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Head-on Collision of Ion-acoustic Multi-Solitons in e-p-i Plasma

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Abstract *The propagation and interaction between ion acoustic multi-solitons in an unmagnetized multicomponent plasma consisting of fluid hot ions, positrons and both hot and cold electrons, are investigated by employing the extended Poincare–Lighthill–Kuo (PLK) method. Two different Kortewege-de Vries (K-dV) equations are derived. The Hirota's method is applied to get the K-dV multi-solitons solution. The phase shift due to the overtaking and head- on collision of the multi-solitons is obtained.*

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1 Introduction

Nonlinear partial differential equation is frequently used to model various important phenomena and dynamical processes in physics, mechanics, chemistry and biology. Seeking for exact solutions of these evolution equations may play an important role in nonlinear theory, particularly in soliton theory. To analyze complete integrable evolution equations one may use various powerful method like the inverse scattering method, the Backlund transformation method, the Darboux transformation method, the Hirota bilinear method, etc. that help in deriving multiple-soliton solutions of these equations. The Hirota bilinear method[1] is one of such methods for the determination of multiple soliton solutions. It is a direct method for a wide class of nonlinear evolution equation. It also determines the multiple singular soliton solutions.

The ion acoustic wave (IAW) is one of the most important nonlinear waves in plasma physics. Washimi and Taniuti[2] have been the first to enlighten on the fact that the propagation of IAWs in a collision-free plasma can be described by the Korteweg-de-Vries (K-dV) equation. The K-dV equation has both solitary wave solutions as well as cnoidal wave solutions in collisionless plasmas without the dissipation and geometry distortion. The reductive perturbative technique (RPT) is generally used to study weakly nonlinear solitary waves and double-layers in plasmas. Many investigators have investigated IA solitary waves (IASWs) in the frame of the Korteweg-de-Vries (K-dV), Kadomtsev–Petviashvili (KP), Zakharov–Kuznetsov (ZK) equations.

Electron-positron (e-p) plasmas, composed of particles of the same mass and opposite charges, are believed to have existed in the early universe.[3–4] They are common in the active galactic nuclei,[5–6] in the polar regions of neutron stars,[7] in the inner regions of the accretion disks surrounding black holes,[8] at the center of our galaxy,[9] in pulsar magnetospheres[7] and plasma heating by intense lasers fields.[10] Nowadays, it has been shown that positrons can be produced in tokamarks due to collisions of runaway electrons with plasma ions or thermal electrons.[11] The fast runaway electrons are the result of disruptions in tokamarks for instance, in the Joint European Torus (JET)[12] and JT-60U.[13] Since in many astrophysical environments there exists a small number of ions with the electrons and positrons, therefore, it is important to study linear and nonlinear behavior of plasma waves in e-p-i plasmas. A lot of research has been carried out to study the e-p and e-p-i plasmas in the past few years.[14–18] For instance, Moolla *et al.*[19] investigated nonlinear low frequency structures in e-p-i plasma. Baluku and Hellberg[20] investigated the effect of nonthermal electrons on IASWs in an e-p-i plasma. Paul *et al.*[21] studied IASWs in an e-p-i plasma.

The aim of the present paper is to investigate the interaction of IA solitons in an e-p-i plasma having two distinct kinds of thermal electrons, with either isothermal or adiabatic fluid ions. It is a well established fact that solitons are solitary waves with the remarkable property that the solitons preserve their form asymptotically even when they undergo a collision. The terminology "Soliton" was first pioneered by Zabusky and Kruskal.[22] The fundamental "microscopic" properties of solitons interaction are (i) Soliton collisions are elastic, i.e., the interaction does not change the soliton amplitudes; (ii) After the interaction, each soliton gets an additional phase shift; (iii) The total phase shift of a "trial" soliton acquired during a

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certain time interval can be calculated as a sum of the "elementary" phase shifts in pairwise collisions of this soliton with other solitons during this time interval. Overtaking collision and head-on collision are two distinct types of interaction between the solitons in a one-dimensional system. The head-on collision[23] where the angle between the two propagation directions of the two solitons is equal to π has been studied by many researchers.[24–28] The head-on collision between two IASWs has been studied making use of the extended Poincare–Lighthill–Kuo (PLK) method. The resonance phenomena can be interpreted by the solitary wave solutions of two K-dV equations and this has been observed in shallow water wave experiments,[29] plasmas experiments,[30] two core optical fiber[31] and fluid filled elastic tubes.[32] In overtaking collisions, the angle between the two propagation directions of the two solitons is equal to zero. The overtaking collision has been studied by the inverse scattering transformation method.[33] Two-solitons solution represents the interaction of two solitary waves. It can be written in a form such that its relationship to the solitary wave is clearly apparent, and the utility of this special formulation of the solution can be demonstrated in analyzing the structure during interaction of the two-soliton solution of the K-dV equation. After the collision between two solitons with different amplitude, the largest one overtakes the smaller. When the solitons have different amplitudes and thus different velocities, they will be ultimately separated in space. Several researchers[34–35] have investigated the head- on collision and related phase shift of different solitary waves in different plasma models.

2 Basic Equations

Let us consider an unmagnetized multicomponent plasma having fluid ions, positrons, cold electrons and hot electrons of density n_i, n_p, $n_{\rm ec}$, and $n_{\rm eh}$, respectively. The dynamics of the nonlinear IAWs is governed by the following normalized equations

$$\frac{\partial n_i}{\partial t} + \frac{\partial (n_i u_i)}{\partial x} = 0\,, \tag{1}$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} = -\frac{\partial \phi}{\partial x} - \frac{\sigma}{n_i}\frac{\partial p}{\partial x}\,, \tag{2}$$

$$\frac{\partial p}{\partial t} + u_i \frac{\partial p}{\partial x} = -\gamma p \frac{\partial u_i}{\partial x}\,, \tag{3}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_{\rm eh} + n_{\rm ec} - \alpha n_p - (1-\alpha) n_i\,, \tag{4}$$

where u_i, p, and ϕ are the ion fluid velocity, the ion fluid pressure and the electrostatic potential respectively. The variables t, x, n_i, u_i, and ϕ are normalized by the ion plasma frequency $\omega_{\rm pi} = \sqrt{4\pi n_{e0} e^2/m_i}$, the Debye length $\lambda_D = \sqrt{T_{\rm eff}/4\pi n_{e0} e^2}$, the unperturbed equilibrium ion density n_{i0}, the ion acoustic speed $C_s = \sqrt{T_{\rm eff}/m_i}$, and $T_{\rm eff}/e$, respectively. Here e is the electron charge, m_i is the mass of the ions, T_c is the cold electron temperature, T_h is the hot electron temperature. Moreover, we define $\alpha = n_{p0}/n_{e0}$, $T_{\rm eff} = T_c/(\mu + \nu\beta)$, $\mu = n_{\rm ec0}/n_{e0}$, $\nu = n_{\rm eh0}/n_{e0}$, $\beta = T_c/T_h$, $\gamma_1 = T_{\rm eff}/T_p$, $\sigma = T_i/T_{\rm eff}$. Note that $\mu + \nu = 1$. The densities of the two temperature electrons and positrons are given, respectively, by

$$n_{\rm ec} = \mu \exp\Big(\frac{1}{\mu + \nu\beta}\phi\Big)\,, \tag{5}$$

$$n_{\rm eh} = \nu \exp\Big(\frac{\beta}{\mu + \nu\beta}\phi\Big)\,, \tag{6}$$

$$n_p = \exp(-\gamma_1 \phi)\,. \tag{7}$$

Now we assume that two solitons, which are asymptotically far apart in the initial state travel towards each other. After some time they interact, collide, and then depart. We also assume that the solitons have small amplitudes proportional to ϵ (where ϵ is a small parameter characterizing the strength of nonlinearity) and that the interaction between two solitons is weak. Hence we expect that the collision will be quasi-elastic, and causes only shifts of the post collision trajectories (phase shift). In order to analyze the effects of this collision, we employ an extended PLK method by first scaling the independent variables through the following new stretched variables

$$\xi = \epsilon(x - \lambda t) + \epsilon^2 P_0(\eta, \tau) + \epsilon^3 P_1(\xi, \eta, \tau) + \cdots\,, \tag{8}$$

$$\eta = \epsilon(x + \lambda t) + \epsilon^2 Q_0(\xi, \tau) + \epsilon^3 Q_1(\xi, \eta, \tau) + \cdots\,, \tag{9}$$

$$\tau = \epsilon^3 t\,, \tag{10}$$

and expand the independent variables in power series of ϵ as

$$n_i = 1 + \epsilon^2 n_i^{(1)} + \epsilon^3 n_i^{(2)} + \epsilon^4 n_i^{(3)} + \cdots\,, \tag{11}$$

$$u_i = 0 + \epsilon^2 u_i^{(1)} + \epsilon^3 u_i^{(2)} + \epsilon^4 u_i^{(3)} + \cdots\,, \tag{12}$$

$$p = 1 + \epsilon^2 p^{(1)} + \epsilon^3 p^{(2)} + \epsilon^4 p^{(3)} + \cdots\,, \tag{13}$$

$$\phi = 0 + \epsilon^2 \phi^{(1)} + \epsilon^3 \phi^{(2)} + \epsilon^4 \phi^{(3)} + \cdots\,, \tag{14}$$

where λ is the unknown phase velocity of the IASWs.

After some long but straightforward calculation,[36] we get the following K-dV equations

$$\frac{\partial \phi_1^{(1)}}{\partial \tau} + A\phi_1^{(1)} \frac{\partial \phi_1^{(1)}}{\partial \xi} + B\frac{\partial^3 \phi_1^{(1)}}{\partial \xi^3} = 0\,, \tag{15}$$

$$\frac{\partial \phi_2^{(1)}}{\partial \tau} - A\phi_2^{(1)} \frac{\partial \phi_2^{(1)}}{\partial \eta} - B\frac{\partial^3 \phi_2^{(1)}}{\partial \eta^3} = 0\,, \tag{16}$$

where

$$A = \frac{\alpha\gamma_1^2(\lambda^2 - \sigma\gamma)^2}{2\lambda(1-\alpha)} + \frac{3\lambda^2 + \sigma\gamma(\gamma - 2)}{2\lambda(\lambda^2 - \sigma\gamma)} - \frac{(\nu\beta^2 + \mu)(\lambda^2 - \sigma\gamma)^2}{\lambda(1-\alpha)(\nu\beta + \mu)^2}\,, \tag{17}$$

$$B = \frac{(\lambda^2 - \sigma\gamma)^2}{2\lambda(1-\alpha)}\,. \tag{18}$$

Equations (15) and (16) are two side-traveling wave K-dV equations in the reference frames of ξ and η, respectively. Their one soliton solutions are given, respectively, by

$$\phi_1^{(1)} = \phi_A \,\mathrm{sech}^2\Big[\Big(\frac{A\phi_A}{12B}\Big)^{1/2}\Big(\xi - \frac{1}{3}A\phi_A\tau\Big)\Big]\,, \tag{19}$$

$$\phi_2^{(1)} = \phi_B \,\mathrm{sech}^2\Big[\Big(\frac{A\phi_B}{12B}\Big)^{1/2}\Big(\eta + \frac{1}{3}A\phi_B\tau\Big)\Big]\,, \tag{20}$$

where ϕ_A and ϕ_B are the amplitudes of the two solitons in their initial positions. After a head-on collision of the two solitons, the corresponding phase shifts are given by

$$\triangle P_0 = 2\epsilon^2 \frac{D}{C}\Big(\frac{12B\phi_B}{A}\Big)^{1/2}, \tag{21}$$

$$\triangle Q_0 = -2\epsilon^2 \frac{D}{C}\Big(\frac{12B\phi_A}{A}\Big)^{1/2}, \tag{22}$$

where

$$C = 2\lambda, \quad D = A - \frac{2\lambda}{(\lambda^2 - \sigma\gamma)}.$$

(i) ***Two-Solitons***

Each of the K-dV equations given by Eqs. (15) and (16) may admit a multi-solitons solution. We consider here the two-solitons solutions of each K-dV equation and assume that each two-solitons move in the same direction. The fast moving soliton eventually overtakes the slower one. Using the Hirota's method,[37–39] the two-solitons solutions of Eqs. (15) and (16) are given by

$$\phi_1^{(1)} = \frac{12B}{A}\frac{k_1^2 e^{\theta_1} + k_2^2 e^{\theta_2} + a_{12} e^{\theta_1+\theta_2}(k_2^2 e^{\theta_1} + k_1^2 e^{\theta_2}) + 2(k_1-k_2)^2 e^{\theta_1+\theta_2}}{(1 + e^{\theta_1} + e^{\theta_2} + a_{12} e^{\theta_1+\theta_2})^2}, \tag{23}$$

$$\phi_2^{(1)} = \frac{12B}{A}\frac{k_1^2 e^{\phi_1} + k_2^2 e^{\phi_2} + a_{12} e^{\phi_1+\phi_2}(k_2^2 e^{\phi_1} + k_1^2 e^{\phi_2}) + 2(k_1-k_2)^2 e^{\phi_1+\phi_2}}{(1 + e^{\phi_1} + e^{\phi_2} + a_{12} e^{\phi_1+\phi_2})^2}, \tag{24}$$

where

$$\theta_{i=1,2} = \frac{k_i}{B^1/3}\xi - k_i^3\tau + \alpha_i,$$

$$\phi_i = -\frac{k_i}{B^1/3}\eta - k_i^3\tau + \alpha_i,$$

$$a_{12} = (k_1-k_2)^2/(k_1+k_2)^2.$$

Here a_{12} is related to the phase shifts of the overtaking collision. When $\tau \gg 1$, the two-solitons solutions of Eqs. (15) and (16), respectively, transform into the following superposition of two single-soliton solutions

$$\phi_1^{(1)} \approx \frac{6B}{A}\Big[\frac{k_1^2}{2}\operatorname{sech}^2\Big\{\frac{k_1}{2B^{1/3}}(\xi - B^{1/3}k_1^2\tau - \Delta_1')\Big\} + \frac{k_2^2}{2}\operatorname{sech}^2\Big\{\frac{k_2}{2B^{1/3}}(\xi - B^{1/3}k_2^2\tau - \Delta_2')\Big\}\Big], \tag{25}$$

$$\phi_2^{(1)} \approx \frac{6B}{A}\Big[\frac{k_1^2}{2}\operatorname{sech}^2\Big\{\frac{k_1}{2B^{1/3}}(-\eta - B^{1/3}k_1^2\tau - \Delta_1)\Big\} + \frac{k_2^2}{2}\operatorname{sech}^2\Big\{\frac{k_2}{2B^{1/3}}(-\eta - B^{1/3}k_2^2\tau - \Delta_2)\Big\}\Big]. \tag{26}$$

We see from Eqs. (25) and (26) that we are now dealing with four solitons, each pair moving in the same direction. Here Δ_i', $\Delta_i = \pm(2B^{1/3}/k_i)\ln|\sqrt{a_{12}}|$.

(ii) ***Three-Solitons***

The three-solitons solutions of Eqs. (15) and (16) are given by

$$\phi_1^{(1)} = -2\frac{\partial^2}{\partial\xi^2}(\ln[g(\xi,\tau)]), \tag{27}$$

$$\phi_2^{(1)} = -2\frac{\partial^2}{\partial\xi^2}(\ln[g'(\eta,\tau)]), \tag{28}$$

where $(i = 1,2,3)$

$$g(\xi,\tau) = 1 + e^{\theta_1} + e^{\theta_2} + e^{\theta_3} + \alpha_{12}^2 e^{\theta_1+\theta_2} + \alpha_{23}^2 e^{\theta_2+\theta_3} + \alpha_{31}^2 e^{\theta_3+\theta_1} + \alpha_{123}^2 e^{\theta_1+\theta_2+\theta_3}, g'^{\phi_1} + e^{\phi_2} + e^{\phi_3} + \alpha_{12}^2 e^{\phi_1+\phi_2} + \alpha_{23}^2 e^{\phi_2+\phi_3} + \alpha_{31}^2 e^{\phi_3+\phi_1} + \alpha_{123}^2 e^{\phi_1+\phi_2+\phi_3},$$

$$\alpha_{12}^2 = \Big(\frac{k_1-k_2}{k_1+k_2}\Big)^2, \quad \alpha_{23}^2 = \Big(\frac{k_2-k_3}{k_2+k_3}\Big)^2,$$

$$\alpha_{31}^2 = \Big(\frac{k_3-k_1}{k_3+k_1}\Big)^2, \quad \alpha_{123}^2 = \alpha_{12}^2\alpha_{23}^2\alpha_{31}^2,$$

$$\theta_i = \frac{k_i}{B^1/3}\xi - k_i^3\tau + \alpha_i, \quad \phi_i = -\frac{k_i}{B^1/3}\eta - k_i^3\tau + \alpha_i.$$

For $\tau \gg 1$, the solutions of Eqs. (27) and (28) reduce to a superposition of three single-soliton as

$$\phi_1^{(1)} \sim \sum_{i=1}^{3} A_i \operatorname{sech}^2\Big[\frac{k_i}{2B^{1/3}}(\xi - k_i^2B^{1/3}\tau + \delta_i')\Big], \tag{29}$$

$$\phi_2^{(1)} \sim \sum_{i=1}^{3} A_i \operatorname{sech}^2\Big[\frac{k_i}{2B^{1/3}}(-\eta - k_i^2B^{1/3}\tau + \delta_i)\Big], \tag{30}$$

where $A_i = 3Bk_i^2/A$ are the amplitudes,

$$\delta_1' = \delta_1 = \pm\frac{2B^{1/3}}{k_1}\log\Big|\frac{\alpha_{123}}{\alpha_{23}}\Big|,$$

$$\delta_2' = \delta_2 = \pm\frac{2B^{1/3}}{k_2}\log\Big|\frac{\alpha_{123}}{\alpha_{31}}\Big|,$$

$$\delta_3' = \delta_3 = \pm\frac{2B^{1/3}}{k_1}\log\Big|\frac{\alpha_{123}}{\alpha_{12}}\Big|$$

are the phase shifts of the solitons.

3 Numerical Results

It is seen that the phase shifts Δ_1, and Δ_2 are of opposite signs and both of them are proportional to $B^{1/3}$, a result consistent with the one already obtained in the study of head-on collision of two solitons.[40–41] Note that B depends on γ (see Eq. (18)). The phase shifts will also depend on the parameter γ_1. In Figs. 1(a) and 1(b), we have plotted the two-solitons solution $\phi_1^{(1)}$ vs. ξ for the several values of τ. At $\tau = -5$, the larger amplitude soliton is behind small amplitude solitary, and at $\tau = -1$ the two solitons merge giving rise to a single soliton at $\tau = 0$. Finally ($\tau = 1$) they depart from each other. Figures 2(a) and 2(b) display the solitary potential structure $\phi_1^{(1)}$ against ξ for different values of $\gamma_1 = T_{\rm eff}/T_p$ (isothermal and adiabatic cases). It can be seen that the amplitude of the solitons decreases with increasing γ_1. This is attributed to the fact that the increase in the γ_1 parameter causes the coefficient of the nonlinear term of the K-dV equation to decrease. It is found that the soliton with the larger amplitude overtakes the one with smaller amplitude as time goes on.

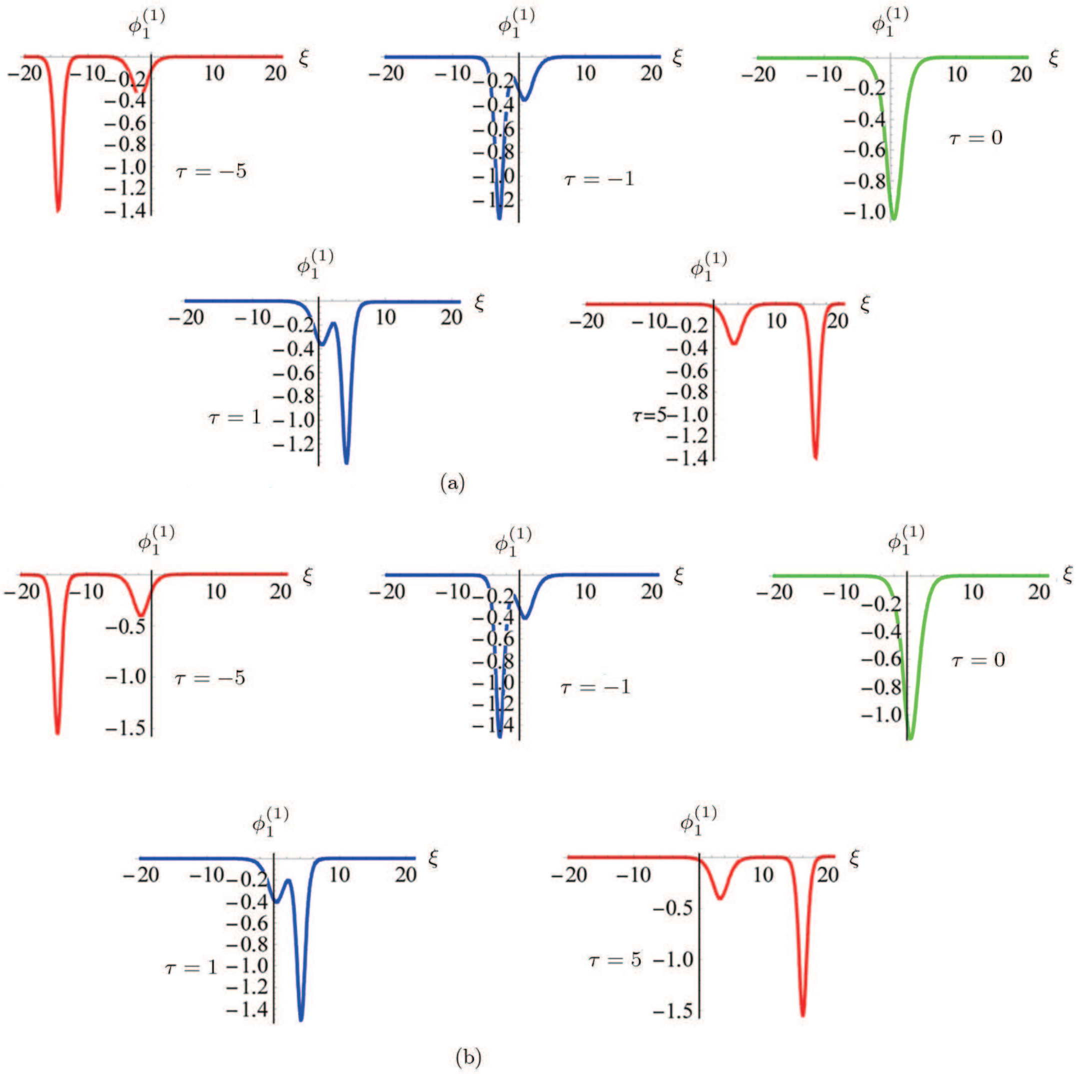

Fig. 1 (a) Variation of the two solitons profiles $\phi^{(1)}$ for different values of τ with $k_1 = 1,\ k_2 = 2, \sigma = 0.04,\ \beta = 0.033,\ \alpha = 0.3,\ \mu = 0.08,\ \gamma_1 = 0.06,\ \alpha_1 = 1,\ \alpha_2 = 1$, and $\gamma = 1$ (isothermal case).(b) Variation of the two solitons profiles $\phi^{(1)}$ for different values of τ with $k_1 = 1,\ k_2 = 2,\ \sigma = 0.04,\ \beta = 0.033,\ \alpha = 0.3,\ \mu = 0.08,\ \gamma_1 = 0.06, \alpha_1 = 1,\ \alpha_2 = 1$, and $\gamma = 3$ (adiabatic case).

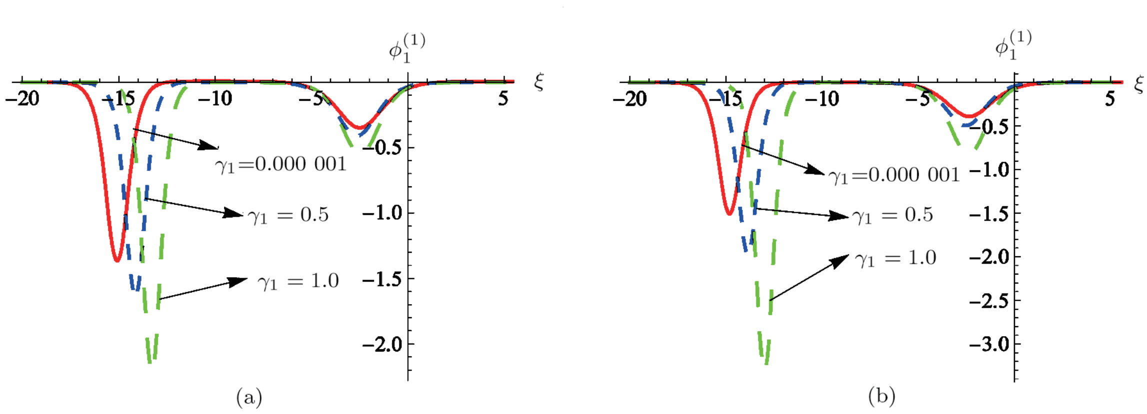

Fig. 2 (a) The two solitons profiles $\phi^{(1)}$ plotted against ξ for different values of γ_1 with $k_1 = 1,\ k_2 = 2,\ \sigma = 0.04,\ \beta = 0.033,\ \alpha = 0.3,\ \mu = 0.0008,\ \tau = -5,\ \alpha_1 = 1,\ \alpha_2 = 1$, and $\gamma = 1$ (isothermal case). (b) The two solitons profiles $\phi^{(1)}$ plotted against ξ for different values of γ_1 with $k_1 = 1,\ k_2 = 2, \sigma = 0.04,\ \beta = 0.033,\ \alpha = 0.3,\ \mu = 0.0008,\ \tau = -5,\ \alpha_1 = 1,\ \alpha_2 = 1$, and $\gamma = 3$ (adiabatic case).

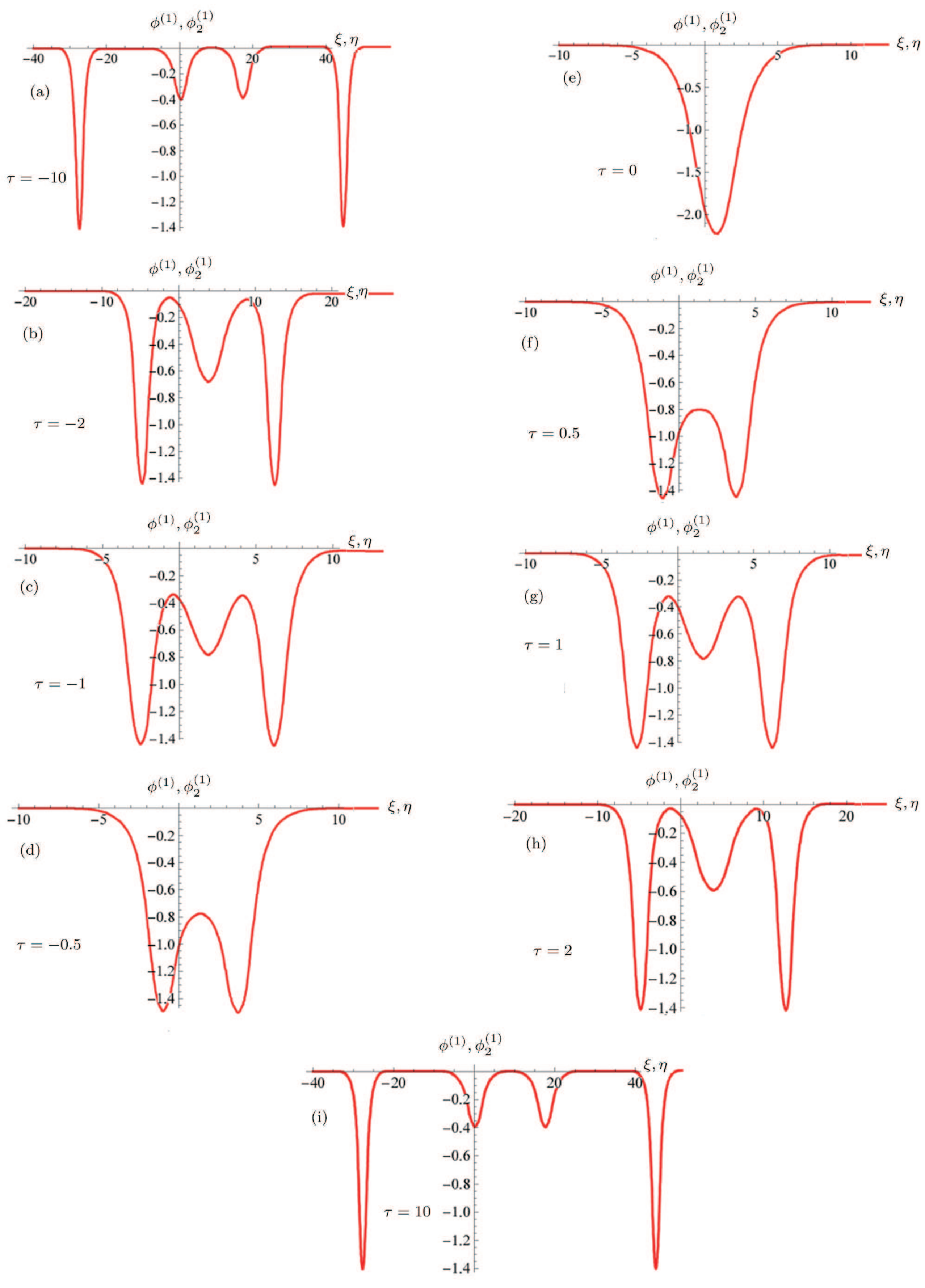

Fig. 3 Variation of the two solitons profile $\phi_1^{(1)}$ and $\phi_2^{(1)}$ for different values of τ (isothermal case).

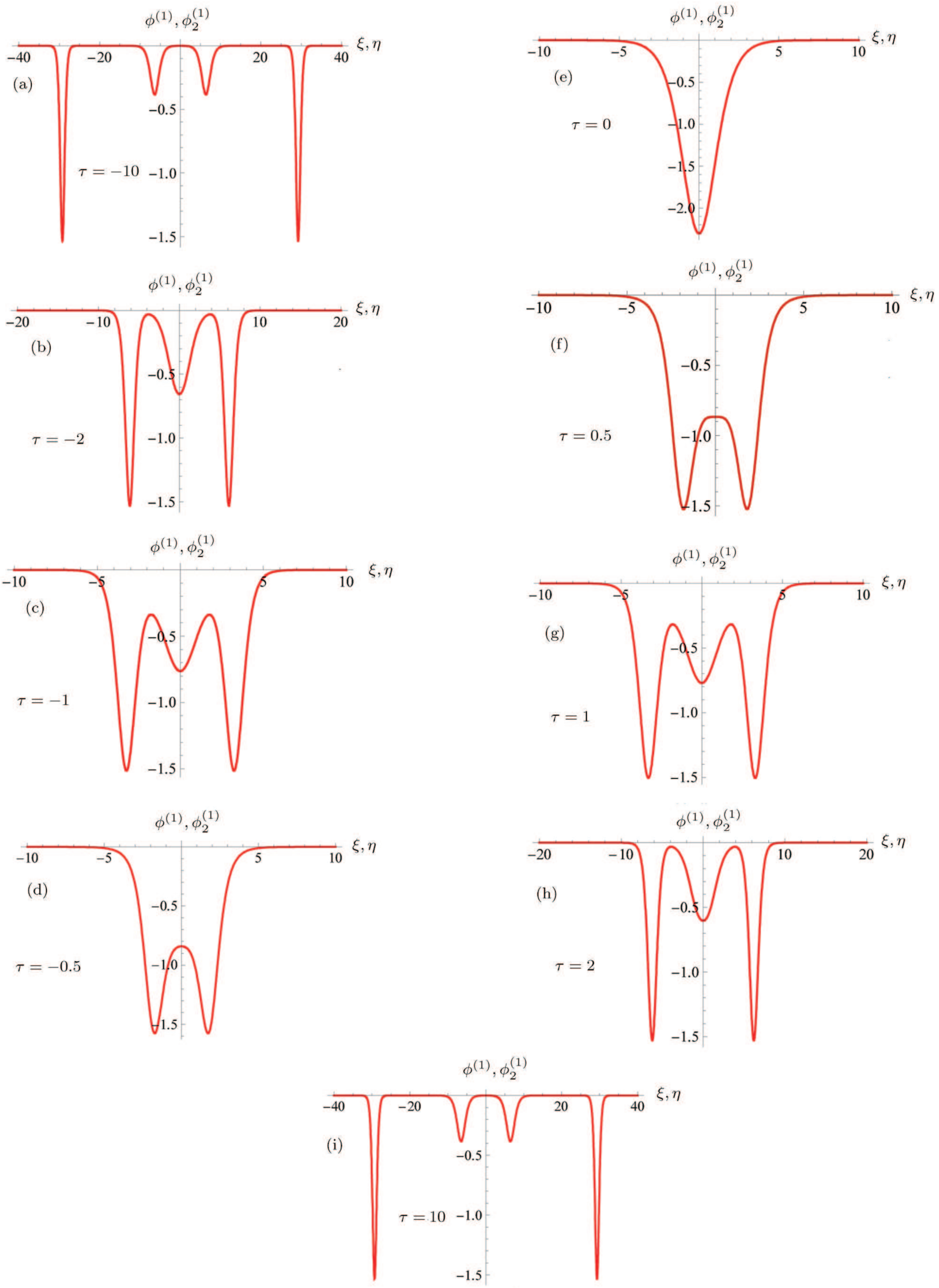

Fig. 4 Variation of the two solitons profile $\phi_1^{(1)}$ and $\phi_2^{(1)}$ for different values of τ (adiabatic case).

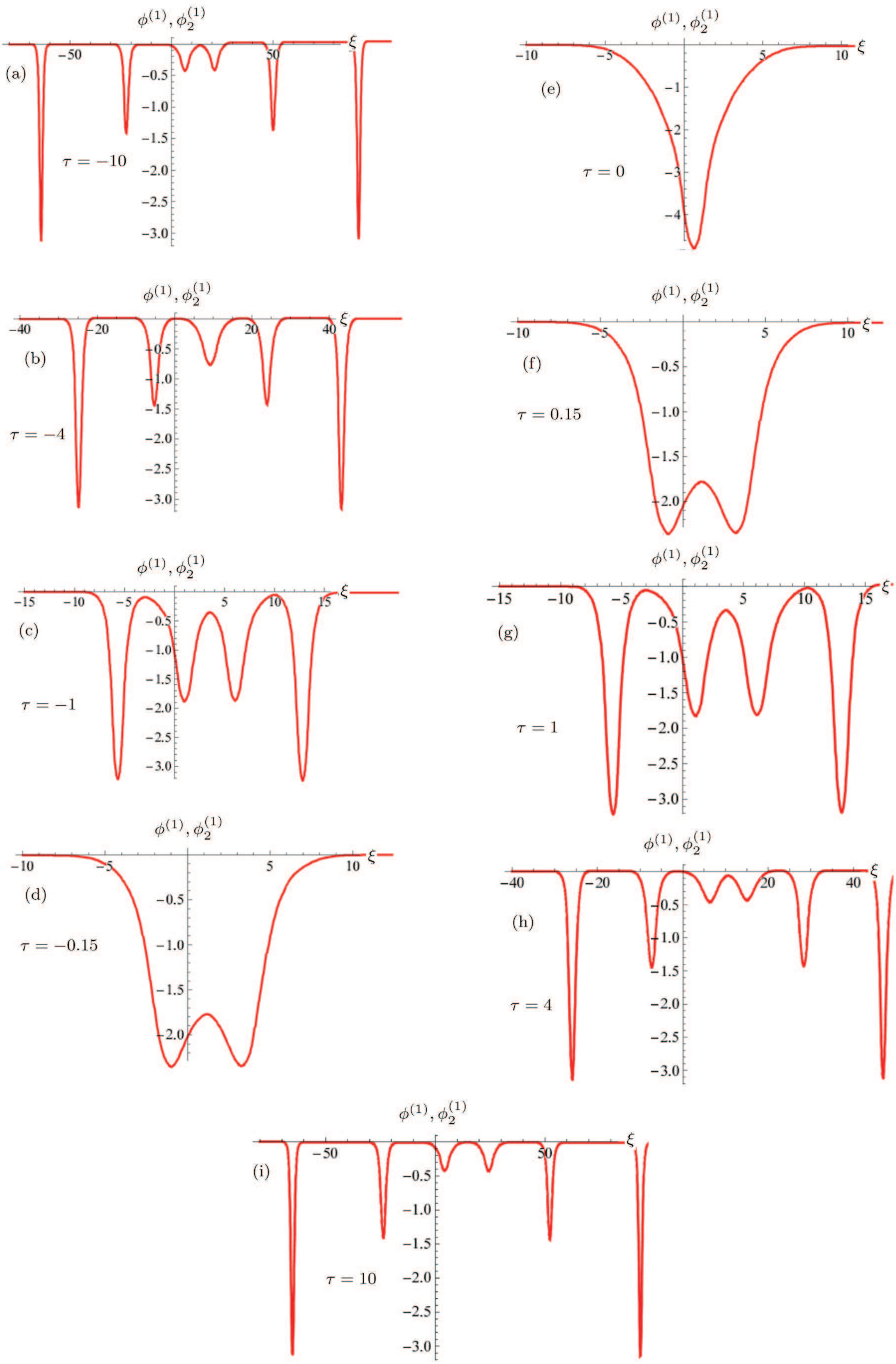

Fig. 5 Variation of the three solitons profile $\phi_1^{(1)}$ and $\phi_2^{(1)}$ for different values of τ (isothermal case).

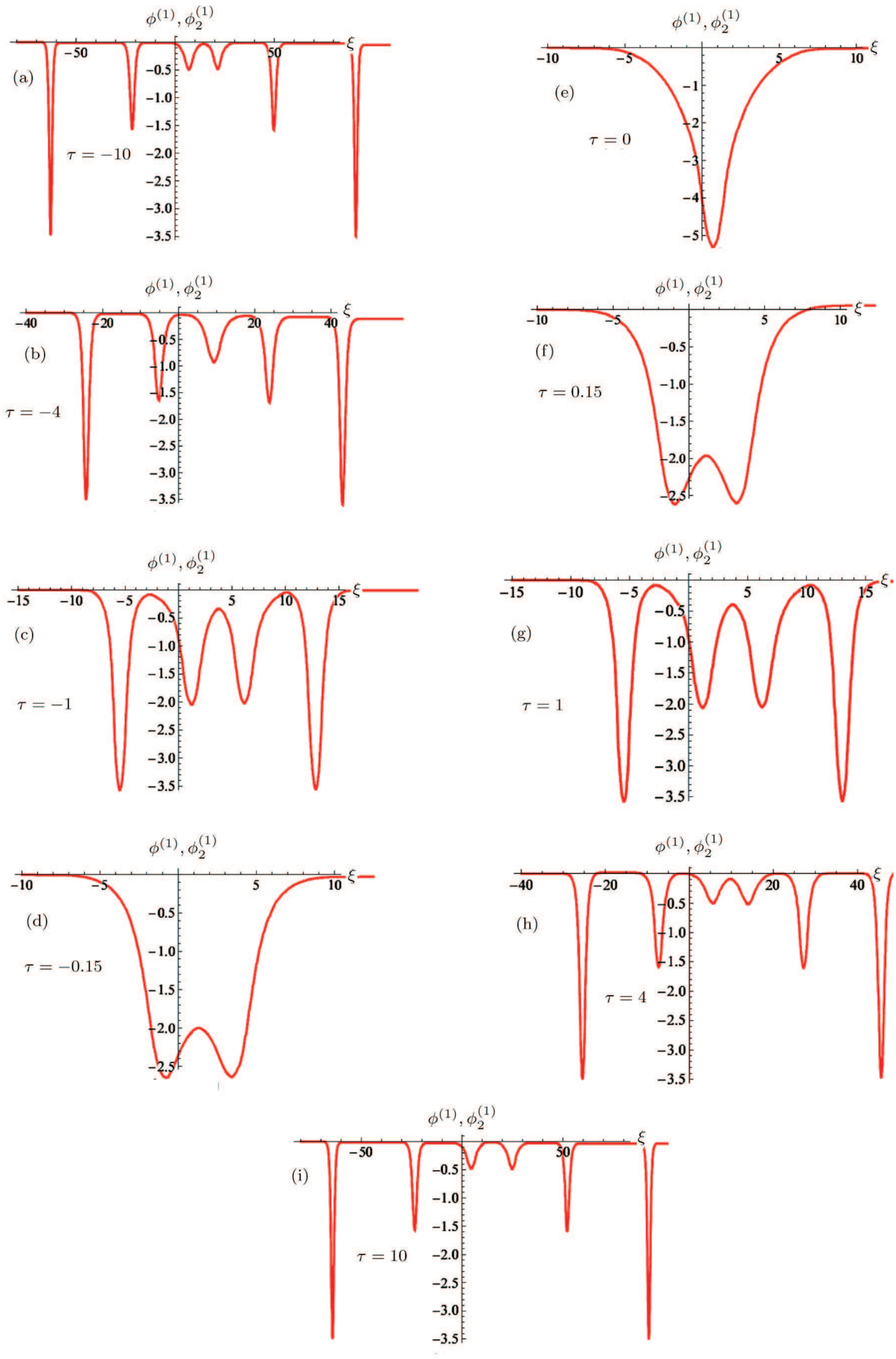

Fig. 6 Variation of the three solitons profile $\phi_1^{(1)}$ and $\phi_2^{(1)}$ for different values of τ (adiabatic case).

The scattering of four solitons is depicted on Figs. 3(a)–3(i). In Fig. 3(a), we see the positions of the solitons at $\tau = -10$. The two solitons on the left hand side are moving to the right and the two solitons on the right hand side are moving towards the left side. As $\tau \to 0$, the fast solitons on each side overtake their slower partners. The overtaking can be seen in Figs. 3(c)–3(d). Figure 3(e) shows the merging of four solitons. Note that Figs. 3(f)–3(i) are mirror images of Figs. 3(d)–3(a), respectively, as may be expected. Here each soliton acquires two phase shift, one due to the head- on collision and the other one due to the overtaking collision, as predicted by Hirota. Qualitatively similar results are obtained in the adiabatic case (Figs. 4(a)–4(i)). The scattering of six solitons is shown in Figs. 5(a)–5(i). In Fig. 5(a), we see the initial positions of the solitons at $\tau = -10$. The three solitons seen on the left hand side are moving to the right and the three solitons on the right hand side are moving towards the left. As $\tau \to 0$, the fast solitons on each side overtake their slower partners. The overtaking process is displayed in Figs. 5(c) and 5(d) leading to a complete merging of the six solitons (Fig. (5e)). Note that Figs. 5(f)–5(i) are mirror images of Figs. 5(a)–5(d). Qualitatively similar results are obtained in the adiabatic case (Figs. 6(a)–6(i)). Figure 7 displays the variation of the phase shifts $\Delta = \Delta_1 + Q_0$ against the density ratio $\alpha = n_{p0}/n_{e0}$ for different values of $\gamma_1 = 10^{-5}$, 0.5 and 1 in the isothermal case ($\gamma = 1$). It can be seen that Δ slightly decreases as α increases. This decrease is more pronounced as γ_1 increases. Qualitatively similar results are obtained in the adiabatic case ($\gamma = 3$) but with a net shift of the curves towards lower values of α (Fig. 8).

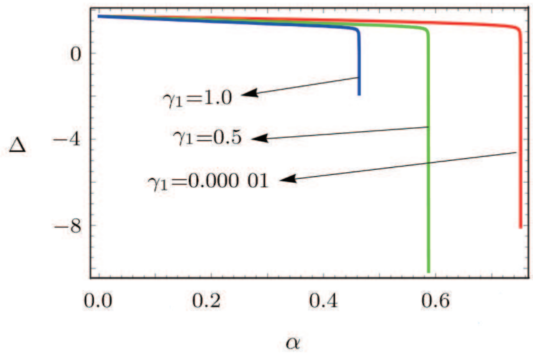

Fig. 7 Variation of the phase shift $\Delta = \Delta_1 + Q_0$ against the parameter α for different values of γ_1 in the isothermal case.

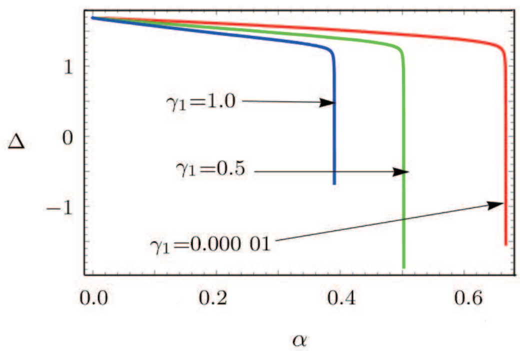

Fig. 8 Variation of the phase shift $\Delta = \Delta_1 + Q_0$ against the parameter α for different values of γ_1 in the adiabatic case.

It may be useful to note that solitary structures can not only be described by fluid theory, but also by phases-space holes of particles. The latter has gained increasing importance in the dynamics of collisionless plasmas. They produce structure of the phase-space distribution and contribute to dissipation of the free plasma energy giving rise to conditions for different instabilities. Electron holes were originally proposed by Bernstein *et al.*[42] as a nonlinear solution of the non magnetized Vlasov–Maxwell system and they showed that, by adding appropriate numbers of particles trapped in the potential-energy troughs, arbitrary traveling wave solutions can be constructed. The dynamics of electron holes has been studied in properly designed numerical simulations.[43−45] It has been demonstrated that the generation mechanism of solitons may be mainly due to the trapping of electrons by the potential well of the waves as shown by the holes that are formed in phase-space plots.

4 Summary

To conclude, we have investigated the propagation and interaction of multi-IA solitons in a plasma consisting of fluid hot ions, positrons and both hot and cold electrons. We have mainly considered collisions of multi-solitons (four, six) using a two step method. We have first derived two different K-dV equations using the PLK method, and then extracted the multi-solitons solution for each K-dV equation using the Hirota approach. The scattering of four and six solitons have then been analyzed and discussed. In the isothermal case, the phase shift due to the head-on collision and overtaking collision slightly decreases as the density ratio $\alpha = n_{p0}/n_{e0}$ increases. This decrease is more pronounced as $\gamma_1 = T_{\rm eff}/T_p$ increases. Qualitatively similar results are obtained in the adiabatic case but with a net shift of the curves towards lower values of α.

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