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# Higher-Order Corrections to Earth's Ionosphere Shocks*

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***Abstract*** *Nonlinear shock wave structures in unmagnetized collisionless viscous plasmas composed fluid of positive (negative) ions and nonthermally electron distribution are examined. For ion shock formation, a reductive perturbation technique applied to derive Burgers equation for lowest-order potential. As the shock amplitude decreasing or enlarging, its steepness and velocity deviate from Burger equation. Burgers type equation with higher order dissipation must be obtained to avoid this deviation. Solution for the compined two equations has been derived using renormalization analysis. Effects of higher-order, positive- negative mass ratio $Q$, electron nonthermal parameter $\delta$ and kinematic viscosities coefficient of positive (negative) ions $\eta_1$ and $\eta_2$ on the electrostatic shocks in Earth's ionosphere are also argued.*

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## 1 Introduction

Research on ion-acoustic waves (IAWs) in ion pair plasmas has gained considerable amount of momentum over the last few years.[1−2] Positive-negative ion have been observed in many astrophysical environments.[3] More specifically, Negative ions are present in D-region altitudes of the ionosphere of Earth in coexistence with electrons as they are formed primarily by electron added to electronegative species.[4] In last few years, ion solitary waves in ion-pair plasmas investigated both numerically and experimentally.[5−6] In past, the nonthermal particles observed in many space environments.[7−9] However, several theoretical studies on nonlinear waves show that, electrons and ions non-thermal distributions are convenient in analyzing observation data in space plasma.[7−12] Cairns *et al.*[9] discussed the nonthermal electron effect on the ion acoustic wave existence. Elwakil *et al.*[2] inspected ion acoustic modulation instability characteristics in plasma having nonthermal electronsand positive-negative ions. It is reported that, the instability conditions affected by nonthermal electron parameter in D and F regions in Earth's ionosphere. Gill *et al.*[13] investigated rarefactive and compressive soliton properties in two polarized ions plasma. Recently, nonthermality effect of positron-electron have been examined on the properties of improved compressive-rarefactive solitons generated in warm ion plasma.[14] On the other hand, there are many theoretical methods for studying nonlinear properties in plasma physics. Reductive perturbation analysis (RPT) aim to study the propagation of small wave amplitude.[15] As the wave amplitude enlarge, the solitary profile sidetrack from the nonlinear equation. In order to beat this deviation, the amplitude modulation of electrostatic nonlinear waves has been studied by many investigators.[16−19] Kalejahi et al.[19] discussed the higher order nonlinear effects in a relativistic plasma. Abdelwahed and El-Shewy[16] improved the soliton shape of solitary wave directly by using algebraic analysis for solving the field equations. Chatterjee *et al.*[17] examined the region of solitary wave existence in nonthermal ions plasma. They discussed the effect of nonthermal ions and electron density on the properties of obtained dressed form. Accordingly, to improve the description of experimental data, the effect of fluctuation of charge of dusty plasma on the dressed nonlinear soliton waves has been investigated.[20] On the other hand, dusty size effect on the dressed soliton like wave amplitude and energy have been discussed in dust plasma containing dust grains and nonthermal ions.[21] Physically, some phenomenon like kinematic viscosity, collisions between plasma components and Landau damping are responsible for shocks formation in Earth ionosphere.[22−24] The shock wave existence and propagation have been investigated experimentally and theoretically.[25−31] Cairns *et al.*[32] investigated laminar ion shocks to explain observations on ion acceleration laser plasmas. Abdelwahed[33] obtained the higher order dissipation in terms of Burger-type equation to modulate the broadband auroral electrostatic shock noise. From theoretical point of view, Kadomtsev–Petviashvili–Burgers (KPB) equation has been derived by

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Hussain *et al.*[34] They discussed the diffraction and dissipation effects on the shock wave structures in epi plasma with kappa distributed positrons and electrons. More specifically, the obliqueness of solitary shocks in a magnetized viscous plasma have been studied.[35] Abdelwahed and el-Shewy[36] studied the nonlinear features of rational and double layer ion acoustic solitary solutions. Furthermore, Masood and Rizvi[37] investigated the viscosity effect of negative and positive ions in dissipative plasma medium in a planar geometry. Also, kinematic viscosity effect on formation of shock waves in asymmetric pair ion plasmas has been examined.[38] It was noted that, kinematic viscosity enhances the amplitude of shock profile. In this study, the higher-order acoustic shock modulation in plasmas with negative-positive ions and nonthermal electrons have been considered. In Sec. 2, the equations of the system is presented, and higher-order Burgers equation is derived in Sec. 3. In Sec. 4, the solutions of the higher-order Burgers-equation are presented. Finally, the results and discussion are recapitulated in Sec. 5.

## 2 Basic Equations

A system of three collisionless unmagnetized plasmas components having viscous fluid of positive (+) and negative (−) ion and nonthermally electron distribution. The normalized equations are given by:

$$\frac{\partial n_\pm}{\partial t} + \frac{\partial}{\partial x}(n_\pm u_\pm) = 0\,, \tag{1}$$

$$\Big(\frac{\partial}{\partial t} + u_+\frac{\partial}{\partial x}\Big)u_+ = -\frac{\partial \phi}{\partial x} + \eta_1\frac{\partial^2 u_+}{\partial x^2}\,, \tag{2}$$

$$\Big(\frac{\partial}{\partial t} + u_-\frac{\partial}{\partial x}\Big)u_- = \frac{Z_-}{Z_+}Q\frac{\partial \phi}{\partial x} + \eta_2\frac{\partial^2 u_-}{\partial x^2}\,, \tag{3}$$

$$\frac{\partial^2\phi}{\partial x^2} = Z_- n_- + n_e - Z_+ n_+\,, \tag{4}$$

$n_e$ is the electrons density obey nonthermally distributed.[9]

$$n_e = \mu\Big(1 - \beta\frac{Z_-}{Z_+}\phi + \beta\Big(\frac{Z_-}{Z_+}\Big)^2\phi^2\Big)\exp\Big(\frac{Z_-}{Z_+}\phi\Big)\,, \tag{5}$$

$$\beta = 4\delta/(1+3\delta)\,, \tag{6}$$

where $\delta$ is the electron nonthermality parameter and $\mu = n_{e0}/n_{+0}$ is the unperturbed electron to positive ion ratio. In Eqs. (1)–(5), $n_+(n_-)$ is positive (negative) ionic density (normalized by $n_{+0}(n_{-0})$, $u_+(u_-)$ is positive (negative) ion fluid velocity normalized by the ion sound speed

$$C_s = (K_B T_e/m_+)^{1/2}$$

and $\phi$ is electrostatic wave potential (normalized by $K_B T_e/eZ_+$). $x$ ($t$) is space (time) coordinate, $x$ is normalized to Debye length of hot electron

$$\lambda_{\rm Di} = (K_B T_e/4\pi e^2 Z_+^2 n_{+0})^{1/2}\,,$$

$t$ normalized by inverse of plasma frequency of cold electron

$$\omega_{\rm pi}^{-1} = (4\pi e^2 Z_+^2 n_{+0}/m_+)^{-1/2}\,,$$

where $K_B$ is the Boltzmann constant. $T_e$ is the electron temperature, $\nu = n_{-0}/n_{+0}$, and the $Q$ (mass ratio) $= m_+/m_-$, where $m_+$ and $m_-$ are positive and negative ion masses, respectively. Also, $Z_\pm$ is ionic charge number. The positive (negative) kinematic viscosity $\eta_+$ ($\eta_-$) are normalized by $\eta_+ = \eta_1/(\omega_{\rm pi}\lambda_{\rm Di}^2)$ and $\eta_- = \eta_2/(\omega_{\rm pi}\lambda_{\rm Di}^2)$. The neutrality condition implies:

$$Z_+ = Z_-\nu + \mu \tag{7}$$

for simplicity we shall consider ($Z_+ = Z_- = 1$).

### 2.1 *Nonlinear Calculations*

The slow stretched co-ordinates in (RPT) method[15] are given by:

$$\tau = \epsilon^{3/2}t\,,\quad \xi = \epsilon^{1/2}(x-\lambda t)\,,$$
$$\eta_1 = \epsilon^{1/2}\eta_1\,,\quad \eta_2 = \epsilon^{1/2}\eta_2\,. \tag{8}$$

The speed of the wave $\lambda$ and $\epsilon$ is a small real parameter. Expanding quantities in Eqs. (1)–(5) about their equilibrium values:

$$\begin{aligned}
n_+ &= 1 + \epsilon n_{+_1} + \epsilon^2 n_{+2} + \epsilon^3 n_{+_3} + \cdots\,,\\
u_+ &= \epsilon u_{+_1} + \epsilon^2 u_{+_2} + \epsilon^3 u_{+_3} + \cdots\,,\\
n_- &= 1 + \epsilon n_{-_1} + \epsilon^2 n_{-_2} + \epsilon^3 n_{-_3} + \cdots\,,\\
u_- &= \epsilon u_{-_1} + \epsilon^2 u_{-_2} + \epsilon^3 u_{-_3} + \cdots\,,\\
\phi &= \epsilon\Phi_1 + \epsilon^2\Phi_2 + \epsilon^3\Phi_3 + \cdots
\end{aligned} \tag{9}$$

The last equations are valid with conditions: $|\xi| \to \infty$, $n_- = n_+ = 1, u_+ = 0$, $\phi = 0$. Using Eqs. (8) and (9) into Eqs. (1)–(6) for quasi-neutrality condition at equilibrium,[39] the lowest-order in $\epsilon$ gives:

$$n_{+1} = \frac{\Phi_1}{\lambda^2}\phi_1\,,\quad u_{+1} = \frac{\Phi_1}{\lambda}\,,$$
$$n_{-1} = -\frac{Q\nu\Phi_1}{\lambda^2}\,,\quad u_{-1} = -\frac{Q\Phi_1}{\lambda}\,. \tag{10}$$

The dispersion form is given by:

$$\frac{(\beta-1)\lambda^2\mu + \nu Q + 1}{\lambda^2} = 0\,. \tag{11}$$

The next equations of $O(\epsilon^2)$ yield:

$$-\lambda\frac{\partial N_{+2}}{\partial\xi} + \frac{\partial U_{+2}}{\partial\xi} + \frac{\partial N_{+1}}{\partial\tau} + N_{+1}\frac{\partial U_{+1}}{\partial\xi} + U_{+1}\frac{\partial N_{+1}}{\partial\xi} = 0\,, \tag{12}$$

$$-\lambda\frac{\partial U_{+2}}{\partial\xi} + \frac{\partial U_{+1}}{\partial\tau} + \frac{\partial\Phi_2}{\partial\xi} + U_{+1}\frac{\partial U_{+1}}{\partial\xi} + \eta_1\frac{\partial^2 U_{+1}}{\partial\xi^2} = 0\,, \tag{13}$$

$$-\lambda\frac{\partial N_{-2}}{\partial\xi} + \nu\frac{\partial U_{-2}}{\partial\xi} + \frac{\partial N_{-1}}{\partial\xi} + N_{-1}\frac{\partial U_{-1}}{\partial\xi} + U_{-1}\frac{\partial N_{-1}}{\partial\xi} = 0\,, \tag{14}$$

$$-\lambda\frac{\partial U_{-2}}{\partial\xi}+\frac{\partial U_{-1}}{\partial\tau}-Q\frac{\partial\Phi_2}{\partial\xi}+U_{-1}\frac{\partial U_{-1}}{\partial\xi}+\eta_2\frac{\partial^2U_{-1}}{\partial\xi^2}=0\,, \tag{15}$$

$$\frac{1}{2}\mu\Phi_1^2+(\mu-\beta\mu)\Phi_2+N_{-2}-N_{+2}=0\,. \tag{16}$$

Put out $N_{+2}$, $N_{-2}$, $\phi_2$, $U_{+2}$, and $U_{-2}$ in Eqs. (12)–(16), one obtained the Burger equation for $\phi_1$:

$$\frac{\partial\Phi_1}{\partial\tau}+A\,\phi_1\frac{\partial\Phi_1}{\partial\xi}+B\frac{\partial^2\Phi_1}{\partial\xi^2}=0\,, \tag{17}$$

where

$$A=-\frac{\lambda^4\mu+3\nu Q^2-3}{2(\lambda+\lambda\nu Q)}\,,$$

$$B=\frac{\eta_1+\eta_2\nu Q}{2\nu Q+2}\,. \tag{18}$$

Equation (17) admits the IA shock wave solution as

$$\Phi_1=\frac{2B}{A}[1+\tanh(\chi)]\,, \tag{19}$$

whose amplitude equals $2B/A$ with $\chi(=\xi-2B\tau)$.

The physical quantities $N_{+2}$, $N_{-2}$, $U_{+2}$ and $U_{-2}$ can be rewritten as:

$$N_{+2}=-\frac{A\Phi_1^2}{\lambda^3}+\frac{\eta_1}{\lambda^3}\frac{\partial\Phi_1}{\partial\xi}-\frac{2B}{\lambda^3}\frac{\partial\Phi_1}{\partial\xi}+\frac{3\Phi_1^2}{2\lambda^4}+\frac{\Phi_2}{\lambda^2}\,,$$

$$U_{+2}=-\frac{A\Phi_1^2}{2\lambda^2}+\frac{\eta_1}{\lambda^2}\frac{\partial\Phi_1}{\partial\xi}-\frac{B}{\lambda^2}\frac{\partial\Phi_1}{\partial\xi}+\frac{\Phi_1^2}{2\lambda^3}+\frac{\Phi_2}{\lambda}\,,$$

$$N_{-2}=\frac{A\nu Q\Phi_1^2}{\lambda^3}+\frac{2B\nu Q}{\lambda^3}\frac{\partial\Phi_1}{\partial\xi}+\frac{3\nu Q^2\Phi_1^2}{2\lambda^4}-\frac{\eta_2\nu Q}{\lambda^3}\frac{\partial\Phi_1}{\partial\xi}-\frac{\nu Q\Phi_2}{\lambda^2}\,,$$

$$U_{-2}=\frac{AQ\Phi_1^2}{2\lambda^2}+\frac{BQ}{\lambda^2}\frac{\partial\Phi_1}{\partial\xi}+\frac{Q^2\Phi_1^2}{2\lambda^3}-\frac{\eta_2 Q}{\lambda^2}\frac{\partial\Phi_1}{\partial\xi}-\frac{Q\Phi_2}{\lambda}\,. \tag{20}$$

Equations for the next order in $\epsilon$ are given by:

$$\frac{\partial N_{+2}}{\partial\tau}-\lambda\frac{\partial N_{+3}}{\partial\xi}+U_{+2}\frac{\partial N_{+1}}{\partial\xi}+U_{+1}\frac{\partial N_{+2}}{\partial\xi}+N_{+2}\frac{\partial U_{+1}}{\partial\xi}+N_{+1}\frac{\partial U_{+2}}{\partial\xi}+\frac{\partial U_{+3}}{\partial\xi}=0\,, \tag{21}$$

$$\frac{\partial U_{+2}}{\partial\tau}+U_{+2}\frac{\partial N_{+1}}{\partial\xi}+U_{+1}\frac{\partial N_{+2}}{\partial\xi}-\lambda\frac{\partial U_{+3}}{\partial\xi}+\eta_1\frac{\partial^2U_{+2}}{\partial\xi^2}+\frac{\partial\Phi_3}{\partial\xi}=0\,, \tag{22}$$

$$\frac{\partial N_{-2}}{\partial\tau}-\lambda\frac{\partial N_{-3}^{(1,0)}}{\partial\xi}+U_{-2}\frac{\partial N_{-1}}{\partial\xi}+U_{-1}\frac{\partial N_{-2}}{\partial\xi}+N_{-2}\frac{\partial U_{-1}}{\partial\xi}+N_{-1}\frac{\partial U_{-2}}{\partial\xi}+\nu\frac{\partial U_{-3}}{\partial\xi}=0\,, \tag{23}$$

$$\frac{\partial U_{-2}}{\partial\tau}+U_{-2}\frac{\partial U_{-1}}{\partial\xi}+U_{-1}\frac{\partial U_{-2}}{\partial\xi}-\lambda\frac{\partial U_{-3}}{\partial\xi}+\eta_2\frac{\partial^2U_{-2}}{\partial\xi^2}-Q\frac{\partial\Phi_3}{\partial\xi}=0\,, \tag{24}$$

$$\frac{1}{6}(3\beta\mu+\mu)\Phi_1 3+(\mu-\beta\mu)\Phi_3+\mu\Phi_2\Phi_1+N_{-3}-N_{+3}=0\,. \tag{25}$$

Eliminating $N_{+2}$, $N_{-2}$, $U_{+2}$, $U_{-2}$ and $\phi_3$ from Eqs. (21)–(25), we get a linearly inhomogeneous Burger type equation for $\phi_1$ and $\phi_2$:

$$\tilde{L}(\phi_1)\phi_2\equiv\frac{\partial\phi_2}{\partial\tau}+A\frac{\partial(\phi_1\phi_2)}{\partial\xi}+B\frac{\partial^2\phi_2}{\partial\xi^2}-S(\phi_1)=0\,, \tag{26}$$

where

$$S(\phi_1)=A_1\frac{\partial^3\phi_1}{\partial\xi^3}+A_2\Big(\phi_1^2\frac{\partial\phi_1}{\partial\xi}\Big)+A_3\Big(\frac{\partial\phi_1}{\partial\xi}\Big)^2+A_4\Big(\phi_1\frac{\partial^2\phi_1}{\partial\xi^2}\Big)\,, \tag{27}$$

where the coefficients $A_i$ (where $i=1,2,\ldots,4$) are given by:

$$A_1=\frac{3B^2(\nu Q+1)-3B\eta_1+\eta_2\nu Q(\eta_2-3B)+\eta_1^2}{2(\lambda+\lambda\nu Q)}\,,$$

$$A_2=\frac{6A^2\lambda^2(\nu Q+1)+20A\lambda\left(\nu Q^2-1\right)+\lambda^6(-(3\beta\mu+\mu))+15\left(\nu Q^3+1\right)}{4\lambda^3(\nu Q+1)}\,,$$

$$A_3=\frac{B\left(3A(\lambda+\lambda\nu Q)+4\nu Q^2-4\right)+\eta_1(4-3A\lambda)-\eta_2\nu Q(3A\lambda+4Q)}{2\lambda^2(\nu Q+1)}\,,$$

$$A_4=\frac{-2B\left(3A(\lambda+\lambda\nu Q)+4\nu Q^2-4\right)+\eta_1(3A\lambda-4)+\eta_2\nu Q(3A\lambda+4Q)}{2\lambda^2(\nu Q+1)}\,. \tag{28}$$

In summary, our model reduced to nonlinear Burger equation (17) for $\phi_1$ and Burger type equation (26) for $\phi_2$; source term (28) is a function $\phi_1$.

## 3 Stationary Solution

By using Eq. (19), Eq. (27) can be transformed into the form:

$$\frac{\mathrm{d}^2\Phi_2}{\mathrm{d}\chi^2}+\frac{\mathrm{d}}{\mathrm{d}\chi}(2\Phi_2\tanh(\chi))=K(\chi)\,, \tag{29}$$

where

$$K(\chi)=\frac{8\tanh(\chi)\mathrm{sech}^2(\chi)(2A_2B^2-AA_4B)}{A^3}\frac{4\ \mathrm{sech}^4(\chi)(-3A^2A_1+AA_3B+2AA_4B-2A_2B^2)}{A^3} \\ +\frac{8\mathrm{sech}^2(\chi)(A^2A_1-AA_4B+2A_2B^2)}{A^3}\,. \tag{30}$$

According to the homogeneous equation of Eq. (29) has two independent solutions, one of them is, $P_3^2(V)=\mathrm{sech}^2(\chi)$, and the other, which can be derived using reduction of order method along with Able's theorem, is given by $Q_3^2(\chi)=[\chi/2+(1/4)\mathrm{sech}(2\chi)]\mathrm{sech}^2(\chi)$. Using the variation of parameters method,[40−41] the particular solution of Eq. (29) can be written as

$$\Phi_2(\chi)=L_1(\chi)P_3^2(\chi)+L_2(\chi)Q_3^2(\chi)\,, \tag{31}$$

where $L_1(\psi)$ and $L_2(\psi)$ are given by

$$L_1(\chi)=\int\frac{Q_3^2(\chi)K(\chi)}{W(P_3^2(\chi),Q_3^2(\chi))}\mathrm{d}\chi\,, \tag{32}$$

$$L_2(\chi)=\int\frac{P_3^2(\chi)K(\chi)}{W(P_3^2(\chi),Q_3^2(\chi))}\mathrm{d}\chi\,, \tag{33}$$

with

$$W(P_3^2,Q_3^2)=P_3^2\frac{\mathrm{d}Q_3^2}{\mathrm{d}\chi}+Q_3^2\frac{\mathrm{d}P_3^2}{\mathrm{d}\chi}=\mathrm{sech}^2(\chi)\,.$$

Then, the formal stationary IA shock solution is given by:

$$\Phi(\chi)=\Phi_1(\chi)+\Phi_2(\chi)=\frac{4B(AA_3-AA_4+4A_2B)}{3A^3}-\frac{4B\chi\mathrm{sech}^2(\chi)(2A_2B-AA_4)}{A^3} \\ +\frac{4\mathrm{sech}^2(\chi)\mathrm{Ln}\,(\cosh(\chi))(-3A^2A_1+AA_3B+2ABA_4-2A_2B^2)}{3A^3} \\ +\frac{2\mathrm{sech}^2(\chi)(-3A^2A_1-AA_3B+4AA_4B-10A_2B^2)}{3A^3}+\frac{2B(\tanh(\chi)+1)}{A}\,. \tag{34}$$

## 4 Model Results and Discussions

Shock waves are studied in unmagnetized viscous positive negative ion plasmas. Numerical values have been introduced for Earth's ionosphere (D- and F-regions).[1−2,22−23] According to the wave dissipation caused by kinematic viscosity, the studied system supports electrostatic shock waves. However, the main essential stimulus was to study the contribution of higher-order electric field structures associated to the shock waves. The comparison of lowest order and higher-order shock noise and associated electric field structures are shown in Fig. 1. It is shown that higher-order decreases both steepness and amplitude of shock form and modulates the associated electric field structure. We have studied the effectuation of plasma parameters such as the population of nonthermal electrons $\delta$, the kinematic viscosities coefficient of positive negative ions $\eta_1$ and $\eta_2$ and the ion mass ratio $Q$ $(=m_+/m_-)$ on the dynamics formation of higher-order broadband electrostatic shock noise. For example, Fig. 2 shows that, the increase of electron nonthermal parameter $\delta$ decreases shock steepness and amplitude. Accordingly, the amplitude and width of associated electric field of shock structure decreased also. On the other hand, the effect of kinematic viscosity coefficients of positive-negative ions $\eta_1$ and $\eta_2$ on higher-order shock profile $\phi$ and associated electric field structures $E_f$ have been examined in Figs. 3–4. It is found that as $\eta_1$ and $\eta_2$ increase the steepness, amplitude of higher-order shock wave and higher-order electric field structures associated to the shock waves. The physical reason for this behavior is that the increase of kinematic viscosity coefficients increases the dissipation and consequently causes strong shock wave and associated electric field structures. Finally, the ratio raise of positive-negative mass $Q$ decreases both wave amplitude and steepness of electrostatic shocks as shown in Fig. 5. In summary, it has been noted that the involvement of higher-order effects, positive-negative mass ratio $Q$, electron nonthermal parameter $\delta$ and kinematic viscosities coefficient of positive and negative ions $\eta_1$ and $\eta_2$ would regulate the countenance of ion electrostatic acoustic shock waves in D- and F-regions of Earth's ionosphere.

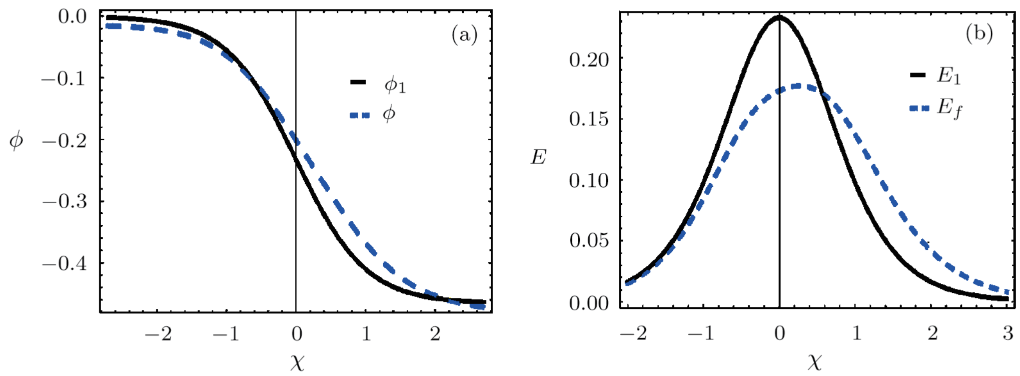


**Fig. 1** The comparison of lowest and higher-order shock profile and associated electric field structures for $Q = 0.03$, $\eta_1 = 0.3$, $\eta_2 = 0.2$, $\nu = 0.5$, and $\delta = 0.2$.

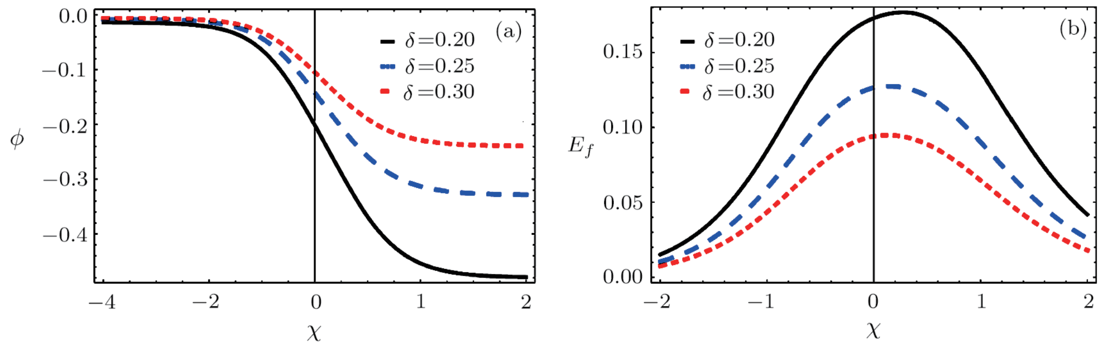


**Fig. 2** Variation of higher-order shock profile $\phi$ and associated electric field structures $E_f$ vs. $\chi$ and $\delta$ for $Q = 0.03$, $\eta_1 = 0.3$, $\eta_2 = 0.2$, $\nu = 0.5$.

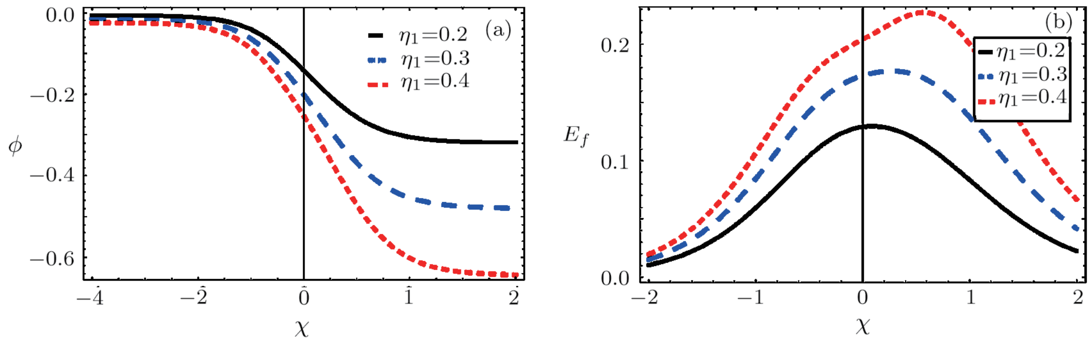


**Fig. 3** Variation of higher-order shock profile $\phi$ and associated electric field structures $E_f$ vs. $\chi$ and $\eta_1$ for $Q = 0.03$, $\eta_2 = 0.2$, $\nu = 0.5$, and $\delta = 0.2$.

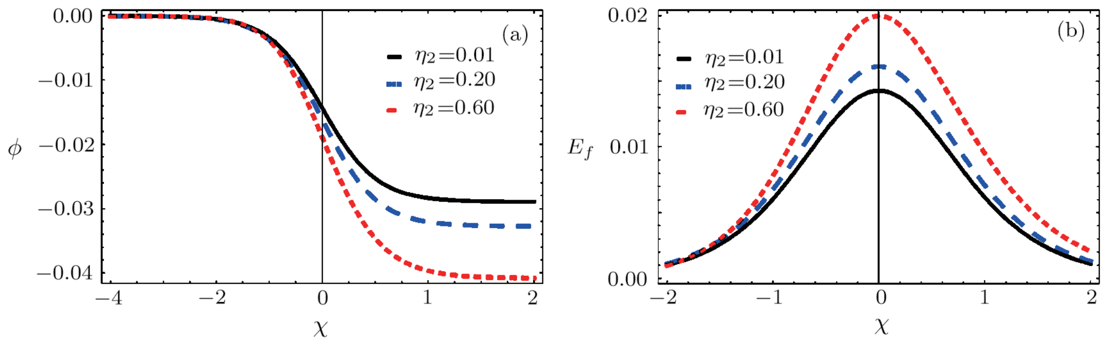


**Fig. 4** Variation of higher-order shock profile $\phi$ and associated electric field structures $E_f$ vs. $\chi$ and $\eta_2$ for $Q = 0.03$, $\eta_1 = 0.03$, $\nu = 0.5$, and $\delta = 0.2$.

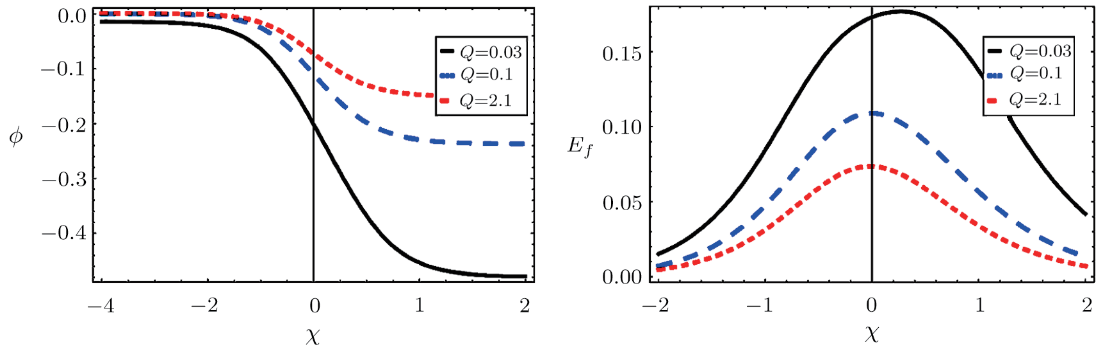


**Fig. 5** Variation of higher-order shock profile $\phi$ and associated electric field structures $E_f$ vs. $\chi$ and $Q$ for $\eta_1 = 0.3$, $\eta_2 = 0.2$, $\nu = 0.5$, and $\delta = 0.2$.

## 5 Conclusion

Ion shock waves are discussed in three collisionless unmagnetized plasmas components having viscous fluid of positive (+) and negative (−) ion and nonthermally electron distribution. The nonlinear Burger equation (17) for lowest order and linear inhomogeneous Burger type equation (26) for higher-order dissipation are obtained. Renormalization technique gives stationary solution for perturbation theory equations. It is emphasized that steepness and amplitude of higher-order shock waves are sensitive to higher-order effects, positive-negative mass ratio $Q$, electron nonthermal parameter $\delta$ and kinematic viscosities coefficient of positive (negative) ions $\eta_1$ ($\eta_2$). It is clear to confirm that the increase (decrease) of $\eta_1$ and $\eta_2$ ($Q$ and $\delta$) can lead to the increase of higher-order broadband electrostatic shock amplitude. The results obtained may be useful in understanding electrostatic shock noise in Earth's ionosphere.

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