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Cylindrical Three-Dimensional Dust-Ion Acoustic Propagation in Plasmas

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Abstract *Wave properties of solitons in an unmagnetized four-component dusty plasma system contains isothermal distributed electrons, mobile ions, and negative-positive dusty grains have been examined. To study DIA wave properties, a reductive perturbation (RP) analysis is used. By a reductive perturbation (RP) analysis under convenient coordinate transformation, the three dimension Kadomtsev-Petviashvili equation in cylindrical coordinates is obtained. The effects of dust grain charge on soliton pulse structures are studied. More specifically, solitary profile depending on the axial, radial, and polar angle coordinates with time is discussed. This investigation may be viable in plasmas of the Earth's mesosphere.*

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1 Introduction

No doubt, studying of nonlinear waves and their structures in plasmas with charged dust grains is one of important activities during the last few decades for explaining many systems in astrophysics (Earth magnetosphere, cometary tails, nebulas, mesosphere),[1–4] as well as in the laboratory.[5–6] Moreover, it has remarkable applications in fiber optics, plasmas of semiconductor, and dust-crystals.[7–9] The charged grains in plasmas change plasma features, and define new types of wave phenomena.[10–13] Many investigations are made to expound the applications of nonlinear properties in astrophysics and space dust plasmas.[14–16] It has been reported that dusty grains in multicomponent plasmas influenced the collective interactions in plasmas[17] and the variation of dust charge modified shock features.[18–20] Many articles discussed the negative dusty plasma applications in space.[21–22] On the other hand, in other studies, both negative-positive grains are taken into account in space plasma[23–24] and in plasma laboratories.[25] On the other hand, a new dust grain model has been approached for cometary plasmas having opposite charges polarity in the depletion of electrons and ions.[26] Later, plasma containing ions, electrons, grains with positively-negatively charged was inspected.[27] It was investigated that, new positively grain component caused the existence of twofold solitary potentials. Furthermore, shock behavior in inhomogeneous plasmas with ionizing source is examined.[28] They reported that, charge polarity fluctuation of dusty charge improved the monotonic characteristics of shock waves. However, many of these studies are regarded to the unbounded planar geometry. This is not true for space and laboratory plasma. So we have taken the non-planar form of cylindrical geometry into account. Several theoretical studies in non-planar geometry on the dust plasmas features have been deliberated.[29–32] A multidimensional cylindrical form of Kadomtsev-Petviashvili equation (CKP) in dusty plasmas with two superthermality distributed temperature ions has been introduced.[32] Finally, El-Bedwehy *et al.* investigated CKP equation in a plasma of two charged dust grains.[33] They examined features of soliton formation that depends on with the polar, radial and axial coordinates.

Our article is to explore the non-planar (cylindrical) three-dimensional DIAWs in four components dust plasma system contains isothermal electrons, mobile ions and negative-positive dust grains. We study the effects of ion to electron number density ratio, negative dust grain to electron density ratio and positive grain with an electron number density ratio on nonlinear wave phase speed as well as on the pulse width and amplitude. In addition, the effect of non-planar geometry on the pulse profile is studied. The organization of this paper is as follows. In Sec. 2, we present model equations. In Sec. 3, the derivation of CKP equation is present. Its solution is given in Sec. 4. Section 5 is devoted for results and discussion.

2 System of Equations

Consider a three-dimensional, unmagnetized dusty plasma system whose constituents is isothermal dis-

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tributed electrons, mobile ions, negative and positive dusty grains. The three-dimensional continuity equations for mobile components are given by

$$\frac{\partial n_i}{\partial t} + \nabla\cdot(n_i u_i) = 0\,,$$
$$\frac{\partial n_n}{\partial t} + \nabla\cdot(n_n u_n) = 0\,,$$
$$\frac{\partial n_p}{\partial t} + \nabla\cdot(n_p u_p) = 0\,. \tag{1}$$

The corresponding momentum equations are,

$$\Big(\frac{\partial}{\partial t} + u_i\cdot\nabla\Big)u_i + \nabla\phi = 0\,,$$
$$\Big(\frac{\partial}{\partial t} + u_n\cdot\nabla\Big)u_n - \mu\nabla\phi = 0\,,$$
$$\Big(\frac{\partial}{\partial t} + u_p\frac{\partial}{\partial x}\Big)u_p + \alpha\nabla\phi = 0\,. \tag{2}$$

These equations are supplemented by Poisson's equation:

$$\nabla^2\phi = \nu n_n - \rho n_i - \eta n_p + \exp(\phi)\,. \tag{3}$$

In the above equations n_j $(j = i, n, p, e)$ are the perturbed number densities and n_{i0}, n_{n0}, n_{p0}, and n_{e0} are the related equilibrium values. Also u_j $(j = i, n, p)$ are the ion, negative and positive dusty plasma velocities, respectively, normalized by the ion sound velocity $(K_B T_e/m_i)^{1/2}$, ϕ is the electrostatic potential and normalized by $(K_B T_e/e)$, time variable t and space coordinate are normalized by inverse of the plasma frequency $\omega_{pe}^{-1} = (m_i/4\pi e^2 n_{e0})^{1/2}$ and electron Debye length $\lambda_d = (K_B T_e/4\pi e^2 n_{e0})^{1/2}$, respectively, $\mu = Z_n m_i/m_n$, $\alpha = Z_p m_i/m_p$, Here, K_B and T_e are Boltzmann constant and temperature of electron, e the electronic charge, m_j $(j = i, n, p)$ denote ion, negative and positive dust masses respectively.

From the charge neutrality condition, we have

$$\nu + 1 = \rho + \eta\,, \tag{4}$$

with

$$\nu = Z_n n_{n0}/n_{e0}\,,\quad \rho = n_{i0}/n_{e0}\,,\quad \eta = Z_p n_{p0}/n_{e0}\,,$$

where Z_n and Z_p are charge numbers of negative and positive grains, respectively.

3 Nonlinear Calculations

To study DIA wave properties, a reductive perturbation (RP) analysis is used.[34] We introduce the new independent variables:[35–36]

$$R = \epsilon^{1/2}(r - \lambda t)\,,\quad \Theta = \epsilon^{-1/2}\theta\,,\quad Z = \epsilon z\,,\quad T = \epsilon^{3/2}t\,, \tag{5}$$

where ϵ is a small parameter measures the degree of perturbation and the λ is the wave propagation velocity. The dependent variables in the model are expanded in the powers of ϵ as

$$n_j = 1 + \epsilon n_j^{(1)} + \epsilon^2 n_j^{(2)} + \cdots\,,$$
$$u_j = \epsilon u_j^{(1)} + \epsilon^2 u_j^{(2)} + \cdots\,,$$
$$v_j = \epsilon^{3/2} v_j^{(1)} + \epsilon^{5/2} v_j^{(2)} + \cdots\,,$$
$$w_j = \epsilon^{3/2} w_j^{(1)} + \epsilon^{5/2} w_j^{(2)} + \cdots\,,$$
$$\phi = \epsilon\phi^{(1)} + \epsilon^2\phi^{(2)} + \cdots \tag{6}$$

where u_j, v_j, and w_j are the ion, negative and positive dust velocities in R, Θ, and Z directions, respectively. Substituting Eqs. (5) and (6) to Eqs. (1)–(3), the first-order in ϵ for ion are

$$n_i^{(1)} = \frac{1}{\lambda^2}\phi^{(1)}\,,\quad u_i^{(1)} = \frac{1}{\lambda}\phi^{(1)}\,,$$
$$\frac{\partial v_i^{(1)}}{\partial R} = \frac{1}{T\lambda^2}\frac{\partial\phi^{(1)}}{\partial\Theta}\,,\quad \frac{\partial w_i^{(1)}}{\partial R} = \frac{1}{\lambda}\frac{\partial\phi^{(1)}}{\partial Z}\,, \tag{7}$$

whereas for negative dust are given by

$$n_n^{(1)} = -\frac{\mu}{\lambda^2}\phi^{(1)}\,,\quad u_n^{(1)} = -\frac{\mu}{\lambda}\phi^{(1)}\,,$$
$$\frac{\partial v_n^{(1)}}{\partial R} = -\frac{\mu}{T\lambda^2}\frac{\partial\phi^{(1)}}{\partial\Theta}\,,\quad \frac{\partial w_n^{(1)}}{\partial R} = -\frac{\mu}{\lambda}\frac{\partial\phi^{(1)}}{\partial Z}\,, \tag{8}$$

and for positive dust are given by

$$n_p^{(1)} = \frac{\alpha}{\lambda^2}\phi^{(1)}\,,\quad u_p^{(1)} = \frac{\alpha}{\lambda}\phi^{(1)}\,,$$
$$\frac{\partial v_p^{(1)}}{\partial R} = \frac{\alpha}{T\lambda^2}\frac{\partial\phi^{(1)}}{\partial\Theta}\,,\quad \frac{\partial w_p^{(1)}}{\partial R} = \frac{\alpha}{\lambda}\frac{\partial\phi^{(1)}}{\partial Z}\,. \tag{9}$$

Poisson equation leads to the compatibility condition:

$$\lambda^2 = \nu\mu + \eta\alpha + \rho\,. \tag{10}$$

At the next order in ε, the evolution equation for the first order perturbed electrostatic potential in the form

$$\frac{\partial}{\partial R}\Big(\frac{\partial\phi^{(1)}}{\partial T} + \frac{\phi^{(1)}}{2T} + A\phi^{(1)}\frac{\partial\phi^{(1)}}{\partial R} + B\frac{\partial^3\phi^{(1)}}{\partial R^3}\Big) + \frac{1}{2\lambda T^2}\frac{\partial^2\phi^{(1)}}{\partial\Theta^2} + \frac{\lambda}{2}\frac{\partial^2\phi^{(1)}}{\partial Z^2} = 0\,. \tag{11}$$

The nonlinear and the dispersion coefficients A and B are given by

$$A = \frac{3}{2\lambda^3}(\rho - \nu\mu^2 + \eta\alpha^2 - \lambda^4/3)\,,\quad B = \frac{\lambda}{2}\,. \tag{12}$$

Equation (11) is a 3D-CKP equation for dust-ion acoustic waves. If we neglect Z and Θ dependence, Eq. (11) is reduced to KdV equation.

4 3D-CKP Solution

In order to solve Eq. (11), the generalized expansion method is used.[37] According to the transformation

$$\eta = L_r R + L_z Z - T\Big(\frac{\lambda}{2}L_r\Theta^2 + U_0\Big)\,,$$

the exact solution of Eq. (11) is given by[31–32,37]

$$\phi = \phi^{(1)} = \phi_0\mathrm{sech}^2\Big[\frac{\eta}{\Delta}\Big]\,, \tag{14}$$

where, the amplitude ϕ_0 and the width Δ are given by

$$\phi_0 = \frac{3\Gamma\lambda}{2A\,L_r^2}\,,\quad \Delta = \sqrt{\frac{8B}{\Gamma\lambda}}\,, \tag{15}$$

with

$$\Gamma = L_r^2 + \frac{2L_r U_0}{\lambda} - 1\,, \tag{16}$$

while L_r and L_z are the direction cosines in R and Z axes, with condition $L_r^2 + L_z^2 = 1$ and U_0 is an arbitrary constant. Also note that the product $\phi_0\Delta^2 = 12L_r^2 B/A$ is independent of U_0 but depends on λ.

5 Results and Discussion

To carry out this study, many conditions are introduced (i) for model components, our assumption is isothermal electrons and cold fluid of ions and dust grains[38] (ii) the gravitational effect is neglected for submicron dusty grains[38–39] (iii) the plasma is of low coupling parameter (iv) inter-grain distance is very small. By applying reductive perturbation theory, introduced by Taniuti and Wei (1968),[34] this plasma model leads to a 3D-CKP equation (11). Now we discuss effects of system parameters on the feature of dust ion acoustic soliton solution using mesospheric parameters.[40–43] The dependence of solitons features i.e. wave phase velocity λ, soliton amplitude and width ϕ_0 and Δ on the various parameters ρ (unperturbed number densities ratio $\rho = n_{i0}/n_{e0}$), μ (normalized ion to negative dust mass ratio $\mu = Z_n m_i/m_n$), α (normalized ion to positive dust mass ratio $\alpha = Z_p m_i/m_p$), η (normalized charge number of positive dust $\eta = Z_p n_{p0}/n_{e0}$) and ν (normalized charge number of negative dust $\nu = Z_n n_{n0}/n_{e0}$) are investigated. Also, the geometric effect on the acoustic waves is considered. Figures 1 and 2 show the variation of the phase velocity λ against ν, η, α and μ. It is clear that λ is elevated with ν, α and μ but is reduced with η.

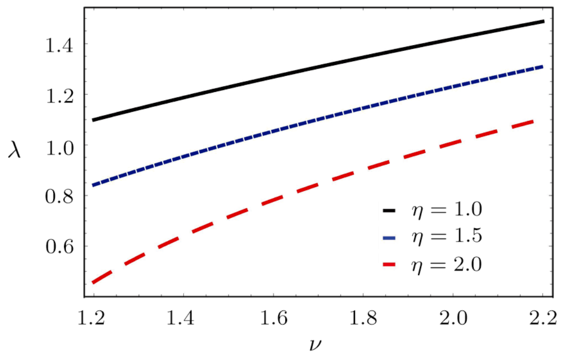

Fig. 1 The change of λ against ν for various values of η and for $\alpha = 0.002$, and $\mu = 0.005$.

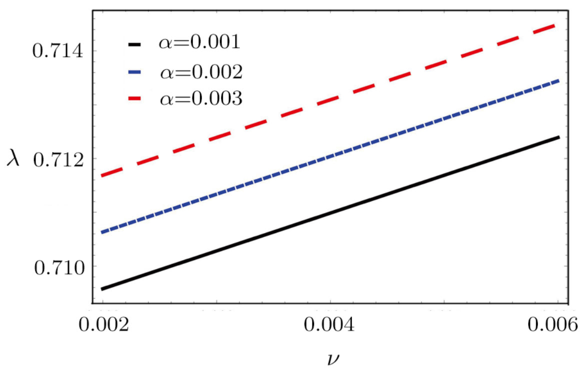

Fig. 2 The change of λ against μ for various values of α and for $\nu = 1$ and $\eta = 1.5$.

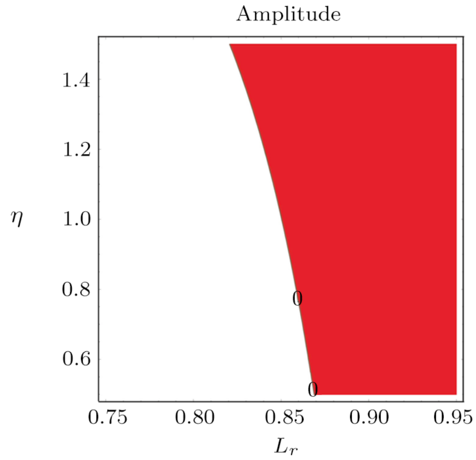

Fig. 3 Variation of the amplitude ϕ_0 against L_r and η for $\alpha = 0.002$, $\mu = 0.005$, and $\nu = 1.5$.

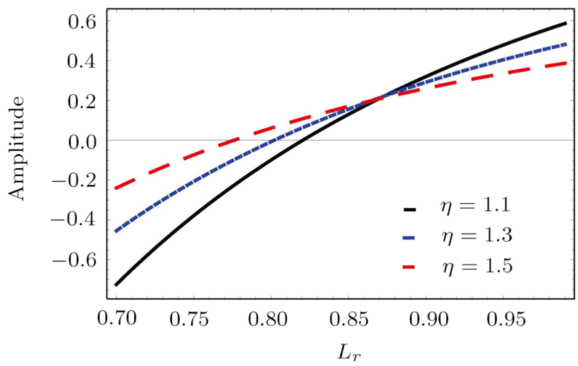

Fig. 4 Change of amplitude ϕ_0 against L_r and η for $\alpha = 0.002$, $\mu = 0.005$, and $\nu = 1.1$.

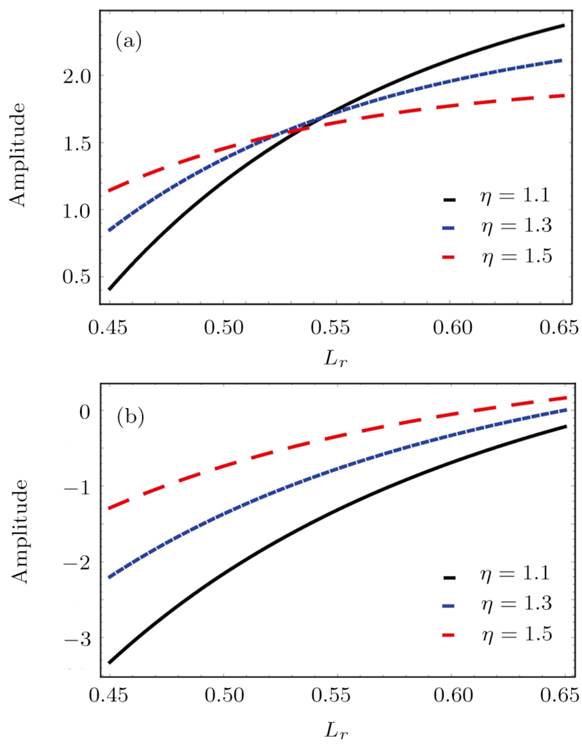

Fig. 5 Change of amplitude ϕ_0 against L_r and η for $\alpha = 0.002$, $\mu = 0.005$, $\nu = 1.1$, and (a) $U_0 = 0.95$; (b) $U_0 = 0.4$.

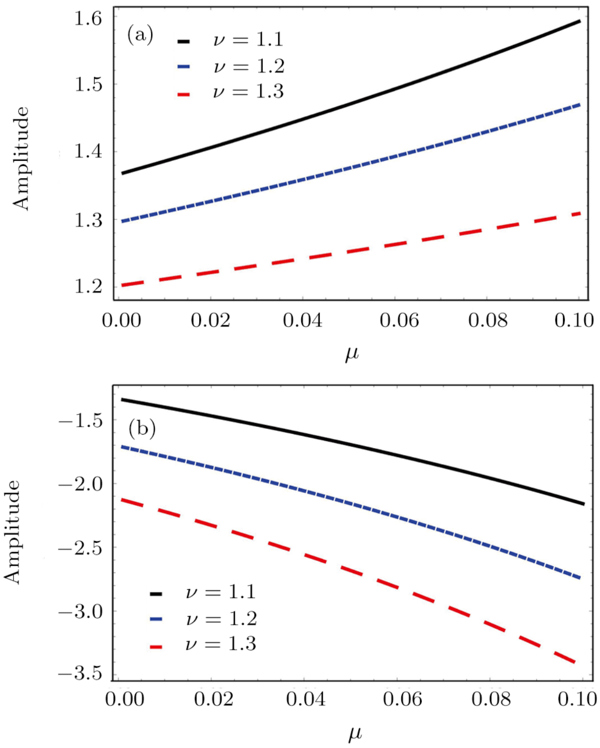

Fig. 6 Change of the amplitude ϕ_0 against μ for different values of ν and for $\alpha = 0.002$, $L_r = 0.5$, $\eta = 1.3$, and (a) $U_0 = 0.95$; (b) $U_0 = 0.4$.

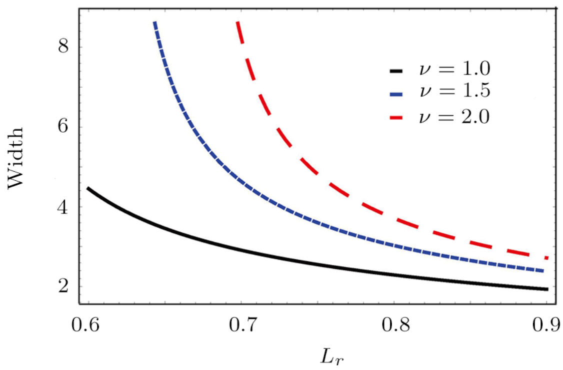

Fig. 7 Change of width Δ against L_r and ν for $\alpha = 0.002$, $\mu = 0.005$, $\eta = 1.5$, and $U_0 = 0.5$.

Figure 3 exhibits the variation of soliton amplitude ϕ_0 with L_r and η. Clearly, this model supports both rarefactive and compressive solitons, depending on sign of nonlinear coefficient A, compressive type exists if $A > 0$ while rarefactive for $A < 0$. More specifically, plots of soliton amplitude ϕ_0 and Δ (width) against η, U_0, L_r, and ν are depicted in Figs. 4–8. The soliton amplitude ϕ_0 of both compressive and rarefactive wave increases with L_r and η, as shown in Fig. 4. Accordingly, L_r and η lead to reduce the pulse amplitude ϕ_0. Also, the soliton amplitude ϕ_0 of compressive soliton decreases with ν but increases with L_r, μ, and η up to a certain value ($L_r = 0.55$), then ϕ_0 begins to decrease with η. While the soliton amplitude ϕ_0 of rarefactive soliton decreases with ν and μ and increases with L_r and η as depicted in Figs. 5 and 6.

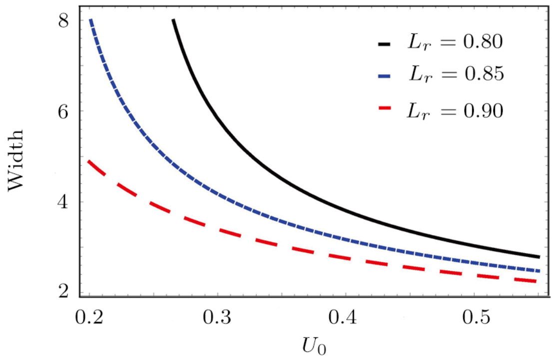

Fig. 8 The change of the width Δ against U_0 for different values of L_r and for $\alpha = 0.002$, $\mu = 0.005$, $\nu = 1.5$, and $\eta = 1.5$.

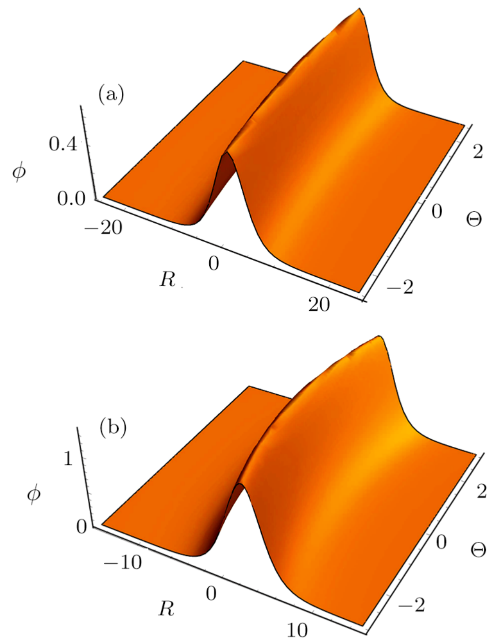

Fig. 9 Three-dimensional solitary wave profile ϕ (14) against R and Θ at $T = 0.5$ and $Z = 0.5$ and for $L_r = 0.9$, $\alpha = 0.002$, $\mu = 0.005$, $\nu = 1.5$, $\eta = 1.5$, and (a) $U_0 = 0.3$; (b) $U_0 = 0.6$.

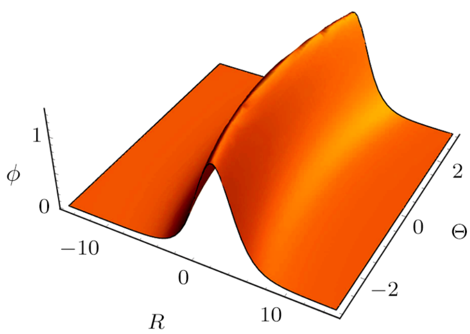

Fig. 10 Three-dimensional solitary wave profile ϕ (14) against R and Θ at $T = 0.5$ and $Z = 0.5$ and for $L_r = 0.9$, $\alpha = 0.002$, $U_0 = 0.5$, $\mu = 0.005$, $\nu = 1.5$, and $\eta = 1.5$.

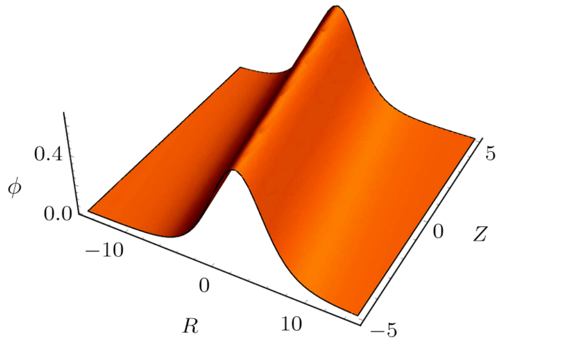

Fig. 11 Three-dimensional solitary wave profile ϕ (14) against R and Z at $T = 0.5$ and $\Theta = 0.2$ and for $L_r = 0.9$, $U_0 = 0.2$, $\alpha = 0.002$, $\mu = 0.005$, $\nu = 1.5$, and $\eta = 1.5$.

On the other hand, Figs. 7 and 8 show the soliton width Δdependence on ν, L_r and U_0. It is seen that Δ increases with ν whereas it decreases with both L_r and U_0. It is shown that ν makes the solitary profile much wider. According to soliton picture.[44−45] Figures 9–11 display ϕ in Eq. (14) with Θ, R and Z, and T. It is noted that, soliton deviates towards the positively radial axis with increasing time and this cannot occur on neglecting Z and Θ coordinates. The DIA soliton is shown in Fig. 9 with R and Θ for two values of $U_0 = 0.3$ and $= 0.6$. It is seen that how DIA amplitude varies with U_0. The one dimensional model cannot explain such behavior.

Summing up, we have studied DIASWs propagation in plasma with isothermal electrons, mobile cold ions, positive and negative dust grains considered cylindrical geometry. The angular and radial dependence have been considered. Employing the (RP) technique, a 3D-CKP equation which describes the evolution of DIASWs has been derived and its localized solution has been obtained by reducing the 3D-CKP to the Korteweg-de Vries equation on employing a simple transformation of the coordinates. The effects of system parameters (ν, η, μ, L_r and U_0) on phase velocity λ, soliton amplitude ϕ_0, and width Δ have been examined numerically. We have shown graphically that these parameters play a vital role in the formation and the features of the DIA mesospheric solitary waves.[38,46−47] These results agree with the mesospheric plasma information, see Refs. [46–47]. Finally, our investigation of the solitary wave properties could be important in understanding nonlinearity features in space, as well as in laboratory and astrophysical environments in mesospheric plasma.

References

[1] D. A. Mendis and M. Rosenberg, Annu. Rev. Astron. Astrophys. **32** (1994) 419.

[2] P. K. Shukla and A. A. Mamun, *Introduction to Dusty plasma Physics*, Institute of Physics, Bristol (2002).

[3] S. K. El-Labany, A. M. Diab, and E. F. El-Shamy, Astrophys. Space Sci. **282** (2002) 595.

[4] H. K Malik., R. Kumar, K. E. Lonngren, and Y. Nishida, Phys. Rev. E **92** (2015) 063107.

[5] R. N. Carlile, S. Geha, J. F. O'Hanlon, and J. C. Stewart, Appl. Phys. Lett. **59** (1991) 1167.

[6] A. Barkan, N. D'Angelo, and R. L. Merlino, Planet. Space Sci. **44** (1996) 239.

[7] J. H. Chu, J. B. Du, and I. Lin, Phys. Rev. Lett. **72** (1994b) 4009.

[8] Y. Hayashi and K. Tachibana, Jpn. J. Appl. Phys. **2** (1994) L804.

[9] H. Thomas, G. E. Morfill, and V. Dammel, Phys. Rev. Lett. **73** (1994) 652.

[10] N. N. Rao, P. K. Shukla, and M. Y. Yu, Planet. Space Sci. **38** (1990) 543.

[11] P. K. Shukla and V. P. Slin, Phys. Scr. **45** (1992) 508.

[12] F. Melandso, Phys. Plasmas **3** (1996) 3890.

[13] B. Sahu and M. Tribechem, Astrophys. Space Sci. **338** (2012) 259.

[14] S. K. El-Labany, W. M. Moslem, and F. M. Safy, Phys. Plasmas **13** (2006) 082903.

[15] R. Malik, H. K. Malik, and S. C. Kaushik, Phys. Plasmas **19** (2012) 032107.

[16] A. M. El-Hanbaly, M. Sallah, E. K. El-Shewy, and H. F. Darweesh, J. Theor. Appl. Phys. **9** (2015) 167.

[17] Y. N. Nejoh, Phys. Plasmas **4** (1997) 2813.

[18] R. A. Tang and J. K. Xue, Phys. Plasmas **9** (2003) 3800.

[19] S. Moolla, R. Bharuthram, and S. Baboolal, Phys. Plasmas **12** (2005) 042310.

[20] T. S. Gill, N. S. Saini, and H. Kaur, Chaos, Solitons and Fractals **28** (2006) 1106.

[21] A. A. Mamun, J. Plasma Phys. **59** (1997) 575.

[22] A. A. Mamun, Astrophys. Space Sci. **268** (1999) 443.

[23] M. Horanyi, Annu. Rev. Astron. Astrophys. **34** (1996) 383.

[24] Y. Nakamura, T. Odagiri, and I. Tsukabayashi, Plasma Phys. Control. Fusion **39** (1997) 105.

[25] P. K. Shukla, Phys. Plasmas **1** (1994) 1362.

[26] A. A. Mamun and P. K. Shukla, Geophys. Res. Lett. **29** (2002) 1518.

[27] A. A. Mamun, Phys. Rev. E **77** (2008) 026406.

[28] S. A. Elwakil, M. A. Zahran, E. K. El-Shewy, and A. E. Mowafy, Adv. Space Res. **48** (2011) 1067.

[29] J. K. Xue, Phys. Lett. A **314** (2003) 479.

[30] A. A. Mamun and P. K. Shukla, Phys. Rev. E **80** (2009) 037401.

[31] W. M. Moslem, U. M. Abdelsalam, R. Sabry, *et al.*, J. Plasma Phys. **76** (2010) 453.

[32] J. Borhanian and M. Shahmansouri, Phys. Plasmas **20** (2013) 013707.

[33] N. A. El-Bedwehy, M. A. El-Attafi, and S. K. El-Labany, Astrophys. Space Sci. **361** (2016) 299.

[34] T. Taniuti and C. C. Wei, J. Phys. Soc. Jpn. **24** (1968) 941.

[35] K. Liu and S. D. Liu, *Atmospheric Dynamics*, Peking University Press, Beijing (1999).

[36] W. M. Moslem, R. Sabry, and P. K. Shukla, Phys. Plasmas **17** (2010) 032305.

[37] R. Sabry, M. A. Zahran, and E. Fan, Phys. Lett. A **326** (2004) 93; R. Sabry and W. F. El-Taibany, Nonlinear Anal. **69** (2008) 2763; R. Sabry, W. M. Moslem, F. Haas, *et al.*, Phys. Plasmas **15** (2008) 122308.

[38] S. K. El-Labany, E. K. El-Shewy, N. A. El-Bedwehy, *et al.*, Indian J. Phys. **91** (2017) 337.

[39] M. S. Barnes, J. H. Keller, J. C. Forster, *et al.*, Phys. Rev. Lett. **68** (1992) 313.

[40] A. M. Zadorozhny, Adv. Space Res. **28** (2001) 1095.

[41] M. Horanyi, Annu. Rev. Astron. Astrophys. **34** (1996) 383.

[42] O. Havnes, J. Trøim, T. Blix, *et al.*, J. Geophys. Res. **101**(1996) 1039.

[43] P.K. Shukla, Phys. Plasmas **11** (2004) 3676.

[44] S. K. El-Labany, W. M. Moslem, W. F. El-Taibany, and M. Mahmoud, Phys. Plasmas **11** (2004) 03303.

[45] T. Dauxois, M. Peyrard, *Physics of Solitons*, Cambridge University Press, Cambridge (2005).

[46] A. A. Mamun, Phys. Rev. E **77** (2008) 026406.

[47] S. K. El-Labany, E. K. El-Shewy, H. N. A. El-Razek, and A. A. El-Rahman, Plasma Phys. Reports **43** (2017) 576.