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Squeezing of Phonons in Photon-Phonon Interaction via Anti-Stokes Light*

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Abstract *A model of the photon-phonon interaction via anti-Stokes light has been proposed to investigate the squeezing properties of phonon mode. The behavior of initial anti-Stokes light with complex statistics has been studied. It is shown that the squeezing of phonon mode can appear under certain conditions.*

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Key words: squeezed state, photon-phonon interaction, anti-Stokes light

Squeezing of light is a purely quantum phenomenon which has no classical counterpart. Squeezed states in quantum optics are distinguished by the property that the quantum fluctuations in one of the field quadratures are smaller than those associated with coherent state or a vacuum. Although a number of theoretical and experimental work have been done on squeezed light,[1] the investigation of squeezed states of phonons in solids especially in dielectric crystals is a new interesting subject in solid state physis and phonon physics, since the quantum properties of crystals are affected by the nonclassical effect of squeezed phonons through the interaction of phonons and other quanta.

In this paper, we shall propose a model of photon-phonon interaction using anti-Stokes light. We shall investigate, according to quantum theory, the squeezing effect of phonons resulted from the interaction of acoustical and light modes in dielectric crystals.

We consider the system of photon-phonon interaction depending on Raman-induce Kerr effect.[2] As shown in Fig. 1, when a Raman-active crystal is simultaneously irradiated by two exciting radiation sources, namely by a pump laser wave (frequency ω_L) and a linearly polarized anti-Stokes wave (frequency ω_a), the nonlinear third order interaction gives rise to a new anti-Stokes wave that is polarized perpendicularly to the wave originally incident on the crystal. We wish to transfer the nonclassical properties of light to acoustic field through photon-phonon interaction.

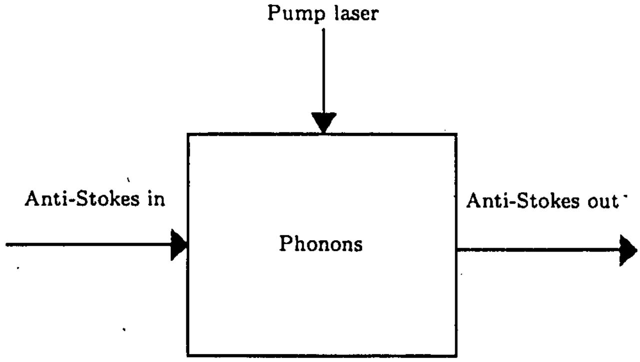

Fig. 1. A model of photon-phonon interaction using anti-Stokes light.

In our analysis, we only consider the transverse optical (TO) phonons in the crystal because they have a stronger coupling with the light field. For the TO phonon field, we make a long

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wavelength approximation.[3] Thus TO phonon mode can be described by a simple harmonic oscillator. In addition, for the pump field we make a constant approximation, thus the pump field can be treated classically. Under the above approximation, the interaction Hamiltonian of the system can be written as[2,3]

$$H = \hbar\Omega a^\dagger a + \hbar\omega_a a_a^\dagger a_a + \hbar R E_L\,\mathrm{e}^{-\mathrm{i}\omega_L t} a_a^\dagger a + \hbar R^* E_L^*\,\mathrm{e}^{\mathrm{i}\omega_L t} a^\dagger a_a\,, \tag{1}$$

where a ($a^\dagger$) and a_a ($a_a^\dagger$) are the annihilation (creation) operators of the phonons and the anti-Stokes mode respectively, Ω is the frequency of the phonon mode, R is the coupling constant between the anti-Stokes mode and the phonon mode, E_L is the classical amplitude of the pumping field.

In the following we just consider the case of no losses in phonon mode and the anti-Stokes mode, which is reasonable for the crystal placed at very low temperature and the system without resonant cavity of anti-Stokes mode.

According to Heisenberg motion equation, we have

$$\mathrm{d}a/\mathrm{d}t = -\mathrm{i}\Omega a - \mathrm{i}R^* E_L^*\,\mathrm{e}^{\mathrm{i}\omega_L t} a_a\,, \qquad \mathrm{d}a_a/\mathrm{d}t = -\mathrm{i}\omega_a a_a - \mathrm{i}R E_L\,\mathrm{e}^{-\mathrm{i}\omega_L t} a\,. \tag{2}$$

Defining the following parameters

$$\beta = |RE_L|\,, \qquad a = (\omega_a - \omega_L - \Omega)/2\,, \qquad b = \sqrt{a^2 + \beta^2}\,, \tag{3}$$

we obtain the solution of Eq. (2)

$$\begin{aligned} a(t) &= \exp(-\mathrm{i}(\Omega + a)t)\{[\cos bt + \mathrm{i}(a/b)\sin bt]a(0) - \mathrm{i}(\beta/b)(\sin bt)a_a(0)\}, \\ a_a(t) &= \exp(-\mathrm{i}(\omega_a - a)t)\{[\cos bt - \mathrm{i}(a/b)\sin bt]a_a(0) - \mathrm{i}(\beta/b)(\sin bt)a(0)\}. \end{aligned} \tag{4}$$

Generally, the initial anti-Stokes field can be described by the normally ordered characteristic function as[4]

$$\chi_a(\eta, 0) = \mathrm{Tr}\,[\rho_a(0)\exp(\eta a_a^\dagger(0))\exp(-\eta^* a_a(0))] \tag{5}$$

or

$$\chi_a(\eta, 0) = \exp[-M_a(0)|\eta|^2 + 0.5(S_a^*(0)\eta^2 + S_a(0)\eta^{*2}) + \eta W_a^*(0) - \eta^* W_a(0)]\,, \tag{6}$$

where $\rho_a(0)$ is the initial density operator, $W_a(0)$ is the coherent signal, and $M_a(0)$ and $S_a(0)$ are related to the noncoherent part of the field. For the pure coherent state, $M_a(0) = S_a(0) = 0$; for the chaotic field $W_a(0) = S_a(0) = 0$; and for a squeezed coherent state, $W_a(0) = 0$ and

$$M_a(0) = 0.5[(4|S_a(0)|^2 + 1)^{1/2} - 1]\,, \quad S_a(0) = \exp(\mathrm{i}\Phi(0))\cosh(r)\sinh(r)\,, \tag{7}$$

here r is the squeezing parameter.

Similarly, we suppose the initial phonons can be described by $\chi(\eta, 0)$ with statistical characteristics $M(0), S(0), W(0)$, and after interaction time t, it is described by $\chi(\eta, t)$ with $M(t)$, $S(t)$ and $W(t)$.

Putting Eq. (4) into the expression of $\chi(\eta, t)$ (similar to Eq. (5)), using the well-known Bake–Hausdorff identity and the expressions of $\chi_a(\eta, 0)$, $\chi(\eta, 0)$, $\chi(\eta, t)$, we obtain

$$\begin{aligned} M(t) &= M(0)[\cos^2 bt + (a^2/b^2)\sin^2 bt] + M_a(0)(\beta^2/b^2)\sin^2 bt\,, \\ S(t) &= \exp(-2\mathrm{i}(\Omega + a)t)\{S(0)[\cos bt + \mathrm{i}(a/b)\sin bt]^2 - S_a(0)(\beta^2/b^2)\sin^2 bt\}, \\ W(t) &= \exp(-\mathrm{i}(\Omega + a)t)\{W(0)[\cos bt + \mathrm{i}(a/b)\sin bt] - W_a(0)(\mathrm{i}\beta/b)\sin bt\}. \end{aligned} \tag{8}$$

For a general state described by Eq. (6), there is a relation as follows:[4]

$$M(t) = Q(t) + N(t)\,, \tag{9}$$

where

$$Q(t) = 0.5[(4|S(t)|^2 + 1)^{1/2} - 1]\,, \tag{10}$$

and $N(t)$ is the noise phonon number which gives the noise level of the phonon mode at interaction time t. If $N(t) = 0$ for $M(t) \neq 0$, $S(t) \neq 0$, the phonon mode is a squeezed coherent state, and in this case $Q(t)$ shows the squeezing level of phonons, that is, the larger $Q(t)$, the higher the level of squeezing.

To discuss the squeezing effect of the phonon mode, we consider the initial state of phonon mode is a coherent state, i.e., $M(0) = S(0) = 0$, and the initial states of anti-Stokes mode are different situations.

(a) $S_a(0) = W_a(0) = 0$, i.e., the initial anti-Stokes light is a chaotic field. From Eqs (8) $\sim$ (10) we obtain

$$N(t) = M(t) = M_a(0)(\beta^2/b^2)\sin^2 bt\,, \qquad S(t) = 0\,,$$
$$W(t) = \exp(-\mathrm{i}(\Omega + a)t)W(0)[\cos bt + \mathrm{i}(a/b)\sin bt]\,. \tag{11}$$

It is clear that the phonon mode is a mixed state of a coherent state with a chaotic field, and the noise level changes periodically as the interaction time, but no squeezing effect presents.

(b) $M_a(0) = S_a(0) = 0$, i.e., the initial anti-Stokes field is a pure coherent state. In this case, we obtain

$$N(t) = M(t) = 0\,, \qquad S(t) = 0\,,$$
$$W(t) = \exp(-\mathrm{i}(\Omega + a)t)\{W(0)[\cos bt + \mathrm{i}(a/b)\sin bt] - W_a(0)(\mathrm{i}\beta/b)\sin bt\}, \tag{12}$$

which means that the phonon mode is also a coherent state.

(c) The initial anti-Stokes field is a squeezed coherent state given by Eq. (7). In this case, we get

$$M(t) = M_a(0)(\beta^2/b^2)\sin^2 bt\,, \quad S(t) = -\exp(-2\mathrm{i}(\Omega + a)t)S_a(0)(\beta^2/b^2)\sin^2 bt\,,$$
$$W(t) = \exp(-\mathrm{i}(\Omega + a)t)W(0)[\cos bt + \mathrm{i}(a/b)\sin bt]\,,$$
$$N(t) = M(t) - 0.5[(4|S(t)|^2 + 1)^{1/2} - 1]\,. \tag{13}$$

Substituting Eq. (7) into Eq. (13), we obtain

$$N(t) = 0.5[(4|S_a(0)|^2+1)^{1/2}-1](\beta^2/b^2)\sin^2 bt - 0.5\{[4|S_a(0)|^2(\beta^4/b^4)\sin^4 bt+1]^{1/2}-1\}. \tag{14}$$

In Fig. 2, solid curves show the noise level of phonon mode, as a function of time given by Eq. (14) for different detuning parameters ($a = 0,\ 0.5\beta,\ 1.0\beta$) and the fixed squeezing parameter $r = 2$ of initial anti-Stokes field. We see that, (a) in the case of non-detuning ($a = 0$), there are some critical times at which it is possible to get pure squeezed states. Those are $\beta t = 1.6,\ 4.7,\ 7.8$, etc. Here we should note that the sharp peaks in the picture ($\beta t = 0,\ 3.14,\ 6.28,\ 9.42$ etc.) correspond to coherent states since $M(t) = S(t) = 0$ at those times. However, in cases (b) and (c) of detuning the squeezing effect of phonon mode cannot appear when the initial Stokes field is a pure squeezed state.

Similarly, we obtain the noise level $N_a(t)$ of anti-Stokes mode,

$$N_a(t) = 0.5[(4|S_a(0)|^2 + 1)^{1/2} - 1][\cos^2 bt + (a^2/b^2)\sin^2 bt] -$$
$$0.5\{\,[4|S_a(0)|^2[\cos^2 bt + (a^2/b^2)\sin^2 bt] + 1]^{1/2} - 1\}\,. \tag{15}$$

Dotted curves in Fig. 2 give $N_a(t)$ versus time βt for $a = 0,\ 0.5\beta,\ 1.0\beta$ and $r = 2$. It is shown that the state of anti-Stokes mode is periodically squeezed. Especially for $a = 0$, the state is not only squeezed ($\beta t = 0,\ 3.14,\ 6.28,\ 9.42$, etc.), but also coherent ($\beta t = 1.6,\ 4.7,\ 7.8$, etc.). When the anti-Stokes mode is coherent, the corresponding state of phonon is just squeezed. Therefore in the case of non-detuning, we can determine the squeezed state of phonons according to the state of anti-Stokes field.

Moreover, let $N(t) = 0$, from Eqs (10), (13) and (14), we find the squeezing level of phonons

$$Q(r) = 0.5\left(\sqrt{1 + \sinh^2(2r)} - 1\right), \tag{16}$$

and from this we know the squeezing level $Q(r)$ increases as the squeezing parameter r of initial light field increases for the time t corresponding to $N(t) = 0$.

In conclusion, the squeezing of phonons can be produced under certain conditions in the model. If the initial anti-Stokes field is a mixed, or chaotic or coherent state, it is impossible to get a squeezed state for phonon mode. Only if the initial anti-Stokes field is a pure squeezed state, can the phonon mode be squeezed for non-detuning case, and the squeezing level of phonons increases as that of initial light field. Moreover the squeezing of phonons can be determined by detecting anti-Stokes field. Although only the lossless is considered, these results will be useful to consider the preparation of phonon squeezed state in practical situations.

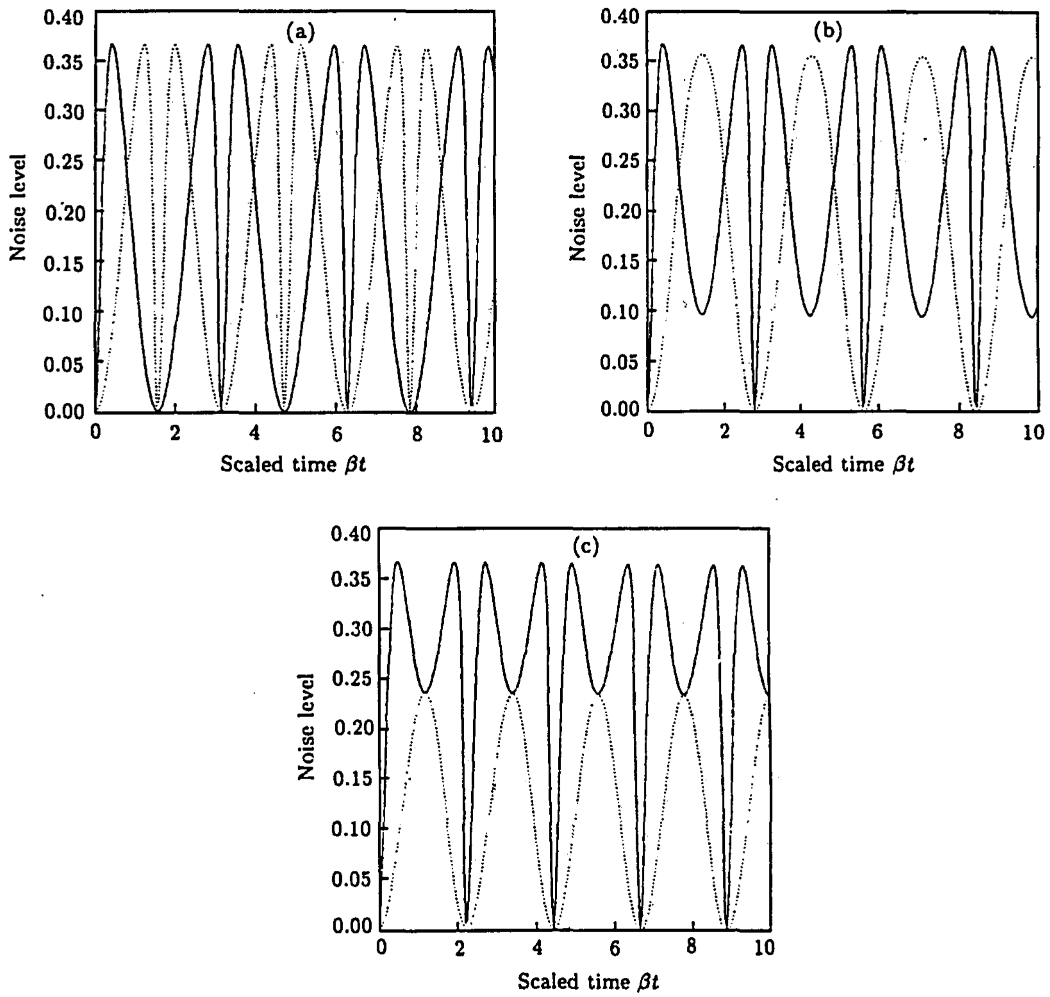

Fig. 2. Solid curves show the noise level $N(t)$ of phonons as a function of time βt for different detuning parameters (a) $a = 0$, (b) $a = 0.5\beta$ and (c) $a = 1.0\beta$, and the fixed squeezed parameter $r = 2$ of the initial anti-Stokes field. Dotted curves give the noise level $N_a(t)$ of anti-Stokes field versus time βt for the same parameters.

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