

# Influence of Dark Energy on Gravitational Time Delay\*

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**Abstract** We investigate the gravitational time delay of light in the Schwarzschild black hole space-time surrounded by quintessence. With the analysis and numerical methods, we find that the gravitational time delay of light in the Schwarzschild black hole space-time surrounded by quintessence increases when the normalization factor  $c$  increases, and that the gravitational time delay also decreases when the quintessential state parameter  $\omega_q$  increases.

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**Key words:** time delay, quintessence

## 1 Introduction

The accelerating expansion of the universe is convincingly supported by a large number of astronomical observations, such as type Ia supernovae,<sup>[1–5]</sup> CMB,<sup>[6–8]</sup> and large scale structure.<sup>[9–12]</sup> The expansion implies the valuable contribution of matter with negative pressure to the evolution of the universe. The origin of accelerating expansion is regarded that the universe is dominated by an exotic component with the negative pressure called “dark energy”, which constitutes 70 percent of the energy density of the universe. There are several candidates for dark energy: The first is the cosmological constant,<sup>[13,14]</sup> and the second is the so-called dynamic candidates, such as Phantom,<sup>[15–18]</sup> quintessence,<sup>[19–21]</sup> K-essence,<sup>[22,23]</sup> and quintom.<sup>[24–26]</sup> The difference of these candidates for dark energy is the size of the parameter  $\omega_q$ , namely the ration of the pressure and energy density of the dark energy. For quintessence, the state equation is given by the relation between the pressure  $p_q$  and the energy density  $\rho_q$ , i.e.  $p_q = \omega_q \rho_q$  at  $\omega_q$  in the range of  $-1 \leq \omega_q \leq -1/3$ , which causes the acceleration. The borderline case of  $\omega_q = -1$  of the extraordinary quintessence covers the cosmological constant term. As has been recognized,<sup>[27–29]</sup> the acceleration is a challenge for a consistent theory of quantum gravity because of the outer horizon, which does not allow one to introduce the observable  $S$ -matrix in terms of asymptotic past and future states. The outer horizon of de Sitter space differs significantly from the inner horizon of a black hole, which has asymptotically flat space far away from the black hole.

A lot of gravitational effects in the solar system and binary systems are well described by means of the Einstein general relativity.<sup>[30,41]</sup> In general relativity, the repulsion necessary to obtain an accelerated expansion of

the universe can be provided by the inclusion of a vacuum energy. This corresponds to the well-known modification of the Einstein equations consisting of the addition of a cosmological term. Observational data suggested the cosmological constant  $\Lambda \sim 10^{-52} \text{ m}^{-2}$ . Alternatively, the vacuum energy can be considered as a dynamical field, such as quintessence.

We all know that a lot of test gravitational effects have supported the general relativity, such as gravitational red-shift, deflection of light, gravitational time delay, perihelion shift and geodetic precession. Kiselev<sup>[42]</sup> constructed a general solution to the spherically symmetric Einstein equations for the quintessence surrounded the Schwarzschild black hole. In this paper we plan to investigate the gravitational frequency-shift and the deflection of light in the Schwarzschild black hole space-time surrounded by the quintessence. It is a very value case to understand the accelerating expansion of the universe and properties of the dark matter and dark energy.

This paper is organized as follows: In Sec. 2, we briefly review the space-time structure of the Schwarzschild black hole surrounded by the quintessence. In Sec. 3, we straightly calculate the gravitational time delay of light in the Schwarzschild black hole space-time surrounded by quintessence. A brief conclusion is given in the last section.

## 2 Schwarzschild Black Hole Space-Time Surrounded by Quintessence

In this section, we briefly review the Schwarzschild black hole space-time surrounded by quintessence.<sup>[42]</sup> Following the notation of Ref. [43], the metric of the Schwarzschild black hole space-time surrounded by quintessence takes the form,

$$ds^2 = e^\nu dt^2 - e^\lambda dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2), \quad (1)$$

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with the function  $\nu = \nu(r)$  and  $\lambda = \lambda(r)$ . If we define the normalization of gravitational constant  $G$  by  $4\pi G = 1$ , the Einstein equations have the form

$$2T_t^t = -e^{-\lambda} \left( \frac{1}{r^2} - \frac{\lambda'}{r} \right) + \frac{1}{r^2}, \quad (2)$$

$$2T_r^r = -e^{-\lambda} \left( \frac{1}{r^2} - \frac{\nu'}{r} \right) + \frac{1}{r^2}, \quad (3)$$

$$2T_\theta^\theta = 2T_\varphi^\varphi = -\frac{1}{2} e^{-\lambda} \left( \nu'' + \frac{\nu'^2}{2} + \frac{\nu' - \lambda'}{r} - \frac{\nu' \lambda'}{2} \right). \quad (4)$$

For quintessence, the state equation is given by the relation between the pressure  $p_q$  and the energy density  $\rho_q$ ,

$$p_q = \omega_q \rho_q, \quad (5)$$

$$\omega_q = \frac{1}{3} \alpha. \quad (6)$$

The quintessential state has

$$-1 < \omega_q < 0 \Rightarrow -3 < \alpha < 0. \quad (7)$$

The additivity and linearity condition which fixes the free parameter of the energy-momentum tensor for the matter implies

$$T_t^t = T_r^r = \rho_q, \quad (8)$$

$$T_\theta^\theta = T_\varphi^\varphi = -\frac{1}{2} \rho_q (3\omega_q + 1). \quad (9)$$

So the spherically symmetric metric of the Schwarzschild black hole space-time surrounded by quintessence is

$$ds^2 = \left( 1 - \frac{2M}{r} - \frac{c}{r^{3\omega_q+1}} \right) dt^2 - \left( 1 - \frac{2M}{r} - \frac{c}{r^{3\omega_q+1}} \right)^{-1} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2), \quad (10)$$

where  $M$  is the mass of the black hole,  $\omega_q$  is the quintessential state parameter,  $c$  is the normalization factor, which depends on

$$\rho_q = -\frac{c}{2} \frac{3\omega_q}{r^{3(1+\omega_q)}},$$

and  $\rho_q$  is the density quintessence. The above exact spherically symmetric solution for Einstein equations describes the black hole surrounded the quintessential matter with the energy-momentum tensor, which satisfies the additivity and linearity condition in accordance with Eqs. (8) and (9).

### 3 Gravitational Time Delay

The time delay in the gravitational field is a famous evidence of Einstein general relativity theory. In order to consider the standard measurement of the gravitational time delay of light in the Schwarzschild black hole space-time surrounded by quintessence, the time interval between the emission of the first pulse and the reception of the reflected pulse must be measured. Here we assume that  $r_S$  and  $r_R$  are the distances between the Sun and the Earth and the reflector, respectively. The time interval between the emission and return of the pulse as measured is

$$\Delta t = 2t(r_S, r_A) + 2t(r_R, r_A). \quad (11)$$

We construct the Lagrangian of the motion equation of light rays

$$L = \left( 1 - \frac{2M}{r} - \frac{c}{r^{3\omega_q+1}} \right) \dot{t}^2 - \left( 1 - \frac{2M}{r} - \frac{c}{r^{3\omega_q+1}} \right)^{-1} \dot{r}^2 - r^2 \dot{\varphi}^2, \quad (12)$$

where a dot stands for differentiation with respect to the affine parameter  $\gamma$ , here we restrict the motion to the equatorial plane. Because the metric (10) is independent of  $t$  and  $\varphi$ , so

$$E \equiv \left( 1 - \frac{2M}{r} - \frac{c}{r^{3\omega_q+1}} \right) \frac{dt}{d\gamma}, \quad J \equiv r^2 \frac{d\varphi}{d\gamma}. \quad (13)$$

The energy and angular momentum of the particle along the orbit, respectively, are conserved. For the light the Lagrangian (12) vanishes, so we have

$$\frac{d\varphi}{dr} = \pm \frac{1}{r^2} \left[ \frac{1}{k^2} - \frac{1 - (2M/r) - (c/r^{3\omega_q+1})}{r^2} \right]^{-1/2}, \quad (14)$$

where  $k = J/E$ . The sign  $\pm$  is related to increasing/decreasing  $r$ . The distance of closest approach  $r_A$  is defined by

$$\left. \frac{dr}{d\varphi} \right|_{r=r_A} = \frac{1}{k^2} - \frac{1 - (2M/r_A) - (c/r_A^{3\omega_q+1})}{r_A^2} = 0. \quad (15)$$

From Eqs. (13) and (14), we get

$$\frac{dt}{dr} = \pm \frac{1}{k[1 - (2M/r) - (c/r^{3\omega_q+1})]} \left[ \frac{1}{k^2} - \frac{1 - (2M/r) - (c/r^{3\omega_q+1})}{r^2} \right]^{-1/2} \quad (16)$$

and we integrate the above equation, so

$$t(r, r_A) = \int_{r_A}^r \frac{dr}{k[1 - (2M/r) - (c/r^{3\omega_q+1})]} \left[ \frac{1}{k^2} - \frac{1 - (2M/r) - (c/r^{3\omega_q+1})}{r^2} \right]^{-1/2}, \quad (17)$$

where we replace  $k$  by  $r_A$  using Eq. (15)

$$t(r, r_A) = \int_{r_A}^r \frac{dr}{1 - (2M/r) - (c/r^{3\omega_q+1})} \left\{ 1 - \frac{[1 - (2M/r) - (c/r^{3\omega_q+1})]r_A^2}{[1 - (2M/r_A) - (cr_A^{3\omega_q+1})]r^2} \right\}^{-1/2}. \quad (18)$$

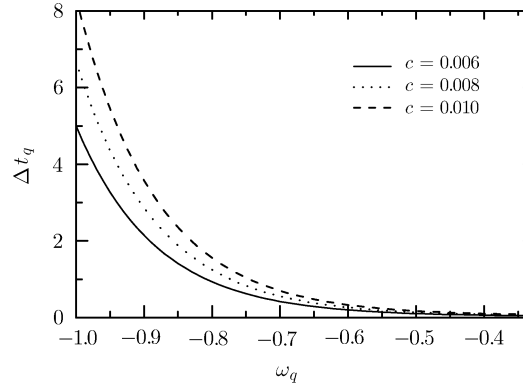
The result of this integration in the Schwarzschild space-time can be found in Ref. [44]. For small mass  $M$  we neglect the interaction term of  $M$  and  $c$ , so the first-order correction to time delay which produces by quintessence is

$$t_q(r, r_A) = c \left[ \frac{r}{\sqrt{r^2 - r_A^2}} + \frac{r^{-3\omega_q}}{\sqrt{r^2 - r_A^2}} - \frac{r_A^2 r (r^{-1-3\omega_q} - r_A^{-1-3\omega_q})}{2\sqrt{(r^2 - r_A^2)^3}} \right]. \quad (19)$$

So the time interval (11) between the emission and return of the pulse as measured is

$$\Delta t_q = 2c \left[ \frac{r_S}{\sqrt{r_S^2 - r_A^2}} + \frac{r_S^{-3\omega_q}}{\sqrt{r_S^2 - r_A^2}} - \frac{r_A^2 r_S (r_S^{-1-3\omega_q} - r_A^{-1-3\omega_q})}{2\sqrt{(r_S^2 - r_A^2)^3}} \right. \\ \left. + \frac{r_R}{\sqrt{r_R^2 - r_A^2}} + \frac{r_R^{-3\omega_q}}{\sqrt{r_R^2 - r_A^2}} - \frac{r_A^2 r_R (r_R^{-1-3\omega_q} - r_A^{-1-3\omega_q})}{2\sqrt{(r_R^2 - r_A^2)^3}} \right]. \quad (20)$$

In order to straightly analyze the gravitational time delay by the quintessence, we numerically investigate Eq. (20), as shown in Fig. 1. We can find that the gravitational time delay of the light increases when normalization factor  $c$  increases, and that the gravitational time delay also decreases when the quintessential state parameter  $\omega_q$  increases. We also find that the effluence of the normalization factor  $c$  will vanish when the quintessential state parameter  $\omega_q$  increases enough.



**Fig. 1** Gravitational time delay  $\Delta t_q$  of light in the Schwarzschild space-time surrounded by the quintessence for  $r_R = 2r_S = 10r_A$ ,  $r_A = 1$ , the horizon axis is  $-1 < \omega_q < -1/3$ .

## 4 Conclusion

In this paper we have investigated gravitational time delay  $\Delta t_q$  of light in the Schwarzschild space-time surrounded by the quintessence. Under the condition of  $c/r^{3\omega_q+1} \ll 2M/r$  and neglecting the interaction of  $c$  and  $M$ , we found that the gravitational time delay of the light in the Schwarzschild black hole space-time surrounded by quintessence increases when the value of the normalization factor  $c$  increases, and that the gravitational time delay also decreases when the quintessential state parameter  $\omega_q$  increases, which is shown in Fig. 1. At the same time, we can find that the effluence of the normalization factor  $c$  will vanish when the quintessential state parameter  $\omega_q$  increases enough.

## References

- [1] S. Perlmutter, *et al.*, *Astrophys. J.* **517** (1999) 565.
- [2] R.A. Knop, *et al.*, *Astrophys. J.* **598** (2003) 102.
- [3] A.G. Riess, *et al.*, *Astrophys. J.* **607** (2004) 665.
- [4] A.G. Riess, *et al.*, *Astrophys. J.* **116** (1998) 1009 (preprint Astro-ph/9805201).
- [5] X. Zhang and F.Q. Wu, *Phys. Rev. D* **72** (2005) 043524.
- [6] P. De Bernardis, *Nature (London)* **404** (2000) 955.
- [7] N.W. Halverson, *et al.*, *Astrophys. J.* **568** (2002) 38.
- [8] R. Lamon and R. Durrur, *Phys. Rev. D* **73** (2006) 023507.
- [9] D.J. Bacon, *Mon. Not. R. Astron. Soc.* **318** (2000) 625.
- [10] D.J. Bacon, *Mon. Not. R. Astron. Soc.* **344** (2003) 673.
- [11] C. Takeshi, *Astrophys. J.* **509** (1998) 74 (preprint gr-qc/9903094).
- [12] M. Tegmark and A.S. Michael, *Phys. Rev. D* **69** (2004) 103501.

- [13] T. Padmanabham, Phys. Rep. **380** (2003) 235.
- [14] J.S. Alcaniz, Phys. Rev. D **69** (2004) 083501.
- [15] R.R. Caldwell and D. Rahul, Phys. Lett. B **545** (2002) 23.
- [16] L.P. Chimento and R. Lazkoz, Phys. Rev. Lett. **91** (2003) 211301.
- [17] R.R. Caldwell, D. Rahul, and J.S. Paul, Phys. Rev. Lett. **80** (1998) 1582.
- [18] Sahniv and L.M. Wang, Phys. Rev. D **62** (2000) 103517.
- [19] S. Capozziello, *et al.*, Class. Quantum Grav. **23** (2006) 1205.
- [20] A. Vikman, Phys. Rev. D **71** (2005) 023515.
- [21] R. Rogerio and A.F. Joshua, astro-ph/0611242v1 (2006).
- [22] T. Chiba, O. Takahiro, and Y. Masahide, Phys. Rev. D **62** (2000) 023511.
- [23] R.J. Scherrer, Phys. Rev. Lett. **93** (2004) 011301.
- [24] H. Wei, R.G. Cai, and D.F. Zeng, Class. Quantum Grav. **22** (2005) 3189.
- [25] G.B. Zhao, T.Q. Xia, M.Z. Li, B. Feng, and X.M. Zhang, Phys. Rev. D **72** (2005) 123515.
- [26] B. Feng, T.Q. Xia, and M.Z. Li, Phys. Lett. B **20** (2006) 2075.
- [27] S. Hellerman and O. Takahiro, JHEP **06** (2001) 003 [hep-ph/0104180].
- [28] T. Banks and Fischler, preprint hep-th/0102077 (2001).
- [29] E. Witten, preprint hep-th/0106109 (2001).
- [30] K. Valeria, K. Juttaand, and L. Claus, preprint gr-qc/0602002 (2006).
- [31] J.H. Chen and Y.J. Wang, Class. Quantum Grav. **20** (2003) 3897.
- [32] J.H. Chen and Y.J. Wang, Acta. Phys. Sin. **50** (2001) 1833 (in Chinese).
- [33] Y.J. Wang and Z.M. Tang, Acta. Phys. Sin. **50** (2001) 1829 (in Chinese).
- [34] Y.J. Wang and Z.M. Tang, Acta. Phys. Sin. **50** (2001) 2284 (in Chinese).
- [35] Y.J. Wang and D. Cao, Chin. Phys. **13** (2004) 579.
- [36] J.H. Chen and Y.J. Wang, Chin. Phys. **12** (2003) 836.
- [37] J.H. Chen and Y.J. Wang, Chin. Phys. **13** (2004) 583.
- [38] J.H. Chen and Y.J. Wang, Chin. Phys. **14** (2005) 1282.
- [39] J.H. Chen and Y.J. Wang, Chin. Phys. **15** (2006) 1705.
- [40] Y.J. Wang and Z.M. Tang, Chin. Phys. **10** (2001) 679.
- [41] J.D. Anderson, *et al.*, Phys. Rev. D **65** (2002) 082004.
- [42] V.V. Kiselev, Class. Quantum Grav. **20** (2003) 1187.
- [43] L.D. Landauand and E.M. Lifshitz, *Field Theory*, Nauka, Moscow (1978) p. 357.
- [44] J.B. Hartle, *Gravitation, An Introduction to Einstein's General Relativity*, Addison Wesley, San Francisco (2003) p. 329.