 Communications in Theoretical Physics

Commun. Theor. Phys. 75 (2023) 117401 (13pp) https://doi.org/10.1088/1572-9494/acf27c

Topical Review

Theoretical aspects of holographic dark energy

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Received 1 July 2023, revised 10 August 2023
Accepted for publication 22 August 2023
Published 13 October 2023

Abstract

We review the theoretical aspects of holographic dark energy (HDE) in this paper. Making use of the holographic principle (HP) and the dimensional analysis, we derive the core formula of the original HDE (OHDE) model, in which the future event horizon is chosen as the characteristic length scale. Then, we describe the basic properties and the corresponding theoretical studies of the OHDE model, as well as the effect of adding dark sector interaction in the OHDE model. Moreover, we introduce all four types of HDE models that originate from HP, including (1) HDE models with the other characteristic length scale; (2) HDE models with extended Hubble scale; (3) HDE models with dark sector interaction; (4) HDE models with modified black hole entropy. Finally, we introduce the well-known Hubble tension problem, as well as the attempts to alleviate this problem under the framework of HDE. From the perspective of theory, the core formula of HDE is obtained by combining the HP and the dimensional analysis, instead of adding a DE term into the Lagrangian. Therefore, HDE remarkably differs from any other theory of DE. From the perspective of observation, HDE can fit various astronomical data well and has the potential to alleviate the Hubble tension problem. These features make HDE a very competitive dark energy scenario.

Keywords: holographic principle, dark energy, holographic dark energy

1. Introduction

The holographic principle (HP) [1, 2], which was inspired by the black hole thermodynamics [3, 4], reveals that all the physical quantities located in a volume of space can be represented by some physical quantities located on the boundary of that space. After the discovery of the Anti-de Sitter/Conformal field theories (AdS/CFT) correspondence [5], it is widely believed that the HP should be a fundamental principle of quantum gravity. So far, the HP has been applied to various fields of physics, including nuclear physics [6], condensed matter physics [7], theoretical physics [8] and cosmology [9].

In this paper we focus on the dark energy (DE) problem [10, 11]. The most popular theoretical model is the ΛCDM model, which includes a cosmological constant Λ and a cold dark matter (CDM) component. But the ΛCDM model has two cosmological constant problems [12–20]: (a) Why $\rho_\Lambda \approx 0$? (b) Why $\rho_\Lambda \sim \rho_m$ now? In the past 25 years, hundreds of DE models have been proposed; however, so far the nature of DE is still a mystery.

In essence, the DE problem should be an issue of quantum gravity. Since the HP is the most fundamental principle of quantum gravity, it may also has great potential to solve the DE problem. In 2004, by applying the HP to the DE problem, One of the present authors (Miao Li) proposed a new DE model, i.e. holographic dark energy (HDE) model [21]. The DE energy density ρ_{de} of this model only relies on two physical quantities: (1) the

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reduced Planck mass $M_p \equiv \sqrt{1/8\pi G}$, where G is the Newton constant; (2) the cosmological length scale L, which is chosen as the future event horizon of the Universe [21]. Note that this model is the first DE model inspired by the HP [22]. From now on, we will call it the original HDE (OHDE) model.

So far, the idea of applying the HP to the DE problem has drawn a lot of attention:

1. To explain the origin of HDE, many different theoretical mechanisms are proposed;
2. To consider the interaction between dark sectors, the interacting HDE models are studied;
3. A lot of other HDE models are proposed, where different forms of L are taken into account.
4. Some attempts are made, to alleviate the Hubble tension problem under the framework of HDE.

In this paper, all the topics mentioned above will be reviewed. We assume today's scale factor $a_0=1$, where the subscript '0' always indicates the present value of the physical quantity. In addition, we use the metric convention $(-,+,+,+)$, as well as the natural units $c=\hbar=1$.

2. The basic of cosmology

This section introduces the basics of cosmology, including the Friedmann–Lemaître–Robertson–Walker (FLRW) cosmology, as well as the DE problem.

2.1. Friedmann–Lemaître–Robertson–Walker cosmology

Modern cosmology has two cornerstones. The first cornerstone is general relativity (GR), whose core is the Einstein field equation

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu}. \tag{1}$$

Note that $G_{\mu\nu}$ is the Einstein tensor, $R_{\mu\nu}$ is the Ricci tensor, R is the Ricci scalar, $g_{\mu\nu}$ is the metric, and $T_{\mu\nu}$ is the energy-momentum tensor. In addition, $T_{\mu\nu}=(\rho+p)u_\mu u_\nu + g_{\mu\nu}p$, where ρ and p are the total energy density and the total pressure of all the components, respectively.

The second cornerstone is the cosmological principle, i.e. the Universe is homogeneous and isotropic on large scales. It means that the Universe should be described by the FLRW metric

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega_2^2\right]. \tag{2}$$

Note that t is the cosmic time, $a(t)$ is the scale factor, r is the spatial radius coordinate, Ω_2 is the two-dimensional unit sphere volume, and the quantity k characterizes the curvature of three-dimensional space.

Based on equations (1) and (2), two Friedmann equations can be obtained

$$3M_p^2H^2 = \rho - \frac{3M_p^2k}{a^2}, \tag{3}$$

$$\frac{\ddot{a}}{a} = -\frac{\rho+3p}{6M_p^2}. \tag{4}$$

Here $H \equiv \dot{a}/a$ is the Hubble parameter, which denotes the expansion rate of the Universe.

From these two Friedmann equations, one can see that the pressure p affects the expansion of the Universe: if $p > -\rho/3$, the Universe will decelerate; if $p < -\rho/3$, the Universe will accelerate. Moreover, if all the components in the Universe were determined, the expansion history of the Universe would be determined too.

2.2. Dark energy problem

Let us start from a short introduction to the history of the DE problem. In 1917, to maintain a static Universe, Einstein added a cosmological constant Λ in the Einstein field equations [23]. Afterwards, because of the discovery of cosmic expansion, Einstein declared that this was the biggest mistake he made in his whole career. In 1967, Zel'dovich reintroduced the cosmological constant by taking the vacuum fluctuations into account [24]. In 1998, Two astronomical teams discovered the accelerating expansion of the Universe [10, 11], which declares the return of DE.

As is well known, the Universe has four main components: baryon matter, DM, radiation, and DE. So the first Friedmann equation satisfies

$$H(z) = H_0\sqrt{\Omega_{r0}(1+z)^4 + \Omega_{b0}(1+z)^3 + \Omega_{\rm dm0}(1+z)^3 + \Omega_{k0}(1+z)^2 + \Omega_{\rm de0}X(z)}. \tag{5}$$

Here $H_0 = 100h(\mathrm{km\cdot s^{-1}\cdot Mpc^{-1}})$ denotes the present value of the Hubble parameter, h denotes the Hubble constant, $z=a^{-1}-1$ denotes the redshift, $\Omega_{i0} \equiv \rho_{i0}/\rho_{c0} = \rho_{i0}/3H_0^2M_p^2$ denote the present fractional densities of various component. Note that the total fractional matter density $\Omega_m = \Omega_b + \Omega_{\rm dm}$, the fractional energy density of spatial curvature $\rho_k \equiv -3M_p^2k/a^2$, and the DE density function

$$X \equiv \frac{\rho_{\rm de}(z)}{\rho_{\rm de0}} = \exp\left[3\int_0^z dz'\frac{1+w(z')}{1+z'}\right], \tag{6}$$

where $w \equiv p_{\rm de}/\rho_{\rm de}$ is the DE equation of state (EoS) [25–30]. As mentioned above, in the past 25 years hundreds of DE models have been proposed; different DE models will yield different forms of EoS.

In 1989, Weinberg published a review article of DE, which divided various DE models into five categories [31]:

1. Symmetry. This category includes many theoretical attempts, such as no-scale supersymmetry [32] and complexification of coordinates [33].
2. Anthropic principle. The key idea is a multiverse, where different DE energy densities can be realized [34, 35]. We live in a universe with the observed DE density, because it allows long enough time for galaxy formation. The discovery of string landscape [36, 37] support this idea.
3. Tuning mechanisms. This category introduces a scalar field which can reduce the DE energy density. Some models of this category result in vanishing Newton's constant [38, 39].
4. Modified gravity. By modifying the left side of Einstein's field equations, modified gravity can also explain cosmic acceleration. There are a large number of modified gravity models, such as unimodular gravity [40, 41] and massive gravity [42].
5. Quantum gravity. Making use of the Hartle-Hawking wave function of the Universe [43], a small DE energy density is predicted [44].

Afterward, some new theoretical DE models were proposed. Therefore, three new categories can be added [18]:

6. Holographic principle. This is the Key point of this review.
7. Back-reaction of gravity. Under the frame of general relativity, inhomogeneities of the Universe can backreact on the FLRW background [45].
8. Phenomenological models. It is argued that DE can be described by scalar fields with various potentials or kinetic terms [14].

In this paper, we just focus on the sixth category, i.e. holographic principle.

3. Original holographic dark energy model

In this section, we introduce how to apply the HP to the DE Problem.

3.1. General formula of HDE energy density

Now let us take into account the Universe with a characteristic length scale L. Based on the HP, one can conclude that the DE energy density $\rho_{\rm de}$ can be described by some physical quantities on the boundary of the Universe. Obviously, one can only use the reduced Planck mass M_p and the cosmological length scale L to construct $\rho_{\rm de}$. Making use of the dimensional analysis, we can obtain

$$\rho_{\rm de} = C_1 M_p^4 + C_2 M_p^2 L^{-2} + C_3 L^{-4} + \dots \tag{7}$$

where C_1, C_2, C_3 are dimensionless constant parameters. Note that the first term is 10^{120} times larger than the cosmological observations [12], so this term should be deleted (For a more theoretical analysis, see [46]). Moreover, compared with the second term, the third and the other terms are negligible, so these terms should be deleted, too.

Therefore, the expression of $\rho_{\rm de}$ can be written as

$$\rho_{\rm de} = 3C^2 M_p^2 L^{-2}, \tag{8}$$

where C is the dimensionless constant parameter, too. It must be stressed that equation (8) is the general formula of HDE energy density. In other words, all the DE models of the sixth category can give an energy density form that is the same as equation (8).

3.2. The original HDE model

After deriving the general formula of the HDE energy density, one needs to choose the specific form of the characteristic length scale L. The simplest choice, i.e. the Hubble scale $L = 1/H$ [47, 48], will yield a wrong EoS of DE [49]. Besides, the particle horizon is not a good choice either, because it cannot yield cosmic acceleration.

In 2004, Li suggested that the characteristic length scale L should be chosen as the future event horizon [21]

$$L = a\int_t^\infty \frac{{\rm d}t'}{a} = a\int_a^\infty \frac{{\rm d}a'}{Ha'^2}. \tag{9}$$

This is the first HDE model that can yield cosmic acceleration, So we call it the original HDE (OHDE) model.

For the OHDE model, the Friedmann equation satisfies

$$3M_p^2 H^2 = \rho_{\rm de} + \rho_m, \tag{10}$$

or equivalently,

$$E(z) \equiv \frac{H(z)}{H_0} = \left(\frac{\Omega_{m0}(1+z)^3}{1-\Omega_{\rm de}(z)}\right)^{1/2}. \tag{11}$$

Here

$$\Omega_{\rm de} \equiv \frac{\rho_{\rm de}}{\rho_c} = \frac{C^2}{L^2 H^2}, \tag{12}$$

where $\rho_c \equiv 3M_p^2 H^2$ is the critical density of the Universe. Taking the derivative of $\Omega_{\rm de}$, and making use of equation (9), one can obtain

$$\Omega'_{\rm de} = 2\Omega_{\rm de}\left(-\frac{H'}{H} - 1 + \frac{\sqrt{\Omega_{\rm de}}}{C}\right), \tag{13}$$

where the prime denotes derivative with respect to $\ln a$. From equation (10), we have

$$-\frac{H'}{H} = \frac{3}{2} - \frac{\Omega_{\rm de}}{2} - \frac{\Omega_{\rm de}^{3/2}}{C}. \tag{14}$$

Based on equations (13) and (14), one can obtain

$$\frac{{\rm d}\Omega_{\rm de}}{{\rm d}z} = -\frac{\Omega_{\rm de}(1-\Omega_{\rm de})}{1+z}\left(1 + \frac{2\sqrt{\Omega_{\rm de}}}{C}\right). \tag{15}$$

This equation describes the dynamical evolution of the OHDE model. Since $0 < \Omega_{\rm de} < 1$, $\mathrm{d}\Omega_{\rm de}/\mathrm{d}z$ is always negative, namely the fraction density of HDE always increases along with redshift $z \to -1$. Based on equations (15) and (11), one can obtain the redshift evolution of Hubble parameter $H(z)$ of the OHDE model.

3.3. Important properties of the OHDE model

- EoS

Energy conservation tells us that

$$\rho'_m + 3\rho_m = 0, \tag{16}$$

$$\rho'_{\rm de} + 3(1+w)\rho_{\rm de} = 0. \tag{17}$$

Based on equations (8) and (17), one can obtain the EoS of the OHDE model

$$w = -\frac{1}{3} - \frac{2\sqrt{\Omega_{\rm de}}}{3C}. \tag{18}$$

In the early Universe with $\Omega_{\rm de} \ll 1$, $w \simeq -1/3$, thus $\Omega_{\rm de} \sim a^{-2}$. In the late Universe with $\Omega_{\rm de} \simeq 1$, $w \simeq -1/3 - 2/3C$, thus cosmic acceleration will be yielded as long as $C > 0$. Moreover, if $C = 1$, $w = -1$, then HDE will be close to the cosmological constant; if $C > 1$, $w > -1$, then HDE will be a quintessence DE [50]; if $C < 1$, $w < -1$ thus HDE will be a phantom DE [51–53].

- The Coincidence Problem

The coincidence problem is equivalent to a problem of why the ratio between the DE density and the radiation density is so tiny at the beginning of the radiation-dominated epoch [54].

Let us consider the inflation epoch, which has two main components: the HDE and the inflation energy. Note that the inflation energy is almost constant during the inflation epoch, and then decayed into radiation after the inflation.

If the inflation energy scale is 10^{14} Gev, the ratio between $\rho_{\rm de}$ and ρ_r is about 10^{-52} [21]. During the inflation epoch, the HDE is diluted as $\Omega_{\rm de} \sim a^{-2}$, this is equivalent to $\exp(-2N)$ with $N = 60$. This means that the OHDE model can explain the coincidence problem, as long as the inflation epoch lasts for 60 e-folds [55].

4. Theoretical motivations for the OHDE model

In addition to the dimensional analysis mentioned above, some other theoretical motivations can also lead to the general formula of HDE energy density. Here we review some related research works.

4.1. Entanglement entropy

It is argued that vacuum entanglement energy associated with the entanglement entropy of the Universe can be viewed as the origin of DE [56]. In the quantum field theory, the entanglement entropy of the vacuum with a horizon can be written as

$$S_{\rm Ent} = \frac{\varrho R_h^2}{l^2}, \tag{19}$$

where ϱ is the dimensionless constant parameter, $R_h = a\int_t^\infty \frac{\mathrm{d}t'}{a}$ denotes the future event horizon, and l denotes the ultraviolet cutoff from quantum gravity. The entanglement energy satisfies

$$\mathrm{d}E_{\rm Ent} = T_{\rm Ent}\,\mathrm{d}S_{\rm Ent}, \tag{20}$$

where $T_{\rm Ent} = 1/(2\pi R_h)$ is the Gibbons–Hawking temperature. Integrating equation (20), one can get

$$E_{\rm Ent} = \frac{\varrho N_{\rm dof} R_h}{\pi l^2}, \tag{21}$$

where $N_{\rm dof}$ is the number of light fields present in the vacuum. Thus the DE energy density is

$$\rho_{\rm de} = 3C^2 M_p^2 R_h^{-2}, \tag{22}$$

where $C = \frac{\sqrt{\beta N_{\rm dof}}}{2\pi l M_p}$, $\beta \sim 0.3$, $N_{\rm dof} \sim 10^2$ and $l \sim 1/M_p$ [57]. One can see that this DE energy density has the same form with equation (8).

4.2. Holographic gas

As is known, a system that appears nonperturbative may be described by weakly interacting quasi-particle excitations. Moreover, it is argued that the quasi-particle excitations of such a system may be described by a gas of holographic particles [58], with modified degeneracy

$$w = w_0 k^A V^B M_p^{3B-A}, \tag{23}$$

where V is the volume of the system, w_0, A, and B are dimensionless constants. Note that with the temperature $T \propto V^{-1/3}$ and the entropy $S \propto V^{2/3}$, one can obtain the relationship $B = (A+2)/3$. Therefore, the corresponding energy density of the system is

$$\rho = \frac{A+3}{A+4}\frac{ST}{V}. \tag{24}$$

For the Universe with a radius R, $S = 8\pi^2 M_p^2 R^2$ and $T = 1/(2\pi R)$, thus one can obtain

$$\rho = 3C^2 M_p^2 R^{-2}, \tag{25}$$

where

$$C^2 = \frac{A+3}{A+4}. \tag{26}$$

It is clear that this DE energy density has the same form as equation (8).

4.3. Casimir energy

As is well known, Casimir energy is a core prediction of quantum field theory [59–62]. It is argued that the Casimir energy in a static de Sitter space may be viewed as the origin of DE [63, 64].

The Casimir energy satisfies

$$E_{\text{Casimir}} = \frac{1}{2}\sum_{\omega}|\omega|. \tag{27}$$

Making use of the heat kernel method with ζ function regularization, It can be calculated as

$$E_{\text{Casimir}} = \frac{3}{8\pi}(\ln \mu^2 - \gamma - \Gamma'(-1/2)/\Gamma(-1/2)) \times \left(\frac{L}{l_p^2} - \frac{1}{L}\ln\left(\frac{2L}{l_p^2}\right)\right) + \mathcal{O}(1/L), \tag{28}$$

where L is the de Sitter radius, γ is the Euler constant and $\Gamma'(-1/2) \simeq -3.48$. Note that the dominant term scales as $E_{\text{Casimir}} \sim L/l_p^2$, then the energy density scales as $\rho_{\text{Casimir}} \sim M_p^2 L^{-2}$, which is just the same as equation (8).

4.4. Entropic force

In 2010, Verlinde conjectured that gravity may be essential an entropic force [65]. Based on this idea, [66] suggested that the entropy change of the future event horizon should be considered together with the entropy change of the test holographic screen.

Let us consider a test particle with physical radial coordinate R. Based on Verlinde's proposal, the energy associated with the future event horizon R_h satisfies

$$E_h \sim N_h T_h \sim R_h/G, \tag{29}$$

where $N_h \sim R_h^2/G$ is the number of degrees of freedom on the horizon, $T_h \sim 1/R_h$ is the Gibbons–Hawking temperature. Note that the energy of the horizon induces a force to a test particle of order $F_h \sim GE_h m/R^2$, which can be integrated to obtain a potential

$$V_h \tilde{} - \frac{R_h m}{R} = -C^2 m/2. \tag{30}$$

Using the standard argument leading to Newtonian cosmology, this potential term will show up in the Friedmann equation as a DE component $\rho_{\text{de}} = 3c^2 M_p^2 R_h^{-2}$. Again, this energy density is the same as equation (8).

4.5. Action principle

Finally, we introduce how to derive the general formula of HDE from the action principle [67].

Consider the action

$$S = \frac{1}{16\pi G}\int \mathrm{d}t[\sqrt{-g}(R - \frac{2C}{a^2(t)L^2(t)}) - \lambda(t)(\dot{L}(t) + \frac{N(t)}{a(t)})] + S_m, \tag{31}$$

where $\sqrt{-g} = Na^3$, R denotes the Ricci scalar, and S_m denotes the action of matter. $\lambda(t)$ is a Lagrange multiplier, which forces the cut-off in the energy density; in addition, $\dot{L}(t) + N(t)/a(t)) = 0$ is a local variant of the definition of the event horizon. After taking the variations of N, a, λ, L and redefining $N\mathrm{d}t$ as $\mathrm{d}t$, one can obtain

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{C}{3a^2L^2} + \frac{\lambda}{6a^4} + \frac{8\pi}{3}\rho_m,$$
$$\frac{2\ddot{a}a + \dot{a}^2 + k}{a^2} = \frac{C}{3a^2L^2} - \frac{\lambda}{6a^4} - 8\pi p_m, \tag{32}$$

and

$$\dot{L} = -\frac{1}{a}, \quad L = \int_t^\infty \frac{\mathrm{d}t'}{a(t')} + L(a = \infty),$$
$$\dot{\lambda} = -\frac{4ac}{L^3}, \quad \lambda = -\int_0^t \mathrm{d}t' \frac{4a(t')C}{L^3(t')} + \lambda(a = 0). \tag{33}$$

Since $L(a \to \infty) = 0$, aL is just the future event horizon. Moreover, one can obtain the DE energy density

$$\rho_{\text{de}} = \frac{1}{8\pi G}\left(\frac{C}{a^2L^2} + \frac{\lambda}{2a^4}\right). \tag{34}$$

In addition to the expression of equation (8), this DE energy density has a new term $\frac{\lambda}{2a^4}$, which can be interpreted as dark radiation [68].

5. OHDE model with dark sector interaction

The interaction between dark sectors is a hot topic in the field of DE [69]. In this section, we introduce the research works about the OHDE model with dark sector interaction.

5.1. Dynamical evolution

Consider the OHDE model with dark sector interaction in a non-flat Universe, the first Friedmann equation is

$$3M_p^2 H^2 = \rho_{\text{dm}} + \rho_b + \rho_r + \rho_k + \rho_{\text{de}}. \tag{35}$$

In addition, the energy density of DM and HDE satisfy

$$\dot{\rho_{\text{dm}}} + 3H\rho_{\text{dm}} = Q, \tag{36}$$
$$\dot{\rho_{\text{de}}} + 3H(1 + w)\rho_{\text{de}} = -Q, \tag{37}$$

where Q phenomenologically describes the interaction term.

It should be mentioned that Q cannot be derived from the first principle, and the most common form of Q is

$$Q = H(\Gamma_1\rho_{\text{dm}} + \Gamma_2\rho_{\text{de}}), \tag{38}$$

where Γ_1, Γ_2 are dimensionless constant parameters. For the specific form of Q, three choices are often made in the previous literature, i.e. $\Gamma_2 = 0$, $\Gamma_1 = 0$ and $\Gamma_1 = \Gamma_2 = \Gamma_3$, which leads to three most common interaction form

$$Q_1 = H\Gamma_1\rho_{\text{dm}}; \quad Q_2 = H\Gamma_2\rho_{\text{de}};$$
$$Q_3 = H\Gamma_3(\rho_{\text{dm}} + \rho_{\text{de}}). \tag{39}$$

Some other interaction forms were also proposed, e.g. $Q = H\Gamma\rho_{\text{dm}}\rho_{\text{de}}/(\rho_{\text{dm}} + \rho_{\text{de}})$ [70], $Q = H\Gamma\rho_{\text{dm}}^{\xi_1}\rho_{\text{de}}^{\xi_2}/\rho_c^{\xi_1+\xi_2-1}$ [71], $Q = \Gamma(\dot{\rho_{\text{dm}}} + \dot{\rho_{\text{de}}})$ [72], and so on.

Based on the energy conservation equations for all the energy components in the Universe, we obtain,

$$p_{\text{de}} = -\frac{2}{3}\frac{\dot{H}}{H^2}\rho_c - \rho_c - \frac{1}{3}\rho_r + \frac{1}{3}\rho_k. \tag{40}$$

Substituting this expression of $p_{\rm de}$ into equation (90), one can get

$$2(\Omega_{\rm de}-1)\frac{\dot{H}}{H}+\dot{\Omega}_{\rm de}+H(3\Omega_{\rm de}-3+\Omega_k-\Omega_r)=-H\Omega_I, \tag{41}$$

which is a derivative equation of $\dot{H}$ and $\dot{\Omega}_{\rm de}$, where

$$\Omega_I\equiv\frac{Q}{H(z)\rho_c}. \tag{42}$$

In a non-flat Universe, L takes the form

$$L=ar(t), \tag{43}$$

where

$$\int_0^{r(t)}\frac{{\rm d}r}{\sqrt{1-kr^2}}=\int_t^{+\infty}\frac{{\rm d}t}{a(t)}. \tag{44}$$

Note that equation (43) can give another derivative equation of $\dot{H}$ and $\dot{\Omega}_{\rm de}$

$$\frac{\dot{\Omega}_{\rm de}}{2\Omega_{\rm de}}+H+\frac{\dot{H}}{H}=\sqrt{\frac{\Omega_{\rm de}H^2}{c^2}-\frac{k}{a^2}}. \tag{45}$$

Based on the equations (41) and (45), one can obtain

$$\frac{1}{E(z)}\frac{{\rm d}E(z)}{{\rm d}z}=-\frac{\Omega_{\rm de}}{1+z}\times\left(\frac{\Omega_k-\Omega_r-3+\Omega_I}{2\Omega_{\rm de}}+\frac{1}{2}+\sqrt{\frac{\Omega_{\rm de}}{C^2}+\Omega_k}\right), \tag{46}$$

$$\frac{{\rm d}\Omega_{\rm de}}{{\rm d}z}=-\frac{2\Omega_{\rm de}(1-\Omega_{\rm de})}{1+z}\times\left(\sqrt{\frac{\Omega_{\rm de}}{C^2}+\Omega_k}+\frac{1}{2}-\frac{\Omega_k-\Omega_r+\Omega_I}{2(1-\Omega_{\rm de})}\right). \tag{47}$$

These two equations describe the dynamical evolution of the IHDE model in a non-flat Universe.

5.2. Equation of state

Then, we discuss the EoS w of the IHDE model. For simplicity, we only consider the case of a flat Universe. Let us take into account the interaction between matter and HDE, then

$$\dot{\rho_m}+3H\rho_m=Q, \tag{48}$$

$$\dot{\rho_{\rm de}}+3H(1+w)\rho_{\rm de}=-Q. \tag{49}$$

Consider the ratio of energy densities [73]

$$r\equiv\frac{\rho_m}{\rho_{\rm de}}. \tag{50}$$

Based on the equations (48) and (49), we can obtain

$$\dot{r}=3Hrw+\frac{(1+r)Q}{\rho_{\rm de}}. \tag{51}$$

It is clear that

$$r=\frac{1-\Omega_{\rm de}}{\Omega_{\rm de}};\quad \dot{r}=-\frac{\dot{\Omega_{\rm de}}}{\Omega_{\rm de}^2}, \tag{52}$$

then we have

$$w=-\frac{\Omega_{\rm de}{}'}{3\Omega_{\rm de}(1-\Omega_{\rm de})}-\frac{Q}{3H(1-\Omega_{\rm de})\rho_{\rm de}}. \tag{53}$$

It should be mentioned that, if DE decays into pressureless matter (i.e. $Q>0$), it will yield a more negative w.

For the OHDE model, Equation (47) leads to

$$\Omega_{\rm de}{}'=2\Omega_{\rm de}(1-\Omega_{\rm de})\left(\frac{\sqrt{\Omega_{\rm de}}}{C}+\frac{1}{2}-\frac{Q}{2H(1-\Omega_{\rm de})\rho_c}\right). \tag{54}$$

Making use of the equations (54) and (53), one can obtain

$$w=-\frac{1}{3}-\frac{2\sqrt{\Omega_{\rm de}}}{3C}-\frac{Q}{3H\rho_{\rm de}}. \tag{55}$$

[73] considered a interaction form $Q=3b^2H\rho_c$, then

$$w=-\frac{1}{3}-\frac{2\sqrt{\Omega_{\rm de}}}{3C}-\frac{b^2}{\Omega_{\rm de}}. \tag{56}$$

It should be mentioned that, if the following two conditions

$$\frac{2\Omega_{\rm de0}}{3}\left(1-\frac{\sqrt{\Omega_{\rm de0}}}{C}\right)<b^2<\frac{8C^2}{81}, \tag{57}$$

$$\sqrt{\Omega_{\rm de0}}<C<\frac{2\sqrt{\Omega_{\rm de0}}}{3\Omega_{\rm de0}-1}. \tag{58}$$

are satisfied, one can get $w<-1$. In other words, the IHDE model can accommodate a transition from a quintessence DE to a phantom DE. This conclusion holds true for the case of a Universe with spatial curvature [74].

5.3. Alleviation of coincidence problem

Now we discuss the coincidence problem under the frame of HDE. For the OHDE model without DM/DE interaction, equation (51) can be reduced to

$$\frac{{\rm d}\ln r}{{\rm d}x}=3w, \tag{59}$$

where $x\equiv\ln a$. For the case of constant w, we get

$$r=r_0a^{3w}. \tag{60}$$

One can see that $r\sim O(1)$ only when t is around t_0, so the coincidence problem still exists for this case.

The inclusion of the DM/DE interaction can make a big difference. For example, [75] choosing $Q=\Gamma\rho_{\rm de}$, then get

$$\dot{r}=3Hr\left(w+\frac{1+r}{r}\frac{\Gamma}{3H}\right). \tag{61}$$

In addition, by choosing the Hubble scale $1/H$ as the characteristic length scale L, one gets [75]

$$w = -\frac{1+r}{r}\frac{\Gamma}{3H}. \tag{62}$$

Based on equations (61) and (62), one can obtain

$$\dot{r} = 0. \tag{63}$$

This means that the coincidence problem can be solved by appropriately choosing the interaction term Q and the characteristic length scale L.

It should be mentioned that, for the case of the OHDE model, adding the DM/DE interaction alone cannot remove the coincidence problem completely. However, [76] demonstrated that for the IHDE model with an appropriate interacting term, setting $\dot{r} = 0$ will give a positive solution of r that has a stable constant solution, whose value is close to the current measured value. Therefore, the inclusion of DM/DE interaction can ensure r varies with time slowly, thus greatly alleviating the coincidence problem [77–79].

5.4. Generalized second law of thermodynamics

It is believed that there is a deep connection between GR and thermodynamics [80–84]. In the following, we will discuss the generalized second law of thermodynamics under the frame of the IHDE model.

For an IHDE mode with an interaction term $Q = \Gamma\rho_{\rm de}$, one can define the effective EoS

$$w_{\rm de}^{\rm eff} = w + \frac{\Gamma}{3H}, \quad w_m^{\rm eff} = -\frac{1}{r}\frac{\Gamma}{3H}. \tag{64}$$

The continuity equations satisfy

$$\dot{\rho_m} + 3H(1 + w_m^{\rm eff})\rho_m = 0, \tag{65}$$

$$\dot{\rho_{\rm de}} + 3H(1 + w_{\rm de}^{\rm eff})\rho_{\rm de} = 0. \tag{66}$$

Moreover, the entropy of the Universe inside the future event horizon takes the forms

$$\mathrm{d}S_m = \frac{1}{T}(\mathrm{d}E_m + p_m \mathrm{d}V), \tag{67}$$

$$\mathrm{d}S_{\rm de} = \frac{1}{T}(\mathrm{d}E_{\rm de} + p_{\rm de}\mathrm{d}V). \tag{68}$$

Here the temperature $T = \frac{1}{2\pi L}$, the volume $V = \frac{4\pi L^3}{3}$,

$$E_m = \frac{4\pi L^3}{3}\rho_m, \quad p_m = w_m^{\rm eff}\rho_m, \tag{69}$$

$$E_{\rm de} = \frac{4\pi L^3}{3}\rho_{\rm de}, \quad p_{\rm de} = w_{\rm de}^{\rm eff}\rho_{\rm de}. \tag{70}$$

Note that the entropy of horizon is $S_L = \pi L^2$, so

$$\mathrm{d}S_L = 2\pi L \cdot \mathrm{d}L. \tag{71}$$

The validity of generalized second law of thermodynamics has been tested under the frame of the IHDE model. For example, by adopting the parameters $\Omega_{\rm de0} = 0.73$, $\Omega_{k0} = 0.01$, $C = 0.1$ and $b^2 = 0.2$, Setare studied this topic and found that [85]

$$\frac{\mathrm{d}}{\mathrm{d}x}(S_m + S_{\rm de} + S_L) = \frac{M_p^2}{H^2} \times\left(-10.88 + \frac{1482.88 - 167.42q}{H^2}\right) + \frac{1.33}{H^2}, \tag{72}$$

where q is the deceleration parameter. If $q \leqslant 8.85 - H^2/15.4$, then $\frac{\mathrm{d}}{\mathrm{d}x}(S_m + S_{\rm de} + S_L) \geqslant 0$. Therefore, the generalized second law of thermodynamics could be respected if the special range of q is chosen.

6. Four types of holographic dark energy models

All the sections above only focus on the OHDE model. In fact, there are four types of HDE models.

To show the differences among the four types of HDE models, let us consider a universe that has DE and characteristic length scale L. As pointed out by Cohen *et al* [46], the energy density of this universe cannot exceed the energy density of a black hole. Therefore, the IR cutoff (characteristic length scale L) and UV cutoff (vacuum quantum zero point energy Λ) should satisfy

$$L^3\Lambda^3 \leqslant (S_{\rm BH})^{3/4}, \tag{73}$$

where $S_{\rm BH}$ is the black hole entropy. Making use of the Bekenstein formula of black hole entropy $S_{\rm BH} \propto A \propto L^2$, and noting that vacuum energy density $\rho_{\rm de} = \Lambda^4$, one can derive

$$\rho_{\rm de} = 3C^2 M_p^2 L^{-2}. \tag{74}$$

This is the core formula for HDE.

As mentioned above, the simplest choice, i.e. the Hubble scale $L = 1/H$, can not yield cosmic acceleration. In the past 20 years, a lot of HDE models have been proposed. These theoretical models can be divided into four categories: (1) HDE models with the other characteristic length scale; (2) HDE models with extended Hubble scale; (3) HDE models with dark sector interaction; (4) HDE models with modified black hole entropy. In this section, we will introduce these four types of HDE models.

6.1. HDE models with other characteristic length scale

This type of HDE model chooses the other characteristic length scale, which has nothing to do with the Hubble scale, as the IR cutoff. It is clear that the OHDE model belongs to this category. Another well-known HDE model of this category is the agegraphic dark energy (ADE) model.

[86, 87] suggested that one can choose the time of the Universe as the IR cutoff, which is the core idea of ADE model.

The first version of ADE [86] adopted the physical time t as the IR cutoff. But this version of ADE model cannot evolve from a sub-dominate component to a dominate component. Soon after, a realistic model of ADE was proposed [87]. It is suggested that one can adopt the conformal time of the Universe as the IR cutoff. In this new version, the energy density of ADE satisfies

$$\rho_{\rm de} = \frac{3n^2 M_p^2}{\eta^2}\,, \tag{75}$$

where η is the conformal time

$$\eta = \int \frac{{\rm d}t}{a} = \int \frac{{\rm d}a}{a^2 H}\,. \tag{76}$$

The fractional energy density is

$$\Omega_{\rm de} = \frac{n^2}{H^2\eta^2}\,. \tag{77}$$

The evolution equation of $\Omega_{\rm de}$ is

$$\Omega'_{\rm de} = \Omega_{\rm de}(1-\Omega_{\rm de})\left(3 - \frac{2}{n}\frac{\sqrt{\Omega_{\rm de}}}{a}\right). \tag{78}$$

In addition, the EoS of ADE is

$$w = -1 + \frac{2}{3n}\frac{\sqrt{\Omega_{\rm de}}}{a}\,. \tag{79}$$

In a matter-dominated Universe, $\eta \propto \sqrt{a}$. Based on equation (75), one can get $\rho_{\rm de} \propto 1/a$. Based on the continuity equation, one can get $w = -2/3$. Compare this result to equation (79)), one obtains that,

$$\Omega_{\rm de} = \frac{n^2 a^2}{4}\,. \tag{80}$$

It is clear that the fractional energy density of ADE in the matter-dominated era is determined. Therefore, there is no coincidence problem in the ADE model.

For the studies of other HDE models of this category, see [88, 89].

6.2. HDE models with extended Hubble scale

This type of HDE model chooses the combination of the Hubble scale and its time derivatives as the IR cutoff.

A well-known HDE model of this category is the Ricci dark energy (RDE) model [90, 91]. In the FLRW cosmology, the Ricci scalar is

$$R = -6\left(\dot{H} + 2H^2 + \frac{k}{a^2}\right). \tag{81}$$

Adopting the Ricci curvature as the IR cutoff, one can obtain the energy density of RDE

$$\rho_{\rm de} = -\frac{\alpha}{16\pi}R = \frac{3\alpha}{8\pi}\left(\dot{H} + 2H^2 + \frac{k}{a^2}\right). \tag{82}$$

Thus, the first Friedmann equation satisfies

$$H^2 = \frac{8\pi G}{3}\rho_{m0}{\rm e}^{-3x} + (\alpha - 1)k{\rm e}^{-2x} + \alpha\left(\frac{1}{2}\frac{{\rm d}H^2}{{\rm d}x} + 2H^2\right), \tag{83}$$

where $x \equiv \ln a$. This equation can be written as

$$E^2(a) = \Omega_{m0}a^{-3} + \Omega_{k0}a^{-2} + \frac{\alpha}{2-\alpha}\Omega_{m0}a^{-3} + f_0 a^{-(4-\frac{2}{\alpha})}\,, \tag{84}$$

where f_0 is an integration constant, which can be fixed by using the condition $E_0 = 1$:

$$f_0 = 1 - \Omega_{k0} - \frac{2}{2-\alpha}\Omega_{m0}\,. \tag{85}$$

Based on equation (84), one can get

$$\Omega_{\rm de} = \frac{\alpha}{2-\alpha}\Omega_{m0}a^{-3} + f_0 a^{-(4-\frac{2}{\alpha})}\,. \tag{86}$$

In addition, the EoS of RDE satisfies

$$w = -1 + \frac{(1+z)}{3}\frac{{\rm d}\ln\Omega_{\rm de}}{{\rm d}z}\,. \tag{87}$$

If $\alpha = 1/2$, RDE will behave as a cosmological constant plus a DM. If $1/2 \leqslant \alpha < 1$, RDE will behave as a quintessence DE. If $\alpha < 1/2$, RDE will start from a quintessence DE and evolve to a phantom DE.

For the studies of other HDE models of this category, see [92–96].

6.3. HDE models with dark sector interaction

This type of HDE model chooses the Hubble scale as the IR cutoff, while the interaction between dark matter and dark energy is taken into account.

In a non-flat Universe, the first Friedmann equation is

$$3M_p^2 H^2 = \rho_{\rm dm} + \rho_b + \rho_r + \rho_k + \rho_{\rm de}. \tag{88}$$

After taking into account the interaction between dark sectors, the energy density of DM and HDE satisfy

$$\dot{\rho_{\rm dm}} + 3H\rho_{\rm dm} = Q, \tag{89}$$

$$\dot{\rho_{\rm de}} + 3H(1+w)\rho_{\rm de} = -Q, \tag{90}$$

where Q describes the energy flow between dark matter and dark energy.

If there is no energy flow between dark matter and dark energy, i.e. $Q = 0$, choosing the Hubble scale as IR cutoff will give a wrong EoS of HDE, which yields a universe without cosmic acceleration. However, the introduction of dark sector interaction can change the dynamical evolution equation of HDE, as well as the EoS of HDE. Therefore, after adopting an appropriate form of Q, choosing the Hubble scale as the IR cutoff can also yield cosmic acceleration.

For more details about the HDE models with dark sector interaction, see the review article [69] and the references therein.

6.4. HDE Models with modified black hole entropy

This type of HDE model chooses the Hubble scale as the IR cutoff, while the formula of black hole entropy is modified. The most popular HDE model in this category is the Tsallis holographic dark Energy (THDE) model.

In [97], Tavayef *et al* proposed the THDE model. This model is based on a modified entropy area relation, which is suggested by Tsallis and Cirto [98],

$$S_\delta = \gamma A^\delta, \tag{91}$$

where δ is an unknown constant and γ is a non-additivity parameter. Based on the holographic principle, one can derive a relation among the system entropy S, the IR cutoff L and UV cutoff Λ [99]

$$L^3\Lambda^3 \leqslant (S)^{3/4}. \tag{92}$$

Combining equations (91) and (92), one can obtain

$$\Lambda^4 \leqslant (\gamma(4\pi)^\delta)L^{2\delta-4}. \tag{93}$$

Note that Λ^4 denotes the vacuum energy density. Based on this inequality, the energy density of THDE can be written as [97]

$$\rho_{\rm de} = BL^{2\delta-4}, \tag{94}$$

where B is a constant model parameter. Moreover, [97] proved that, for a flat FLRW universe filled by THDE and pressureless matter, choosing the Hubble horizon as the IR cutoff will yield cosmic acceleration.

In recent years, the THDE model has drawn a lot of attention [100–105]. In addition to many theoretical explorations and observational constraints, this model has also been studied in various modified gravity theories, such as Brans Dicke theory [106] and Brane cosmology [107].

It should be mentioned that there are some other theoretical attempts at the entropy-corrected HDE models, such as the Barrow holographic dark energy [108–110], the Renyi holographic dark energy [111, 112] and the Kaniadakis holographic dark energy [113]. For the studies of other HDE models of this category, see [114–117].

7. Hubble tension problem and holographic dark energy

In the recent years, the Hubble tension problem has become one of the biggest challenges of cosmology [118]. In this section, we introduce the Hubble tension problem, as well as the attempts to alleviate this problem under the framework of HDE.

7.1. Hubble tension problem

Since the 21st century, it was widely believed that the simplest cosmological model, i.e. the ΛCDM model, is most favored by various astronomical observations. Therefore, the ΛCDM model was also called the standard model of cosmology. However, in recent years, it is found that under the framework of the ΛCDM model, the high redshift cosmic microwave background (CMB) observations and the low redshift cepheid observations will give very different measurement results of the Hubble constant H_0.

For example, under the framework of the ΛCDM model, the Planck 2018 data, which is the last release from the Planck satellite measurements of the CMB anisotropies, gave $H_0 = 67.4 \pm 0.5\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ [119]. On the other side, under the framework of the ΛCDM model, based on the analysis of cepheids in 42 Type Ia supernova host galaxies, Riess *et al* gave $H_0 = 73.04 \pm 1.04\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ [120]. It is clear that these two measurement results of H_0 have a very big tension. It must be emphasized that the difference between the H_0 measurement results given by these two observations has exceeded the 5σ confidence level (CL). In other words, the Hubble constant tension between the early time and late time measurements of the Universe has exceeded 5σ CL.

Therefore, there is an impossible triangle among the high redshift CMB observations, the low redshift cepheid observations, and the ΛCDM model. In other words, at least one of the three factors is wrong. If not due to the systematic errors of the CMB and the cepheid observations, the Hubble constant tension will reveal an exciting possibility: what we need is new physics beyond the standard model of cosmology.

7.2. Alleviation of Hubble tension problem under the framework of HDE

A lot of theoretical attempts have been made to alleviate the Hubble tension problem [118], such as early dark energy [121, 122], late dark energy [123, 124], modified gravity [125, 126], sterile neutrino [127] and dark sector interaction [128]. In this review, we only focus on one kind of late dark energy, i.e. HDE.

Some literature has discussed the possibility of alleviating the Hubble tension problem under the framework of the OHDE model. For example, [129] found that, after taking into account the OHDE model and sterile neutrino, the combined data of Planck 2015 + BAO + JLA + R16 will give $H_0 = 70.7 \pm 1.1\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$; for this case, the difference with low redshift cepheid observations is reduced by 1.5σ. In addition, [130] found that, based on the OHDE model, the combined data of Planck 2018 + BAO + R19 gives $H_0 = 73.12 \pm 1.14\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$, which has no tension with low redshift cepheid observations.

In addition, the case of Tsallis holographic dark energy has also been studied. [131] found that, for this model, Planck 2018 + BAO + BBN + CC + Pantheon gives $H_0 = 69.8 \pm 1.8\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$, which alleviates the Hubble tension at 1.5σ CL.

These studies show that the HDE model has the potential to alleviate the Hubble tension problem.

8. Summary

As the most important principle of quantum gravity, HP has the great potential to solve the DE problem. In this paper, we reviewed previous theoretical attempts at applying the HP to the DE problem.

Based on the HP and the dimensional analysis, we gave the general formula of HDE energy density, i.e. $\rho_{\rm de} = 3C^2M_p^2L^{-2}$. Then, we introduced the OHDE model, which chooses the future event horizon as the characteristic length scale. Next, we introduced various theoretical motivations that can lead to the general formula of HDE. Moreover, we introduced the research works about the IHDE models, which consider the interaction between dark sectors. Moreover, we introduce all four types of HDE models that originate from HP, including (1) HDE models with other characteristic length scale; (2) HDE models with extended Hubble scale; (3) HDE models with dark sector interaction; (4) HDE models with modified black hole entropy. Finally, we introduce the well-known Hubble tension problem, as well as the attempts to alleviate this problem under the framework of HDE.

From the perspective of theory, the core formula of HDE is obtained by combining the HP and the dimensional analysis, instead of adding a DE term into the Lagrangian. Therefore, HDE remarkably differs from any other theory of DE. From the perspective of observation, HDE can fit various astronomical data well, and have the potential to alleviate the Hubble tension problem. These features make HDE a very competitive dark energy scenario.

Recent theoretical developments show that spacetime itself may be emergent from the entanglement entropy [132, 133]. This discovery will bring new insight to the theoretical explorations of HDE, as well as the theoretical studies of applying the HP to cosmology [134, 135]. In addition, S. Nojiri *et al* proved that the holographic approach can be used to describe the early-time acceleration and the late-time acceleration of our Universe in a unified manner [136]. For more details, see [137, 138].

Acknowledgments

We are grateful to Prof. Yi Wang for helpful discussions. SW is supported by the Guangdong Province Science and Technology Innovation Program under Grant No. 2020A1414040009. ML is supported by the National Natural Science Foundation of China under Grant No. 11 275 247 and No. 11 335 012.

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