

Complete Analysis of Four-Photon χ -Type Entangled State via Cross-Kerr Nonlinearity*

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Abstract We propose an efficient method to construct an optical four-photon $|\chi\rangle$ state analyzer via the cross-Kerr nonlinearity combined with linear optical elements. In this protocol, two four-qubit parity-check gates and two controlled phase gates are employed. We show that all the 16 orthogonal four-qubit $|\chi\rangle$ states can be completely discriminated with our apparatus. The scheme is feasible and realizable with current technology. It may have useful potential applications in quantum information processing which based on $|\chi\rangle$ state.

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Key words: $|\chi\rangle$ state analysis, parity-check gate, controlled phase gate, cross-Kerr nonlinearity

1 Introduction

Quantum entanglement is the particular resource in the quantum world, which is widely used in quantum communication (QC) and quantum information processing (QIP).^[1–8] For two-qubit system, there is only one type of maximal entanglement, i.e., the Bell states, whose features were well investigated in the past decades. With the growth of the number of particles constituting the system, the structure of entanglement becomes more diverse. There are two types of tripartite entanglement, the Greenberger–Horne–Zeilinger (GHZ) state and the W state, which cannot be transformed into each other with only local operations and classical communication (LOCC). Due to its crucial application in QIP, the characterization of entanglement in multiparticle systems is still under intense research. There are several new types of entanglement in addition to these two well known states when the number of particles is larger than 3, one of which is a genuine four-qubit entangled state, called as the $|\chi\rangle$ state.^[9]

The $|\chi\rangle$ state was introduced in 2006 by Yeo and Chua, written as

$$|\chi^{00}\rangle_{1234} = \frac{1}{2\sqrt{2}}(|HHHH\rangle - |HHVV\rangle - |HVHV\rangle + |HV VH\rangle + |VHHV\rangle + |VHVH\rangle + |VVHH\rangle + |VVVV\rangle)_{1234}. \quad (1)$$

Here we use $|H\rangle$ and $|V\rangle$ to represent the horizontal and vertical polarization states, respectively. This new entanglement is neither reducible to a pair of Bell states nor LOCC equivalent to either the GHZ or W state. It per-

fectly violates a new Bell inequality which cannot be violated by any of the four-qubit GHZ state, the W state or the linear cluster state.^[10] Moreover, research has shown that this genuinely entangled state has the maximum entanglement between particles (1, 3) and (2, 4), and between (1, 2) and (3, 4), simultaneously. The sixteen states constitute a complete set of entangled orthogonal basis

$$|\chi^{ij}\rangle = \sigma_1^i \sigma_3^j |\chi^{00}\rangle_{1234}. \quad (2)$$

The subscripts 1 and 3 represent the photons on which the unitary operations perform. And $i, j \in \{0, 1, 2, 3\}$,

$$\begin{aligned} \sigma^0 &= |H\rangle\langle H| + |V\rangle\langle V|, \quad \sigma^1 = |V\rangle\langle H| + |H\rangle\langle V|, \\ \sigma^2 &= |V\rangle\langle H| - |H\rangle\langle V|, \quad \sigma^3 = |H\rangle\langle H| - |V\rangle\langle V| \end{aligned} \quad (3)$$

are the four single-qubit unitary operations.

Owing to its interesting entanglement properties and potential applications in fundamental tests of quantum physics, the $|\chi\rangle$ state attracts much attention in recent years. It was firstly introduced for faithfully teleporting an arbitrary two-qubit state and dense coding.^[9] In 2008, the $|\chi\rangle$ state was also used as the quantum channel for a quantum secure direct communication protocol.^[11] Subsequently, this state also had applications in the quantum splitting,^[12] deterministic secure quantum communication,^[13] quantum secret sharing,^[14] quantum networks^[15] and so on.

Besides its application in quantum communication protocols, researches also focus on the preparation of this genuine entanglement in different physical systems. In 2008, Wang and Yang presented a simple scheme to generate this type of state in an ion-trap system.^[16] This task

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can also be completed in a cavity quantum electrodynamic (QED) system efficiently.^[17] There were also many protocols for generating $|\chi\rangle$ state in different systems.^[18–24] In addition to state preparation, which is one of the key steps in QIP, the discrimination of a set of orthogonal basis is also important. In Ref. [16], the discrimination between 16 $|\chi\rangle$ states was proposed in the ion-trap system. The distinguishing of them was also investigated in the flux qubit system.^[24] Generally speaking, the photon is considered as the ideal candidate for QC in virtue of its high-speed transmission, excellent low-noise properties, and maneuverability. It is significative to investigate the preparation and analysis of $|\chi\rangle$ states in the optical system. In 2009, Wang and Zhang proposed an experimental scheme to generate the four-photon entangled $|\chi\rangle$ state based on linear optics and conventional photon detectors.^[25] Later, a scheme based on nonlinearity was presented.^[26] In 2013, a nearly deterministic scheme for generating χ -type entangled states with weak cross-Kerr nonlinearities was also proposed.^[27] However, limited by the natural properties of photons, the study of analysis of $|\chi\rangle$ states was very limited. The complete analysis of these 16 $|\chi\rangle$ states is necessary and meaningful for the QIP that based on them.

So far, there were many state discrimination schemes for photonic system.^[28–40] It was proved that the complete analysis of a set of orthogonal basis is fundamentally impossible by only linear evolution and local projective measurement, both for qubits and qudits.^[41–44] The interaction between different particles which can be realized resorting to the nonlinearity, is essential to the complete state analysis of entangled basis. In previous schemes, the cross-Kerr nonlinearity was widely selected to assist the interactions.^[26–27,30,34,36,39] In this paper, we present a method to distinguish the 16 orthogonal $|\chi\rangle$ states in the photonic system utilizing the parity-check gate (PG) and controlled phase gate (CPG), both of which can be realized with the cross-Kerr nonlinearity. The success probability of our scheme is 100% in principle with ideal quantum gates. This letter is organized as follows: Firstly, we introduce the principle of cross-Kerr effect, the PG and CPG based on it. Then, we demonstrate the complete discrimination of the 16 orthogonal $|\chi\rangle$ states. A discuss and summary is given in the last section.

2 The Parity-Check Gate and Controlled Phase Gate

In this section, we firstly introduce the photon number quantum nondemolition detector (QND) measurement constructed with the cross-Kerr nonlinearity, then present the realization of the PG and CPG.

2.1 The Principle of Photon Number QND

As we know, the cross-Kerr nonlinearity is an interaction between a signal photon state $|\psi\rangle_s$ and a probe

coherent state $|\alpha\rangle_p$ with the Hamiltonian^[45–46] of

$$H = \hbar\chi a_s^\dagger a_s a_p^\dagger a_p. \quad (4)$$

The a_s (a_p) and a_s^\dagger (a_p^\dagger) are the destruction operator and creation operator for the signal (coherent) state, respectively. χ represents the coupling strength of the nonlinearity and it depends on the property of nonlinear material. Here, supposing the signal state is a superposition of the Fock states as

$$|\psi\rangle_s = a|0\rangle_s + b|1\rangle_s + c|2\rangle_s + \dots, \quad (5)$$

the effect of the cross-Kerr nonlinearity can be described as

$$U|\psi\rangle_s|\alpha\rangle_p = a|0\rangle_s|\alpha\rangle_p + b|1\rangle_s|\alpha e^{i\theta}\rangle_p + c|2\rangle_s|\alpha e^{2i\theta}\rangle_p + \dots \quad (6)$$

Here $\theta = \chi t$ (t is the interaction time) and the coherent state will pick up a phase shift, which is proportional to the number of photons in the signal state. As the number of photons can be read by X quadrature measurement on the coherent state, this QND can be used for constructing the PG via the cross-Kerr nonlinearity.

2.2 The Implementation of Parity-Check Gate

The principle of our PG for four-photon system is shown in Fig. 1, where $\pm(\pi/2)$ represent that the cross-Kerr nonlinear medium, which will make the coherent state $|\alpha\rangle$ pick up a phase shift $\pm(\pi/2)$ when it couples with one photon in the medium. It can distinguish the even-parity states that contain an even number of $|V\rangle$ from the odd-parity states that contain an odd number of $|V\rangle$ for four-photon system.

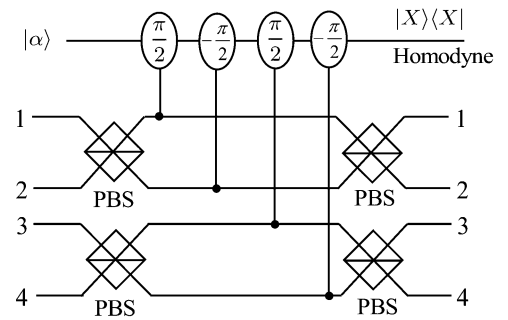


Fig. 1 Schematic diagram of the four-photon polarization parity-check gate (PG). The polarizing beam splitter (PBS) transmits the horizontal states $|H\rangle$ and reflects the vertical ones $|V\rangle$. The device is used to distinguish the even-parity states which contain an even number of $|V\rangle$ with the phase shift 0 ($\pm 2\pi$ are just the phase shift 0 for the coherent state) from the odd-parity states that contain an odd number of $|V\rangle$ with the phase shift $\pm\pi$ on coherent probe beam. This device can detect the parity information without destroying the photons.

Assume that the input four-qubit state is

$$|\varphi\rangle_{\text{in}} = \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)^{\otimes 4}. \quad (7)$$

After the interaction between the four-photon state and coherent state, the whole state becomes

$$|\varphi\rangle_{\text{out}} = \frac{1}{4}[(|HHHH\rangle + |HHVV\rangle + |VVHH\rangle + |VVVV\rangle + |HV VH\rangle + |VHHV\rangle)|\alpha\rangle + |HVHV\rangle|\alpha e^{-i2\pi}\rangle + |VHVH\rangle|\alpha e^{i2\pi}\rangle + (|HHHV\rangle + |HVHH\rangle + |VVHV\rangle + |HVVV\rangle)|\alpha e^{-i\pi}\rangle + (|HHVH\rangle + |VHHH\rangle + |VHVV\rangle + |VVVH\rangle)|\alpha e^{i\pi}\rangle]. \quad (8)$$

It is worth noting that $\pm 2\pi$ are just the same phase shifts as 0 for the coherent state, which cannot be distinguished by the X quadrature measurement. Thus an X quadrature measurement on the coherent beam can be used to discriminate the even-parity states from the odd-parity states according to the phase shift of coherent state. As a result, the parity of the state can be obtained without destroying the photonic state.

2.3 The Effect of Controlled Phase Gate

The CPG is an important element in our analyzer, which can be realized via the π cross-Kerr nonlinearity.^[47–48] Here we consider an arbitrary two-photon polarization state

$$|\varphi\rangle_{12} = \alpha|HH\rangle + \beta|HV\rangle + \gamma|VH\rangle + \delta|VV\rangle. \quad (9)$$

After the cross-Kerr interaction, it evolves as^[47]

$$U|\varphi\rangle_{12} = \alpha|HH\rangle + \beta|HV\rangle + \gamma|VH\rangle + \delta e^{i\theta}|VV\rangle. \quad (10)$$

By choosing $\theta = \pi$, the action of π cross-Kerr nonlinearity will generate a π phase shift on the $|VV\rangle$ term,^[48] then $|\varphi\rangle_{12}$ turns into

$$U|\varphi\rangle_{12} = \alpha|HH\rangle + \beta|HV\rangle + \gamma|VH\rangle - \delta|VV\rangle. \quad (11)$$

It just produces a π phase shift on the $|VV\rangle$ term while keeps the other terms unchanged.

3 Complete Analysis of the Four-Photon Genuine Entanglement

Now we start to distinguish the 16 orthogonal entangled states using the PG and CPG mentioned above. The principle is shown in Fig. 2. In order to simplify the writing form, we ignore the global phase that makes no influence on final result and preserve the relative phase expressed with “ \pm ” sign. At the same time, we also use $|\chi_{cd}^{ab}\rangle$ to represent two separate states $|\chi^{ab}\rangle$ and $|\chi^{cd}\rangle$ corresponding to the “ \pm ” sign, respectively. So, the 16 orthogonal states can be succinctly written as

$$\begin{aligned} |\chi_{30}^{00}\rangle &= \frac{1}{\sqrt{8}}[|H\rangle(|HHH\rangle - |HVV\rangle - |VHV\rangle + |VVH\rangle) \\ &\quad \pm |V\rangle(|VVV\rangle + |VHH\rangle + |HVV\rangle + |HHV\rangle)]_{1234}, \\ |\chi_{22}^{12}\rangle &= \frac{1}{\sqrt{8}}[|H\rangle(|HHH\rangle - |HVV\rangle + |VHV\rangle - |VVH\rangle) \\ &\quad \pm |V\rangle(|VVV\rangle - |VHH\rangle - |HVV\rangle + |HHV\rangle)]_{1234}, \end{aligned}$$

$$\begin{aligned} &\pm |V\rangle(|VVV\rangle + |VHH\rangle - |HVV\rangle - |HHV\rangle)]_{1234}, \\ |\chi_{11}^{21}\rangle &= \frac{1}{\sqrt{8}}[|H\rangle(|HHH\rangle + |HVV\rangle + |VHV\rangle + |VVH\rangle) \\ &\quad \pm |V\rangle(|VVV\rangle - |VHH\rangle - |HVV\rangle + |HHV\rangle)]_{1234}, \\ |\chi_{03}^{33}\rangle &= \frac{1}{\sqrt{8}}[|H\rangle(|HHH\rangle + |HVV\rangle - |VHV\rangle - |VVH\rangle) \\ &\quad \pm |V\rangle(|VVV\rangle - |VHH\rangle + |HVV\rangle - |HHV\rangle)]_{1234}, \\ |\chi_{32}^{02}\rangle &= \frac{1}{\sqrt{8}}[|H\rangle(|HHV\rangle + |HVV\rangle - |VHH\rangle - |VVV\rangle) \\ &\quad \pm |V\rangle(|VVH\rangle - |VHV\rangle + |HVV\rangle - |HHH\rangle)]_{1234}, \\ |\chi_{20}^{10}\rangle &= \frac{1}{\sqrt{8}}[|H\rangle(|HHV\rangle + |HVV\rangle + |VHH\rangle + |VVV\rangle) \\ &\quad \pm |V\rangle(|VVH\rangle - |VHV\rangle - |HVV\rangle + |HHH\rangle)]_{1234}, \\ |\chi_{13}^{23}\rangle &= \frac{1}{\sqrt{8}}[|H\rangle(|HHV\rangle - |HVV\rangle + |VHH\rangle - |VVV\rangle) \\ &\quad \pm |V\rangle(|VVH\rangle + |VHV\rangle - |HVV\rangle - |HHH\rangle)]_{1234}, \\ |\chi_{01}^{31}\rangle &= \frac{1}{\sqrt{8}}[|H\rangle(|HHV\rangle - |HVV\rangle - |VHH\rangle + |VVV\rangle) \\ &\quad \pm |V\rangle(|VVH\rangle + |VHV\rangle + |HVV\rangle + |HHH\rangle)]_{1234}. \quad (12) \end{aligned}$$

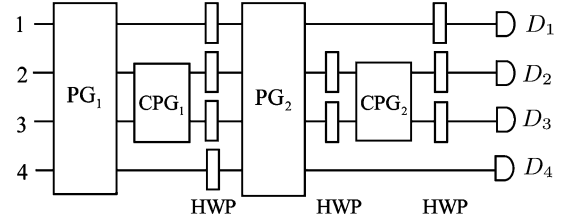


Fig. 2 Schematic diagram of the $|\chi\rangle$ state analyzer. PG₁ and PG₂ are four-qubit polarization parity-check gates. CPG₁ and CPG₂ are controlled phase gates acting on photons 2 and 3. HWPs and single photon detectors (D_1, D_2, D_3, D_4) are used to perform Hadamard operations and measurement with $\{H, V\}$ basis, respectively. The sixteen orthogonal $|\chi\rangle$ states can be completely discriminated with this apparatus.

It is easy to find out that these states can be divided into two groups, the even-parity group and the odd-parity one, according to the number of the $|V\rangle$ states. In other words, with the help of PG₁ we can separate $\{|\chi_{30}^{00}\rangle, |\chi_{22}^{12}\rangle, |\chi_{11}^{21}\rangle, |\chi_{03}^{33}\rangle\}$ from $\{|\chi_{32}^{02}\rangle, |\chi_{20}^{10}\rangle, |\chi_{13}^{23}\rangle, |\chi_{01}^{31}\rangle\}$ nondestructively. The CPG₁ is performed on photons 2 and 3, HWPs are half-wave plates which effect the Hadamard operation

$$|H\rangle \rightarrow \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle), \quad |V\rangle \rightarrow \frac{1}{\sqrt{2}}(|H\rangle - |V\rangle). \quad (13)$$

After the CPG₁ and four HWPs shown in our analyzer, the 16 orthogonal states before PG₂ are

$$\begin{aligned} |\chi_{03}^{11}\rangle &\rightarrow \frac{1}{\sqrt{8}}[(|HH\rangle + |VV\rangle)(|HH\rangle + |VV\rangle) \\ &\quad \pm (|HV\rangle - |VH\rangle)(|HV\rangle - |VH\rangle)]_{1234}, \end{aligned}$$

$$\begin{aligned}
|\chi_{30}^{22}\rangle &\rightarrow \frac{1}{\sqrt{8}}[(|HV\rangle + |VH\rangle)(|HV\rangle + |VH\rangle) \\
&\quad \pm (|HH\rangle - |VV\rangle)(|HH\rangle - |VV\rangle)]_{1234}, \\
|\chi_{13}^{01}\rangle &\rightarrow \frac{1}{\sqrt{8}}[(|HH\rangle - |VV\rangle)(|HH\rangle + |VV\rangle) \\
&\quad \pm (|HV\rangle + |VH\rangle)(|HV\rangle - |VH\rangle)]_{1234}, \\
|\chi_{20}^{32}\rangle &\rightarrow \frac{1}{\sqrt{8}}[(|HV\rangle - |VH\rangle)(|HV\rangle + |VH\rangle) \\
&\quad \pm (|HH\rangle + |VV\rangle)(|HH\rangle - |VV\rangle)]_{1234}, \\
|\chi_{12}^{00}\rangle &\rightarrow \frac{1}{\sqrt{8}}[(|HV\rangle - |VH\rangle)(|HH\rangle - |VV\rangle) \\
&\quad \pm (|HH\rangle + |VV\rangle)(|HV\rangle + |VH\rangle)]_{1234}, \\
|\chi_{21}^{33}\rangle &\rightarrow \frac{1}{\sqrt{8}}[(|HH\rangle - |VV\rangle)(|HV\rangle - |VH\rangle) \\
&\quad \pm (|HV\rangle + |VH\rangle)(|HH\rangle + |VV\rangle)]_{1234}, \\
|\chi_{02}^{10}\rangle &\rightarrow \frac{1}{\sqrt{8}}[(|HV\rangle + |VH\rangle)(|HH\rangle - |VV\rangle) \\
&\quad \pm (|HH\rangle - |VV\rangle)(|HV\rangle + |VH\rangle)]_{1234}, \\
|\chi_{31}^{23}\rangle &\rightarrow \frac{1}{\sqrt{8}}[(|HH\rangle + |VV\rangle)(|HV\rangle - |VH\rangle) \\
&\quad \pm (|HV\rangle - |VH\rangle)(|HH\rangle + |VV\rangle)]_{1234}. \quad (14)
\end{aligned}$$

Coincidentally, these 16 evolutionary states can also be divided into another two groups by PG_2 , i.e., $\{|\chi_{03}^{11}\rangle, |\chi_{30}^{22}\rangle, |\chi_{13}^{01}\rangle, |\chi_{20}^{32}\rangle\}$ are even-parity states and $\{|\chi_{12}^{00}\rangle, |\chi_{21}^{33}\rangle, |\chi_{02}^{10}\rangle, |\chi_{31}^{23}\rangle\}$ are odd-parity states. Thus, the 16 states are classed into four groups according to the results of the two parity-check gates. The detail is shown in Table 1.

Table 1 The states and their corresponding parity-check results.

| Group | States | PG_1 | PG_2 |
|-------|--|--------|--------|
| 1 | $ \chi_{03}^{03}\rangle, \chi_{11}^{11}\rangle, \chi_{22}^{22}\rangle, \chi_{30}^{30}\rangle$ | even | even |
| 2 | $ \chi_{00}^{00}\rangle, \chi_{12}^{12}\rangle, \chi_{21}^{21}\rangle, \chi_{33}^{33}\rangle$ | even | odd |
| 3 | $ \chi_{01}^{01}\rangle, \chi_{13}^{13}\rangle, \chi_{20}^{20}\rangle, \chi_{32}^{32}\rangle$ | odd | even |
| 4 | $ \chi_{02}^{02}\rangle, \chi_{10}^{10}\rangle, \chi_{23}^{23}\rangle, \chi_{31}^{31}\rangle$ | odd | odd |

Then, the four photons go through another two HWPs and CPG_2 . After the three HWPs behind CPG_2 , states turn to

$$\begin{aligned}
|\chi_{20}^{10}\rangle &\rightarrow \frac{1}{2}[(|HHHH\rangle - |HVVV\rangle) \\
&\quad \pm (|VHHV\rangle + |VVVH\rangle)]_{1234}, \\
|\chi_{21}^{11}\rangle &\rightarrow \frac{1}{2}[(|HHHH\rangle + |HVVV\rangle) \\
&\quad \pm (|VHHV\rangle - |VVVH\rangle)]_{1234}, \\
|\chi_{30}^{00}\rangle &\rightarrow \frac{1}{2}[(|HHHV\rangle + |HVVH\rangle) \\
&\quad \pm (|VHHH\rangle - |VVVV\rangle)]_{1234}, \\
|\chi_{31}^{01}\rangle &\rightarrow \frac{1}{2}[(|HHHV\rangle - |HVVH\rangle)
\end{aligned}$$

$$\begin{aligned}
&\quad \pm (|VHHH\rangle + |VVVV\rangle)]_{1234}, \\
|\chi_{32}^{02}\rangle &\rightarrow \frac{1}{2}[(|HHVV\rangle - |HVHH\rangle) \\
&\quad \pm (|VHVH\rangle + |VVHV\rangle)]_{1234}, \\
|\chi_{33}^{03}\rangle &\rightarrow \frac{1}{2}[(|HHVV\rangle + |HVHH\rangle) \\
&\quad \pm (|VHVH\rangle - |VVHV\rangle)]_{1234}, \\
|\chi_{22}^{12}\rangle &\rightarrow \frac{1}{2}[(|HHVH\rangle + |HVHV\rangle) \\
&\quad \pm (|VHVV\rangle - |VVHH\rangle)]_{1234}, \\
|\chi_{23}^{13}\rangle &\rightarrow \frac{1}{2}[(|HHVH\rangle - |HVHV\rangle) \\
&\quad \pm (|VHVV\rangle + |VVHH\rangle)]_{1234}. \quad (15)
\end{aligned}$$

According to above expression, the 16 potential measurement results can be divided into four classes with the four photons measured in $|H\rangle/|V\rangle$ basis,

$$\begin{aligned}
\text{I:} & \quad |HHVV\rangle, |HVHH\rangle, |VHVH\rangle, |VVHV\rangle; \\
\text{II:} & \quad |HHHH\rangle, |HVVV\rangle, |VHHV\rangle, |VVVH\rangle; \\
\text{III:} & \quad |HHHV\rangle, |HVVH\rangle, |VHHH\rangle, |VVVV\rangle; \\
\text{IV:} & \quad |HHVH\rangle, |HVHV\rangle, |VHVV\rangle, |VVHH\rangle. \quad (16)
\end{aligned}$$

We can easily find that the four states in each group shown in Table 1 can be distinguished by these measurement results, as shown in Table 2. So far, the 16 states are completely discriminated.

Table 2 The corresponding state of the parity-check results and single-photon measurement results.

| Group | 1 | 2 | 3 | 4 |
|-------|--------------------------|--------------------------|--------------------------|--------------------------|
| I | $ \chi_{03}^{03}\rangle$ | $ \chi_{33}^{33}\rangle$ | $ \chi_{32}^{32}\rangle$ | $ \chi_{02}^{02}\rangle$ |
| II | $ \chi_{11}^{11}\rangle$ | $ \chi_{21}^{21}\rangle$ | $ \chi_{20}^{20}\rangle$ | $ \chi_{10}^{10}\rangle$ |
| III | $ \chi_{30}^{30}\rangle$ | $ \chi_{00}^{00}\rangle$ | $ \chi_{01}^{01}\rangle$ | $ \chi_{31}^{31}\rangle$ |
| IV | $ \chi_{22}^{22}\rangle$ | $ \chi_{12}^{12}\rangle$ | $ \chi_{13}^{13}\rangle$ | $ \chi_{23}^{23}\rangle$ |

4 Discussion and Summary

Obviously, the key elements of our $|\chi\rangle$ state analyzer are the PG and CPG, both of whose performance dominate the total success probability of our scheme. Considering the PG, perhaps the main challenge is that the phase shift of cross-Kerr nonlinearity should achieve $\theta = \pi/2$ while the realization of efficient cross-Kerr nonlinearity is still a hard problem in the area of optical single-photon.^[49] Fortunately, the researchers attempt to amplify the strength of cross-phase modulation by the assistance of various appropriate systems and methods.^[50–53] In 2003, Hofmann *et al.* showed how to obtain a nonlinear phase shift of π by utilizing a single two-level atom in a one-sided cavity with high efficiency.^[50] In 2011, Feizpour *et al.* showed that the cross-Kerr phase shift can be strengthened to an observable value with the help of weak-value amplification.^[52] These researches show that

it is promising to use the cross-Kerr effect to perform the parity-check gate required in our scheme. Moreover, we can also resort to other kinds of photon-photon nonlinear interaction to accomplish the parity-check detections, such as a Rydberg atom ensemble,^[54] a cavity waveguide^[55] and so on.^[37–38,40]

The CPG is a universal two-qubit gate which can be realized by using a cross-Kerr interaction involving the vertically polarized modes only.^[47] In 2007, Zou *et al.* presented a scheme to implement the two-qubit CPG with linear optics.^[56] Then, a more efficient and convenient two-qubit CPG implemented with cross-Kerr nonlinearity was proposed in 2011.^[57] Encouragingly, many significant works have been accomplished on the realization of π phase shift required in the CPG.^[48,58–60] In 2000, an important method was proposed to gain the large nonlinear interaction for two slow light pulses.^[58] In 2005, Friedler *et al.* put forward that the π phase shift is realizable while a weak probe pulse interacting with a signal pulse in a par-

ticular situation.^[48] In 2006, Chen *et al.* made it possible to achieve the phase shift of π via the light-storage technique experimentally.^[59] Moreover, in the year of 2009, Sun *et al.* presented a way to realize the π phase shift based on EIT and spin-orbit coupling, they also demonstrated that it can be used to implement the CPG with high fidelity.^[60] Actually, the CPG with π phase shift has already been used for the GHZ state and cluster state analysis efficiently in recent years.^[61–62] As mentioned above, all of these significant researches make us more confident on the reliability and feasibility of the proposed scheme.

In summary, we have presented a faithful method for complete analysis of the 16 orthogonal $|\chi\rangle$ states in photonic system. Our scheme has a principle success probability 100% with the ideal gates. As the measurement of quantum entangled state is an essential part of quantum information processing, we believe that our scheme will have useful applications in the future.

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