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Investigation of Weakly Relativistic Ponderomotive Effects on Self-Focusing During Interaction of High Power Elliptical Laser Beam with Plasma

Keshav Walia,[1,*] Deepak Tripathi,[2] and Yachna Tyagi[2]

[1]Department of Physics, DAV University Jalandhar, India

[2]Department of Physics, AIAS, Amity University Noida, India

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Abstract *This paper presents an investigation of weakly relativistic ponderomotive effects on self-focusing during interaction of high power elliptical laser beam with plasma. The nonlinear differential equations for the beam width parameters of elliptical laser beam have set up by using Wentzal–Krammers–Brillouin (WKB) and paraxial approximations. These equations have been solved numerically by using fourth order Runge–Kutta method to study the variation of these beam width parameters against normalized distance of propagation. Effects of variation in laser beam intensity, plasma density and electron temperature on the beam width parameters are also analyzed.*

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Key words: self-focusing, elliptical laser beam, weakly relativistic ponderomotive effects, plasma density, electron temperature

1 Introduction

There has been considerable interest in the nonlinear interaction of high power laser beams with plasmas on account of its wide range of applications such as laser driven fusion, charged particle acceleration, X-ray lasers etc.[1−12] In order to achieve success in these applications, it is highly desirable that laser beam must propagate up to several Rayleigh lengths inside plasma. So, it becomes very important to guide a laser beam over several Rayleigh lengths inside plasma. Optical guiding of laser beam in plasma can be achieved through various nonlinear effects. The most important nonlinear effect is presence of relativistic and ponderomotive nonlinearities.[13] Whenever beam has an intensity gradient and very high intensity, then redistribution of electrons takes place from high field region to low field region resulting in modification in plasma density. Effective dielectric permittivity of electrons gets modified on account of induced density variation, the wavefront of laser beam acquires a curvature and tens to focus. This is known as famous ponderomotive self-focusing mechanism.[14] The relativistic nonlinearity becomes operative, when the power of beam is of the order of 10^{16} W/cm^2. In such case, speed of electrons becomes comparable to speed of light resulting in increase in mass of electrons. This further causes modification in refractive index. Hence, transverse gradient of refractive index of plasma gets established thereby imposing a curvature on the wavefront. This results in relativistic self-focusing as pointed out many years ago by Hora.[15] In the laser-plasma interaction, process of self-focusing has been the focus of attention as it effects many other nonlinear processes.[16−28] The self-focusing/defocusing in the electromagnetically induced transparency atomic medium is well known experimentally as well as theoretically.[29−30] In the atomic like medium, laser induced index gratings can be used for creating high enough refractive index, which further leads to production of dipole-mode solitons.[31−32] These solitons are produced due to balanced interaction between spatial diffraction and cross-kerr non-linearity.

In-fact relativistic self-focusing occurs almost instantaneously in the time of the order of a period of the optical oscillation, while the ponderomotive self-focusing arises later in time on account of motion of plasma from the axis of beam. So, the ponderomotive self-focusing only adds to relativistic self-focusing. Relativistic nonlinearity is operative, when laser pulse duration τ is less than relaxation time of electrons τ_e i.e. $\tau < \tau_e$, whereas relativistic-ponderomotive nonlinearity is operative, when laser pulse duration τ is in between relaxation time of electrons τ_e and relaxation time of ions τ_i, i.e. $\tau_e < \tau < \tau_i$. The relativistic ponderomotive force is used for several practical phenomena such as electron cavitation, X-ray generation, intense magnetic generation, electron acceleration etc.[33−41] Self-focusing phenomenon has been studied in detail both theoretically as well as experimentally in presence of relativistic and ponderomotive nonlinearities alone as well as under combined effect of both nonlinearities. Most of the work on laser-plasma interaction in different regimes is carried out by taking into account cylindrical gaussian beams.[42−44] Since, beams produced by many laser systems are more elliptical than circular in cross-section. So,

*E-mail: keshavwalia86@gmail.com

it is worthwhile to study this realistic situation. So, motivation of present work is to investigate weakly relativistic ponderomotive effects on self-focusing during interaction of high power elliptical laser beam with plasma.

In the present paper, weakly relativistic ponderomotive effects on self-focusing during interaction of high power elliptical laser beam with plasma are investigated. In Sec. 2, second order differential equations governing the evolution of spot size of laser beam have been set up with the help of Wentzal–Kramers–Brillouin (WKB) approximation and paraxial ray approximation. The detailed discussion and conclusions drawn from the results of present investigation have been summarized in Secs. 3 and 4.

2 Evolution of Spot Size of Laser Beam

A high power laser beam with elliptical cross-section is considered to be propagating in underdense plasma along z-direction. The initial field distribution of laser beam is given by

$$E = E_0 \exp\Big[-\Big(\frac{x^2}{2a^2}+\frac{y^2}{2b^2}\Big)\Big]\,, \tag{1}$$

where, a and b correspond to initial dimensions of the laser beam in x and y directions respectively and E_0 is the axial amplitude of the beam.

The slowly varying electric field E of the beam satisfied the following wave equation

$$\nabla^2 E + \frac{\omega^2}{c^2}\epsilon E = 0\,. \tag{2}$$

Equation (2) can be derived directly from Maxwell's equations by neglecting the term $\nabla(\nabla\cdot E)$ provided that $(c^2/\omega^2)|(1/\epsilon)\nabla^2 \ln\epsilon| \ll 1$. Here ω, ϵ, and c represent the laser frequency, effective dielectric function for the plasma and speed of light respectively.

Further, the relativistic-ponderomotive force on electrons modifies the electron density. The relativistic-ponderomotive force on electrons is given by

$$F_{\rm pe} = -m_0c^2\nabla(\gamma-1)\,, \tag{3}$$

where m_0 represents electron mass in absence of external field and γ is relativistic factor represented by

$$\gamma = \sqrt{1+\alpha EE^*}\,, \tag{4}$$

where $\alpha = e^2/m_0^2c^2\omega^2$. Following Nikham *et al.*,[45] the dielectric function for plasma may be represented as

$$\epsilon = 1-\frac{\omega_p^2}{\gamma\omega^2}\exp\Big(-\frac{m_0c^2}{T_e}(\gamma-1)\Big)\,, \tag{5}$$

where $\omega_p = \sqrt{4\pi n_0 e^2/m_0}$ represents electron plasma frequency. $-e$, T_e and m_0 correspond to electronic charge, temperature of plasma electrons, and electron mass in absence of external field. Also, modified density of plasma electrons n_e in presence of external electric field is given by

$$n_e = n_0\exp\Big(-\frac{m_0c^2}{T_e}(\gamma-1)\Big)\,, \tag{6}$$

where n_0 represents density of plasma electrons in absence of external field. In general, one can formally express the effective dielectric function of plasma as

$$\epsilon = \epsilon_0 + \Phi(EE^*)\,, \tag{7}$$

where $\epsilon_0 = 1-\omega_p^2/\omega^2$ and $\Phi(EE^*) = (\omega_p^2/\omega^2)[1-n_e/\gamma n_0]$ are linear and non-linear parts of the dielectric function respectively.

Now, following Refs. [42–44], the solution of E can be written as,

$$E = E_0\exp[\mathrm{i}(\omega t - k(S+z))]\,, \tag{8}$$

$$E_0^2 = \frac{E_{00}^2}{f_1f_2}\exp\Big(\frac{-x^2}{a^2f_1^2}\Big)\exp\Big(\frac{-y^2}{b^2f_2^2}\Big)\,, \tag{9}$$

$$S = \frac{1}{2}x^2\frac{1}{f_1}\frac{\mathrm{d}f_1}{\mathrm{d}z}+\frac{1}{2}y^2\frac{1}{f_2}\frac{\mathrm{d}f_2}{\mathrm{d}z}+\Phi_0(z)\,, \tag{10}$$

$$k = \frac{\omega}{c}\epsilon_0^{1/2}\,, \tag{11}$$

where, E_0 is the real function of x, y and z. S represents the eikonal for the laser beam, $\Phi_0(z)$ represents phase shift, whose value will not be needed explicitly in further analysis. Here, f_1 and f_1 represent the dimensionless beam width parameters of semi-major and semi-minor axis of elliptical laser beam along x and y directions respectively and satisfy the following differential equations.

$$\frac{\mathrm{d}^2f_1}{\mathrm{d}\eta^2} = \frac{1}{f_1^3}-\Big(\frac{\omega_pa}{c}\Big)^2\frac{\alpha E_{00}^2}{2f_1^2f_2}\exp\Big[-\frac{m_0c^2}{T_e}\Big(\sqrt{1+\frac{\alpha E_{00}^2}{f_1f_2}}-1\Big)\Big]\Big(1+\frac{\alpha E_{00}^2}{f_1f_2}\Big)^{-3/2}\Big(1+\frac{m_0c^2}{T_e}\sqrt{1+\frac{\alpha E_{00}^2}{f_1f_2}}\Big)\,, \tag{12}$$

$$\frac{\mathrm{d}^2f_2}{\mathrm{d}\eta^2} = \frac{a^4}{b^4f_2^3}-\Big(\frac{\omega_pa}{c}\Big)^2\frac{a^2\alpha E_{00}^2}{2b^2f_2^2f_1}\exp\Big[-\frac{m_0c^2}{T_e}\Big(\sqrt{1+\frac{\alpha E_{00}^2}{f_1f_2}}-1\Big)\Big]\Big(1+\frac{\alpha E_{00}^2}{f_1f_2}\Big)^{-3/2}\Big(1+\frac{m_0c^2}{T_e}\sqrt{1+\frac{\alpha E_{00}^2}{f_1f_2}}\Big)\,. \tag{13}$$

Initial conditions of plane wavefront are, $f_1 = f_2 = 1$ and $\mathrm{d}f_1/\mathrm{d}\eta = \mathrm{d}f_2/\mathrm{d}\eta = 0$ at $\eta = 0$, where η represents dimensionless distance of propagation.

3 Discussion

Equations (12) and (13) are the nonlinear differential equations governing the behavior of dimensionless beam width parameters f_1 and f_2 of elliptical laser beam as a function of dimensionless distance of propagation η. The first term on right hand side of Eqs. (12) and (13) represents the diffraction phenomenon and second term which arises on account of weakly relativistic ponderomotive nonlinearity represents the nonlinear refraction. The diffraction term is responsible for divergence of the beam, while the nonlinear term is responsible for self-focusing. The relative magnitude of these terms determines the focusing/defocusing of the beam. When the magnitude of first term is greater than second term, the beam diverges, while the opposite is true when the second term exceeds the first one. The differential Eqs. (12) and (13) have been solved numerically for the following set of parameters:

$\omega = 1.778 \times 10^{15}$ rad/sec; $\alpha E_{00}^2 = 0.1, 0.2, 0.3$; $\omega_p/\omega = 0.1, 0.2, 0.3$; $T_e = 50$ KeV, 75 KeV, 100 KeV.

The results are presented in the form of graphs as discussed below.

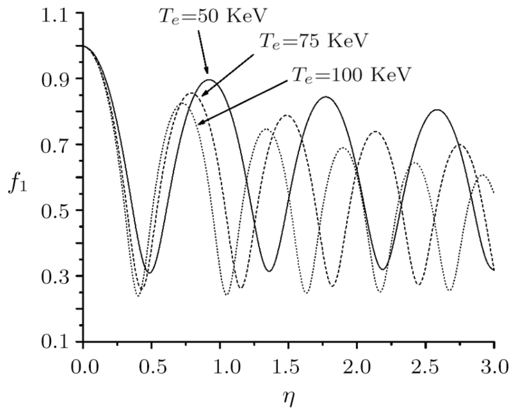

Fig. 1 Variation of beam width parameter f_1 versus normalized distance of propagation η at different values of plasma temperature ($T_e = 50$ KeV, 75 KeV, 100 KeV) keeping all other parameters fixed. Solid line corresponds to $T_e = 50$ KeV, dashed line corresponds to $T_e = 75$ KeV, and dotted line corresponds to $T_e = 100$ KeV.

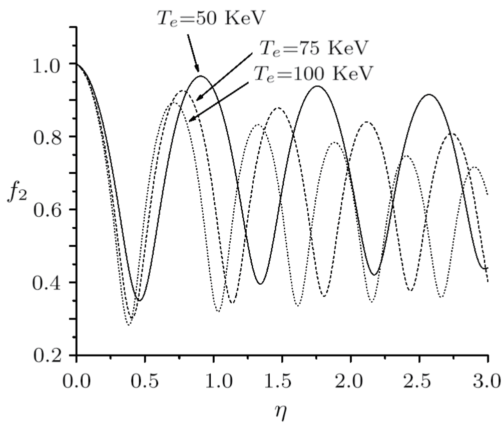

Fig. 2 Variation of beam width parameter f_2 versus normalized distance of propagation η at different values of plasma temperature ($T_e = 50$ KeV, 75 KeV, 100 KeV) keeping all other parameters fixed. Solid line corresponds to $T_e = 50$ KeV, dashed line corresponds to $T_e = 75$ KeV, and dotted line corresponds to $T_e = 100$ KeV.

Figures 1 and 2 describe the variation of beam width parameters f_1 and f_2 versus normalized distance of propagation at different values of plasma temperature ($T_e = 50$ KeV, 75 KeV, 100 KeV) keeping all other parameters fixed. Solid line corresponds to $T_e = 50$ KeV, dashed line corresponds to $T_e = 75$ KeV and dotted line corresponds to $T_e = 100$ KeV. Oscillatory self-focusing with distance of propagation is observed in all the three cases on account of saturating nature of dielectric function. It is further observed from the figure that self-focusing length of the beam decreases with increase in plasma temperature. This is due to the fact that the nonlinear refractive term, which is sensitive to plasma temperature becomes relatively stronger than diffractive term at higher values of plasma temperature resulting in decrease in self-focusing length of the beam.

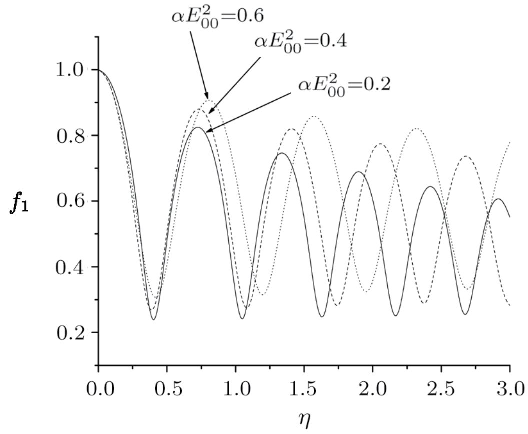

Fig. 3 Variation of beam width parameter f_1 with normalized distance of propagation η at different values of intensities ($\alpha E_{00}^2 = 0.2, 0.4, 0.6$) keeping all other parameters fixed. Solid line corresponds to $\alpha E_{00}^2 = 0.2$, dashed line corresponds to $\alpha E_{00}^2 = 0.4$ and dotted line corresponds to $\alpha E_{00}^2 = 0.6$.

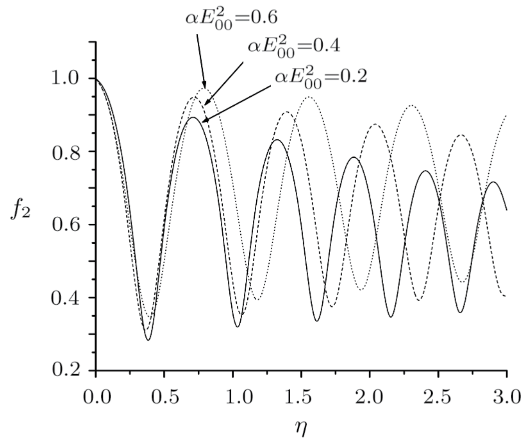

Fig. 4 Variation of beam width parameter f_2 with normalized distance of propagation η at different values of intensities ($\alpha E_{00}^2 = 0.2, 0.4, 0.6$) keeping all other parameters fixed. Solid line corresponds to $\alpha E_{00}^2 = 0.2$, dashed line corresponds to $\alpha E_{00}^2 = 0.4$ and dotted line corresponds to $\alpha E_{00}^2 = 0.6$.

Figures 3 and 4 describe the variation of beam width parameters f_1 and f_2 with normalized distance of propagation η at different values of intensities ($\alpha E_{00}^2 = 0.2, 0.4, 0.6$) keeping all other parameters fixed. Solid line corresponds to $\alpha E_{00}^2 = 0.2$, dashed line corresponds to $\alpha E_{00}^2 = 0.4$ and dotted line corresponds to $\alpha E_{00}^2 = 0.6$. It is observed from the figure that the extent of self-focusing of the beam decreases with increase in laser beam intensity. This is due to reason that nonlinear refractive term on right hand side of Eqs. (12) and (13) becomes relatively weaker than diffractive term at higher values of intensity resulting in decrease in extent of self-focusing of the beam.

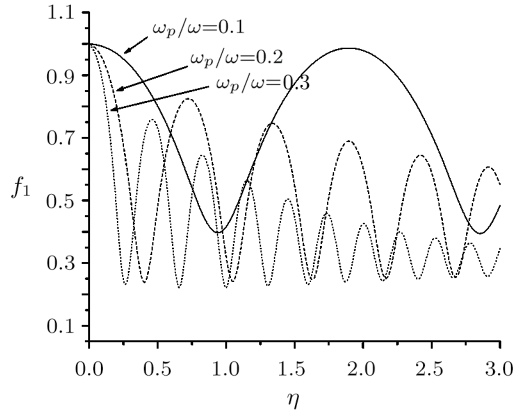

Fig. 5 Variation of beam width parameter f_1 with normalized distance of propagation η at different values of plasma density ($\omega_p/\omega_0 = 0.1, 0.2, 0.3$) keeping all other parameters fixed. Solid line corresponds to $\omega_p/\omega_0 = 0.1$, dashed line corresponds to $\omega_p/\omega_0 = 0.2$ and dotted line corresponds to $\omega_p/\omega_0 = 0.3$.

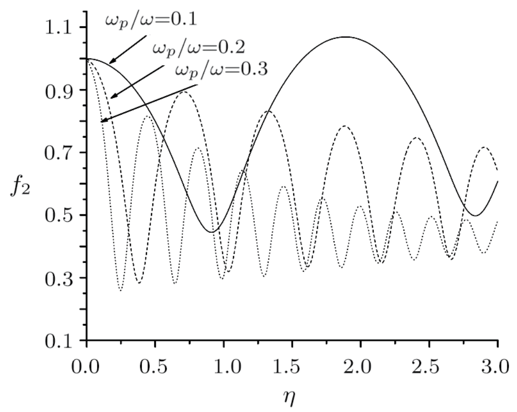

Fig. 6 Variation of beam width parameter f_2 with normalized distance of propagation η at different values of plasma density ($\omega_p/\omega_0 = 0.1, 0.2, 0.3$) keeping all other parameters fixed. Solid line corresponds to $\omega_p/\omega_0 = 0.1$, dashed line corresponds to $\omega_p/\omega_0 = 0.2$, and dotted line corresponds to $\omega_p/\omega_0 = 0.3$.

Figures 5 and 6 present the variation of beam width parameters f_1 and f_2 with the normalized distance of propagation η at different values of plasma density ($\omega_p/\omega_0 = 0.1, 0.2, 0.3$) keeping all other parameters fixed. Solid line corresponds to $\omega_p/\omega_0 = 0.1$, dashed line corresponds to $\omega_p/\omega_0 = 0.2$ and dotted line corresponds to $\omega_p/\omega_0 = 0.3$. It is observed from the figure that with the increase in plasma density, minimum values of beam width parameters shift towards lower values of η, i.e. increase in plasma density causes enhanced self-focusing of laser beam. This is on account of fact that nonlinear refractive term being sensitive to plasma density becomes relatively strongly than diffractive term as the value of plasma density is increased.

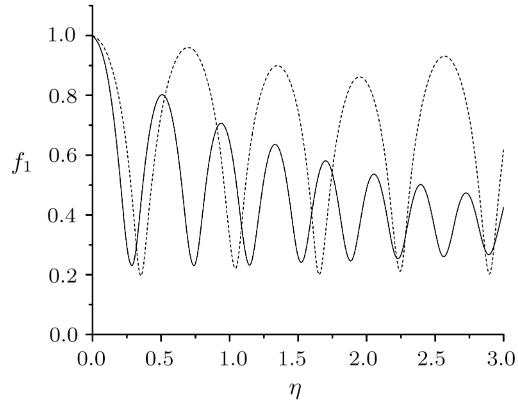

Fig. 7 Variation of beam width parameters f_1 versus normalized distance of propagation η in the weakly relativistic and ponderomotive regime and only relativistic regime. Solid line corresponds to weakly relativistic and ponderomotive case, while dashed line corresponds to relativistic case only.

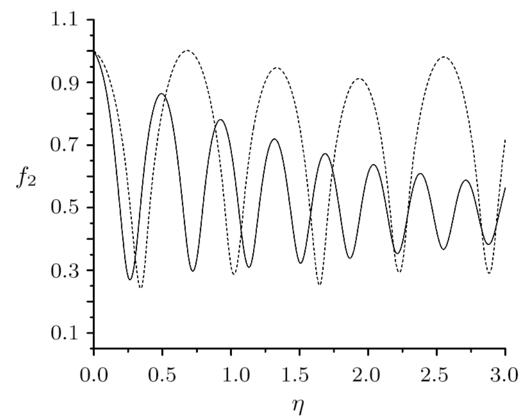

Fig. 8 Variation of beam width parameter f_2 versus normalized distance of propagation η in the weakly relativistic and ponderomotive regime and only relativistic regime. Solid line corresponds to weakly relativistic and ponderomotive case, while dashed line corresponds to relativistic case only.

Figures 7 and 8 describe the variation of beam width parameters f_1 and f_2 versus normalized distance of propagation η in the weakly relativistic and ponderomotive regime and only relativistic regime. Solid line corresponds to weakly relativistic and ponderomotive case, while dashed line corresponds to relativistic case only. It is observed from the figure that when both relativistic and ponderomotive nonlinearities are present, then self-focusing starts acting at lower values of η as compared to relativistic case only i.e. extent of self-focusing of the beam increases, when both relativistic and ponderomotive nonlinearities are considered. This is due to the fact that relativistic self-focusing takes place instantaneously in the time of the order of a period of the optical oscillation, while ponderomotive self-focusing arises later on account of motion of plasma electrons from the high field region to low field region. So, ponderomotive self-focusing only adds to relativistic self-focusing resulting in increase in extent of self-focusing of beam.

4 Conclusion

In the present work, weakly relativistic ponderomotive effects on self-focusing during interaction of high power elliptical laser beam with plasma is investigated by making use of WKB and paraxial approximations. Following important observations are made from present analysis:

(i) Self-focusing length of the beam decreases with increase in plasma temperature and plasma density.

(ii) Larger values of laser beam intensity weaken the extent of self-focusing of the beam.

(iii) Stronger self-focusing is observed, when both ponderomotive and relativistic nonlinearities are operative as compared to only relativistic nonlinearity.

The results of present analysis are useful in understanding the physics of high power laser driven fusion.

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