

The Effect of Magnetic Field on an Asymmetrical Gaussian Potential Quantum Well Qubit*

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Abstract Under the influence of an applied magnetic field (MF), the eigenenergies and the eigenfunctions of the ground and the first excited states (GFES) are obtained by using a variational method of the Pekar type (VMPT) in a strong electron-LO-phonon coupling asymmetrical Gaussian potential quantum well (AGPQW). This AGPQW system may be employed as a two-level qubit. The numerical results have indicated (i) that when the electron situates in the superposition state of the GFES, we obtain the time evolution and the coordinate change of the electron probability density in the AGPQW, (ii) that due to the presence of the asymmetrical potential in the growth direction of the AGPQW, the probability density shows double-peak configuration, whereas there is only one peak if the confinement is a two dimensional symmetric one in the xy plane of the AGPQW, (iii) that the oscillatory period is a decreasing function of the cyclotron frequency of the MF, the height of the AGPQW and the polaron radius, (iv) and that as the range of the confinement potential (RCP) decreases the oscillatory period will decrease firstly and then increase and it will take a minimum when $R = -0.234$ nm.

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1 Introduction

The progress of low-dimensional semiconductor structures, such as quantum dot (QD), quantum well (QW), etc has stimulated greatly by the nanofabrication technology.^[1–4] Due to their potential applications in quantum computing, which combines computer science with quantum mechanics and is a fast growing research field.^[5] Quantum two-level systems with long coherence time are candidates for qubit, which is the basic component of quantum computation and quantum information process.^[6–9] In the present work, we employ the asymmetrical Gaussian potential quantum well's (AGPQW's) ground and the first excited states (GFES) as a two level system, then we will investigate the qubit's temporal and spatial distribution and the properties of its oscillatory period. However, quantum systems are very vulnerable and the external fields, such as magnetic field (MF), etc., will destroy the quantum coherence for information storing.^[10] In fact, such a topic has been addressed very recently. Uchiyama *et al.* pointed out that both the localization and the inter QW transfer of excitons could be regulated by an external MF, which provided the basis for functional devices operating without any wiring.^[11] Nam *et al.* obtained the expression for the electron distribution function in QWs with parabolic potential in the presence of MF.^[12] Jin studied theoretically the effect of the in-plane MF on two-dimensional electron gas transport in GaAs/InGaAs single QW structure, which showed that the in-plane MF

led to an anisotropic scattering probability.^[13] Yesilgul *et al.* studied the effects of the MF on impurity binding energy in a $\text{Ga}_{1-x}\text{In}_x\text{N}_y\text{As}_{1-y}/\text{GaAs}$ QW.^[14] Moreover, the structure of AGPQW possessed novel optical properties, which attracted researchers' interests. The influences of the height of the AGPQW, the range of the confinement potential (RCP), and the applied electric field on the optical coefficients were investigated by the effective mass approximation and the compact density matrix method or the perturbation theory in the AGPQW.^[15–17] However, the influences of MF on the AGPQW' qubit were neglected in the above works. This article is to study the effects of MF on the AGPQW' qubit, which is the motivation of this work.

2 Theoretical Model

The electron under consideration is moving in a polar AGPQW crystal, and is interacting with bulk LO phonons, under the influence of an MF along the z-direction with vector potential of $\mathbf{A} = \mathbf{B}(-y/2, x/2, 0)$.^[18] Within the framework of effective mass approximation, the Hamiltonian of the electron-phonon interaction system can be written as

$$H = \frac{1}{2m} \left(p_x - \frac{\bar{\beta}^2}{4} y \right)^2 + \frac{1}{2m} \left(p_y + \frac{\bar{\beta}^2}{4} x \right)^2 + \frac{p_z^2}{2m} + \sum_{\mathbf{q}} \hbar \omega_{\text{LO}} a_{\mathbf{q}}^\dagger a_{\mathbf{q}}$$

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$$+ \sum_{\mathbf{q}} [V_{\mathbf{q}} a_{\mathbf{q}} \exp(i\mathbf{q} \cdot \mathbf{r}) + \text{h.c.}] + V(z), \quad (1)$$

where

$$V(z) = \begin{cases} -V_0 \exp(-\frac{z^2}{2R^2}), & z \geq 0, \\ \infty, & z < 0, \end{cases} \quad (2)$$

where $\bar{\beta}^2 = 2eB/c$, is the band mass of the electron, $a_{\mathbf{q}}^\dagger(a_{\mathbf{q}})$ denotes the creation (annihilation) operator of the bulk LO phonon with wave vector \mathbf{q} , and \mathbf{p} and $\mathbf{r} = (x, y, z) = (\rho, z)$ are the momentum and the position vector of the electron. $V(z)$ is the z -directional potential that represents the growth direction of the AGPQW.^[15–17] V_0 and R are the height of the AGPQW and the RCP, respectively. $V_{\mathbf{q}}$ and α in Eq. (1) are

$$V_{\mathbf{q}} = i \left(\frac{\hbar\omega_{\text{LO}}}{q} \right) \left(\frac{h}{2m\omega_{\text{LO}}} \right)^{1/4} \left(\frac{4\pi\alpha}{V} \right)^{1/2},$$

$$\alpha = \left(\frac{e^2}{2\hbar\omega_{\text{LO}}} \right) \left(\frac{2m\omega_{\text{LO}}}{\hbar} \right)^{1/2} \left(\frac{1}{\varepsilon_{\infty}} - \frac{1}{\varepsilon_0} \right). \quad (3)$$

Following the variational method of the Pekar type (VMPT),^[19] the trial wavefunction of the strong-coupling polaron can be separated into two parts, which individually describes the electron and the phonon, the trial wavefunction can be written as

$$|\psi\rangle = |\varphi\rangle U |0_{\text{ph}}\rangle, \quad (4)$$

where $|\varphi\rangle$ depends only on the electron coordinate, $|0_{\text{ph}}\rangle$ represents the phonon's vacuum state with $a_{\mathbf{q}}|0_{\text{ph}}\rangle$, and $U|0_{\text{ph}}\rangle$ is the coherent state of the phonon,

$$U = \exp \left[\sum_{\mathbf{q}} (a_{\mathbf{q}}^\dagger f_{\mathbf{q}} - a_{\mathbf{q}} f_{\mathbf{q}}^*) \right], \quad (5)$$

where $f_{\mathbf{q}}(f_{\mathbf{q}}^*)$ is the variational function, the trial GFES wavefunctions of the electron-phonon system may be chosen as

$$|\varphi_0\rangle = |0\rangle|0_{\text{ph}}\rangle = \pi^{-3/4} \lambda_0^{3/2} \exp \left[-\frac{\lambda_0^2 r^2}{2} \right] |0_{\text{ph}}\rangle, \quad (6)$$

$$|\varphi_1\rangle = |1\rangle|0_{\text{ph}}\rangle = \left(\frac{\pi^3}{4} \right)^{-1/4} \lambda_1^{5/2} r \cos \theta$$

$$\times \exp \left(-\frac{\lambda_1^2 r^2}{2} \right) \exp(\pm i\phi) |0_{\text{ph}}\rangle, \quad (7)$$

where λ_0 and λ_1 are the variational parameters, ϕ is the phase angle. And Eqs. (6) and (7) satisfy the following normalized relations:

$$\langle \varphi_0 | \varphi_0 \rangle = 1, \quad \langle \varphi_0 | \varphi_1 \rangle = 0, \quad \langle \varphi_1 | \varphi_1 \rangle = 1, \quad (8)$$

By minimizing the expectation value of the Hamiltonian, we then obtain the energies of the polaron's GFES by $E_0 = \langle \varphi_0 | H' | \varphi_0 \rangle$ and $E_1 = \langle \varphi_1 | H' | \varphi_1 \rangle$. The calculated results can be written as

$$E_0(\lambda_0) = \frac{3\hbar^2}{4m} \lambda_0^2 - V_0 \left(1 + \frac{1}{2\lambda_0^2 R^2} \right)^{-1/2} + \frac{m\omega_c^2}{8\lambda_0^2}$$

$$- \frac{\sqrt{2}}{\sqrt{\pi}} \alpha \hbar\omega_{\text{LO}} \lambda_0 r_0, \quad (9)$$

$$E_1(\lambda_1) = \frac{5\hbar^2}{4m} \lambda_1^2 - V_0 \left(1 + \frac{1}{2\lambda_1^2 R^2} \right)^{-3/2} + \frac{m\omega_c^2}{8\lambda_1^2}$$

$$- \frac{3\sqrt{2}}{4\sqrt{\pi}} \alpha \hbar\omega_{\text{LO}} \lambda_1 r_0, \quad (10)$$

where $r_0 = (\hbar/2m\omega_{\text{LO}})^{1/2}$ is the polaron radius. $\omega_c = eB/mc$ is the cyclotron frequency of the MF. We can obtain λ_0 and λ_1 by using the variational method, then, we will get the eigen levels and eigenfunctions. So, a two-level system as a single qubit is built up. The superposition state can be expressed as

$$|\psi_{01}\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = \frac{1}{\sqrt{2}}(\psi_0(r) + \psi_1(r)). \quad (11)$$

The time evolution of the superposition state of the electron can be written as

$$\psi_{01}(r, t) = \frac{1}{\sqrt{2}} \psi_0(r) \exp \left(-\frac{iE_0 t}{\hbar} \right)$$

$$+ \frac{1}{\sqrt{2}} \psi_1(r) \exp \left(-\frac{iE_1 t}{\hbar} \right). \quad (12)$$

The probability density of the electron in the AGPQW is in the following form

$$Q(r, t) = |\psi_{01}(r, t)|^2 = \frac{1}{2} [|\psi_0(r)|^2 + |\psi_1(r)|^2$$

$$+ \psi_0^*(r) \psi_1(r) \exp(i\omega_{01} t)$$

$$+ \psi_0(r) \psi_1^*(r) \exp(-i\omega_{01} t)], \quad (13)$$

where $\omega_{01} = (E_1 - E_0)/\hbar$ is the transition frequency between the GFES. The oscillatory period of the probability density is

$$T_0 = \frac{h}{E_1 - E_0}. \quad (14)$$

3 Numerical Results and Discussion

In this work, numerical calculations for RbCl crystal have been performed, and the material parameters used in the calculation^[20] are $\hbar\omega_{\text{LO}} = 22.317$ meV, $m = 0.432m_0$, $\alpha = 3.81$. The numerical results are presented in Figs. 1–5.

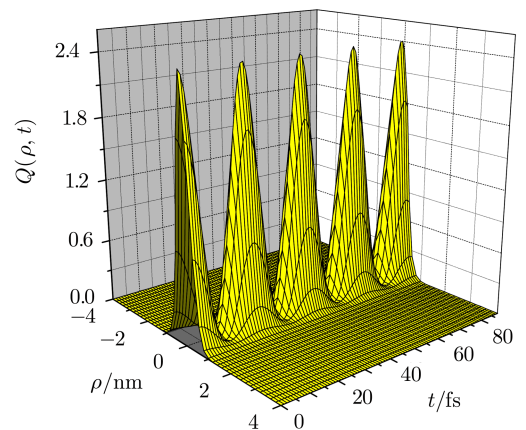


Fig. 1 The probability density $Q(\rho, t)$ versus the time t and the coordinate ρ .

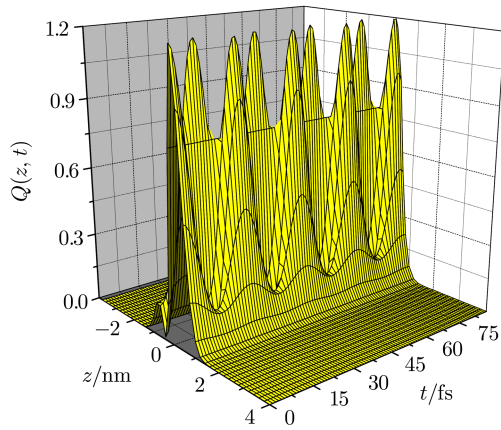


Fig. 2 The probability density $Q(z, t)$ versus the time t and the coordinate z .

Figure 1 plots the probability density changing with the time and the coordinate ρ when electrons are in the superposition state for $\omega_c = 1.0 \times 10^{13}$ Hz, $V_0 = 5.0$ meV, $R = 1.0$ nm, $r_0 = 2.0$ nm, $z = 0.35$ nm, and $\cos \theta = 1$. It is observed that the probability density oscillates in the RbCl AGPQW with an oscillating period $T_0 = 21.639$ fs. It is also shown that the distribution of probability density with the coordinate ρ . Moreover, there is only one peak if the confinement is a two-dimensional symmetric structure in the x - y plane of the AGPQW. This result is similar to the case of the parabolic QD in Refs. [21–22].

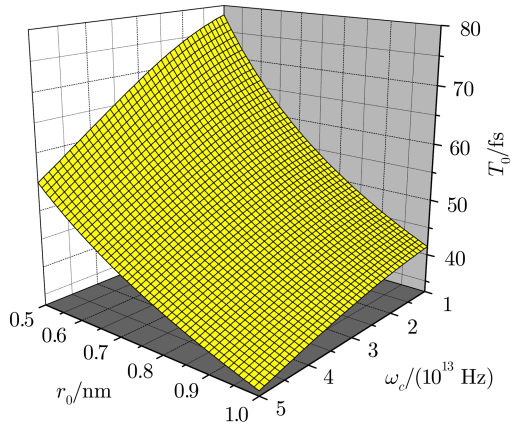


Fig. 3 The oscillating period versus the cyclotron frequency of a MF and the polaron radius.

In order to show the influences of the asymmetrical Gaussian potential on the coordinate better, we also present the distribution of probability density versus the time and the coordinate z in Fig. 2 when electrons are in the superposition state for $\omega_c = 1.0 \times 10^{13}$ Hz, $V_0 = 5.0$ meV, $R = 1.0$ nm, $r_0 = 2.0$ nm, $\rho = 0.35$ nm and $\cos \theta = 1$. Due to the presence of an asymmetrical potential in the growth direction of the AGPQW, the probability density shows double peaks configuration. This result is consistent with results of the asymmetric QD^[23] and

quantum rod.^[24] It is also indicated that the probability density oscillates in the RbCl AGPQW with an oscillating period $T_0 = 21.639$ fs, whose magnitude agrees with the related quantity's magnitude of Ref. [25].

Figure 3 indicates the oscillating period as a function of the polaron radius and the MF's cyclotron frequency for $V_0 = 5.0$ meV and $R = 1.0$ nm. Figure 4 presents the oscillating period versus the height of the AGPQW and the cyclotron frequency for $r_0 = 2.0$ nm and $R = 1.0$ nm. It is shown that the oscillating period is a decreasing function of the cyclotron frequency, the height of the AGPQW and the polaron radius. This is because the influences of these physical quantities on the first excited state are weaker than those on the ground state. Besides, the decrease of the first excited state is less than that of the ground state with increasing these physical quantities. So the increasing of the energy space between the GFES will lead to the reduction of the oscillating period.

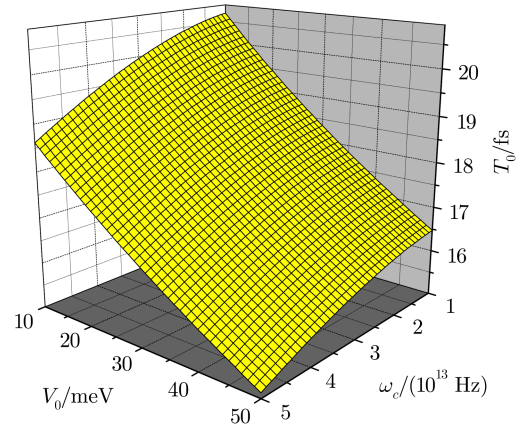


Fig. 4 The oscillating period versus the height of the AGPQW and the MF's cyclotron frequency.

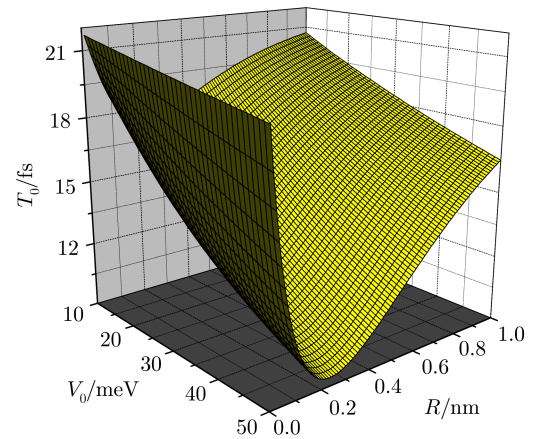


Fig. 5 The oscillating period versus the height of the AGPQW and the RCP.

The oscillating period as a function of the height of the AGPQW and the RCP is depicted in Fig. 5 for $r_0 = 2.0$ nm

and $\omega_c = 1.0 \times 10^{13}$ Hz. From Fig. 5, we also find the oscillatory period decreases with increasing the height of the AGPQW. Besides, as the RCP decreases we can see that it will decrease firstly and then increase. And it takes a minimum when $R = 0.234$ nm. The reason is that when the RCP is larger than the critical value, the energy space will increase as the RCP decreases, which will lead to the decrease of the oscillating period; besides, when the RCP is smaller than the critical value, there is only one confined quantum state (the ground state) in the AGPQW and it will increase as the RCP decreases, however, the excited state is not sensitive to the variation of the RCP. As a result, the energy gap will decrease with decreasing the RCP.

4 Conclusion

The probability density and the oscillating period have been obtained in the AGPQW under an applied MF by utilizing VMPT. The numerical results have shown that (i) the probability density oscillates with a certain period, (ii) due to the presence of the asymmetrical potential in the growth direction of the AGPQW, the electron probability density shows double-peak configuration, whereas there is only one peak if the confinement is a two-dimensional symmetric one in the xy plane, (iii) the oscillating period will decrease with increasing the cyclotron frequency, the height of the AGPQW and the polaron radius, (iv) as the RCP decreases, the oscillatory period will decrease firstly and then increase and it will take a minimum when $R = 0.234$ nm.

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