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The Sharma–Tasso–Olver–Burgers equation: its conservation laws and kink solitons

K Hosseini[1], A Akbulut[2], D Baleanu[3,4] and S Salahshour[5]

[1] Department of Mathematics, Near East University TRNC, Mersin 10, Turkey
[2] Department of Mathematics-Computer, Art-Science Faculty, Eskisehir Osmangazi University, Eskisehir, Turkey
[3] Department of Mathematics, Faculty of Arts and Sciences, Cankaya University, Ankara 06530, Turkey
[4] Institute of Space Sciences, Magurele-Bucharest, Romania
[5] Faculty of Engineering and Natural Sciences, Bahcesehir University, Istanbul, Turkey

E-mail: kamyar_hosseini@yahoo.com

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Abstract

The present paper deals with the Sharma–Tasso–Olver–Burgers equation (STOBE) and its conservation laws and kink solitons. More precisely, the formal Lagrangian, Lie symmetries, and adjoint equations of the STOBE are firstly constructed to retrieve its conservation laws. Kink solitons of the STOBE are then extracted through adopting a series of newly well-designed approaches such as Kudryashov and exponential methods. Diverse graphs in 2 and 3D postures are formally portrayed to reveal the dynamical features of kink solitons. According to the authors' knowledge, the outcomes of the current investigation are new and have been listed for the first time.

Keywords: Sharma–Tasso–Olver–Burgers equation, conservation laws, Kudryashov and exponential methods, kink solitons, dynamical features

(Some figures may appear in colour only in the online journal)

1. Introduction

Partial differential equations (PDEs) particularly their nonlinear regimes are applied to model a wide variety of phenomena in the extensive areas of science and engineering. As useful tools for simulating many nonlinear phenomena, such models play a pivotal role in progressing the real world. During the past few decades, one of the major goals has been the construction of novel approaches to extract solitons for PDEs. In the last years, several well-organized methods have been proposed to derive solitons of PDEs, the hyperbolic function method [1–4], the modified Jacobi methods [5–8], the Kudryashov method [9–14], and the exponential method [15–20], are samples to point out.

It is noteworthy that all conservation laws of PDEs do not include physical meanings, however, such laws are essential to explore the integrability and reduction of PDEs [21, 22]. Conservation laws of PDEs can be found by using a wide range of methods such as the multiplier approach, the Noether approach, the new conservation theorem, and so on. Of these, the new conservation theorem was first established by Ibragimov and is associated with the formal Lagrangian, Lie symmetries, and adjoint equations of PDEs. Conservation laws can be derived for every symmetry of PDEs and the obtained conservation laws are referred to as trivial or non-trivial [23–29].

In the present study, the authors deal with the following Sharma–Tasso–Olver–Burgers equation [30]

$$u_t + c_1(3u_x^2 + 3u^2u_x + 3uu_{xx} + u_{xxx}) + c_2(2uu_x + u_{xx}) = 0, \tag{1}$$

and derive its conservation laws and kink solitons. As is clear from the name of equation (1), such a model consists of the

Sharma–Tasso–Olver and Burgers equations. These equations have been the main concern of a lot of research works. Or-Roshid and Rashidi [31] employed an exponential method to derive the solitons of these equations. In another investigation, multiple solitons of these equations were obtained by Wazwaz in [32] through the simplified Hirota's method. Very recently, Hu *et al* [33] constructed soliton, lump, and interaction solutions of a 2D-STOBE using a series of systematic ansatzes.

Kudryashov and exponential methods, as privileged approaches, have been applied by many scholars to retrieve solitons of PDEs. Particularly, the effectiveness of these methods has been demonstrated by Hosseini *et al* in several papers. Hosseini *et al* [34] derived solitons of the cubic-quartic nonlinear Schrödinger equation using the Kudryashov method. The exponential method was utilized in [35] by Hosseini *et al* to acquire solitons of the unstable nonlinear Schrödinger equation.

The rest of the current study is as follows: In section 2, the conservation theorem and the foundation of Kudryashov and exponential methods are given. In section 3, the formal Lagrangian, Lie symmetries, and adjoint equations of the STOBE are established to derive its conservation laws. In section 4, Kudryashov and exponential methods are adopted to seek solitons of the STOBE. Section 4 further gives diverse graphs in 2 and 3D postures to demonstrate the dynamical features of kink solitons. The achievements of the present paper are provided in section 5.

2. The conservation theorem and methods

In the current section, the conservation theorem and the foundation of Kudryashov and exponential methods are formally given.

2.1. The conservation theorem

To start, suppose that a PDE can be expressed as

$$P(u,\, u_x,\, u_t,\ldots) = 0, \tag{2}$$

where P is a polynomial. The Lie point symmetry generator of equation (2) is given by

$$X = \xi^x(x,\, t,\, u)\frac{\partial}{\partial x} + \xi^t(x,\, t,\, u)\frac{\partial}{\partial t} + \eta(x,\, t,\, u)\frac{\partial}{\partial u}, \tag{3}$$

where $\xi^x(x,\, t,\, u)$, $\xi^t(x,\, t,\, u)$, and $\eta(x,\, t,\, u)$ are known as the infinitesimals. The kth prolongation of equation (3) is retrieved by

$$X^{(k)} = X + \eta_i^{(1)}\frac{\partial}{\partial u_i^{\alpha}} + \ldots + \eta_{i_1 i_2 \ldots i_k}^{(k)}\frac{\partial}{\partial u_{i_1 i_2 \ldots i_k}},\ k \geqslant 1,$$

where

$$\eta_i^{(1)} = D_i\eta - (D_i\xi^j)u_j,$$
$$\eta_{i_1 i_2 \ldots i_k}^{(k)} = D_{i_k}\eta_{i_1 i_2 \ldots i_{k-1}}^{(k-1)} - (D_{i_k}\xi^j)u_{i_1 i_2 \ldots i_{k-1}j}.$$

Above $i,\, j = 1,\, 2$ and $i_l = 1$ for $l = 1,\, 2,\ldots,k$. The total derivative operator is indicated by D_i.

The formal Lagrangian is given by multiplying equation (2) by w as follows

$$L = wP,$$

where w is the adjoint variable. It is noteworthy that the adjoint equation is retrieved by

$$P^* = \frac{\delta L}{\delta u}, \tag{4}$$

where $\frac{\delta}{\delta u}$ is the variational derivative.

If the solution of equation (4) is found, then, a finite number of conservation laws for equation (2) are derived.

Theorem 1. *Every Lie point, Lie-Bäcklund, and nonlocal symmetry of equation (2) results in a conservation law. The components of the conserved vector are given by [29]*

$$T^i = \xi^i L + W\frac{\delta L}{\delta u_i} + \sum_{s\geqslant 1} D_{i_1}\ldots D_{i_s}(W)\frac{\delta L}{\delta u_{i i_1 i_2 \ldots i_s}}, \tag{5}$$

where $W = \eta - \xi^j u_j$, and ξ^i and η are the infinitesimal functions. The conserved vectors generated by equation (5) consist of the arbitrary solutions of the adjoint equation. Consequently, one derives a finite number of conservation laws for equation (2) by w [36].

Theorem 2. *Derived conserved vectors using (5) are conservation laws of equation (2) if*

$$D_i(T^i) = 0.$$

Here, D_i is referred to as the total derivative [37].

2.2. Foundation of methods

The Kudryashov method applies the following series [9, 10]

$$U(\epsilon) = a_0 + a_1 K(\epsilon) + a_2 K^2(\epsilon) + \ldots + a_N K^N(\epsilon),\ a_N \neq 0, \tag{6}$$

as the nontrivial solution of

$$O(U(\epsilon),\, U'(\epsilon),\, U''(\epsilon),\ldots) = 0. \tag{7}$$

Above, a_i, $i = 0,\, 1,\ldots,N$ are unknowns, N is retrieved by the balance approach, and

$$K(\epsilon) = \frac{1}{1 + da^{\epsilon}}$$

is the solution of

$$K'(\epsilon) = K(\epsilon)(K(\epsilon) - 1)\ln(a).$$

Equations (6) and (7) together result in a nonlinear system which by solving it, solitons of equation (7) are derived.

The exponential method investigates a solution for equation (7) as [15, 16]

$$U(\epsilon) = \frac{a_0 + a_1 a^{\epsilon} + a_2 a^{2\epsilon} + \ldots + a_N a^{N\epsilon}}{b_0 + b_1 a^{\epsilon} + b_2 a^{2\epsilon} + \ldots + b_N a^{N\epsilon}},\quad a_N \neq 0,\ b_N \neq 0, \tag{8}$$

where the unknowns are acquired later and $N \in \mathbb{Z}^{+}$.

Again, equations (7) and (8) together yield a nonlinear system which by solving it, solitons of equation (7) are obtained.

3. The STOBE and its conservation laws

In the present section, the conservation theorem is applied to the STOBE to derive its conservation laws. First, the formal Lagrangian is derived in the following form

$$L = w(u_t + c_1(3u_x^2 + 3u^2u_x + 3uu_{xx} + u_{xxx}) + c_2(2uu_x + u_{xx})), \tag{9}$$

where w is the adjoint variable.

The adjoint equation is acquired with the aid of the variational derivative as

$$F^* = -w_t + c_1(-3u^2w_x + 3uw_{xx} - w_{xxx}) + c_2(w_{xx} - 2uw_x). \tag{10}$$

If u is replaced by w in equation (10), then, equation (1) is not obtained. Thus, equation (1) is not self-adjoint. In such a case, one can say that $w = 1$ is a solution of equation (10).

A one-parameter Lie group for equation (1) is given by

$$x \to x + \varepsilon\xi^x(x, t, u),$$

$$t \to t + \varepsilon\xi^t(x, t, u),$$

$$u \to u + \varepsilon\eta(x, t, u),$$

where ε is the group parameter, and consequently equation (1) admits following Lie point symmetry generator

$$X = \xi^x\frac{\partial}{\partial x} + \xi^t\frac{\partial}{\partial t} + \eta\frac{\partial}{\partial u}.$$

Obviously, X must be satisfied the Lie symmetry condition as follow

$$X^{(3)}(u_t + c_1(3u_x^2 + 3u^2u_x + 3uu_{xx} + u_{xxx}) + c_2(2uu_x + u_{xx})) = 0.$$

The third prolongation of the Lie point symmetry generator is given by

$$X^{(3)} = \xi^x\frac{\partial}{\partial x} + \xi^t\frac{\partial}{\partial t} + \eta\frac{\partial}{\partial u} + \eta^x\frac{\partial}{\partial u_x} + \eta^t\frac{\partial}{\partial u_t} + \eta^{xx}\frac{\partial}{\partial u_{xx}} + \eta^{xxx}\frac{\partial}{\partial u_{xxx}},$$

where

$$\eta^x = D_x(\eta) - u_xD_x(\xi^x) - u_tD_x(\xi^t),$$

$$\eta^t = D_t(\eta) - u_xD_t(\xi^x) - u_tD_t(\xi^t),$$

$$\eta^{xx} = D_x(\eta^x) - u_{xx}D_x(\xi^x) - u_{xt}D_x(\xi^t),$$

$$\eta^{xxx} = D_x(\eta^{xx}) - u_{xxx}D_x(\xi^x) - u_{xxt}D_x(\xi^t).$$

If we apply $X^{(3)}$ to equation (1), an equivalent condition is obtained as

Table 1. The commutator table for the above symmetries.

$[X_i, X_j]$	X_1	X_2	X_3
X_1	0	0	$X_1 - \frac{2c_2^2}{9c_1}X_2$
X_2	0	0	$\frac{X_2}{3}$
X_3	$-X_1 + \frac{2c_2^2}{9c_1}X_2$	$-\frac{X_2}{3}$	0

$$\eta^t + c_1(6u_x\eta^x + 6u\eta u_x + 3u^2\eta^x + 3\eta u_{xx} + 3u\eta^{xx} + \eta^{xxx}) + c_2(2\eta u_x + 2u\eta^x + \eta^{xx}) = 0.$$

Now, after some operations, we find the infinitesimal functions as follows

$$\xi^t = k_1 + k_3t,\ \ \xi^x = k_2 + k_3\left(\frac{3c_1x - 2c_2^2t}{9c_1}\right),$$

$$\eta = k_3\left(\frac{-3c_1u - c_2}{9c_1}\right),$$

where k_1, k_2, and k_3 are arbitrary constants. Finally, the Lie point symmetry generators of equation (1) are given by

$$X_1 = \frac{\partial}{\partial t},\ X_2 = \frac{\partial}{\partial x},\ X_3 = \left(\frac{3c_1x - 2c_2^2t}{9c_1}\right)\frac{\partial}{\partial x} + t\frac{\partial}{\partial t} + \left(\frac{-3c_1u - c_2}{9c_1}\right)\frac{\partial}{\partial u}. \tag{11}$$

The commutator table for the above symmetries (see equation (11)) has been given in table 1.

Now, conservation laws of the STOBE for all founded Lie point symmetry generators are derived. Conservation laws formulae for equation (1) are as follows

$$T^x = \xi^xL + W\left[\frac{\partial L}{\partial u_x} - D_x\left(\frac{\partial L}{\partial u_{xx}}\right) + D_x^2\left(\frac{\partial L}{\partial u_{xxx}}\right)\right] + D_x(W)\left[\frac{\partial L}{\partial u_{xx}} - D_x\left(\frac{\partial L}{\partial u_{xxx}}\right)\right] + D_x^2(W)\left[\frac{\partial L}{\partial u_{xxx}}\right], \tag{12}$$

$$T^t = \xi^tL + W\left(\frac{\partial L}{\partial u_t}\right). \tag{13}$$

Case 1. If we employ (12) and (13) to equation (9) with the use of X_1, we acquire the following conservation laws

$$T_1^x = -3c_1(u_twu_x + u_twu^2 + u_{xt}uw) - 2c_2u_twu - c_2u_{xt}w + c_1(3u_tuw_x - u_tw_{xx} + u_{xt}w_x - u_{xxt}w) + c_2u_tw_x,$$

$$T_1^t = 3c_1(u^2u_xw + wuu_{xx} + wu_x^2) + c_2w(2uu_x + u_{xx}) + c_1wu_{xxx}.$$

Due to the satisfaction of the divergence condition, these conservation laws are called local conservation laws. Such conservation laws are infinite trivial conservation laws. In this case, we have

$$D_x(T_1^x) + D_t(T_1^t) = u_tw_t - u_tw_t = 0.$$

It is noted that for $w = 1$, from T_1^x and T_1^t, one can find

$$\widetilde{T_1^x} = -3c_1u^2u_t - 3c_1uu_{xt} - 3c_1u_xu_t - 2c_2uu_t - c_2u_{xt} - c_1u_{xxt}, \tag{14}$$

$$\widetilde{T_1^t} = 3c_1u^2u_x + 3c_1uu_{xx} + 3c_1u_x^2 + 2c_2uu_x + c_2u_{xx} + c_1u_{xxx}. \quad (15)$$

Finite conserved vectors (14) and (15) satisfy the divergence condition, so they are trivial conservation laws.

Case 2. If we apply (12) and (13) to equation (9) with the aid of X_2, we find the following local conservation laws

$$T_2^x = c_1u_{xx}w_x - c_1u_xw_{xx} + u_xw_x(3c_1u + c_2) + wu_t,$$
$$T_2^t = -wu_x.$$

Owing to the satisfaction of the divergence condition, these conservation laws are called local conservation laws. Such conservation laws are infinite trivial conservation laws. In this case, we have

$$D_x(T_2^x) + D_t(T_2^t) = -w_x(3c_1u^2u_x + 2c_2uu_x) + u_x(3c_1u^2w_x + 2c_2uw_x) = 0.$$

It is notable that for $w = 1$, one can get

$$\widetilde{T_2^x} = u_t,$$
$$\widetilde{T_2^t} = -u_x.$$

The above finite conserved vectors satisfy the divergence condition, consequently, they are trivial conservation laws.

Case 3. If we employ (12) and (13) to equation (9) with the use of X_3, we obtain the following conservation laws

$$\begin{aligned}T_3^x = {}& 3tc_1uu_tw_x - \frac{2c_2^2tuu_xw_x}{3} - \frac{2c_2^2twu_t}{9\alpha} \\ & -3c_1twu^2u_t - 2c_2twuu_t - 3c_1twu_xu_t \\ & +c_1xuu_xw_x - \frac{2c_2^3tu_xw_x}{9c_1} + c_1u^2w_x + \frac{2c_2uw_x}{3} \\ & -c_1wu_{xx} - \frac{c_1uw_{xx}}{3} - \frac{c_2w_{xx}}{9} \\ & -3c_1wuu_x - \frac{2c_2^2uw}{9c_1} + \frac{c_2xu_xw_x}{3} + c_2tu_tw_x + \frac{2c_1u_xw_x}{3} \\ & +\frac{xwu_t}{3} - c_1wu^3 - c_2wu^2 \\ & -c_2wu_x + \frac{c_2^2w_x}{9c_1} - 3c_1tuwu_{xt} + \frac{2c_2^2tu_xw_{xx}}{9} - \frac{2c_2^2tw_xu_{xx}}{9} \\ & -\frac{c_1xu_xw_{xx}}{3} - c_1tu_tw_{xx} \\ & -c_2twu_{xt} + \frac{c_1xw_xu_{xx}}{3} + c_1tw_xu_{xt} - c_1twu_{xxt},\end{aligned}$$

$$\begin{aligned}T_3^t = {}& 3c_1twu_x^2 + 3c_1twu^2u_x + 3c_1tuwu_{xx} + c_1twu_{xxx} \\ & + 2c_2tuwu_x + c_2twu_{xx} \\ & - \frac{uw}{3} - \frac{c_2w}{9c_1} - \frac{xwu_x}{3} + \frac{2c_2^2twu_x}{9c_1}.\end{aligned}$$

Due to the satisfaction of the divergence condition, these conservation laws are called local conservation laws. Such conservation laws are infinite trivial conservation laws.

It is noted that for $w = 1$, one can find

$$\begin{aligned}\widetilde{T_3^x} = {}& -c_1tu_{xxt} - 3c_1uu_x - \frac{2c_2^2u}{9c_1} - c_2u_x - c_2tu_{xt} \\ & -c_1u_{xx} - c_1u^3 - c_2u^2 - \frac{2c_2^2tu_t}{9c_1} \\ & -3c_1tu^2u_t - 2c_2tuu_t - 3c_1tu_xu_t - 3c_1tuu_{xt} + \frac{uu_t}{3},\end{aligned}$$

$$\begin{aligned}\widetilde{T_3^t} = {}& 3c_1tu^2u_x + 2c_2tuu_x + 3c_1tuu_{xx} + 3c_1tu_x^2 \\ & +c_1tu_{xxx} + c_2tu_{xx} - \frac{u}{3} - \frac{c_2}{9c_1} - \frac{xu_x}{3} + \frac{2c_2^2tu_x}{9c_1}.\end{aligned}$$

Such conserved vectors satisfy the divergence condition, so, they are trivial conservation laws.

4. The STOBE and its solitons

In the present section, Kudryashov and exponential methods are adopted to seek solitons of the STOBE. The present section further gives diverse graphs in 2 and 3D postures to demonstrate the dynamical features of kink solitons. To start, we establish a transformation as follows

$$u(x, t) = U(\epsilon),\ \ \epsilon = x - wt, \quad (16)$$

which w is the soliton velocity. Equations (16) and (1) together result in

$$\begin{aligned}& c_1\frac{d^3U(\epsilon)}{d\epsilon^3} + c_2\frac{d^2U(\epsilon)}{d\epsilon^2} - w\frac{dU(\epsilon)}{d\epsilon} \\ & +3c_1U(\epsilon)\frac{d^2U(\epsilon)}{d\epsilon^2} + 3c_1\left(\frac{dU(\epsilon)}{d\epsilon}\right)^2 \\ & +3c_1U^2(\epsilon)\frac{dU(\epsilon)}{d\epsilon} + 2c_2U(\epsilon)\frac{dU(\epsilon)}{d\epsilon} = 0. \quad (17)\end{aligned}$$

From $\frac{d^3U(\epsilon)}{d\epsilon^3}$ and $U^2(\epsilon)\frac{dU(\epsilon)}{d\epsilon}$ in equation (17), one can get [10] $N + 3 = 3N + 1 \Rightarrow N = 1$.

4.1. Kudryashov method

Equation (6) and $N = 1$ offer taking the following solution

$$U(\epsilon) = a_0 + a_1K(\epsilon),\ \ a_1 \neq 0, \quad (18)$$

for equation (17). From equations (17) and (18), we will attain a nonlinear system as

$$\begin{aligned}& 6(\ln(a))^2c_1 + 9\ln(a)a_1c_1 + 3a_1^2c_1 = 0, \\ & -12(\ln(a))^2c_1 + 6\ln(a)a_0c_1 - 15\ln(a)a_1c_1 \\ & +6a_0a_1c_1 - 3a_1^2c_1 + 2\ln(a)c_2 \\ & +2a_1c_2 = 0, \\ & 7(\ln(a))^2c_1 - 9\ln(a)a_0c_1 + 6\ln(a)a_1c_1 + 3a_0^2c_1 \\ & -6a_0a_1c_1 - 3\ln(a)c_2 \\ & +2a_0c_2 - 2a_1c_2 - w = 0, \\ & -(\ln(a))^2c_1 + 3\ln(a)a_0c_1 - 3a_0^2c_1 + \ln(a)c_2 \\ & -2a_0c_2 + w = 0,\end{aligned}$$

where its solution yields

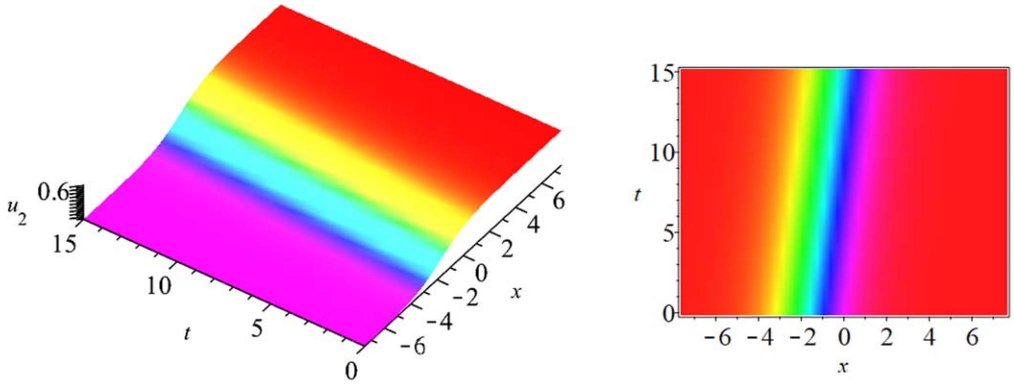

Figure 1. The kink soliton $u_2(x, t)$ for $c_1 = 0.15$, $c_2 = 0.15$, $d = 5$, and $a = 2.7$.

Case 1:

$$a_1 = -\ln(a),\ \ w = (\ln(a))^2 c_1 - 3\ln(a)a_0c_1 + 3a_0^2c_1 - \ln(a)c_2 + 2a_0c_2.$$

Thus, the following soliton to the STOBE is acquired

$$u_1(x, t) = a_0 - \frac{\ln(a)}{1 + da^{x-((\ln(a))^2c_1 - 3\ln(a)a_0c_1 + 3a_0^2c_1 - \ln(a)c_2 + 2a_0c_2)t}}.$$

Case 2:

$$a_0 = \frac{3c_1\ln(a) - c_2}{3c_1},\ \ a_1 = -2\ln(a),$$
$$w = \frac{3(\ln(a))^2c_1^2 - c_2^2}{3c_1}.$$

Thus, the following soliton to the STOBE is derived

$$u_2(x, t) = \frac{3c_1\ln(a) - c_2}{3c_1} - \frac{2\ln(a)}{1 + da^{x-\frac{3(\ln(a))^2c_1^2 - c_2^2}{3c_1}t}}.$$

The dynamical features of the kink soliton $u_2(x, t)$ are given in figure 1 for $c_1 = 0.15$, $c_2 = 0.15$, $d = 5$, and $a = 2.7$.

4.2. Exponential method

Taking $N = 1$ in equation (8) leads to

$$U(\epsilon) = \frac{a_0 + a_1a^\epsilon}{b_0 + b_1a^\epsilon},\ \ a_1 \neq 0,\ \ b_1 \neq 0. \tag{19}$$

From equations (17) and (19), we will achieve a nonlinear system as

$$\begin{aligned}
&-(\ln(a))^2a_0b_0^2b_1c_1 + (\ln(a))^2a_1b_0^3c_1 - 3\ln(a)a_0^2b_0b_1c_1 \\
&+3\ln(a)a_0a_1b_0^2c_1 - \ln(a)a_0b_0^2b_1c_2 + \ln(a)a_1b_0^3c_2 \\
&+wa_0b_0^2b_1 - wa_1b_0^3 - 3a_0^3b_1c_1 \\
&\quad +3a_0^2a_1b_0c_1 - 2a_0^2b_0b_1c_2 + 2a_0a_1b_0^2c_2 = 0,
\end{aligned}$$

$$\begin{aligned}
&4(\ln(a))^2a_0b_0b_1^2c_1 - 4(\ln(a))^2a_1b_0^2b_1c_1 \\
&\quad +6\ln(a)a_0^2b_1^2c_1 - 12\ln(a)a_0a_1b_0b_1c_1 \\
&\quad +6\ln(a)a_1^2b_0^2c_1 + 2wa_0b_0b_1^2 - 2wa_1b_0^2b_1 \\
&\quad -6a_0^2a_1b_1c_1 - 2a_0^2b_1^2c_2 \\
&\quad +6a_0a_1^2b_0c_1 + 2a_1^2b_0^2c_2 = 0,
\end{aligned}$$

$$\begin{aligned}
&-(\ln(a))^2a_0b_1^3c_1 + (\ln(a))^2a_1b_0b_1^2c_1 \\
&+3\ln(a)a_0a_1b_1^2c_1 + \ln(a)a_0b_1^3c_2 - 3\ln(a)a_1^2b_0b_1c_1 \\
&-\ln(a)a_1b_0b_1^2c_2 + wa_0b_1^3 - wa_1b_0b_1^2 - 3a_0a_1^2b_1c_1 \\
&-2a_0a_1b_1^2c_2 + 3a_1^3b_0c_1 + 2a_1^2b_0b_1c_2 = 0,
\end{aligned}$$

where its solution results in

$$a_0 = -\frac{b_0(3\ln(a)c_1 + c_2)}{3c_1},\ \ a_1 = \frac{b_1(3\ln(a)c_1 - c_2)}{3c_1},$$
$$w = \frac{3(\ln(a))^2c_1^2 - c_2^2}{3c_1}.$$

Thus, the following soliton to the STOBE is acquired

$$u_1(x, t) = \frac{a_0 + a_1a^{x-\frac{3(\ln(a))^2c_1^2 - c_2^2}{3c_1}t}}{b_0 + b_1a^{x-\frac{3(\ln(a))^2c_1^2 - c_2^2}{3c_1}t}},$$

where

$$a_0 = -\frac{b_0(3\ln(a)c_1 + c_2)}{3c_1},\ \ a_1 = \frac{b_1(3\ln(a)c_1 - c_2)}{3c_1}.$$

Taking $N = 2$ in equation (8) yields

$$U(\epsilon) = \frac{a_0 + a_1a^\epsilon + a_2a^{2\epsilon}}{b_0 + b_1a^\epsilon + b_2a^{2\epsilon}},\ \ a_2 \neq 0,\ \ b_2 \neq 0. \tag{20}$$

Equations (17) and (20) together result in a nonlinear system whose solution gives

Case 1:

$$a_0 = -\frac{b_0(3\ln(a)c_1 + c_2)}{3c_1},\ \ a_2 = \frac{b_2(3\ln(a)c_1 - c_2)}{3c_1},$$
$$b_1 = -\frac{3a_1c_1}{c_2},\ \ w = \frac{3(\ln(a))^2c_1^2 - c_2^2}{3c_1}.$$

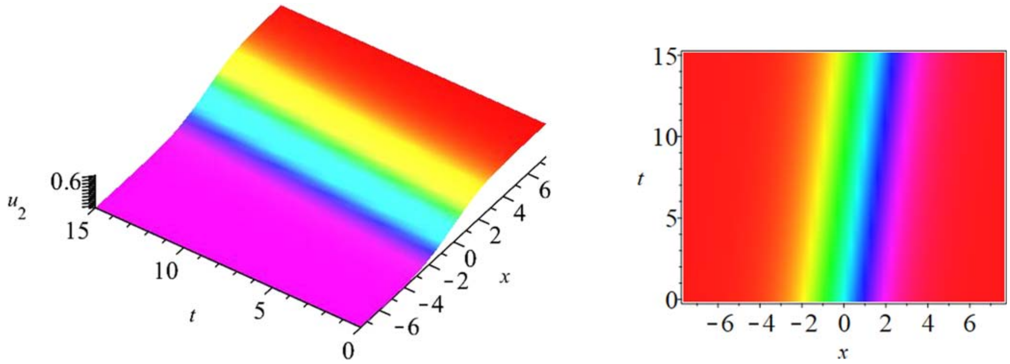

Figure 2. The kink soliton $u_2(x, t)$ for $a_1 = -1$, $b_0 = 1$, $b_2 = 1$, $c_1 = 0.15$, $c_2 = 0.15$, and $a = 2.7$.

Thus, the following soliton to the STOBE is derived

$$u_2(x, t) = \frac{a_0 + a_1 a^{x - \frac{3(\ln(a))^2 c_1^2 - c_2^2}{3c_1}t} + a_2 a^{2\left(x - \frac{3(\ln(a))^2 c_1^2 - c_2^2}{3c_1}t\right)}}{b_0 + b_1 a^{x - \frac{3(\ln(a))^2 c_1^2 - c_2^2}{3c_1}t} + b_2 a^{2\left(x - \frac{3(\ln(a))^2 c_1^2 - c_2^2}{3c_1}t\right)}},$$

where

$$a_0 = -\frac{b_0(3\ln(a)c_1 + c_2)}{3c_1},\ a_2 = \frac{b_2(3\ln(a)c_1 - c_2)}{3c_1},$$
$$b_1 = -\frac{3a_1c_1}{c_2}.$$

Case 2:

$$a_0 = -\frac{b_0(3\ln(a)c_1 + c_2)}{3c_1},\ a_1 = -\frac{b_1c_2}{3c_1},$$
$$a_2 = -\frac{b_1^2(3\ln(a)c_1 - c_2)}{6b_0c_1},$$
$$b_2 = -\frac{b_1^2}{2b_0},\ w = \frac{3(\ln(a))^2 c_1^2 - c_2^2}{3c_1}.$$

Consequently, the following exact solution to the STOBE is obtained

$$u_3(x, t) = \frac{a_0 + a_1 a^{x - \frac{3(\ln(a))^2 c_1^2 - c_2^2}{3c_1}t} + a_2 a^{2\left(x - \frac{3(\ln(a))^2 c_1^2 - c_2^2}{3c_1}t\right)}}{b_0 + b_1 a^{x - \frac{3(\ln(a))^2 c_1^2 - c_2^2}{3c_1}t} + b_2 a^{2\left(x - \frac{3(\ln(a))^2 c_1^2 - c_2^2}{3c_1}t\right)}},$$

where
$$a_0 = -\frac{b_0(3\ln(a)c_1 + c_2)}{3c_1},\ a_1 = -\frac{b_1c_2}{3c_1},$$
$$a_2 = -\frac{b_1^2(3\ln(a)c_1 - c_2)}{6b_0c_1},\ b_2 = -\frac{b_1^2}{2b_0}.$$

Figure 2 gives the dynamical features of the kink soliton $u_2(x, t)$ for $a_1 = -1$, $b_0 = 1$, $b_2 = 1$, $c_1 = 0.15$, $c_2 = 0.15$, and $a = 2.7$. Furthermore, the physical behaviors of u_2^{KM} and u_2^{EM} for above parameters have been given in figure 3 when $t = 0$.

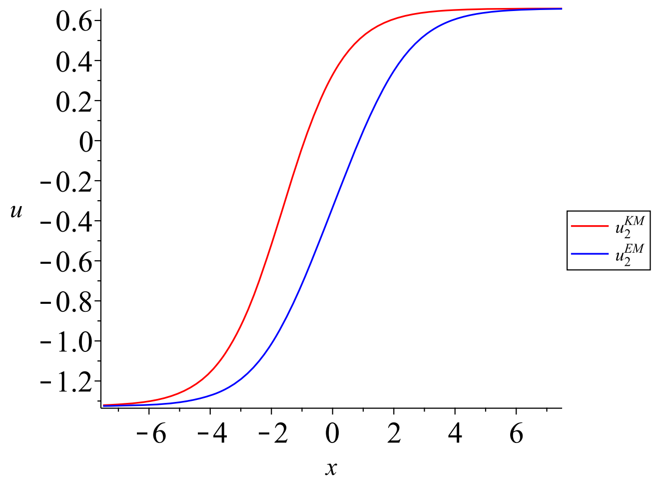

Figure 3. u_2^{KM} and u_2^{EM} for above parameters when $t = 0$.

Remark 1. According to the authors' knowledge, the outcomes of the current investigation are new and have been listed for the first time.

Remark 2. The authors successfully used a symbolic computation system to check the correctness of the outcomes of the current paper.

5. Conclusion

The principal aim of the current paper was to explore a newly well-established model known as the Sharma–Tasso–Olver–Burgers equation and derive its conservation laws and kink solitons. The study proceeded systematically by constructing the formal Lagrangian, Lie symmetries, and adjoint equations of STOBE to acquire its conservation laws. Besides, kink solitons of the STOBE were formally established using Kudryashov and exponential methods. Various plots in 2 and 3D postures were graphically represented to observe the dynamical characteristics of kink solitons. Based on information from the authors, the outcomes of the current investigation are new and have been listed for the first time. The authors' suggestion for future works is employing newly well-organized methods [38–44] to acquire other wave structures of STOBE.

Declaration of competing interest

The authors declare no conflict of interest.

ORCID iDs

A Akbulut https://orcid.org/0000-0003-2448-2481
S Salahshour https://orcid.org/0000-0003-1390-3551

References

[1] Xie F, Yan Z and Zhang H 2001 Explicit and exact traveling wave solutions of Whitham–Broer–Kaup shallow water equations *Phys. Lett.* A **285** 76–80
[2] Bai C 2001 Exact solutions for nonlinear partial differential equation: a new approach *Phys. Lett.* A **288** 191–5
[3] Seadawy A R, Kumar D, Hosseini K and Samadani F 2018 The system of equations for the ion sound and Langmuir waves and its new exact solutions *Results Phys.* **9** 1631–4
[4] Rezazadeh H, Kumar D, Sulaiman T A and Bulut H 2019 New complex hyperbolic and trigonometric solutions for the generalized conformable fractional Gardner equation *Modern Phys. Lett.* B **33** 1950196
[5] Ma H C, Zhang Z P and Deng A P 2012 A new periodic solution to Jacobi elliptic functions of MKdV equation and BBM equation *Acta Math. Appl. Sin.* **28** 409–15
[6] El-Sheikh M M A, Seadawy A R, Ahmed H M, Arnous A H and Rabie W B 2020 Dispersive and propagation of shallow water waves as a higher order nonlinear Boussinesq-like dynamical wave equations *Physica* A **537** 122662
[7] Hosseini K, Mirzazadeh M, Vahidi J and Asghari R 2020 Optical wave structures to the Fokas–Lenells equation *Optik* **207** 164450
[8] Hosseini K, Salahshour S, Mirzazadeh M, Ahmadian A, Baleanu D and Khoshrang A 2021 The (2 + 1)-dimensional Heisenberg ferromagnetic spin chain equation: its solitons and Jacobi elliptic function solutions *Eur. Phys. J. Plus* **136** 206
[9] Kudryashov N A 2012 One method for finding exact solutions of nonlinear differential equations *Commun. Nonlinear Sci. Numer. Simul.* **17** 2248–53
[10] Ege S M and Misirli E 2014 The modified Kudryashov method for solving some fractional-order nonlinear equations *Adv. Differ. Equ.* **2014** 135
[11] Hosseini K, Bekir A and Kaplan M 2017 New exact traveling wave solutions of the Tzitzéica-type evolution equations arising in non-linear optics *J. Mod. Opt.* **64** 1688–92
[12] Kumar D, Seadawy A R and Joardar A K 2018 Modified Kudryashov method via new exact solutions for some conformable fractional differential equations arising in mathematical biology *Chin. J. Phys.* **56** 75–85
[13] Kumar D, Darvishi M T and Joardar A K 2018 Modified Kudryashov method and its application to the fractional version of the variety of Boussinesq-like equations in shallow water *Opt. Quantum Electron.* **50** 128
[14] Hosseini K, Korkmaz A, Bekir A, Samadani F, Zabihi A and Topsakal M 2021 New wave form solutions of nonlinear conformable time-fractional Zoomeron equation in (2+1)-dimensions *Waves Random Complex Media* **31** 228–38
[15] He J H and Wu X H 2006 Exp-function method for nonlinear wave equations *Chaos Solitons Fractals* **30** 700–8
[16] Ali A T and Hassan E R 2010 General $\exp_a$ function method for nonlinear evolution equations *Appl. Math. Comput.* **217** 451–9
[17] Hosseini K, Mirzazadeh M, Zhou Q, Liu Y and Moradi M 2019 Analytic study on chirped optical solitons in nonlinear metamaterials with higher order effects *Laser Phys.* **29** 095402
[18] Hosseini K, Osman M S, Mirzazadeh M and Rabiei F 2020 Investigation of different wave structures to the generalized third-order nonlinear Scrödinger equation *Optik* **206** 164259
[19] Hosseini K, Mirzazadeh M, Rabiei F, Baskonus H M and Yel G 2020 Dark optical solitons to the Biswas–Arshed equation with high order dispersions and absence of self-phase modulation *Optik* **209** 164576
[20] Hosseini K, Ansari R, Zabihi A, Shafaroody A and Mirzazadeh M 2020 Optical solitons and modulation instability of the resonant nonlinear Schrödinger equations in (3 + 1)-dimensions *Optik* **209** 164584
[21] Yaşar E 2010 On the conservation laws and invariant solutions of the mKdV equation *J. Math. Anal. Appl.* **363** 174–81
[22] Olver P J 1993 *Application of Lie Groups to Differential Equations* (New York: Springer)
[23] Naz R, Mahomed F M and Mason D P 2008 Comparison of different approaches to conservation laws for some partial differential equations in fluid mechanics *Appl. Math. Comput.* **205** 212–30
[24] Celik N, Seadawy A R, Ozkan Y S and Yasar E 2021 A model of solitary waves in a nonlinear elastic circular rod: abundant different type exact solutions and conservation laws *Chaos Solitons Fractals* **143** 110486
[25] Wang G, Kara A H, Fakhar K, Vega-Guzmand J and Biswas A 2016 Group analysis, exact solutions and conservation laws of a generalized fifth order KdV equation *Chaos Solitons Fractals* **86** 8–15
[26] Ibragimov N H, Khamitova R, Avdonina E D and Galiakberova L R 2015 Conservation laws and solutions of a quantum drift-diffusion model for semiconductors *Int. J. Non Linear Mech.* **77** 69–73
[27] Ibragimov N H and Kolsrud T 2004 Lagrangian approach to evolution equations: symmetries and conservation laws *Nonlinear Dyn.* **36** 29–40
[28] Akbulut A and Taşcan F 2017 Lie symmetries, symmetry reductions and conservation laws of time fractional modified Korteweg–de Vries (mkdv) equation *Chaos, Solitons Fractals* **100** 1–6
[29] Ibragimov N H 2007 A new conservation theorem *J. Math. Anal. Appl.* **333** 311–28
[30] Yan Z and Lou S 2020 Soliton molecules in Sharma–Tasso–Olver–Burgers equation *Appl. Math. Lett.* **104** 106271
[31] Or-Roshid H and Rashidi M M 2017 Multi-soliton fusion phenomenon of Burgers equation and fission, fusion phenomenon of Sharma–Tasso–Olver equation *J. Ocean Eng. Sci.* **2** 120–6
[32] Wazwaz A M 2019 Multiple complex soliton solutions for the integrable KdV, fifth-order Lax, modified KdV, Burgers, and Sharma–Tasso–Olver equations *Chin. J. Phys.* **59** 372–8
[33] Hu X, Miao Z and Lin S 2021 Solitons molecules, lump and interaction solutions to a (2 + 1)-dimensional Sharma–Tasso–Olver–Burgers equation *Chin. J. Phys.* **74** 175–83

[34] Hosseini K, Samadani F, Kumar D and Faridi M 2018 New optical solitons of cubic-quartic nonlinear Schrödinger equation *Optik* **157** 1101–5
[35] Hosseini K, Zabihi A, Samadani F and Ansari R 2018 New explicit exact solutions of the unstable nonlinear Schrödinger's equation using the exp_a and hyperbolic function methods *Opt. Quantum Electron.* **50** 82
[36] Akbulut A and Taşcan F 2018 On symmetries, conservation laws and exact solutions of the nonlinear Schrödinger–Hirota equation *Waves Random Complex Media* **28** 389–98
[37] Akbulut A and Taşcan F 2017 Application of conservation theorem and modified extended tanh-function method to (1 + 1)-dimensional nonlinear coupled Klein–Gordon–Zakharov equation *Chaos Solitons Fractals* **104** 33–40
[38] Kaabar M K A, Kaplan M and Siri Z 2021 New exact soliton solutions of the (3 + 1)-dimensional conformable Wazwaz–Benjamin–Bona–Mahony equation via two novel techniques *J. Funct. Spaces* **2021** 4659905
[39] Kaabar M K A, Martínez F, Gómez-Aguilar J F, Ghanbari B, Kaplan M and Günerhan H 2021 New approximate analytical solutions for the nonlinear fractional Schrödinger equation with second-order spatio-temporal dispersion via double Laplace transform method *Math. Methods Appl. Sci.* **44** 11138–56
[40] Zada L, Nawaz R, Nisar K S, Tahir M, Yavuz M, Kaabar M K A and Martínez F 2021 New approximate-analytical solutions to partial differential equations via auxiliary function method *Partial Differ. Equ. Appl. Math.* **4** 100045
[41] Bi Y, Zhang Z, Liu Q and Liu T 2021 Research on nonlinear waves of blood flow in arterial vessels *Commun. Nonlinear Sci. Numer. Simul.* **102** 105918
[42] Roshid H O, Noor N F M, Khatun M S, Baskonus H M and Belgacem F B M 2021 Breather, multi-shock waves and localized excitation structure solutions to the extended BKP–Boussinesq equation *Commun. Nonlinear Sci. Numer. Simul.* **101** 105867
[43] Gao W, Yel G, Baskonus H M and Cattani C 2020 Complex solitons in the conformable (2 + 1)-dimensional Ablowitz–Kaup–Newell–Segur equation *AIMS Math.* **5** 507–21
[44] Kumar A, Ilhan E, Ciancio A, Yel G and Baskonus H M 2021 Extractions of some new travelling wave solutions to the conformable Date–Jimbo–Kashiwara–Miwa equation *AIMS Math.* **6** 4238–64