

# On the emergence of gravitational dynamics from tensor networks

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## Abstract

Tensor networks are used to describe the ground state wavefunction of the quantum many-body system. Recently, it has been shown that a tensor network can generate the anti-de Sitter (AdS) geometry by using the entanglement renormalization approach, which provides a new way to realize bulk reconstruction in the AdS/conformal field theory correspondence. However, whether the dynamical connections can be found between the tensor network and gravity is an important unsolved problem. In this paper, we give a novel proposal to integrate ideas from tensor networks, entanglement entropy, canonical quantization of quantum gravity and the holographic principle and argue that the gravitational dynamics can be generated from a tensor network if the wave function of the latter satisfies the Wheeler–DeWitt equation.

Keywords: AdS/CFT correspondence, tensor network, emergent gravity

## 1. Introduction

In the study of quantum many-body physics, a tensor network becomes a natural language in which the wave function of the system is described by a series of tensors that comprise a network. Each tensor can be viewed as a building block of the wave function and the connections between tensors are captured by quantum entanglement among the particles. Typically, the total Hilbert space of a quantum many-body system is too large to handle due to the large number of particles and their microstates. An efficient way to deal with the problem is to utilize the idea of a real space renormalization group (RG) to make the number of coarse-grained effective degrees of freedom (d.o.f.) reduce dramatically. The RG approach for a tensor network is called the multi-scale entanglement renormalization (MERA), which was shown very powerful both for theoretical and numerical calculations [1].

Recently, important progress made was that the MERA tensor network can be used to construct the bulk anti-de Sitter (AdS) geometry. More specifically, a discrete-time slice of AdS geometry can emerge from the coarse graining of some tensor network at the quantum critical point, which can be viewed as a tensor network realization or bulk reconstruction of the AdS/conformal field theory (CFT) correspondence [2].

Soon after, different kinds of tensor networks have been investigated to mimic the properties of the holographic duality, such as tensor networks with quantum error correction [3], and random tensor networks [4], with the attempt to reconstruct the bulk AdS geometry and matter fields by using the information of the boundary quantum theory such as the correlation functions and entanglement entropy.

Indeed, in the construction of bulk geometry, quantum entanglement or entanglement entropy was shown to play a vital important role. Related progresses include the proposal of calculating the entanglement entropy of the boundary CFT from the minimal surface in bulk AdS [5], the proposal that spacetime can emerge from the quantum entanglement of boundary CFT, in which disentangling CFTs in two boundary regions can make the bulk spacetime disconnected [6], the study of reconstructing bulk AdS geometry from the entanglement wedge of the boundary CFT [7], the proposal of a surface growth approach for bulk reconstruction [8, 9] and so on. Among these approaches, the essential point is to find out the underlying connections between the dynamics of the non-gravitational system (such as the CFT) and that of the spacetime geometry, i.e. the gravitational dynamics. Similar situations occurred in the study of analogue gravity [10], and it was not until recently that the dynamical connections between acoustic black holes in fluid (one kind of analogue gravity) and real black holes have been revealed [11, 12].

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As for the tensor network approach of building spacetime geometry is concerned, the crucial question is whether the gravitational dynamics, namely, Einstein's equation can also be constructed (or generated) from the tensor network, and hence, from the non-gravitational quantum many-body systems. Recently, the Einstein equation associated with the Bruhat-Tits tree geometry has been shown can emerge from the  $p$ -adic CFT tensor network [13]. In the present paper, we present a novel proposal to combine the key ideas from the tensor networks, entanglement entropy, canonical quantization of quantum gravity and holographic principle together and argue that Einstein's equation can be generated from the tensor network if the Schrödinger equation that is satisfied by the wave function of the tensor network can be rewritten as the Wheeler–DeWitt equation.

## 2. Equivalence between the wave functions

Considering a quantum many-body system (with  $N$  particles) in  $d$ -dimensional flat spacetime, its ground state wave function  $|\Psi\rangle$  can be expressed as

$$|\Psi\rangle = \sum_{a_1 \cdots a_N} T_{a_1 \cdots a_N} |a_1\rangle \otimes \cdots \otimes |a_N\rangle, \quad (1)$$

where  $|a_j\rangle$  is the basis of the  $j$ -th particle, and  $T_{a_1 \cdots a_N}$  can be viewed as the coefficients of an  $N$ -rank tensor. In the tensor network representation, the tensor  $T_{a_1 \cdots a_N}$  can be reduced into a network comprised by  $N_T$  number of tensors  $t_{b_1 \cdots b_n}$  with less rank, namely,  $n < N$ , and for simplicity, we require each index  $b_j$  takes the same  $q$  number of values.

Note that  $|\Psi\rangle$  can also be written into the Euclidean path integral form as [14]

$$\Psi[\phi(x)] = \mathcal{N}^{-\frac{1}{2}} \int \prod_{0 < \tau < \infty} D\phi(\tau, x) \delta(\phi(0, x) - \phi(x)) e^{-I_E[\phi]}, \quad (2)$$

where  $\phi(x)$  are fields,  $I_E[\phi]$  is the Euclidean action of the many-body system and  $\mathcal{N}^{-\frac{1}{2}}$  is the normalization factor, respectively.

On the other hand, the wave function  $\Psi_G[h_{IJ}, \varphi]$  of a spacetime can also be expressed as the Euclidean path integral [15]

$$\Psi_G[h_{IJ}, \varphi] = \mathcal{N}^{-\frac{1}{2}} \int_C D[g] D[\varphi] e^{-I_E[g, \varphi]}, \quad (3)$$

where  $\mathcal{N}^{-\frac{1}{2}}$  is the normalization factor and  $C$  indicates a class of spacetimes with a compact boundary such that the induced metric  $h_{IJ}$  and matter fields  $\varphi$  satisfy the given boundary conditions. For example, in the asymptotically AdS spacetime, the boundary condition can be chosen as the spacelike hypersurface (time slice) with an induced metric  $h_{IJ}$  and a matter field  $\varphi$  on it, and the integral region is all possible geometries which are asymptotically AdS spacetime.

Recall that the fundamental equation of the AdS/CFT correspondence is the equivalence between the partition function (generating functional) of the bulk gravity and that of the boundary CFT [16–18]

$$Z_{\text{AdS}} = Z_{\text{CFT}}. \quad (4)$$

For general spacetime backgrounds, the holographic principle indicates that the partition function of the bulk theory should be equal to that of the boundary theory. Furthermore, the wave functionals  $\Psi[\phi(x)]$  and  $\Psi_G[h_{IJ}, \varphi]$  contain all of the information of their corresponding systems respectively and they will reduce to the associated partition functions when the initial states are chosen as the  $\delta$  function source. Therefore, if a quantum many-body system is holographically dual to a gravitational theory living in higher dimensional spacetime, we conjecture that the relation

$$\Psi[\phi(x)] = \Psi_G[h_{IJ}, \varphi] \quad (5)$$

is held. Equation(5) can be viewed as a generalization of equation (4), and it is a bridge to connect the dynamics of the two sides.

## 3. Transformation of d.o.f. from the tensor network to the metric

If a tensor network can describe gravity, a crucial question to ask is how are the d.o.f. of the former mapping to those of the latter, such as the relation of field/operator duality and subregion-subregion duality in the AdS/CFT correspondence [19]. Interestingly, we found that the entanglement entropy obtained from the tensor network plays a vitally important role. To see this, note that the open indices of a tensor network correspond to the physical d.o.f. of the quantum many-body system [20], for an arbitrarily given region (without open indices inside it) in the tensor network with volume  $V$  (denoted as region A) and boundary  $\partial V$ , the number of external indices (legs)  $m$  of the region are proportional to its boundary area  $\Sigma_A$ , when each leg has  $q$  number of excitations, there are  $q^m$  microstates on  $\Sigma_A$ , and hence the associated entropy is  $S_A = k_B \ln q^m = m k_B \ln q \propto m \propto \Sigma_A$ , which just describes the entanglement entropy between regions  $V$  and  $\bar{V}$  (denoted as region B) and obeys the area law.

Moreover, assuming that the tensor network forms a  $d-1$ -dimensional flat space (note that this space is not necessarily the real space), the entanglement renormalization method together with the holographic entanglement entropy proposal suggests that  $S_A$  is a quarter of the area  $A$  of a bulk co-dimensional-2 minimal surface  $\lambda_A$  with boundary  $\partial V$

$$S_A = \frac{A}{4l_p^{d-1}}, \quad (6)$$

where  $A = \int \sqrt{\tilde{\lambda}} d^{d-1}\xi$ , with  $\tilde{\lambda}_{ij}$  and  $\xi$  the induced metric and coordinate on  $\lambda_A$ , and  $l_p$  is the fundamental length scale of the  $d+1$ -dimensional spacetime. We regard this geometry as the one that emerged from the tensor network. Nevertheless, for dimensional analysis, it is expected that  $l_p^{d-1}$  be the gravitational coupling constant  $G_{d+1}$ . On the other hand,  $S_A$  is calculated from the von Neumann entropy

$$S_A = -\text{tr} \rho_A \ln \rho_A, \quad (7)$$

where  $\rho_A$  is the reduced density matrix of subsystem A, which can be expressed as

$$\begin{aligned}\rho_A &= \int \mathcal{D}\phi_B \Psi^*[\phi_B \oplus \phi_A] \Psi[\phi_B \oplus \phi'_A] \\ &= \frac{1}{\mathcal{N}} \int_{\phi(\vec{x}, 0^-) = \phi'^A}^{\phi(\vec{x}, 0^+) = \phi_A} \mathcal{D}\phi e^{-I_E[\phi]},\end{aligned}\quad (8)$$

in which  $\phi_B$  are fields belonging to the region B and  $\text{tr}\rho_A = 1$ .

An important hint from equation (6) is that it governs the transformation of the d.o.f. from the tensor network to the induced geometry on  $\lambda_A$ , namely,  $\tilde{\lambda}_{ij}$ . In addition,  $\tilde{\lambda}_{ij}$  is obtained from the induced geometry  $\tilde{h}_{IJ}$  on a time slice  $\Sigma_t$  via  $\tilde{\lambda}_{ij} = \frac{\partial y^I}{\partial \xi^i} \frac{\partial y^J}{\partial \xi^j} \tilde{h}_{IJ}$  (for static minimal surface), where  $y^I$  is the coordinate on  $\Sigma_t$ . Consequently, the d.o.f. of the tensor network are transformed to those of induced geometry  $\Sigma_t$ , namely,  $\tilde{h}_{IJ}$ , which indicates a similar relation with equation (5)

$$\Psi[\phi(x)] = \Psi[\phi[\tilde{h}_{IJ}]], \quad (9)$$

since the wave functional  $\Psi[\phi(x)]$  is a scalar function, equation (9) is just a change of variables in the wave function. Clearly, if  $\tilde{h}_{IJ}$  can represent the real spacetime geometry, equations (9) and (5) are the same.

To see more clearly how the d.o.f. of the tensor network and the emerged geometry are connected with each other, let us consider a variation on both sides. From equations (6), (7), we have

$$\delta S_A = -\text{tr}(\delta\rho_A \ln \rho_A) = -\frac{1}{8G_{d+1}} \int \sqrt{\tilde{\lambda}} d^{d-1}\xi \tilde{\lambda}_{ij} \delta\tilde{\lambda}^{ij}. \quad (10)$$

When the variation is caused by a single operator perturbation, namely,

$$I_E^{(0)} \rightarrow I_E = I_E^{(0)} + g_s \int d^d x \mathcal{O}(x), \quad (11)$$

where  $g_s$  is the coupling constant, the density matrix changes as

$$\begin{aligned}\delta\rho_A &= \rho_A - \rho_A^{(0)} \\ &= \frac{1}{\mathcal{N}} \int_{\phi(\vec{x}, 0^-) = \phi'^A}^{\phi(\vec{x}, 0^+) = \phi_A} \mathcal{D}\phi e^{-I_E^{(0)} - g_s \int d^d x \mathcal{O}(x)} \\ &\quad - \frac{1}{\mathcal{N}^{(0)}} \int_{\phi(\vec{x}, 0^-) = \phi'^A}^{\phi(\vec{x}, 0^+) = \phi_A} \mathcal{D}\phi e^{-I_E^{(0)}},\end{aligned}\quad (12)$$

where

$$\begin{aligned}\mathcal{N} &= \int \mathcal{D}\chi \int_{\phi(\vec{x}, 0^-) = \phi(\vec{x}, 0^+) = \chi} \mathcal{D}\phi e^{-I_E^{(0)} - g_s \int d^d x \mathcal{O}(x)} \\ &= \mathcal{N}^{(0)} \left( 1 - g_s \int d^d x \langle \mathcal{O}(x) \rangle \right. \\ &\quad \left. + \frac{g_s^2}{2} \int d^d x \int d^d x' \langle \mathcal{O}(x) \mathcal{O}(x') \rangle + \dots \right),\end{aligned}\quad (13)$$

then at the first order perturbation,

$$\delta\rho_A = g_s \rho_A^{(0)} \left( \int d^d x \langle \mathcal{O}(x) \rangle - \int d^d x \mathcal{O}(x) \right). \quad (14)$$

Substituting equations (14) into (10), one can then obtain the explicit relationship between  $\mathcal{O}(x)$  and  $\delta\tilde{\lambda}^{ij}$ . A simple example is that  $I_E^{(0)}$  is the action of a massless scalar field

$\frac{1}{2} \int d^d x \partial_\mu \phi \partial^\mu \phi$ , and the single trace operator  $\mathcal{O}(x)$  can be chosen as  $\phi^2$  with the coupling constant  $g_s = \frac{1}{2}m^2$ .

#### 4. Emergence of the gravitational dynamics

So that the emergent induced metric  $\tilde{h}_{IJ}$  can describe the real spacetime geometry, it needs to satisfy the gravitational dynamical equation, namely, Einstein's equation or its equivalent form. Considering a real  $d+1$ -dimensional stationary spacetime with

$$ds^2 = -N^2 dt^2 + h_{IJ}(N^I dt + dx^I)(N^J dt + dx^J), \quad (15)$$

the Hamiltonian formalism gives the Hamiltonian and the momentum constraints, and in the canonical quantization, they become the Wheeler–DeWitt equation and the quantum momentum constraint equation that the wave function of the spacetime to satisfy [21]

$$\left\{ -G_{IJLK} \frac{\delta^2}{\delta h_{IJ} \delta h_{KL}} - \sqrt{-h} (^{(d)}R - 2\Lambda) \right\} \Psi[h_{IJ}] = 0, \quad (16)$$

$$\left\{ \frac{\delta}{\delta h_{IJ}} \Psi[h_{IJ}] \right\}_{|I} = 0, \quad (17)$$

where  $G_{IJLK} = h^{-1/2} \left( \frac{1}{2} (h_{IK} h_{JL} + h_{IL} h_{JK}) - \frac{1}{d-1} h_{IJ} h_{KL} \right)$  is the supermetric and  $^{(d)}R$  is  $d$ -dimensional Ricci curvature constructed from the induced metric  $h_{IJ}$ , and here we only consider the bulk to be the vacuum, namely, without the matter fields.

Furthermore, the ground state wave function  $\Psi[\varphi(x)]$  described by a tensor network satisfies the Schrödinger equation  $\hat{\mathcal{H}}\Psi[\varphi(x)] = 0$ . Therefore, from equations (9) and (5), if a tensor network can describe a real spacetime, its associated Schrödinger equation should be able to be rewritten in the same form as the Wheeler–DeWitt equation, i.e.

$$\begin{aligned}\hat{\mathcal{H}}\Psi[\varphi(x)] &= \left\{ -\tilde{G}_{IJLK} \frac{\delta^2}{\delta \tilde{h}_{IJ} \delta \tilde{h}_{KL}} \right. \\ &\quad \left. - \sqrt{-\tilde{h}} (^{(d)}\tilde{R} - 2\tilde{\Lambda}) \right\} \Psi[\varphi[\tilde{h}_{IJ}]] = 0,\end{aligned}\quad (18)$$

$$\left\{ \frac{\delta}{\delta \tilde{h}_{IJ}} \Psi[\varphi[\tilde{h}_{IJ}]] \right\}_{|I} = 0, \quad (19)$$

which means that  $\tilde{h}_{IJ}$  can describe a real spacetime metric  $h_{IJ}$ , where  $\hat{\mathcal{H}}$  is the Hamiltonian density of the quantum many-body system.

#### 5. Projecting the Wheeler–DeWitt equation on the AdS boundary

In the original holographic approach, the CFT or quantum field theory (QFT) is located at the asymptotical spatial boundary. Therefore, the Hamiltonian  $\hat{\mathcal{H}}$  of the tensor network should be expressed as an operator in terms of variables on the spatial boundary. This can be done by

projecting the Wheeler–DeWitt equation (16) on the AdS boundary. Let's consider the  $d+1$ -dimensional static AdS spacetime with metric

$$\begin{aligned} ds^2 &= g_{AB} dX^A dX^B = -N^2(r) dt^2 + h_{IJ}(r) dy^I dy^J \\ &= -N^2(r) dt^2 + h_{rr}(r) dr^2 + h(r) dx_i^2, \end{aligned} \quad (20)$$

where the asymptotical spatial boundary  $\Sigma_r$  (with induced metric  $\gamma_{\alpha\beta}$  and coordinate  $x^\alpha$ ) is located at  $r \rightarrow \infty$ . Also, denoting the spatial boundary of the time slice  $\Sigma_t$  to be  $\mathcal{B}_t$  (with induced metric  $\sigma_{ab}$  and coordinate  $\theta^a$ ), which is the region of taking  $\Sigma_t$  to  $r \rightarrow \infty$ . The Ricci tensor  ${}^{(d)}R_{IJ} = -(d-1)h_{IJ}/L^2$ , which gives  ${}^{(d)}R - 2\Lambda = 0$  for static AdS spacetime case, where  $L$  is the curvature radius of the AdS spacetime. Then the Wheeler–DeWitt equation (16) becomes

$$\left\{ -G_{IJK} \frac{\delta^2}{\delta h_{IJ} \delta h_{KL}} \right\} \Psi[h_{IJ}] = 0. \quad (21)$$

Furthermore, the displacement on  $\Sigma_r$  is

$$dX^A = \frac{\partial X^A}{\partial t} dt + \frac{\partial X^A}{\partial \theta^a} d\theta^a = N n^A dt + e_a^A d\theta^a, \quad (22)$$

where  $n^A$  is the timelike unit normal vector of  $\Sigma_t$ , then the induced line element on  $\Sigma_r$  is

$$\begin{aligned} ds^2 &= g_{AB} (N n^A dt + e_a^A d\theta^a) (N n^B dt + e_b^B d\theta^b) \\ &= -N^2 dt^2 + g_{AB} e_a^A e_b^B d\theta^a d\theta^b \\ &= -N^2 dt^2 + \sigma_{ab} d\theta^a d\theta^b \\ &\equiv \gamma_{\alpha\beta} dx^\alpha dx^\beta, \end{aligned} \quad (23)$$

which gives  $\sqrt{-\gamma} = N\sqrt{\sigma}$ . In addition,  $h_{IJ}$  and  $\sigma_{ab}$  are related by  $\sigma_{ab} = h_{IJ} \frac{\partial y^I}{\partial \theta^a} \frac{\partial y^J}{\partial \theta^b} \equiv h_{IJ} e_a^I e_b^J$ , then  $\frac{\delta}{\delta h_{IJ}} = e_a^I e_b^J \frac{\delta}{\delta \sigma_{ab}}$ . Note that for the asymptotically AdS spacetime case, the boundary d.o.f. and the radial d.o.f. can be decoupled in the wave functional as  $\Psi[h_{IJ}] = e^{f(r)} \Psi[\sigma_{ab}]$ , where  $f(r)$  is related to the Liouville action [22]. Therefore, the Wheeler–DeWitt equation (16) can be rewritten as

$$\begin{aligned} &\left\{ -G_{IJK} \frac{\delta^2}{\delta h_{IJ} \delta h_{KL}} \right\} \Psi[h_{IJ}] \\ &= \frac{1}{\sqrt{h}} \left( \frac{1}{2} (h_{IK} h_{JL} + h_{IL} h_{JK}) - \frac{1}{d-1} h_{IJ} h_{KL} \right) \\ &\quad e_a^I e_b^J e_c^K e_d^L \frac{\delta^2}{\delta \sigma_{ab} \delta \sigma_{cd}} \Psi[\sigma_{ab}] \\ &= \frac{1}{\sqrt{h}} \left( \frac{1}{2} (\sigma_{ac} \sigma_{bd} + \sigma_{ad} \sigma_{bc}) - \frac{1}{d-1} \sigma_{ab} \sigma_{cd} \right) \\ &\quad \times \frac{\delta^2}{\delta \sigma_{ab} \delta \sigma_{cd}} \Psi[\sigma_{ab}] \\ &= 0. \end{aligned} \quad (24)$$

While at a fixed time, the induced metric  $\gamma_{\alpha\beta}$  on the AdS boundary reduces to  $\sigma_{ab}$ , namely,  $\gamma_{ab} = \sigma_{ab} = h_{IJ} e_a^I e_b^J$ .

Consequently, equation (24) is equivalent to

$$\begin{aligned} &\frac{1}{\sqrt{h}} \left( \frac{1}{2} (\gamma_{ac} \gamma_{bd} + \gamma_{ad} \gamma_{bc}) - \frac{1}{d-1} \gamma_{ab} \gamma_{cd} \right) \\ &\quad \times \frac{\delta^2}{\delta \gamma_{ab} \delta \gamma_{cd}} \Psi[\gamma_{ab}] = 0, \end{aligned} \quad (25)$$

which is the Wheeler–DeWitt equation on the AdS boundary.

According to our proposal, so that a tensor network can generate a real AdS spacetime, its associated Schrödinger equation should be expressed as

$$\begin{aligned} \hat{\mathcal{H}} \Psi[\varphi(x)] &= \frac{1}{\sqrt{\tilde{h}}} \left( \frac{1}{2} (\tilde{\gamma}_{ac} \tilde{\gamma}_{bd} + \tilde{\gamma}_{ad} \tilde{\gamma}_{bc}) \right. \\ &\quad \left. - \frac{1}{d-1} \tilde{\gamma}_{ab} \tilde{\gamma}_{cd} \right) \frac{\delta^2}{\delta \tilde{\gamma}_{ab} \delta \tilde{\gamma}_{cd}} \Psi[\tilde{\gamma}_{ab}] = 0. \end{aligned} \quad (26)$$

Clearly, not every tensor network can satisfy equation (26) and it is nontrivial to find out concrete examples of quantum many-body systems (concrete  $\hat{\mathcal{H}}$ ) which can describe gravity. Besides, it is known that not all of the wave functions of CFTs are dual to classical gravity geometry. A fundamental question is what kinds of conditions a wave function must satisfy in order to have a classical gravity duality. We propose that the answers lie in the Wheeler–DeWitt equation.

## 6. Conclusions and discussions

The gauge/gravity dualities have provided us with very powerful tools to study the CFT or QFT from their dual gravitational theories in bulk. However, the study in the inverse direction, i.e. from boundary to the bulk, is not so clear and straightforward. The essential questions are how to construct the bulk geometry, and especially, how to construct or generate the bulk gravitational dynamics from the information of the boundary QFT. In this paper, we studied the bulk reconstruction of the AdS spacetime from tensor networks on the boundary and proposed a novel approach to generate the bulk gravitational dynamics by combining the ideas of the holographic entanglement entropy and the canonical quantization of quantum gravity and argue that there is a connection between the boundary Schrödinger equation and the Wheeler–DeWitt equation in the bulk gravity side. Our approach deepens the understanding of the gauge/gravity duality and makes its formalism more complete. The approach also supports the emergent picture of gravity. There remains a lot of work to do, such as finding (or constructing) appropriate tensor network models to generate the desired gravitational backgrounds and extending our method to stationary spacetime cases.

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