposition of both the coherent entangled state  $|\alpha, x\rangle$  and the state  $|\eta\rangle$ , which implies the importance of con-

structing entangled state representation. Needless to say, the state  $|\alpha, x\rangle$  itself can be served as a quantum channel between senders and receivers. We hope to discuss the further applications of  $|\alpha, x\rangle$  in a forthcoming paper.

## References

- [1] D.P. DiVincenzo, *Science* **270** (1995) 255; L.K. Grover, *Phys. Rev. Lett.* **79** (1997) 325.
- [2] N.R. Zhou, *et al.*, *Opt. Commun.* **254** (2005) 380; N.R. Zhou, G.H. Zeng, and J. Xiong, *Electron. Lett.* **40** (2004) 1149.
- [3] C.H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, and W.K. Wothers, *Phys. Rev. Lett.* **70** (1993) 1895.
- [4] S.L. Braunstein and H.J. Kimble, *Phys. Rev. Lett.* **80** (1998) 869.
- [5] D. Bouwmeester, *et al.*, *Nature (London)* **390** (1997) 575.
- [6] A. Furusawa, *et al.*, *Science* **282** (1998) 706.
- [7] C.H. Bennett and S.J. Wiesner, *Phys. Rev. Lett.* **69** (1992) 2881.
- [8] S.L. Braunstein and H.J. Kimble, *Phys. Rev. A* **61** (2000) 042302.
- [9] D. Boschi, *et al.*, *Phys. Rev. Lett.* **80** (1998) 1121.
- [10] I. Marcikic, *et al.*, *Nature (London)* **421** (2003) 509.
- [11] L. Vaidman, *Phys. Rev. A* **49** (1994) 1473.
- [12] G.J. Milburn and S.L. Braunstein, *Phys. Rev. A* **60** (1999) 937.
- [13] J. Janszky, M. Koniorczyk, and V. Gábris, *Phys. Rev. A* **64** (2001) 034302.
- [14] T. Ide, H. Hofmann, T. Kobayashi, and A. Furusawa, *Phys. Rev. A* **65** (2001) 012313.
- [15] M. Zukowski, A. Zeilinger, M.A. Horne, and A. Ekert, *Phys. Rev. Lett.* **71** (1993) 4287; J.W. Pan, D. Bouwmeester, H. Weifurter, and A. Zeilinger, *Phys. Rev. Lett.* **80** (1998) 3891; S. Bose, V. Vedral, and P.L. Knight, *Phys. Rev. A* **57** (1998) 822.
- [16] A. Einstein, B. Podolsky, and N. Rosen, *Phys. Rev.* **47** (1935) 777.
- [17] H.Y. Fan, *Phys. Lett. A* **294** (2002) 253.
- [18] H.Y. Fan and J.R. Klauder, *Phys. Rev. A* **47** (1994) 704.
- [19] H.Y. Fan and B.Z. Chen, *Phys. Rev. A* **53** (1996) 2948.
- [20] H.Y. Fan and X. Ye, *Phys. Rev. A* **51** (1995) 3343.
- [21] H.Y. Fan and X.T. Liang, *Commun. Theor. Phys. (Beijing, China)* **44** (2005) 833.
- [22] H.Y. Fan and H.L. Lu, *J. Phys. A: Math. Gen.* **37** (2004) 10993.
- [23] H.Y. Fan and H.L. Lu, *Int. J. Mod. Phys. B* **19** (2005) 799.
- [24] H.Y. Fan and X.B. Tang, *J. Opt. B: Quantum Semiclass. Opt.* **7** (2005) S765.
- [25] R.J. Glauber, *Phys. Rev.* **130** (1963) 2529; **131** (1963) 2766; J.R. Klauder and B.S. Skagerstam, *Coherence States*, World Scientific, Singapore (1985).
- [26] X.F. Xu, *Chin. Phys. (Beijing, China)* **15** (2006) 235.
- [27] A. Ekert and R. Jozsa, *Rev. Mod. Phys.* **68** (1996) 733; D.P. DiVincenzo, *Science* **270** (1995) 255; C.A. Huches, *et al.*, *Phys. Rev. A* **56** (1997) 1163.