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Controlling Soliton Collisions of Condensates by Time-Dependence of Both Interactions and External Potential*

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Abstract *Considering time-dependence of both interactions and external potential, we analytically study the collisional behaviors of two bright solitons in Bose–Einstein condensates by using Darboux transformation. It is found that for a closed external potential, the soliton-soliton distance is decreased with nonlinearly increased interactions, while the amplitude of each soliton increases and its width decreases. For linearly increased interactions but nonlinearly decreased external potential, especially, the atom transfer between two solitons is observed, different from previous theory of no atom transfer in solitons collision in a fixed external potential. In addition, it is shown that the collisional type, such as head-on, "chase", or collision period between two solitons, can be controlled by tuning both interactions and external potential. The predicted phenomena can be observed under the condition of the current experiments and open possibilities for future application in atoms transport.*

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Key words: Bose–Einstein condensates, bright solitons, time-dependent interactions, time-dependent external potential

1 Introduction

Bright[1−3] and dark solions[4−10] in Bose–Einstein condensates (BECs) have been observed experimentally. The dark solitons were generated for the repulsive atomic interactions of BECs in the magnetic trap[4] or the optical trap.[10] The bright solitons were obtained for attractive atomic interactions in the optical trap,[1−2] while the bright gap solitons were observed for the repulsive atomic interactions in the weaker periodic potential.[3] Theoretically, the properties of the BECs are usually described by the Gross-Pitaevskii (GP) equation based on the mean-field approximation.[11−45] In the GP equation, the controlling macroscopical parameters are the s-wave scattering length (SL) a_s and the external potential.

The atomic interaction in BECs is represented by the SL.[46−47] The SL can be tuned by means of Feshbach resonance (FR) in the experiment, and even it is possible to change the sign of the SL from positive to negative.[48−49] The external potential, such as a harmonic potential, is usually considered to be fixed in the experiments.[8] It was found that the collisional properties of solitons in BECs depend strongly on the atomic interactions and external potential.[7−9,20,29] For the repulsive interactions and fixed attractive harmonic potential, it was shown that the solitons did not pass through each other at the collision point but there was a momentum exchange between them.[7−8] For the attractive interactions and fixed repulsive harmonic potential, it was predicted that the solitons can pass through each other and accomplish the collision.[20] In these two cases, the collisions between the solitons were elastic, and there was no atoms transfer between solitons.[8,20] Meanwhile, the collision points can be controlled by tuning the interactions through FR.[20] In fact, it is noticed that the external harmonic potential can be modulated experimentally,[50−51] which offers a good opportunity for manipulation of soliton collision in BECs. This means that the transverse trapping frequencies ω_x may be time-dependent, which should be taken into account for exploring the collisional behaviors of solitions.

In this paper, considering time-dependence of both interactions and external harmonic potential, we analytically study the collisional behaviors of two bright solitons in BECs by using Darboux transformation. It is found that there is atom transfer between two solitons with linear increased interactions and nonlinear decreased external potential, and two solitons can merge into one soliton with stronger attractive interactions. These phenomenon open possibilities for future application of BECs in atoms transport. Furthermore, the predicted phenomena, such as head-on, "chase" or collision period between two bright

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solitons in BECs, can be observed under the condition of the current experiments.

2 Molel

At the mean-field level, the evolution of BECs is described by the GP equation

$$\mathrm{i}\hbar\frac{\partial\Psi(r,\tau)}{\partial\tau}=\Big[-\frac{\hbar^2}{2M}\nabla^2+V_{\rm ext}+G|\Psi|^2\Big]\Psi\,,\qquad(1)$$

where M and Ψ are the atomic mass and the macroscopic wave functions of BECs, and $G=4\pi\hbar^2a_s/M$. The normalization of wave functions is taken as $\int\mathrm{d}r|\Psi|^2=N$ with N the atomic number. Experimentally, the tuning SL can be achieved by FR[48–49]

$$a_s(\tau)=a_{bg}\Big(1-\frac{\triangle}{B(\tau)-B}\Big)\,,\qquad(2)$$

where a_{bg} is the SL far from the resonance; $B(\tau)$ is the time-dependent external magnetic field; B and $\triangle$ are the resonance position and width, respectively. In the real experiments, various forms of the time dependence of $B(\tau)$ have been explored.[49] In theoretical studies, several forms of time-dependent SL have been proposed, such as the exponential function,[52–53] the periodic function,[54–58] and so on. We here choose that the SL is the linear, exponential, and periodic functions of the time, as discussed in Refs. [17] and [18]. And the time-dependent harmonic external potentials is given by[59–60]

$$V_{\rm ext}(r)=\frac{M}{2}[\omega_\perp^2(Y^2+Z^2)+\omega_1^2(\tau)x^2]\,.\qquad(3)$$

Here, $\omega_\perp$ and ω_1 are the radial and transverse trapping frequencies, respectively. When $\omega_\perp\gg\omega_1$, it results into a cigar-shaped BECs.

To study the dynamics of multisolitons of one-dimensional (1D) BECs, we consider the solutions

$$\Psi=\frac{\sqrt{N}}{\sqrt{\pi}a_\perp}u(X,\tau)\exp\Big(-\mathrm{i}\omega_\perp\tau-\frac{Y^2+Z^2}{2a_\perp^2}\Big)\,,\qquad(4)$$

with $a_\perp=\sqrt{\hbar/M\omega_\perp}$. Substituting Eq. (4) into Eq. (1), an effective 1D form is given by

$$\mathrm{i}\hbar\frac{\partial u(X,\tau)}{\partial\tau}=-\frac{\hbar^2}{2M}\frac{\partial^2u(X,\tau)}{\partial X^2}+\frac{GN}{2\pi a_\perp^2}|u|^2u+\frac{M\omega_1^2(\tau)}{2}X^2u\,.\qquad(5)$$

Subsequently, we introduce some dimensionless variables $X=a_\perp x$, $\tau=2t/\omega_\perp$, and $\psi=\sqrt{a_\perp}u$, so the Eq. (5) is reduced into

$$\mathrm{i}\psi_t+\psi_{xx}+g(t)|\psi|^2\psi+f(t)x^2\psi=0\,,\qquad(6)$$

where $g(t)=2a_s(t)N/a_\perp$ and $f(t)=-\omega_1^2(t)/\omega_\perp^2$.

3 Darboux Transformation and Exact Soliton Solutions

To obtain the exact solutions of a 1D nonlinear Schrödinger Eq. (6), we make use of the Darboux transformation.[21,61–63] The seed solution of Eq. (6) can be chosen as

$$\psi_0=\sqrt{g}Q\exp\Big[\mathrm{i}\int2g^2\,\mathrm{d}t\Big]\,,\qquad(7)$$

where $Q=\exp[-\mathrm{i}A(t)x^2-\mathrm{i}B(t)x-\mathrm{i}C(t)]$. Substituting Eq. (7) into Eq. (6), we have

$$A-\frac{g_t}{4g}=0\,,\qquad f(t)=A_t-4A^2\,,$$
$$B_t-4AB=0\,,\qquad C_t-B^2=0\,.\qquad(8)$$

Subsequently, we introduce the following Lax-pair

$$\Phi_x=U\Phi\,,\quad\Phi_t=V\Phi\,,\qquad(9)$$

where $\Phi=(\phi_1,\phi_2)^{\rm T}$, the superscript "T" denotes the matrix transpose. Here,

$$U=\begin{pmatrix}\lambda & p\\ -\bar p & -\lambda\end{pmatrix},\quad V=\begin{pmatrix}v_{11} & v_{12}\\ v_{21} & -v_{11}\end{pmatrix},$$

with

$$p=\sqrt{g(t)}\psi\bar Q\,,\quad v_{11}=2\mathrm{i}\lambda^2+\mathrm{i}g|\psi|^2+(4Ax+2B)\lambda\,,$$
$$v_{12}=2\mathrm{i}\sqrt{g}\psi\bar Q\lambda+\mathrm{i}\sqrt{g}\psi_x\bar Q+(2Ax+B)\sqrt{g}\psi\bar Q\,,$$
$$v_{21}=-2\mathrm{i}\sqrt{g}\bar\psi Q\lambda-\mathrm{i}\sqrt{g}\bar\psi_xQ-(2Ax+B)\sqrt{g}\bar\psi Q$$

(the overbar denotes the complex conjugate). From the compatibility condition $\partial^2\Phi/\partial x\partial t=\partial^2\Phi/\partial t\partial x$, we have $U_t-V_x+UV-VU=0$. By using the Darboux transformation, N soliton solution is given by

$$\psi_n=\psi_0+2\sum_{n=1}^{N}(\lambda_n+\bar\lambda_n)\frac{\phi_1[n,\lambda_n]\bar\phi_2[n,\lambda_n]Q}{\sqrt{g}\Phi[n,\lambda_n]^T\bar\Phi[n,\lambda_n]}\,,\qquad(10)$$

with

$$\Phi[n,\lambda]=(\lambda I-S[n-1])\cdots(\lambda I-S[1])\Phi[1,\lambda]\,.$$

Here

$$S_{l_1l_2}[n']=(\lambda_{n'}+\bar\lambda_{n'})\frac{\phi_{l_1}[n',\lambda_{n'}]\bar\phi_{l_2}[n',\lambda_{n'}]}{|\phi_1[n',\lambda_{n'}]|^2+|\phi_2[n',\lambda_{n'}]|^2}-\bar\lambda_{n'}\delta_{l_1l_2}\,,$$
$$l_1,l_2=1,2\,,\quad n'=1,2,\ldots,n-1,\quad n=2,3,\ldots,N\,.$$

To get the collisional behaviors, we here chose $N=2$. So, the exact 2-order solution of Eq. (6) is given by

$$\psi_2=\psi_0\Big(1+\frac{2F_1}{F_2}\Big)\,,\qquad(11)$$

where

$$F_1=(\lambda_{01}+\lambda_{02})^2(h_1+h_2)(k_1+k_2)-4\lambda_{01}\lambda_{02}(h_1k_1+h_2k_2+2h_3k_3+2\sqrt{\lambda_{01}^2-1}\sqrt{\lambda_{02}^2-1})\sin\varphi_1\sin\varphi_2)\,,$$

$$F_2=2\lambda_{02}\big(\lambda_{02}^2-\lambda_{01}^2)(h_1+h_2)(k_3+\mathrm{i}\sqrt{\lambda_{02}^2-1}\sin\varphi_2\big)+2\lambda_{01}\big(\lambda_{01}^2-\lambda_{02}^2)(k_1+k_2)(h_3+\mathrm{i}\sqrt{\lambda_{01}^2-1})\sin\varphi_1\big),$$

with

$$h_1=\cosh\theta_1-\cos\varphi_1\,,\quad k_1=\cosh\theta_2-\cos\varphi_2\,,$$
$$h_2=(2\lambda_{01}^2-1)\cosh\theta_1+2\lambda_{01}\sqrt{\lambda_{01}^2-1}\sinh\theta_1-\cos\varphi_1\,,$$
$$h_3=-\lambda_{01}\cosh\theta_1-\sqrt{\lambda_{01}^2-1}\sinh\theta_1+\lambda_{01}\cos\varphi_1\,,$$
$$k_2=(2\lambda_{02}^2-1)\cosh\theta_2+2\lambda_{02}\sqrt{\lambda_{02}^2-1}\sinh\theta_2-\cos\varphi_2\,,$$

$$k_3 = -\lambda_{02}\cosh\theta_2 - \sqrt{\lambda_{02}^2 - 1}\sinh\theta_2 + \lambda_{02}\cos\varphi_2 \,.$$

Here

$$\theta_i = 2\sqrt{\lambda_{0i}^2 - 1}\int(4Ax + 2B)g\,\mathrm{d}t \,,$$

$$\varphi_i = 4\sqrt{\lambda_{0i}^2 - 1}\int g^2\,\mathrm{d}t \,,$$

and $\lambda_i = \lambda_{0i}g$ ($i = 1, 2$, and λ_{0i} is constant)

4 Results and Discussion

As a typical example, we consider a BEC consisting of ^{7}Li. Based on the currently experimental conditions, the radial frequency is chosen as $\omega_\perp = \pi \times 100$ Hz, and the time and space units correspond to 6.4 ms and 5.4 μm in reality, respectively.[2] The FR position and width of ^{7}Li atoms are 725 G and 2 G, respectively.[2,49] The a_{bg} and atoms number are chosen as $4a_B$ (here a_B is Bohr radius)[49] and 10^3,[64] respectively.

4.1 *External Potential is Closed*

Here, we consider a ^{7}Li condensate is formed in a magneto-optical trap.[2] The external trap is turned off after the solitons was created. In this case, the strength of external potential is $f(t) = 0$. Then, we propose that the time-dependent SL is $a_s(t) = -51/(30 - ct)a_B$.

Figure 1 shows the corresponding space-time distributions of two bright solitons. From Fig. 1(a), one can see that two sharp peaks appear at $t = 0$ for a constant attractive interactions ($c = 0$). The width, amplitude, and position of each soliton keep unchanged with time. Such a localized bright soliton is mainly attributed to the constant interactions. For the increased interactions ($c = 0.5$), the space-time distributions of two solitons is shown in Fig. 1(b). With the increasing time, the width of each soliton decreases and its amplitude increases. The similar results can be found in Ref. [33]. Meanwhile, two bright solitons are heading towards each other, a collision taking place. The distance between two solitons becomes smaller. These results illustrate that the width and amplitude of each soliton, and the distance between two bright solitons can be manipulated by tuning the interactions.

The validity of the GP equation relies on the condition that the system be dilute and weakly interacting, $d|a_s(t)|^3 \ll 1$, where d is the average density of the condensate. In the real experiment of ^{7}Li atoms, the typical value of the atomic densities is 10^{13} cm^{-3}. In the Fig. 1, the absolute value of the SL is $|a_s(t)|_{\max} = 2.55a_B$, so that $d|a_s(t)|^3 < 10^{-8} \ll 1$ and thus GP equation is valid. Thereby, we can design an experimental protocol to control the distance between two solitons in BECs. Here, we briefly discuss the experimental setups. Firstly, we create a condensate in a magneto-optical trap and tune the magnetic field strength of FR technology $B(t) = 726.41$ G. Then, we close the magneto-optical trap and manipulate the magnetic field $B(t)$ ramped down from 726.41 G to 726.22 G. While the time $t \approx 128$ ms, one sees that the distance between two bright solitons decreases and two bright solitons take place a collision.

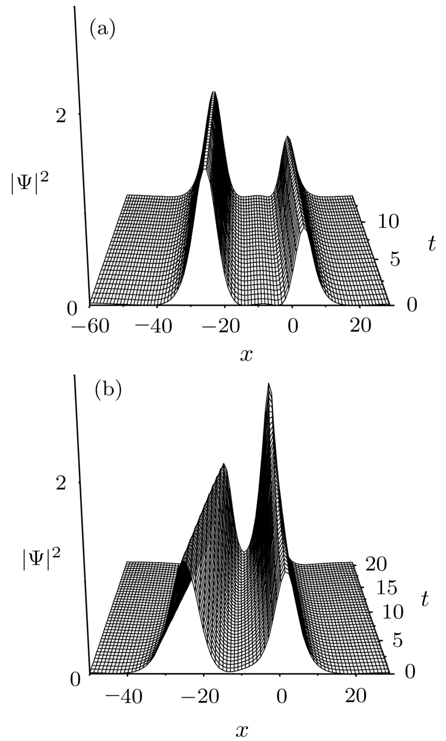

Fig. 1 The space-time distributions of two bright solitons in a BEC with the closed external potential and the attractive interactions of (a) constant ($c = 0$), (b) increase ($c = 0.5$). The other parameters used are $\lambda_{01} = 2.0$ and $\lambda_{02} = 2.5$.

4.2 *External Potential is Fixed*

We consider a ^{7}Li condensate is firstly created in a magneto-optical trap then loaded to a fixed harmonic potential.[2] To obtain the collisional behaviors between two solitons with a fixed harmonic potential, we choose the strength of harmonic potential $f(t) = -c^2/4$ and the time-dependent SL $a_s(t) = -5.1\exp(ct)a_B$.[21]

Figure 2 shows the collisional behaviors between two bright solitons in a condensate with the fixed external potential. At the initial time ($t = 0$), there appear two bright solitons. When the time increases to $t = 2$, one can see that the amplitude of each soliton increases and its width decreases. Meanwhile, the right bright soliton moves along the negative direction of x-axis, and the left bright soliton moves along the positive direction of x-axis. The distance between the two solitons becomes smaller. When the interactions further increases, the two bright solitons take place a head-on collision at $t = 10$. While $t = 15$, interestingly, it is observed that two solitons interact very strongly and merge into one soliton with a high peak and the narrowest width, which illustrates that two bright solitons only exist at the weaker atomic interactions. After the fusion, the soliton moves along the negative direction of x-axis due to the repulsive force of the external potential (see $t = 17$). In Fig. 2, the absolute value of the SL is $|a_s(t)| = 153a_B$ at $t = 17$, so that $d|a_s(t)|^3 < 10^{-2} \ll 1$ and thus GP equation is still valid. Thereby, we can briefly conclude that the experimental protocols are: firstly, a condensate containing two bright solitons is created in the magneto-optical trap; then, we load it to a fixed harmonic potential; finally, we manipulate the magnetic field $B(t)$ of FR technology exponentially ramped down as $\exp(-t/L)$ (here $L = 96$ ms) from 725.88 G to 725.08 G. When the time $t = 96$ ms, one can see that two solitons interact very strongly and merge into one soliton.

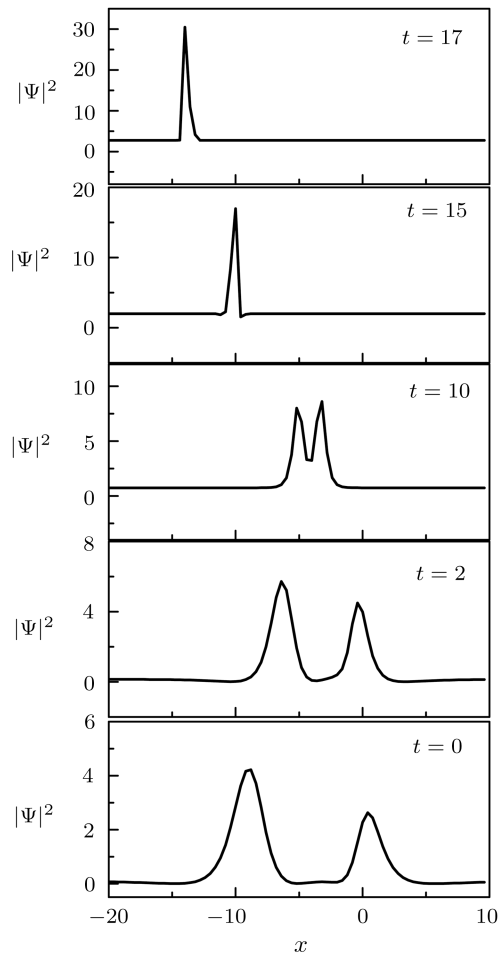

Fig. 2 The head-on collision between two bright solitons and the fusion of two bright solitons in a BEC with the exponential increased attractive interactions and the fixed external potential. The parameter used is $c = 0.2$. The other parameters used are the same with Fig. 1.

4.3 *External Potential is Time-Dependent*

In previous theory, the time-dependent external potential can generally be regarded as nonlinear decreased or periodic.[14,53] For studying the collisional behaviors between two solitons with a nonlinear decreased external potential, we choose the strength of external potential $f(t) = -c^2/(\sqrt{2} + \sqrt{2}ct)^2$ and the time-dependent SL $a_s(t) = -5.1(1 + ct)a_B$.

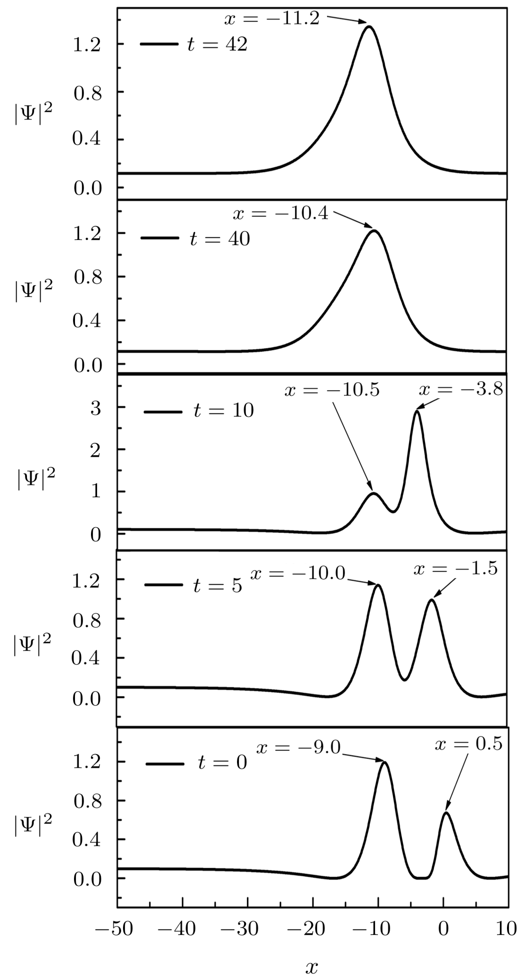

Fig. 3 The "chase" after collision between two bright solitons and the fusion of two bright solitons in a BEC with linearly increased attractive interactions and nonlinearly decreased external potential. The parameters used are $\lambda_{01} = 1.2$, $\lambda_{02} = 2.5$, and $c = 0.02$.

The corresponding collision properties between two bright solitons are plotted in Fig. 3. At the initial time ($t = 0$), the left bright soliton is at $x = -9.0$ and the right bright soliton is at $x = 0.5$. When $t = 5$, the left soliton is at $x = -10.0$ and the right solitons is at $x = -1.5$. It illustrates that both two solitons move along the negative direction of x-axis. And the velocity of the right soliton is bigger than that of the left soliton. That is to say, the right soliton chases after the left one. At $t = 10$, the left soliton is at $x = -10.5$ and the right one is at $x = -3.8$. This indicates that the propagating velocity of the left (right) soliton decreases (increases) due to the stronger attractive interactions between the two bright solitons. The two bright solitons interact and exhibit a "chase" collision. Meanwhile, the amplitude of right soliton increases but the amplitude of left soliton decreases, which means that there is atom transfer from left soliton to right soliton. While the time $t = 40$, all of atoms in left soliton have transferred to the right soliton, and two bright soliton merge into one soliton at $x = -10.4$. After the fusion, the soliton moves along the negative direction of x-axis due to the repulsive force of the external potential (as $t = 42$).

For exploring the collisional behaviors between two solitons with an external potential varying periodically with time, we propose that the external potential strength is $f(t) = -0.5w^2[\sin(wt)+0.5+0.5\cos^2(wt)]/[2+\sin(wt)]^2$ and the SL varies periodically with the time, $a_s(t) = -5.1[1 + 0.5\sin(wt)]a_B$. Meanwhile, we introduce T to represent the period of SL, i.e., $T = 2\pi/w$.

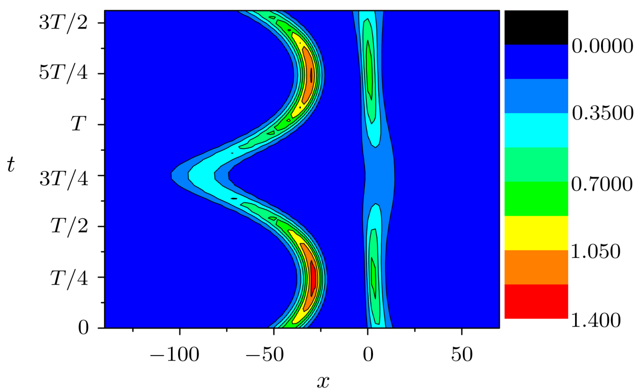

Fig. 4 (Color online) The periodic collisions between two bright solitons in a BEC with periodic varied both attractive interactions and external potential. The parameter used is $w = 0.1$. The other parameters used are the same with Fig. 1. T is the period of time-dependent interactions, and when $w = 0.1$, $T = 62.8$.

The corresponding collisional behaviors between two bright solitons are shown in Fig. 4. One can see that the two bright solitons are far apart at the initial time. While the time varies from 0 to $T/4$, the absolute value of SL increases. With the increasing SL, the interactions become stronger. As a result, the amplitude of each soliton increases and its width decreases. Meanwhile, the left bright soliton moves rightward, the distance between two solitons becomes smaller. Two solitons exhibit a collision at $t = T/4$. While the time t varies from $T/4$ to $3T/4$, the absolute value of the SL decreases, and the interactions become smaller. After collision, the left soliton moves along the negative direction of x-axis, but the right soliton is still not moved. So the distance between solitons increases. Meanwhile, the amplitude of each soliton decreases and its width increases. With the time going on, interestingly, the next similar collision can be observed at $t = 5T/4$. Thus, these collisions are periodic, and the frequencies of collisions depend strongly on the frequencies of interactions. In Figs. 3 and 4, the SL is small, the validity of GP equation is still satisfied. Thereby, the "chase" collision and periodic collisions between the two bright solitons can be observed under the current experimental conditions by means of tuning the both SL and transverse trapping frequencies ω_x.

5 Conclusions

In summary, we analytically study the collisional behaviors between two solitons in BECs, taken into account the time-dependence of both interactions and external potential. Applying the Darboux transformation, we obtain the exact two bright solitons solution. By numerically calculating, we find that the width and amplitude of each soliton, the distance between two bright solitons can be controlled by tuning the interactions. Especially, when the interactions exponential increases and the external potential is fixed, two bright solitons exhibit a head-on collision. While the interaction linear increases and the external potential nonlinearly decreases, two bright solitons show a "chase" collision and there is atom transform between two solitons. When the both interactions and external potential vary periodically with the time, the collisional orbit between two bright solions is also periodic, and the collisional frequencies depend strongly on the frequencies of the interactions. Moreover, two bright solitons can merge into one soliton with stronger attractive interactions. These phenomenon open possibilities for future application of BECs in atoms transport. We believe that these results will stimulate experiments to manipulate multisolitons in the BECs.

References

[1] K.E. Strecker, G.B. Partridge, A.G. Truscott, and R.G. Hulet, Nature (London) **417** (2002) 150.

[2] L. Khaykovich, F. Schreck, G. Ferrari, T. Bourdel, J. Cubizolles, L.D. Carr, Y. Castin, and C. Salomon, Science **296** (2002) 1290.

[3] B. Eiermann, T. Anker, M. Albiez, M. Taglieber, P. Treutlein, K.P. Marzlin, and M.K. Oberthaler, Phys. Rev. Lett. **92** (2004) 230401.

[4] S. Burger, K. Bongs, S. Dettmer, W. Ertmer, K. Sengstock, A. Sanpera, G.V. Shlyapnikov, and M. Lewenstein, Phys. Rev. Lett. **83** (1999) 5198.

[5] J. Denschlag, J.E. Simsarian, D.L. Feder, C.W. Clark, L.A. Collins, J. Cubizolles, L. Deng, E.W. Hagley, K. Helmerson, W.P. Reinhardt, S.L. Rolston, B.I. Schneider, and W.D. Phillips, Science **287** (2000) 97.

[6] B.P. Anderson, P.C. Haljan, C.A. Regal, D.L. Feder, L.A. Collins, C.W. Clark, and E.A. Cornell, Phys. Rev. Lett. **86** (2001) 2926.

[7] S. Stellmer, C. Becker, P. Soltan-Panahi, E.M. Richter, S. Dörscher, M. Baumert, J. Kronjäger, K. Bongs, and K. Sengstock, Phys. Rev. Lett. **101** (2008) 120406.

[8] A. Weller, J.P. Ronzheimer, C. Gross, J. Esteve, M.K. Oberthaler, D.J. Frantzeskakis, G. Theocharis, and P.G. Kevrekidis, Phys. Rev. Lett. **101** (2008) 130401.

[9] L. Shomroni, E. Lahoud, S. Levy, and J. Steinhauer, Nat. Phys. **5** (2009) 193.

[10] C. Becker, S. Stellmer, P. Soltan-Panahi, S. Dörscher, M. Baumert, E. Richter, J. Kronjäger, K. Bongs, and K. Sengstock, Nat. Phys. **4** (2008) 496.

[11] P.G. Kevrekidis, D.J. Frantzeskakis, and R. Carretero-Gonzá lez, *Emergent Nonlinear Phenomena in Bose–Einstein Condensates*, Springer-Verlag, Berlin (2008).

[12] J. Belmonte-Beitia, V.M. Pérez-García, V. Vekslerchik, and V.V. Konotop, Phys. Rev. Lett. **100** (2008) 164102.

[13] V.N. Serkin, A. Hasegawa, and T.L. Belyaeva, Phys. Rev. Lett. **98** (2007) 074102.

[14] V.N. Serkin, A. Hasegawa, and T.L. Belyaeva, Phys. Rev. A **81** (2010) 023610.

[15] V.N. Serkin, A. Hasegawa, and T.L. Belyaeva, J. Mod. Optic. **57** (2010) 1456.

[16] V.R. Kumar, R. Radha, and P.K. Panigrahi, Phys. Rev. A **77** (2008) 023611.

[17] A. Mohamadou, E. Wamba, S.Y. Doka, T.B. Ekogo, and T.C. Kofane, Phys. Rev. A **84** (2011) 023602.

[18] U.A. Khawaja, J. Phys. A: Math. Theor. **42** (2009) 265206.

[19] Z.Y. Yang, L.C. Zhao, T. Zhang, X.Q. Feng, and R.H. Yue, Phys. Rev. A **83** (2011) 066602.

[20] D.S. Wang, X.F. Zhang, P. Zhao, and W.M. Liu, J. Phys. B: At. Mol. Opt. Phys. **42** (2009) 245303.

[21] Z.X. Liang, Z.D. Zhang, and W.M. Liu, Phys. Rev. Lett. **94** (2005) 050402.

[22] E. Kengne and P.K. Talla, J. Phys. B: At. Mol. Opt. Phys. **39** (2006) 3679.

[23] F.K. Abdullaev and J. Garnier, Phys. Rev. A **75** (2007) 033603.

[24] Y.C. Wen, P.M. Zhang, and S.J. Yang, Commun. Theor. Phys. **56** (2011) 1031.

[25] D.L. Wang, X.H. Yan, and W.M. Liu, Phys. Rev. E **78** (2008) 026606.

[26] W.X. Zhang, D.L. Wang, Z.M. He, F.J. Wang, and J.W. Ding, Phys. Lett. A **273** (2008) 4407.

[27] Z.M. He, D.L. Wang, W.X. Zhang, F.J. Wang, and J.W. Ding, Chin. Phys. B **17** (2008) 3640.

[28] Z.Q. Zhang, D.L. Wang, X.Q. Luo, and J.W. Ding, Commun. Theor. Phys. **56** (2011) 440.

[29] B. Li, X.F. Zhang, Y.Q. Li, and W.M. Liu, J. Phys. B: At. Mol. Opt. Phys. **44** (2011) 175301.

[30] H.N. Xuan and M. Zuo, Commun. Theor. Phys. **56** (2011) 1035.

[31] H.Y. Yu, L.X. Pan, J.R. Yan, and J.Q. Tang, J. Phys. B: At. Mol. Opt. Phys. **42** (2009) 025301.

[32] S.J. Wang, C.L. Jia, D. Zhao, H.G. Luo, and J.H. An, Phys. Rev. A **68** (2003) 015601.

[33] X.F. Zhang, Q. Yang, J.F. Zhang, X.Z. Chen, and W.M. Liu, Phys. Rev. A **77** (2008) 023613.

[34] B. Wu, J. Liu, and Q. Niu, Phys. Rev. Lett. **88** (2002) 034101.

[35] Z. Chen and B. Wu, Phys. Rev. A **81** (2010) 043611.

[36] L. Wu, L. Li, and J.F. Zhang, Phys. Rev. A **80** (2009) 013617.

[37] J.K. Xue and P. Peng, Chin. Phys. B **15** (2006) 1149.

[38] C.W. Zhang, J. Liu, M.G. Raizen, and Q. Niu, Phys. Rev. Lett. **92** (2004) 054101.

[39] G.X. Huang, M.G. Velarde, and V.A. Makarov, Phys. Rev. A **64** (2001) 013617.

[40] X.Q. Luo, D.L. Wang, Z.Q. Zhang, J.W. Ding, and W.M. Liu, Phys. Rev. A **84** (2011) 033803.

[41] G.X. Huang, Chin. Phys. B **13** (2004) 1866.

[42] F.Kh. Abdullaev, J.G. Caputo, R.A. Kraenkel, and B.A. Malomed, Phys. Rev. A **67** (2003) 013605.

[43] D.S. Wang, X.H. Hu, and W.M. Liu, Phys. Rev. A **82** (2010) 023612.

[44] D.S. Wang, X.H. Hu, J.P. Hu, and W.M. Liu, Phys. Rev. A **81** (2010) 025604.

[45] D.S. Wang, S.W. Song, B. Xiong, and W.M. Liu, Phys. Rev. A **84** (2011) 053607.

[46] F. Dalfovo, S. Giorgini, L.P. Pitaevskii, and S. Stringari, Rev. Mod. Phys. **71** (1999) 463.

[47] A.J. Leggett, Rev. Mod. Phys. **73** (2001) 307.

[48] S. Lnouye, M.R. Andrews, J. Stenger, H.J. Miesner, D.M. Stamper-Kurn, and W. Ketterle, Nature (London) **392** (1998) 151.

[49] C. Chin, R. Grimm, P. Julienne, and E. Tiesinga, Rev. Mod. Phys. **82** (2010) 1225.

[50] C. Raman, M. Köhl, R. Onofrio, D.S. Durfee, C.E. Kuklewicz, Z. Hadzibabic, and W. Ketterle, Phys. Rev. Lett. **83** (1999) 2502.

[51] R. Onofrio, C. Raman, J.M. Vogels, J.R. Abo-Shaeer, A.P. Chikkatur, and W. Ketterle, Phys. Rev. Lett. **85** (2000) 2228.

[52] F.K. Abdullaev, A.M. Kamchatnov, V.V. Konotop, and V.A. Brazhnyi, Phys. Rev. Lett. **90** (2003) 230402.

[53] V.M. Pérez-García, V.V. Konotop, and V.A. Brazhnyi, Phys. Rev. Lett. **92** (2004) 220403.

[54] H. Saito and M. Ueda, Phys. Rev. Lett. **90** (2003) 040403.

[55] D.E. Pelinovsky, P.G. Kevrekidis, and D.J. Frantzeskakis, Phys. Rev. Lett. **91** (2003) 240201.

[56] G.D. Montesinos, V.M. Pérez-García, and H. Michinel, Phys. Rev. Lett. **92** (2004) 133901.

[57] V.V. Konotop and P. Pacciani, Phys. Rev. Lett. **94** (2005) 240405.

[58] F.K. Abdullaev and J. Garnier, Phys. Rev. E **75** (2007) 033603.

[59] R. Atre, P.K. Panigrahi, and G.S. Agarwal, Phys. Rev. E **73** (2006) 056611.

[60] G. Theocharis, Z. Rapti, P.G. Kevrekidis, D.J. Frantzeskakis, and V.V. Konotop, Phys. Rev. A **67** (2003) 003610.

[61] Z.X. Liang, Z.D. Zhang, and W.M. Liu, Mod. Phys. Lett. A **21** (2006) 383.

[62] Z.D. Li, Q.Y. Li, L. Li, and W.M. Liu, Phys. Rev. E **76** (2007) 026605.

[63] Z.D. Li, Q.Y. Li, P.B. He, Z.G. Bai, and Y.B. Sun, Ann. Phys. **322** (2007) 2945.

[64] J.M. Gerton, D. Strekalov, I. Prodan, and R.G. Hulet, Nature (London) **408** (2000) 692.