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Self-organized Criticality in an Earthquake Model on Random Network

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Abstract *A simplified Olami–Feder–Christensen model on a random network has been studied. We propose a new toppling rule — when there is an unstable site toppling, the energy of the site is redistributed to its nearest neighbors randomly not averagely. The simulation results indicate that the model displays self-organized criticality when the system is conservative, and the avalanche size probability distribution of the system obeys finite size scaling. When the system is nonconservative, the model does not display scaling behavior. Simulation results of our model with different nearest neighbors q is also compared, which indicates that the spatial topology does not alter the critical behavior of the system.*

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1 Introduction

In 1987, Bak, Tang, and Wiensenfield introduced a concept of self-organized criticality (SOC),[1] which was proposed as a possible explanation for the widespread occurrence in nature of long-rang correlations in space and time. The main characteristic of SOC model is that without the fine tuning of parameters, the system can reach a steady state which is characterized by a power-law distribution of the size of avalanches. The study of the SOC systems to a great extent has been based on simulations using cellular automaton models. A number of simple models have been developed to test the applicability of SOC to a variety of complex interacting dynamical systems, such as sand piles and earthquakes.[2−10] The majority of these simulations have been limited to conservative models. At first, it was suggested that the necessary condition for SOC was indeed a conservation law. This seems to be the situation for SOC models where perturbation is done locally as in the original BTW model.[11] But in recent years, it has been shown that nonconservative earthquake model with a global perturbation displays SOC.

Earthquakes may be the most dramatic example of SOC that can be seen on earth. The relevance of SOC to earthquakes was first pointed out by Bak and Tang,[12] Sornette and Sornette,[13] Ito and Matcusaki.[14] Most of the time the crust of the earth is at rest, or quiescent. These periods of stasis are punctuated by sudden, thus far unpredictable, bursts, or earthquake.[9]

In 1992, Olami, Feder, and Christensen proposed a simplified earthquake model (OFC model) on a two-dimensional regular square lattice,[15,16] where the OFC model displays criticality. Later, earthquake model has been applied on some other networks, such as the RN OFC model,[17−20] where each site interacts with randomly chosen sites instead of its nearest neighbors at each update on the square lattice, displays critical behavior only when the system is conservative. Lise and Paczuski have proposed an earthquake model on quenched random graph,[3] where all sites have the same number of nearest neighbors chosen randomly except two sites, which have fewer neighbors, displays criticality even the system is nonconservative. Recently, the earthquake model on small world networks has been investigated,[5,6] and it displays criticality under some conditions. However, the presence of criticality in the nonconservative OFC model has still been controversial.[21−23] It has been verified that the avalanche distribution of the OFC model on the lattice does not display finite size scaling.

The toppling rule of the earthquake model on different networks that has been studied is that when there is a site toppling, the force of the site is redistributed to all its nearest neighboring sites averagely. In this paper, we propose our earthquake model: on a random network with N sites, where every site has the same number of neighbors, q, with a number of defects which have $q-1$ neighbors. We introduce a new toppling rule — the energy of the toppling is redistributed to all its neighbors randomly. The system evolves into a critical state after a transient period. We numerically investigate the probability distribution of the avalanche size of the system in detail. And we measure the exponent characterizing the probability distribution.

2 Model

The random network is defined as a set of Nsites connected by bonds randomly. Two connected sites are denoted as "nearest neighbors". The number of nearest

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neighbors of every site is the same q (self-connections and duplicate edges are excluded).

The dynamical process of our model is as follows.

(i) With each site of the network is associated a real variable F_i. Initially, the force at each site is chosen randomly from a uniform distribution between 0 and $F_{\rm th}$, and $F_{\rm th} = 1$.

(ii) If any $F_i \geq F_{\rm th}$ then redistribute the force on F_i to its neighbors randomly according to the following rule

$$F_j \to F_j + \alpha_j F_i\,, \quad F_i \to 0\,, \tag{1}$$

$$\sum_{j=1}^{q} \alpha_j = \alpha\,, \tag{2}$$

where j denotes the nearest neighbor site of i. The parameter α_j is the fraction that the nearest neighbor site of j gets from the unstable site i. The parameter α controls the level of conservation of the dynamics, where $\alpha = 1$ corresponds to the conservative case, while $\alpha < 1$ implies the model is nonconservative, and the energy decreases during an avalanche.

(iii) Repeat step (ii) until all sites of the network are stable. The sequence of the toppling of the unstable sites forms an avalanche. Define this process as one avalanche and the number of topplings during an earthquake defines its size, s.

(iv) Find out the maximal value of all F_i, $F_{\max}$, and add $F_{\rm th} - F_{\max}$ to all sites and return to step (ii), i.e. all the forces are increased uniformly and simultaneously at the same speed.

(v) Begin step (iv) again and another new avalanche begins.

We find that the system with periodic boundary conditions quickly reach an exactly periodic state in OFC model on the square lattice, the avalanche size distribution function drops very quickly with size,[24,27] i.e. there is no critical behavior if every site has exactly the same number of nearest neighbors. It has been verified that the OFC model on quenched random graph also has no critical behavior if all sites have exactly the same number of nearest neighbor sites q.[3]

In order to observe scaling in the avalanche distribution, it is necessary to introduce some inhomogeneities. In the lattice model this is generally achieved by considering open boundary conditions,[24–26] i.e. the boundary sites have fewer neighbors than other sites. In random graph model it suffices to consider just two sites with $q-1$ neighbors[3] to break the periodicity of the system. A number of inhomogeneities is introduced randomly to ensure that there are a number of sites which have $q-1$ neighbors in our model, here we consider defects whose number scale as $\sqrt{N}$ (analogously to boundary sites in the lattice model). When one of these sites topples according to rule (ii), there will be a fraction $\alpha_j F_i$ lost by the system.

3 Simulations and Results

After a sufficiently long transient time, the system reaches a critical steady state. In the critical state, the properties of the avalanche are studied. The size of an avalanche can be defined in several ways: the number of topplings s, the avalanche time duration t, and the avalanche area a. The duration t of an avalanche is equal to the number of time steps needed for the earthquake to finish; the area a is defined as the number of system sites toppling at least once during an avalanche. We focus on the probability distribution of earthquake sizes s, in a system of size N, $P(s, N)$, when the system is conservative. The statistics are collected in the critical state for 10^6 non-zero avalanches for each system size.

We consider first $\alpha = 1$, the nearest neighbor sites $q = 4$ and the defects $c = \sqrt{N}$, with different system sizes $N = 1024$, 2048, 8192, respectively. In Fig. 1, we find that the distribution of the avalanche sizes has power-law behavior, i.e,

$$p(s, N) \sim s^{-\tau}\,. \tag{3}$$

We can see that with the increasing N, the region of the power-law increases, which is indicative of a critical state. In order to characterize the critical behavior of the model, a finite size scaling (FSS) ansatz is used. One ansatz that can describe critical behavior is the FSS ansatz, which was previously used by OFC model, i.e.,

$$P(s, N) \simeq N^{-\beta} f(s/N^D)\,, \tag{4}$$

where f is a suitable scaling function, and β and D are critical exponents describing the scaling of the distribution function. The critical index D expresses how the finite-size cutoff scales with the system size, while the critical index β is related to the normalization (or rather renormalization) of the distribution function.[13] In Fig. 2, an FSS collapse of $P(s, N)$ for different $N = 1024$, 2048, 8192 with the same value of α, the same q, and defects c is shown. We can see that the probability distribution $P(s, N)$ satisfies the FSS hypothesis reasonably well. The critical exponents derived from the fit of Fig. 2 are $\beta \simeq 1.54$, $D = 1.12$. The FSS hypothesis implies that for asymptotically large N, the value of the exponent is $\tau = \beta/D \simeq 1.38$. The exponent τ we obtain from our model is different from the one for the OFC model in a two-dimensional lattice ($\tau = 1.8$), conservative RN model ($\tau = 1.5$), or the earthquake model on a quenched random graph ($\tau = 1.65$).

Here we also study the influence of the spatial topology of the system. We take the system size $N = 1024$, the defects $c = 32$, in Fig. 3 from left to right; the log-log plot of the distribution is $q = 4$, $q = 6$, respectively. In order to see clearly, the curves for $q = 4$ is shifted in the downward direction. From Fig. 3 we can see that when the system is conservative, different neatest neighbor sites q of the system give parallel lines of distribution. This indicates that the different spatial topology does not alter the critical behavior, specifically the scaling exponents of our model.

We have shown that the critical behavior of our model when the system is conservative, and now we want to know whether the system still displays scaling when the system is nonconservative. Here we take the value of $\alpha = 0.8$ (20% of the force in the toppling site is dissipated), the nearest neighbor sites $q = 4$ and the defects $c = \sqrt{N}$ with different system sizes $N = 1024$, 2048. As shown in Fig. 4, it is clear that the cutoff in the avalanche size distribution does not grow with system size, i.e., no scaling is present when system is nonconservative. This is in contrast to what happens in the quenched random graph, where the earthquake model displays critical behavior even the system is nonconservative.[3]

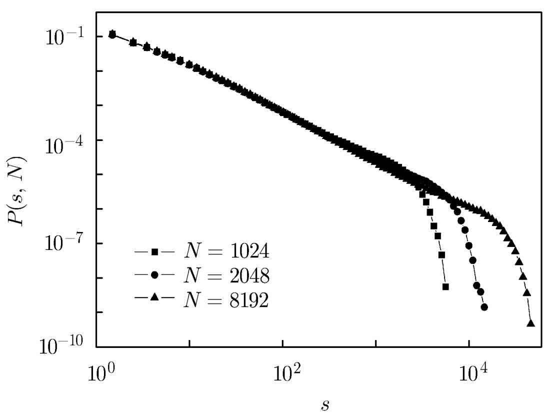

Fig. 1 Log-log plot of the probability distribution $P(s, N)$ for nearest neighbor sites $q = 4$ and defects $c = \sqrt{N}$ with different system sizes when the system is conservative. The data have been binned over exponentially increasing sizes with base 1.1.

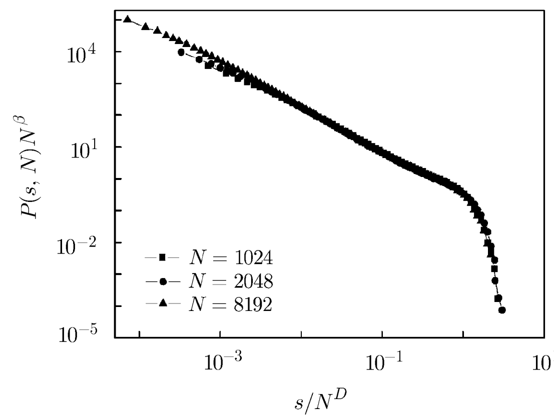

Fig. 2 Data collapse analysis of the case with nearest neighbor sites $q = 4$, the defects $c = \sqrt{N}$ and the system sizes $N = 1024$, 2048, 8192, respectively. The critical exponents are $\beta = 1.54$, $D = 1.12$.

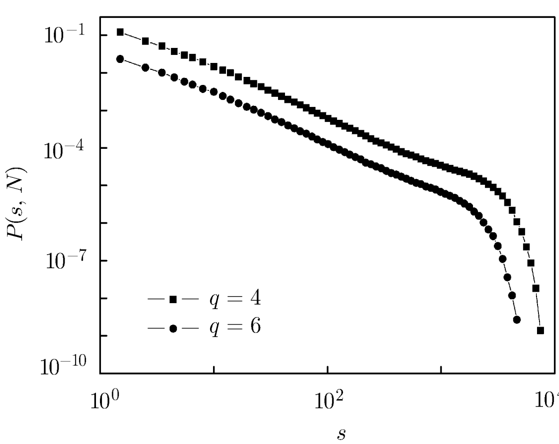

Fig. 3 Log-log plot of the probability distribution $P(s, N)$ for the system size $N = 1024$ and the defects $c = 32$ with different nearest neighbor sites q. For visual clarity, the curves for $q = 4$ is shifted in the downward direction.

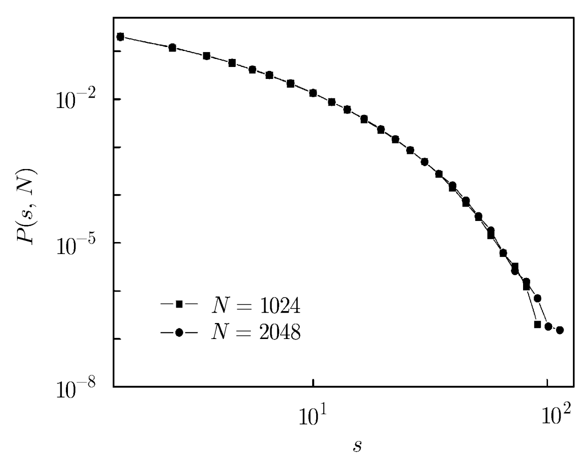

Fig. 4 Log-log plot of the distribution probability $P(s, N)$ for the nearest neighbor sites $q = 4$ and the defects $c = \sqrt{N}$ with different system sizes when 20% of the force in the toppling site is dissipated.

Inhomogeneities are necessary to break the periodicity of the system to ensure that the system can reach critical state. The system with $\sqrt{N}$ defects in our model has been studied here. Different defects of the system may result in different conclusion, and it is difficult to get the maximal number of defects to make sure that the system still displays critical behavior.

4 Conclusion

In this paper, we have presented a new earthquake model based on a random network, on which every site has the same number of nearest neighbors with a number of defects distributed randomly on the random network. The introduction of the inhomogeneities in our model is the necessary condition that the system can reach critical state.

The toppling mechanism of the system is that the force of the unstable site is redistributed to their nearest neighbors randomly. It is shown that when the system is conservative, the probability distribution $P(s,N)$ displays power-law behavior, and $P(s,N)$ satisfies the FSS hypothesis. However, it displays no scaling behavior when the system is nonconservative. This is the same with the RN OFC model. But this is quite different to the model on quenched random graph[3] and the model on square lattice,[12] both of which display criticality even the system is dissipated. It seems that the toppling mechanism of the system has affected the critical behavior of the system. We also compare the critical behavior of our model with different number of nearest neighbors. It is shown that different spatial topology does not alter the critical behavior of the system.

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