

Generalized Two-State Theory for an Atom Laser with Nonlinear Couplings

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(Received October 9, 2001; Revised January 7, 2002)

Abstract We present a generalized two-state theory to investigate the quantum dynamics and statistics of an atom laser with nonlinear couplings. The rotating wave approximate Hamiltonian of the system is proved to be analytically solvable. The fraction of output atoms is then showed to exhibit an interesting collapse and revival phenomenon with respect to the evolution time, a sign of nonlinear couplings. Several nonclassical effects, such as sub-Poissonian distribution, quadrature squeezing effects, second-order cross-correlation and accompanied violation of Cauchy–Schwartz inequality are also revealed for the output matter wave. The initial global phase of the trapped condensate, in weak nonlinear coupling limits, is found to exert an interesting impact on the quantum statistical properties of the propagating atom laser beam.

PACS numbers: 03.75.Fi

Key words: atom laser, nonlinear couplings, nonclassical effects

1 Introduction

Currently there have been many interests in creating an atom laser and exploring its novel properties.^[1–3] In 1997, the MIT-group first realized a pulsed atom laser,^[4] by using an RF pulse to transfer the initially trapped condensate into the untrapped state. Based on this experiment, Sun *et al.* presented an elegant theoretical analysis^[5] for the MIT output coupler of the trapped Bose–Einstein condensate with the rotating wave approximation (RWA), which showed a factorization of dynamical process, i.e., the initially coherent condensate remains in such a state after RF interaction (with an attenuated field amplitude), while the output atomic pulse is also in a coherent state. Later we generalized the work of Sun *et al.* to the non-RWA case and showed that the nonclassical behaviors, such as sub-Poisson distribution and quadrature squeezing effect may exist in the atom laser beam, which provides an optical control on quantum statistics of the output matter wave.^[6] The possibility of realizing the squeezed output from an atomic Bose–Einstein condensate (BEC) was also probed based on the MIT experiment.^[7]

However, these works were all started from a simple linear-coupled model which is only reasonable for very dilute system (see also Ref. [8]), the interesting problems about the impacts of nonlinear atom-atom interactions and the initial state of condensed atoms inside the magnetic trap on the quantum coherence of output atom laser are totally ignored. Besides, the applied quantized input RF field is in practice rather difficult to be produced and manipulated, since one may need some extremely high- Q cavity in the laboratory.^[9] In this paper, based on a generalized two-state theory, we hope to gain some ana-

lytical insights on these problems, especially the possible effects from the elastic binary atomic collisions.^[10,11] The reduced effective Hamiltonian of the system is shown to be exactly solvable, from which several interesting nonclassical effects, as the sight of nonlinear couplings, are then exhibited for the output matter wave. In particular, for the weak nonlinear coupling case, it is also shown that the initial global phase of trapped condensate, which could be considered as a definite one,^[12] has an important influence on the quantum coherence property of the output atom laser beam, which may indicate a possible way to understand and even detect the subtle global phase of the condensate. In the ideal linear coupling case, the previous results in literature^[5] are also exactly restored.

2 Model and Solution

The scheme of an atom laser is most easily investigated by using the two-level model.^[13] For simplicity, we shall assume that the atoms have two states, $|1\rangle$ and $|2\rangle$ with the initial condensation occurring in the trapped state $|1\rangle$. State $|2\rangle$, which has different trapping properties and is typically unconfined by the magnetic trap, is coupled to $|1\rangle$ by an RF light tuned near the $|1\rangle \rightarrow |2\rangle$ transition with the amplitude \mathcal{F} . The possibility of applying only one significantly populated mode for output atoms was also investigated in scheme of Raman transition.^[11] Now we take into account nonlinear atomic interactions, then the general Hamiltonian of the total system can be written as

$$H_T = \int d\mathbf{x} \hat{\psi}_T^\dagger(\mathbf{x}) \left[V_{\text{ex}}(\mathbf{x}) + \frac{\mathbf{p}^2}{2m} \right] \hat{\psi}_T(\mathbf{x}) + \frac{1}{2} \int d\mathbf{x} d\mathbf{x}' \hat{\psi}_T^\dagger(\mathbf{x}) \hat{\psi}_T^\dagger(\mathbf{x}') U(\mathbf{x}-\mathbf{x}') \hat{\psi}_T(\mathbf{x}') \hat{\psi}_T(\mathbf{x}), \quad (1)$$

where $\hat{\psi}_T(\mathbf{x})$ ($\hat{\psi}_T^\dagger(\mathbf{x}')$) annihilates (creates) an atom at position \mathbf{x} , $V_{\text{ex}}(\mathbf{x})$ stands for the external potential and $U(\mathbf{x} - \mathbf{x}')$ is the atomic interactions. By using a short-range interaction $U(\mathbf{x} - \mathbf{x}') = g_0\delta(\mathbf{x} - \mathbf{x}')$ and $\hat{\psi}_T(\mathbf{x}) = a_1\phi_1 + a_2\phi_2$, one can derive from Eq. (1) the second quantized Hamiltonian as

$$H_T = \omega_1 a_1^\dagger a_1 + \omega_2 a_2^\dagger a_2 + (\omega_R a_2^\dagger a_1 + \text{h.c.}) + \frac{1}{2} \sum_{j l k m} \zeta_{j l k m} a_j^\dagger a_l^\dagger a_k a_m, \quad (2)$$

where we use ω_j to denote ω_{jj} ($j = 1, 2$), and

$$\begin{aligned} \omega_{jl} &= \int d\mathbf{x} \phi_j^* \left[V_{\text{ex}}(\mathbf{x}) + \frac{\mathbf{p}^2}{2m} \right] \phi_l, \\ \omega_R &= \omega_{21} + \mathcal{F} \int d\mathbf{x} \phi_2^* e^{i\mathbf{k}\mathbf{x}} \phi_1, \\ \zeta_{j l k m} &= \zeta_0 \int d\mathbf{x} \phi_j^* \phi_l^* \phi_k \phi_m. \end{aligned}$$

However, for the present purpose, it is sufficient to consider the energy conserving terms of collision part in Eq. (2) which are expected to have the most significant influence on the coherence of the output atoms, i.e.,

$$H_{\text{col}} = \frac{1}{2} \zeta_{11} a_1^\dagger a_1^\dagger a_1 a_1 + \frac{1}{2} \zeta_{22} a_2^\dagger a_2^\dagger a_2 a_2 + 2\zeta_{21} a_1^\dagger a_1^\dagger a_2 a_2, \quad (3)$$

where we use ζ_{jl} to denote $\zeta_{j l j l}$ ($j, l = 1, 2$). Besides, one may also assume ω_R to be real by producing a phase-matched coupling. The reduced Hamiltonian could then be exactly solved. Here, to avoid the tedious calculations and grasp the main features of this system, we will only consider the resonant case, namely, $\omega_1 = \omega_2 = \omega$ and $\zeta_{11} = \zeta_{22} = \zeta$, thus the Heisenberg equations about a_1 and a_2 are

$$\frac{\partial}{\partial t} \begin{pmatrix} a_1(t) \\ a_2(t) \end{pmatrix} = \begin{pmatrix} i(\omega + 2\eta_1) & i\omega_R \\ i\omega_R & i(\omega + 2\eta_2) \end{pmatrix} \begin{pmatrix} a_1(t) \\ a_2(t) \end{pmatrix}, \quad (4)$$

where $\eta_j = \zeta(n_j + \varepsilon n_l)$ ($j \neq l$), and $\varepsilon = \epsilon/2\zeta$ with the convenient notation $\zeta_{21} = \zeta_{12} = \epsilon$. Note that the particle number operator $n \equiv n_1 + n_2 = a_1^\dagger a_1 + a_2^\dagger a_2$ is a constant of motion. Now by introducing a new kind of operators^[14]

$$\tilde{a}_{1,2}(t) = \frac{1}{\sqrt{2}} \exp[-i(\omega \pm \omega_R)t] [a_1(t) \pm a_2(t)], \quad (5)$$

which clearly also satisfy the ordinary Boson commutators: $[\tilde{a}_j^\dagger, \tilde{a}_l] = \delta_{jl}$ ($j, l = 1, 2$), one can then make Eq. (4) decoupled as follows:

$$\frac{d\tilde{a}_j(t)}{dt} = i\zeta[(1 + \varepsilon)\tilde{n}_j + 2\tilde{n}_l]\tilde{a}_j(t), \quad (j \neq l), \quad (6)$$

where

$$\tilde{n}_j = \tilde{a}_j^\dagger(t)\tilde{a}_j(t) = \tilde{a}_j^\dagger(0)\tilde{a}_j(0).$$

Here we have neglected the oscillating terms in the nonlinear part (RWA). This decoupled equation can be easily

solved

$$\tilde{a}_j(t) = \exp\{i\zeta t[(1 + \varepsilon)\tilde{n}_j(0) + 2\tilde{n}_l(0)]\} \tilde{a}_j(0), \quad (j \neq l), \quad (7)$$

from which the final solutions could be easily obtained

$$\begin{aligned} a_{1,2}(t) &= \frac{1}{\sqrt{2}} \exp(i\zeta t n) \{ \exp[i(\tilde{\eta}_2 + \omega_R)t] a_{1,2}(0) \\ &\quad \pm \exp[i(\tilde{\eta}_1 - \omega_R)t] a_{2,1}(0) \}, \end{aligned} \quad (8)$$

where

$$\tilde{\eta}_j = \zeta(\tilde{n}_j + \varepsilon\tilde{n}_l), \quad j \neq l, \quad n \equiv n_1 + n_2 = \tilde{n}_1 + \tilde{n}_2.$$

Of course, one could substitute Eq. (5) into Eq. (8) and get the solutions purely in terms of the original operators $a(0)$ and $b(0)$, which is in fact unnecessary for our present purpose. What we should keep in mind is that, if the system is initially in a product of coherent state, i.e., $|\alpha_1, \alpha_2\rangle = |\alpha_1\rangle|\alpha_2\rangle$, it also can be written as

$$|\beta_1\rangle|\beta_2\rangle = \left| \frac{1}{\sqrt{2}}(\alpha_1 + \alpha_2) \right\rangle \left| \frac{1}{\sqrt{2}}(\alpha_1 - \alpha_2) \right\rangle$$

with respect to the new operators $\tilde{a}_{1,2}$. Obviously, if the nonlinear interactions could be ignored ($\zeta, \epsilon \rightarrow 0$), we can exactly restore the previous results of ideal linear model as in Ref. [5].

3 Nonclassical Properties of the Output Atomic Beam

The initial state of the system is theoretically described as $|\varphi(0)\rangle = |0\rangle_2 \otimes |\alpha\rangle_1$, in which $|\alpha\rangle_1$ is a Glauber coherent state of the operator a_1 characterizing the condensed atoms in the trapped state $|1\rangle$, i.e. $a_1|\varphi(0)\rangle = |\alpha|e^{i\theta}|\varphi(0)\rangle$ with the initial global phase θ of the condensate, the state $|0\rangle_2$ represents that the initial untrapped state $|2\rangle$ is a vacuum state since there is no occupying atoms in it. Hence, by using the general solutions obtained above, we can get the fraction of atoms coupled out of the trap after the interaction of the RF pulse of duration t , i.e.,

$$\begin{aligned} \mathcal{N}_2(t) &= \frac{\langle \varphi(0) | a_2^\dagger(t) a_2(t) | \varphi(0) \rangle}{|\alpha|^2} \\ &= \frac{1}{2} \left\{ 1 - \exp\left(\frac{1}{2}|\alpha|^2 \mu_1\right) \cos(2\omega_R t) \right\}, \end{aligned} \quad (9)$$

where

$$\mu_1 = \mu_1(\zeta, \varepsilon, t) = \lambda(i\zeta(1 - \varepsilon)t) + \lambda(-i\zeta(1 - \varepsilon)t),$$

and we have used the well-known normal ordering formula

$$\langle \exp(\tau b^\dagger b) \rangle = : \langle \exp[\lambda(\tau) b^\dagger b] \rangle : , \quad \lambda(\tau) = e^\tau - 1. \quad (10)$$

Obviously, $\lambda(\tau) \sim \tau$ for $\tau \rightarrow 0$. The mean number N_1 of the atoms remaining in the trap can also easily be given as $N_1 = |\alpha|^2(1 - \mathcal{N}_2)$ by noticing that the total particle number is conserved. In comparison of the ideal linear case,^[5] the effects of nonlinear atomic collisions on the fraction of output atoms are only embodied in the appearance of

a μ_1 -dependent exponential factor, the form of which, instead of the simple ideal oscillating behaviors, will lead to an interesting collapse and revival phenomenon with respect to the evolution time, as is well-known in quantum optics (see, e.g., Ref. [13]). The collapse time will depend on the coupling constants and, in particular, the complex dependence on the amplitude $|\alpha|$ for the fraction of output

atoms, besides the probed case of fully linear coupling,^[6] shows another benefit to use an attenuated RF pulse to produce a coherent atom laser in the MIT experiments.^[4]

It is well-known that, in order to decide the statistics properties of a quantum field, we can define the Mandel's Q parameter as^[13]

$$Q_j(t) = \frac{\langle \Delta N_j^2(t) \rangle}{\langle N_j(t) \rangle} - 1 \begin{cases} < 0, & \text{sub-Poisson distribution,} \\ = 0, & \text{Poisson distribution,} \\ > 0, & \text{super-Poisson distribution,} \end{cases} \quad (11)$$

or, for our interested case $\mathcal{N}_2 \neq 0$, we can employ an effective one $\tilde{Q}_2(t) = \langle a_2^{\dagger 2} a_2^2 \rangle - \langle a_2^{\dagger} a_2 \rangle^2$. To grasp the main features of the results, we first consider $\varepsilon = 0$ case and it will be seen that the general results can be obtained through a simple replacement of coupling parameters. Taking into account

$$\begin{aligned} \langle \varphi(0) | a_{1,2}^{\dagger 2}(t) a_{1,2}^2(t) | \varphi(0) \rangle &= \frac{|\alpha|^4}{4} \left\{ 1 \pm 2 \cos(2\omega_R t) [\cos(\zeta t) - 1] \exp\left(\frac{1}{2}|\alpha|^2 \mu_1\right) \right. \\ &\quad \left. + \frac{1}{2} \cos(4\omega_R t) \left[1 + \exp\left(\frac{1}{2}|\alpha|^2 \mu_2\right) \pm 2 \exp\left(\frac{1}{2}|\alpha|^2 \mu_1\right) \right] \pm \exp\left(\frac{1}{2}|\alpha|^2 \mu_1\right) \right\} \end{aligned} \quad (12)$$

with the notation

$$\mu_1 = \mu_1(\zeta, t) = \lambda(i\zeta t) + \lambda(-i\zeta t), \quad \mu_2 = \mu_2(\zeta, t) = \lambda(2i\zeta t) + \lambda(-2i\zeta t),$$

we can then easily obtain

$$\tilde{Q}_b(t) = \frac{|\alpha|^4}{8} \left\{ 1 - \exp(|\alpha|^2 \mu_1) - 2 \cos(2\omega_R t) \mu_1 \exp\left(\frac{1}{2}|\alpha|^2 \mu_1\right) + \cos(4\omega_R t) \left[\exp\left(\frac{1}{2}|\alpha|^2 \mu_2\right) - \exp(|\alpha|^2 \mu_1) \right] \right\}. \quad (13)$$

From Eq. (13), it is easily checked that the output atomic field can exhibit the sub-Poissonian distribution. Note that the unimportant factor $|\alpha|^4/8$ can be dropped off for the evaluation of the sign of $\tilde{Q}_b(t)$.

Now we consider the possible correlations between the two atomic modes, which are characterized by the second-order cross-correlation function

$$\mathcal{Q}_{12}(t) = g_{12}^{(2)}(t) - 1 \begin{cases} > 0, & \text{correlated states,} \\ = 0, & \text{uncorrelated states,} \\ < 0, & \text{anti-correlated states} \end{cases} \quad (14)$$

with

$$g_{12}^{(2)}(t) = \frac{\langle a_1^{\dagger}(t) a_1(t) a_2^{\dagger}(t) a_2(t) \rangle}{\langle a_1^{\dagger}(t) a_1(t) \rangle \langle a_2^{\dagger}(t) a_2(t) \rangle},$$

or, for our interested case $0 < \mathcal{N}_2 < 1$, we can employ an effective one $\tilde{Q}_{12}(t) = \langle a_1^{\dagger} a_1 a_2^{\dagger} a_2 \rangle - \langle a_1^{\dagger} a_1 \rangle \langle a_2^{\dagger} a_2 \rangle$. In particular, there is the Cauchy-Schwartz inequality for a system consisting of two bosonic modes, i.e.,

$$[g_{12}^{(2)}(t)]^2 \leq g_1^{(2)}(t) g_2^{(2)}(t), \quad (15)$$

where

$$g_j^{(2)}(t) = \frac{\langle a_j^{\dagger 2}(t) a_j^2(t) \rangle}{\langle a_j^{\dagger}(t) a_j(t) \rangle}, \quad (j = 1, 2).$$

According to Reid and Walls,^[13] the violations of this inequality can be accompanied by violations of Bell's inequality (nonlocality), and the correlations between the two modes are called nonclassical correlations. This could be characterized by the effective quantity

$$\tilde{I}_0(t) = \langle a_1^{\dagger 2}(t) a_1^2(t) \rangle \langle a_2^{\dagger 2}(t) a_2^2(t) \rangle - \langle a_1^{\dagger}(t) a_1(t) a_2^{\dagger}(t) a_2(t) \rangle^2,$$

which is negative if the inequality is violated.

Taking into account Eqs. (9), (12) and

$$\langle a_1^{\dagger}(t) a_1(t) a_2^{\dagger}(t) a_2(t) \rangle = \frac{1}{8} |\alpha|^4 \cos(4\omega_R t) \left[1 - \exp\left(\frac{1}{2}|\alpha|^2 \mu_2\right) \right], \quad (16)$$

we can easily obtain the following final results

$$\tilde{\mathcal{Q}}_{12}(t) = \exp(|\alpha|^2 \mu_1) - 1 - \cos(4\omega_R t) \left[\exp\left(\frac{1}{2}|\alpha|^2 \mu_2\right) - \exp(|\alpha|^2 \mu_1) \right], \quad (17)$$

where we have dropped off an unimportant global factor $|\alpha|^4/8$, and in the same way, the reduced rule function is (moving a global factor $|\alpha|^8/16$)

$$\begin{aligned} \tilde{I}_0(t) = & \cos(4\omega_R t) \left[1 + \exp\left(\frac{1}{2}|\alpha|^2 \mu_2\right) - 2 \exp(|\alpha|^2 \mu_1) \right] - 4 \cos(2\omega_R t) [\cos(\zeta t) - 1] \exp(|\alpha|^2 \mu_1) \\ & - 4 \cos^2(2\omega_R t) [\cos(\zeta t) - 1]^2 + \cos^2(4\omega_R t) \left[\exp\left(\frac{1}{2}|\alpha|^2 \mu_2\right) - \exp(|\alpha|^2 \mu_1) \right] + 1 \\ & - 2 \cos(4\omega_R t) \cos(2\omega_R t) [\cos(\zeta t) - 1] \exp\left(\frac{1}{2}|\alpha|^2 \mu_1\right) \left[1 + \exp\left(\frac{1}{2}|\alpha|^2 \mu_2\right) \right]. \end{aligned} \quad (18)$$

Obviously, for $\zeta \rightarrow 0$, we have $\tilde{I}_0(t) \rightarrow 0$, i.e., there is no violation of the C-S-I and thus no nonclassical correlations occurring between the two atomic modes, as it should be. In general, one can resort to some numerical method to evaluate the sign of Eq. (18), however, just for the simple $\omega_R t = (n + 1/4)\pi$, ($n = 1, 3, 5$), it is clearly seen that there exist nonclassical correlations between the trapped condensate and the out-coupled atomic field, accompanying a violation of the C-S-I. It is easily checked that, for the general case $\varepsilon \neq 0$, the above results should be still valid simply by making a parameter replacement of $\zeta \rightarrow \zeta(1 - \varepsilon)$.

At last we proceed to analyze the interesting quadrature squeezed effects in the output atom laser beam. According to Ref. [13], the field quadratures X_j and Y_j are defined as

$$X_i = \frac{1}{2}(a + a^\dagger), \quad X_i = \frac{1}{2i}(a - a^\dagger), \quad (i = 1, 2), \quad (19)$$

where we have replaced a_2 as a . Following Bruzek *et al.*,^[15] we introduce the squeezed coefficients

$$S_i = \frac{\langle (\Delta X_i)^2 \rangle - \frac{1}{2} |\langle [X_1, X_2] \rangle|}{\frac{1}{2} |\langle [X_1, X_2] \rangle|}, \quad i = 1, 2, \quad (20)$$

one then gets

$$S_{1a}(t) = 2\langle N_a(t) \rangle + 2 \operatorname{Re} \langle a^2(t) \rangle - (\operatorname{Re} \langle a(t) \rangle)^2, \quad S_{2a}(t) = 2\langle N_a(t) \rangle - 2 \operatorname{Re} \langle a^2(t) \rangle - (\operatorname{Im} \langle a(t) \rangle)^2. \quad (21)$$

Using the general solutions obtained above, we can compute the needed quantities, and the final results are

$$\langle a(t) \rangle = i\alpha \exp\left[\frac{\lambda(i\zeta t) + \lambda(2i\zeta t)}{2} |\alpha|^2\right] \sin(\omega_R t), \quad (22)$$

and

$$\langle a^2(t) \rangle = \frac{1}{2} |\alpha|^2 \left\{ \exp\left[i\zeta t + \frac{\lambda(i\zeta t) + \lambda(4i\zeta t)}{2} |\alpha|^2\right] \cos(2\omega_R t) - \exp[2i\zeta t + \lambda(3i\zeta t) |\alpha|^2] \right\}. \quad (23)$$

Based on these results, one can compute the general results of the squeezed parameters and then resort to the same graphical methods used above to analyze their behaviors with respect to the evolution time. However, to get the main idea about the influence of atomic collisions on the coherence of the output matter wave, here we would like to just analytically consider the interested weak nonlinear coupling, which in the first order approximation of g/ω may simplify the above results as the following form

$$\langle a^\dagger(t)a(t) \rangle = |\alpha|^2 \sin^2(\omega_R t), \quad \langle a^2(t) \rangle = -\alpha^2 \left\{ \sin^2(\omega_R t) + i\zeta t \left[\sin^2(\omega_R t) \left(\frac{1}{2} |\alpha|^2 + 1 \right) - \frac{1}{2} \right] \right\}, \quad (24)$$

and

$$\operatorname{Re} \langle a(t) \rangle = |\alpha| \left(-\frac{3}{2} \zeta t |\alpha|^2 \cos \theta - \sin \theta \right) \sin(\omega_R t), \quad \operatorname{Im} \langle a(t) \rangle = |\alpha| \left(\cos \theta - \frac{3}{2} \zeta t |\alpha|^2 \sin \theta \right) \sin(\omega_R t). \quad (25)$$

Hence the squeezed coefficients of the atomic field can be obtained (in a re-scaled form)

$$s_{ia}(t) = s_i^{(0)}(t) + \zeta t s_i^{(1)}, \quad (i = 1, 2), \quad (26)$$

where $s_i(t) = S_i(t)/|\alpha|^2$ with

$$s_1^{(0)}(t) = 3 \sin^2(\omega_R t) \sin^2(\theta) > 0, \quad s_2^{(0)}(t) = 3 \sin^2(\omega_R t) \cos^2(\theta) > 0, \quad (27)$$

and

$$s_{1,2}^{(1)}(t) = \pm \sin(2\theta) \left[\sin^2(\omega_R t) \left(2 - \frac{3}{2} |\alpha|^2 \right) + 1 \right]. \quad (28)$$

Obviously, when the nonlinear coupling $\zeta \rightarrow 0$, there is no squeezing occurring in the output matter wave, which is true for a very dilute condensate with an attenuated RF amplitude.^[4,7] The above results show that, besides the amplitude of the RF pulse, the initial global phase of the trapped condensate can also affect the squeezing properties of the output atom laser beam. If we have chosen the conditions of evolution time as $\sin(\omega_R t) = 0$, or $\omega_R t = m\pi$ ($m = 0, 1, 2, \dots$), we get

$$\begin{pmatrix} s_{1a}(t) \\ s_{2a}(t) \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \zeta t \sin(2\theta), \quad (29)$$

which means that the X_{2a} component is squeezed for $n\pi < \theta < (n+1/2)\pi$, but the squeezing will happen for the X_{1a} component in the case of $(n-1/2)\pi < \theta < n\pi$. It is clear that the squeezing effect occurs purely due to the intrinsic nonlinearity in this system. The phenomenon that the global phase of the initial trapped condensate has an important impact on the quantum statistics of the out-coupled matter wave, is quite similar to the fully linear coupled case (without RWA) in the presence of a strong input RF field.^[6] In particular, if the initial global condensate phase can be adjusted as

$$\sin(2\theta) = 0, \quad \text{or} \quad \theta = \frac{n}{2}\pi, \quad (n = 0, 1, 2, \dots), \quad (30)$$

there will be never squeezing effect for the output atomic beam in any instance, which may suggest a possible way to diminish the influence of nonclassical effect and therefore realize a steady output of coherent atom laser beam. It should be noted that this condition is completely identical to that one in Ref. [6] which, however, aims to suppress the nonclassical disturbing originated from the non-rotating-wave part in a fully linear coupling model.

4 Conclusion and Outlook

In this paper, we presented a generalized two-state theory to investigate the quantum dynamics and statistics of an atom laser with nonlinear couplings, which generalized our previous linear model to incorporate the binary atom-atom collisions in the two modes of atomic condensate

and the output atomic beam, along with the nonlinear interactions between the two modes. The RWA approximate Hamiltonian of the system was proved to be analytically solved. The fraction of output atoms was shown to exhibit an interesting collapse and revival phenomenon with respect to the evolution time, due to the existence of nonlinear couplings. Several nonclassical effects, such as sub-Poissonian distribution, quadrature squeezing effects, second-order cross-correlation, and accompanied violation of Cauchy-Schwarz inequality were also revealed for the output matter wave. In the special case of linear coupling, the previous results in the literature^[5] could be exactly restored, as it should be.

Having obtained the general solutions of our two-mode system, one can also analyze other interesting quantum phenomena, like the macroscopic entanglement of atoms in two modes (i.e., the collective spin-squeezing^[16–18]), which may indicate possible applications for the atom laser to accomplish some forms of quantum information processing.^[19,20] Of course, our analysis here is just the beginning point to the difficult work of studying the role of nonlinear atom-atom interactions in an atom laser system, since the input RF field strength in our model is weak and then the possible effects from the strong optical coupling^[21] are omitted, which may comprise the topics of further works in future.

Note Added

After this work finished, a new experimental article on the squeezed state in BEC appeared,^[22] the physical mechanism of which in fact can be understood by our method although what they manipulated is a BEC in an optical-trap array.

Acknowledgments

Hui JING thanks Prof. Guang-Can GUO for providing a short-term visiting in Lab of Quantum Communication and Computation (USTC) when part of this work was finished there. Kind help from Dr. Yu-Chun WU and Dr. Zheng-Wei ZHOU is also acknowledged.

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