 Communications in Theoretical Physics

Commun. Theor. Phys. 72 (2020) 015402 (6pp) https://doi.org/10.1088/1572-9494/ab544e

Tsallis agegraphic dark energy model with the sign-changeable interaction

Y D Xu

School of Science, Jiangsu Ocean University, Lianyungang 222005, China

E-mail: 2006000047@jou.edu.cn

Received 5 August 2019, revised 26 September 2019
Accepted for publication 31 October 2019
Published 30 December 2019

Abstract

In this paper, we study the evolution of a spatially flat universe composed of Tsallis agegraphic dark energy (TADE) and a pressureless dark matter (DM), by assuming that there is a sign-changeable interaction between TADE and DM. The density, deceleration parameter and the equation of state parameters (EoS) show satisfactory behaviors in the model. By analysis we find that the accelerated expansion of the universe can be achieved at the late time if model parameters $\delta > 2$ and $-2/3 < \beta < 0$. Also, we investigate the interacting TADE model by means of statefinder diagnostic and w–w' analysis.

Keywords: Tsallis agegraphic dark energy, cosmological evolution, statefinder, w–w' analysis

1. Introduction

Various cosmological observations [1–4] indicate that our universe is undergoing a period of cosmic acceleration. This cosmic acceleration can be explained by dark energy (DE) with negative pressure. DE contributes about 73% of the energy density of the current universe. The simplest candidate for DE is cosmological constant with EoS $w = -1$. Although the model can fit the observational data well, it faces two difficulties, namely, cosmic coincidence and fine-tuning problems. So cosmologists propose other DE candidates.

Agegraphic dark energy (ADE) is an alternative, which has been proposed by Cai [5]. The ADE model is based on the line of quantum fluctuations of spacetime, the so-called Károlyházy relation $\delta t = \lambda t_p^{2/3} t^{1/3}$ [6] and the well-known time-energy uncertainty relation $E_{\delta t^3} \sim t^{-1}$. The energy density of the ADE reads [5]

$$\rho_d = 3n^2 m_p^2 T^{-2}, \tag{1}$$

where $m_p = (8\pi G)^{-1/2}$, n is a constant and T is chosen to be the age of the universe

$$T = \int_0^a \frac{\mathrm{d}a}{Ha}. \tag{2}$$

Here $H \equiv \dot{a}/a$ is the Hubble parameter. The ADE models have been widely studied in the literatures [7–14]. Recently, it is shown by Tsallis and Cirto that the horizon entropy of a black hole may be modified as $S_\delta = \gamma A^\delta$ [15], where γ is an unknown constant and δ denotes the non-additivity parameter. Based on the the Tsallis generalized entropy and the holographic hypothesis, a new DE model, dubbed the Tsallis holographic dark energy (THDE), has been proposed by Tavayef *et al* [16]. The energy density of the THDE is written as

$$\rho_d = BL^{2\delta-4}, \tag{3}$$

where B is an unknown parameter and L is the IR cutoff. Later, using the age of the Universe as IR cutoff [17], proposed TADE model. The energy density of the TADE can be written as

$$\rho_d = 3n^2 m_p^2 T^{2\delta-4} \tag{4}$$

if $\delta = 1$, then it reduces to the ADE model [5].

On the other hand, interactions between DE and DM have been intensively investigated [18–26]. The most familiar interaction is $Q = 3cH\rho$, where c is a coupling constant denoting the transfer strength, and ρ is taken to be the density of DE, DM, or the sum of them. Obviously, these interactions are always positive or negative and hence can not change their signs. Recently, the observational data suggests that the sign of interaction could change its sign in the approximate redshift range of $0.45 \leqslant z \leqslant 0.9$ [27]. This redshift range is coincident with the one of our universe changing from deceleration to acceleration [28, 29]. In view of the above, Wei [30] proposed a sign-changeable interaction $Q = q(\alpha\dot{\rho} + 3\beta H\rho)$, where α, β are both dimensionless constants, and q is the deceleration parameter. One can see that the interaction Q can change its sign when the Universe changes from deceleration ($q > 0$) to acceleration ($q < 0$) and brings different evolution to cosmology. The authors of [31] find a sign change in the cosmological interaction which is described by a running coupling scenario. In [31], they propose a parametrization of the cosmological interaction that changes sign as the scale factor evolves. This change in sign is explored further in [32].

The TADE with usual interaction has been introduced and investigated in [17]. In the present paper, we will investigate the cosmological evolution of the model with the sign-changeable interaction ($Q = 3\beta qH\rho_d$) between TADE and DM. The density parameter of TADE (Ω_d), the deceleration parameter (q), the total EoS (w_{tot}) and the EoS of TADE (w_d) are studied. On the other hand, the differences between the LCDM model and other DE models are attractive, because the LCDM model mostly fits todays observations. Statefinder parameter introduced by Sahni *et al* [33] is proven to be a useful tool to differentiate between various DE models. In addition to statefinder, in view of the characteristic of the EoS for the DE models, w–w' analysis [34] can also be used to distinguish various models. In the present paper, we will also investigate the interacting TADE model by means of the statefinder diagnostic and w–w' analysis.

The paper is organized as follow: in section 2, we introduce the TADE model with the sign-changeable interaction and study its cosmological evolution. In section 3, we apply statefinder diagnostic and w–w' analysis to the interacting TADE model. The conclusions are given in section 4.

2. The TADE model with the sign-changeable interaction

In our scenario, we consider a spatially flat universe in which there are the TADE (ρ_d) and DM (ρ_m). The corresponding Friedmann equation reads

$$H^2 = \frac{1}{3m_p^2}(\rho_m + \rho_d). \tag{5}$$

Introducing the fractional energy densities such as

$$\Omega_m = \frac{\rho_m}{3m_p^2H^2}, \quad \Omega_d = \frac{\rho_d}{3m_p^2H^2} \tag{6}$$

we rewrite the Friedmann equation as

$$\Omega_m + \Omega_d = 1. \tag{7}$$

Using equations (4) and (6)

$$\Omega_d = \frac{n^2T^{2\delta-4}}{H^2}. \tag{8}$$

We assume that DM (ρ_m) and the TADE (ρ_d) exchange energy through an interaction term Q, namely

$$\dot{\rho}_m + 3H\rho_m = Q, \tag{9}$$

$$\dot{\rho}_d + 3H(\rho_d + p_d) = -Q, \tag{10}$$

where Q is assumed as follows

$$Q = 3\beta qH\rho_d \tag{11}$$

with β being a coupling constant [30, 35]. The parameter β is assumed to be negative, since positive β will result in negative ρ_m in the flat universe. Obviously, as the expansion of the Universe changes from from deceleration ($q > 0$) to acceleration ($q < 0$), Q can change its sign from $Q < 0$ to $Q > 0$. For $Q < 0$ ($Q > 0$), there is an energy flow from DM (TADE) to TADE (DM). p_d denotes the pressure of TADE.

The equation of motion for Ω_d can be obtained by differentiating equation (8). The result is

$$\Omega_d' = -2\Omega_d\left(\frac{\dot{H}}{H^2} + \frac{2-\delta}{HT}\right), \tag{12}$$

where

$$T = \left(\frac{\Omega_d H^2}{n^2}\right)^{\frac{1}{2\delta-4}} \tag{13}$$

from equation (8). The dot and the prime denote the derivative with respect to the cosmic time and the derivative with respect to ln a, respectively.

Taking the derivative of equation (5) with respect to the cosmic time and using equations (4), (8)and (9) one can obtain

$$\frac{\dot{H}}{H^2} = -\frac{3}{2}(1-\Omega_d) + \frac{Q}{6m_p^2H^3} + \frac{(\delta-2)\Omega_d}{HT}. \tag{14}$$

From equations (11) and (6) we obtain

$$\frac{Q}{6m_p^2H^3} = \frac{3}{2}\beta q\Omega_d. \tag{15}$$

Here the deceleration parameter is given by

$$q \equiv -\frac{\ddot{a}a}{\dot{a}^2} = -1 - \frac{\dot{H}}{H^2}. \tag{16}$$

Therefore, $\frac{\dot{H}}{H^2}$ can be calculated as

$$\frac{\dot{H}}{H^2} = -\frac{\frac{3}{2}(1-\Omega_d+\beta\Omega_d) + \frac{(2-\delta)\Omega_d}{HT}}{1+\frac{3}{2}\beta\Omega_d}. \tag{17}$$

Substituting equation (17) into equation (12), we rewrite Ω_d' as

$$\Omega_d' = -2\Omega_d\left(-\frac{\frac{3}{2}(1-\Omega_d+\beta\Omega_d) + \frac{(2-\delta)\Omega_d}{HT}}{1+\frac{3}{2}\beta\Omega_d} + \frac{2-\delta}{HT}\right). \tag{18}$$

Using relation $H' = \dot{H}/H$, we obtain

$$H' = -\frac{\frac{3}{2}(1-\Omega_d+\beta\Omega_d)H + (2-\delta)\Omega_d/T}{1+\frac{3}{2}\beta\Omega_d} \tag{19}$$

Substituting equation (17) into equation (16), we get

$$q = \frac{\frac{1}{2} - \frac{3}{2}\Omega_d + \frac{(2-\delta)\Omega_d}{HT}}{1+\frac{3}{2}\beta\Omega_d} \tag{20}$$

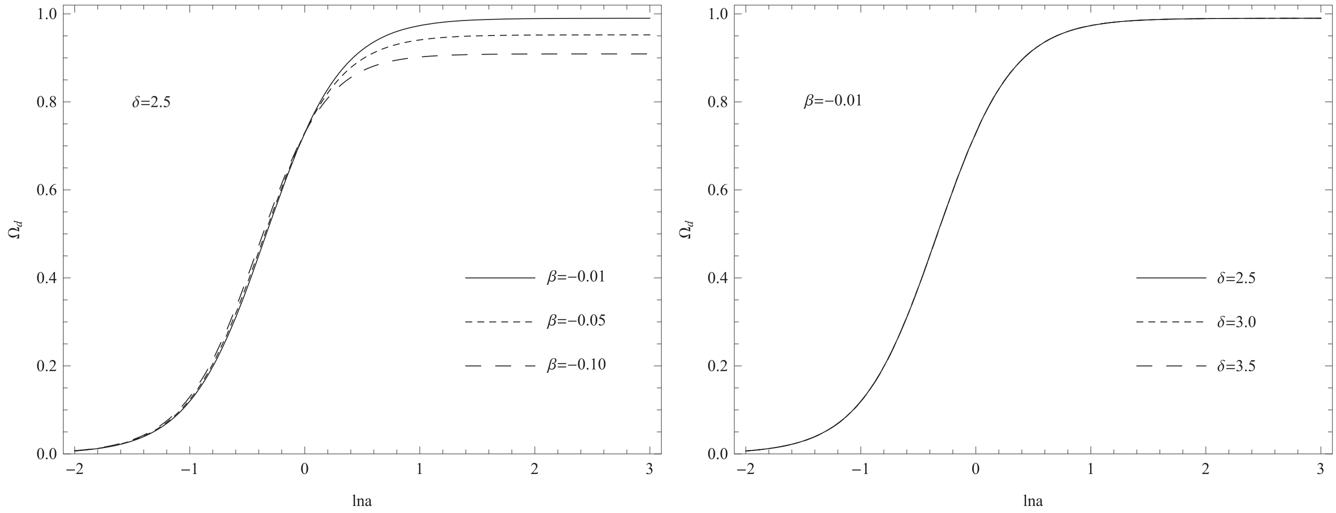

Figure 1. Evolution of Ω_d for different model parameters β and δ. We take $n = 3$.

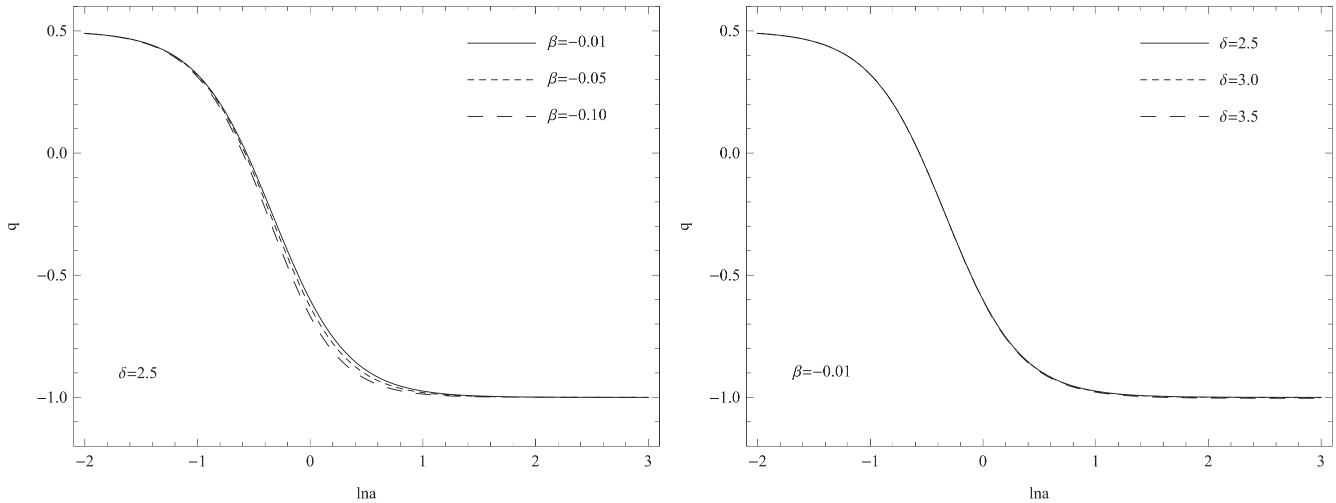

Figure 2. Evolution of the deceleration parameter q for different model parameters β and δ. We take $n = 3$.

The total EoS is given by

$$w_{\mathrm{tot}} = -1 - \frac{2}{3}\frac{\dot{H}}{H^2} = -\frac{1}{3} + \frac{2}{3}q. \tag{21}$$

To accelerate the expansion of our universe, $w_{tot} < -1/3$ is necessary. Obviously, if $\delta > 2$, $1 + \frac{3}{2}\beta\Omega_d > 0$ and $\Omega_d > 1/3$ ($\Omega_d \simeq 0.73$ today), then $w_{\mathrm{tot}} < -1/3$ follows. $1 + \frac{3}{2}\beta\Omega_d$ is always positive provided $-2/3 < \beta < 0$ since $0 \leqslant \Omega_d \leqslant 1$. Therefore, the late time accelerated expansion of the Universe can be achieved if $\delta > 2$ and $-2/3 < \beta < 0$. From equations (4), (8), (10) and (11), we have the EoS of the TADE

$$w_d \equiv \frac{p_d}{\rho_d} = -1 - \beta q - \frac{2\delta - 4}{3HT}. \tag{22}$$

To get illustrations for the behaviors of Ω_d, q, w_{tot} and w_d, we show some numerical plots by using equations (18)–(22) We take $n = 3$, and let β and δ vary. In the numerical integration of equations (18) and (19) the initial condition is taken to be $\Omega_{d0} = 0.73$ and $H_0 = 67$ km s^{-1} Mpc^{-1} to compare with the observation data (the subscript '0' denotes the present value). The results are contained in figures 1–4. In figure 1, we show the evolution of Ω_d for different model parameters β and δ in the case of $Q = 3\beta qH\rho_d$. It is easy to see that for the fixed δ, Ω_d tends to a lower value at the late time when $|\beta|$ is larger. Furthermore, the flat tails of the curves perhaps indicates that there exist the scaling solutions at the late time, which can alleviate the coincidence problem. The evolution of q for different model parameters β and δ in the case of $Q = 3\beta qH\rho_d$ is shown in figure 2, from which we can see that $q \to 1/2$ at the early time and $q \to -1$ at the late time. This implies that our universe changes from deceleration ($q > 0$) to acceleration ($q < 0$) as the Universe expands. For the fixed δ, the Universe starts accelerated expansion earlier when when $|\beta|$ is larger. The evolution of w_{tot} in the case of $Q = 3\beta qH\rho_d$ is shown in figure 3. From figure 3 we see that $w_{\mathrm{tot}} \to 0$ at the early time and $w_{\mathrm{tot}} \to -1$ at the late time. The plots of the evolution w_{tot} and q are similar. This is because $w_{\mathrm{tot}} = -\frac{1}{3} + \frac{2}{3}q$. In figure 4, we show the evolution of w_d for different model parameters β and δ in the case of $Q = 3\beta qH\rho_d$. It is easy to see that the EoS of TADE w_d can cross $w_d = -1$. For the fixed δ, w_d tends to a lower value at

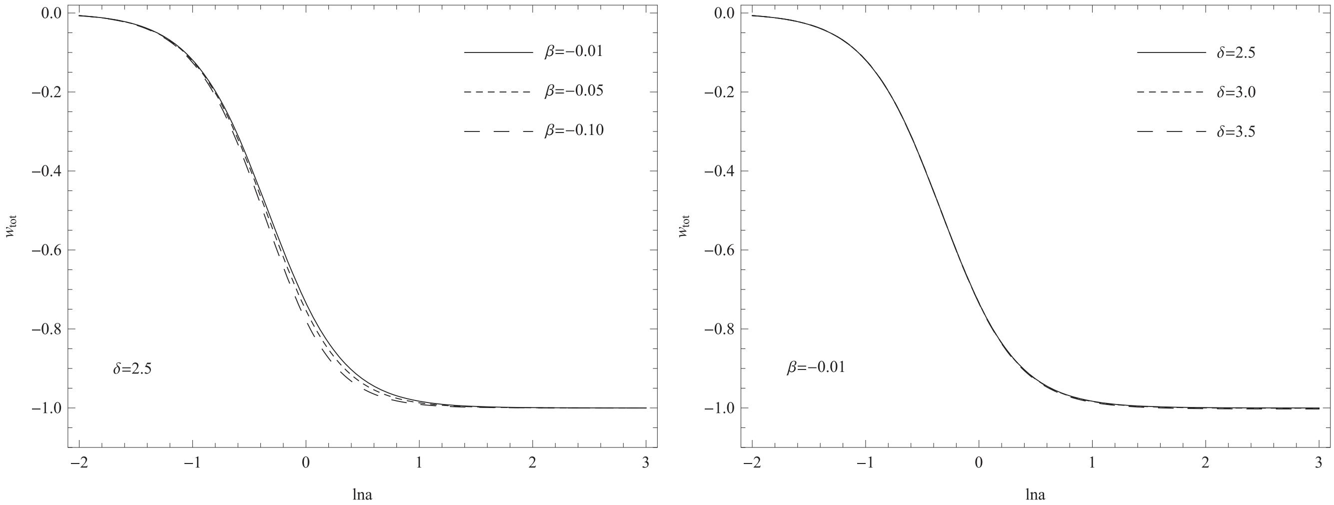

Figure 3. Evolution of w_{tot} for different model parameters β and δ. We take $n = 3$.

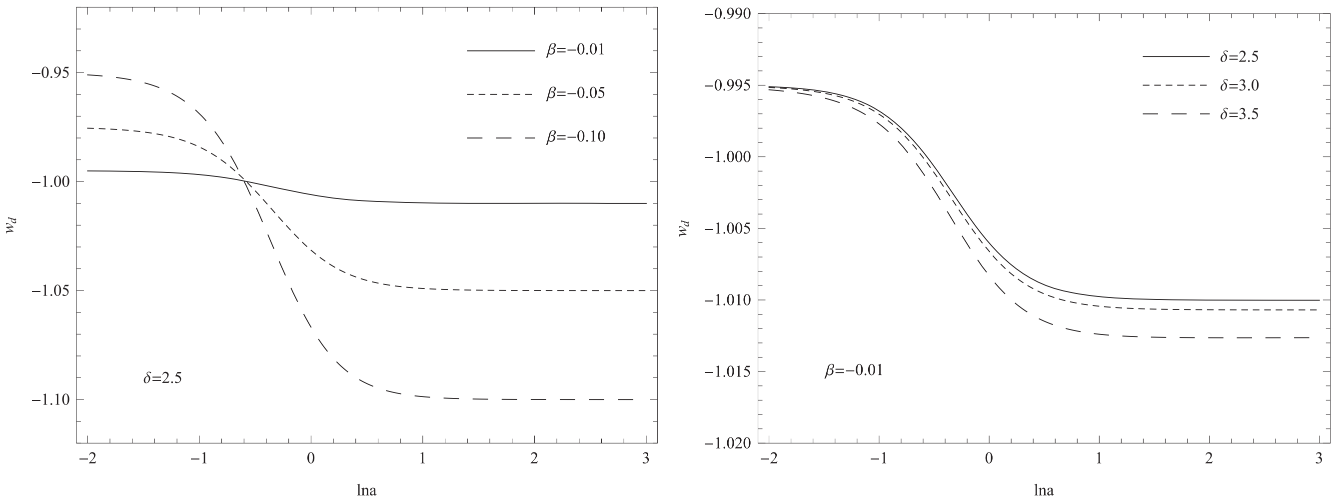

Figure 4. Evolution of w_d for different model parameters β and δ. We take $n = 3$.

the late time when $|\beta|$ is larger. On the other hand, for fixed β, w_d tends to a lower value at the late time when δ is larger.

3. Statefinder diagnostic and w–w' analysis

The statefinder pair $\{r, s\}$ is defined as follows [33]

$$r \equiv \frac{\ddot{a}}{aH^3}, \qquad s \equiv \frac{r-1}{3(q-1/2)}. \tag{23}$$

This diagnostic is constructed from the scale factor $a(t)$ and its derivatives with respect to the cosmic time, up to the third order. For the spatially flat LCDM model, the statefinder pair is the fixed point located at $\{r = 1, s = 0\}$, while the trajectories evolve in the $\{r, s\}$ plane for other DE models. So the parameters $\{r, s\}$ can be used as the diagnostic tool. By far, many cosmological models have been successfully distinguished by the statefinder diagnostic [36–47]. Now we apply the statefinder diagnostic to the interacting TADE model. From equation (23), we have

$$r = \frac{\ddot{H}}{H^3} - 3q - 2, \tag{24}$$

where the deceleration parameter can be expressed as

$$q = \frac{1}{2} + \frac{3}{2}\Omega_d w_d \tag{25}$$

and

$$\frac{\ddot{H}}{H^3} = \frac{9}{2} + \frac{9}{2}\Omega_d w_d(w_d + 2 + \beta q) - \frac{3}{2}\Omega_d w_d' \tag{26}$$

from equations (5), (9) and (10) The statefinder parameter r is written as

$$r = 1 + \frac{9}{2}\Omega_d w_d(1 + w_d + \beta q) - \frac{3}{2}\Omega_d w_d'. \tag{27}$$

Using equations (23), (25) and (27), we obtain the other statefinder parameter s as

$$s = 1 + w_d - \frac{w_d'}{3w_d} + \beta q. \tag{28}$$

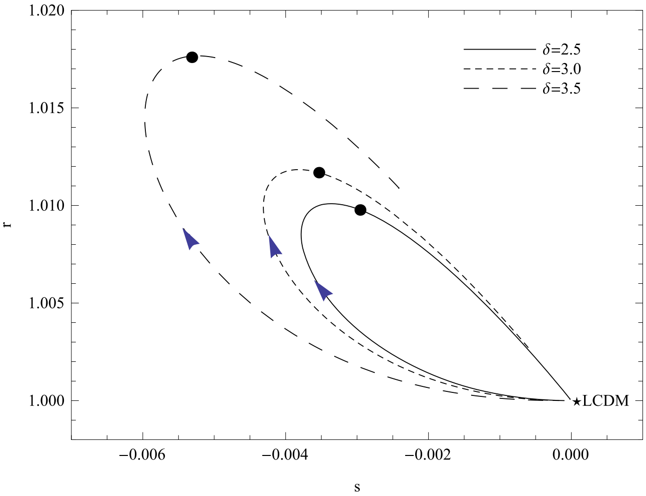

Figure 5. The $r-s$ diagram for different model parameters δ. We fix $n = 3$ and $\beta = -0.01$, and vary $\delta = 2.5, 3.0, 3.5$ respectively. The dots show the current values of the statefinder pair $\{r, s\}$. A star denotes the LCDM fixed point $\{r = 1, s = 0\}$.

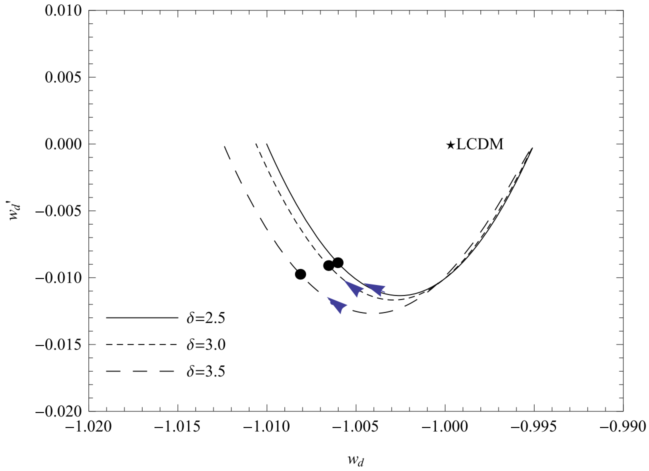

Figure 6. The w_d–w_d' diagram for different model parameters δ. We fix $n = 3$ and $\beta = -0.01$, and vary $\delta = 2.5, 3.0, 3.5$ respectively. The dots show the current values of $\{w_d, w_d'\}$. A star denotes the LCDM fixed point $\{w_d = -1, w_d' = 0\}$.

Differentiating equation (22), we get

$$w_d' = -\beta q' - \frac{2\delta - 4}{3}\left(\frac{1}{HT}\right)'. \tag{29}$$

From equation (25), we have

$$q' = \frac{3}{2}(\Omega_d' w_d + \Omega_d w_d'). \tag{30}$$

Substituting equation (30) into equation (29), we rewrite w_d' as

$$w_d' = \frac{-\frac{3}{2}\beta\Omega_d' w_d - \frac{2\delta - 4}{3}\left(\frac{1}{HT}\right)'}{1 + \frac{3}{2}\beta\Omega_d}, \tag{31}$$

where

$$\left(\frac{1}{HT}\right)' = -\frac{\dot{H}}{H^2}\frac{1}{HT} - \frac{1}{H^2T^2}. \tag{32}$$

In addition to the statefinder r–s, there is other dynamical diagnostic w_d–w_d' which is also used to discriminate DE models. The LCDM model corresponds to a fixed point $\{w_d = -1, w_d' = 0\}$ in the w_d–w_d' plane. In the TADE model with the sign-changeable interaction, w_d and w_d' are given by equations (22) and (31) respectively.

Next, we show the evolution trajectories of this model in the r–s and w_d–w_d' planes for different δ, respectively. We fix model parameters $n = 3$ and $\beta = -0.01$, and let δ be adjustable. In figure 5, we plot the $r(s)$ diagram for different δ. It is interesting to find that there is a loop in the r–s plane. The loop will be larger with the increasing of δ. From figure 5, we can see that the current values of $\{r_0, s_0\}$ are different for various values of δ. The larger the value of δ is, the greater distance from the current value $\{r_0, s_0\}$ to the LCDM fixed point is. Therefore, we can not only distinguish the interacting TADE model from the LCDM model but also discriminate the model with different model parameter δ. The evolutionary trajectories of the w_d–w_d' pair for the model are plotted in figure 6. From figure 6, it can be seen that the differences between various values of parameter can be identified. In addition, the LCDM model can be easily distinguished from the interacting TADE model by w_d–w_d' analysis.

4. Conclusions

In the present paper, we analyze the cosmological model with the sign-changeable interaction ($Q = 3\beta q H\rho_d$) between TADE and DM in flat universe. We show numerically the evolution of Ω_d, q, $w_{\rm tot}$ and w_d for different model parameters, where the initial condition is taken to be $\Omega_{d0} = 0.73$ and $H_0 = 67$ km s^{-1} Mpc^{-1}. We find that the accelerated expansion of the Universe can be achieved if $\delta > 2$ and $-2/3 < \beta < 0$. Furthermore, we apply statefinder diagnostic to the interacting TADE model. It is interesting to find that there is a loop in the r–s plane. The loop will be larger with the increasing of δ. This characteristic can differentiate the model from other DE models. The evolutionary trajectories of the w_d–w_d' pair for the model are plotted. It can be seen that the differences between various values of parameter δ can be identified. The LCDM model can be easily distinguished from the model by w_d–w_d' analysis.

Recently, Arabsalmani and Sahni extended the statefinder to higher-order derivatives of $a(t)$, and called such a diagnostic statefinder hierarchy [48]. The statefinder hierarchy has been applied to some DE models [49–52]. We will use the statefinder hierarchy to diagnose the interacting TADE model in future work.

References

[1] Riess A G *et al* 1998 *Astron. J.* **116** 1009
[2] Perlmutter S *et al* 1999 *Astrophys. J.* **517** 565
[3] Bennett C L *et al* 2003 *Astrophys. J. Suppl.* **148** 1
[4] Tegmark M *et al* 2004 *Phys. Rev.* D **69** 103501
[5] Cai R G 2007 *Phys. Lett.* B **657** 228
[6] Károlyházy F 1966 *Nuovo Cimento* A **42** 390

[7] Wei H and Cai R G 2008 *Phys. Lett.* B **663** 1
[8] Kim K Y, Lee H W and Myung Y S 2008 *Phys. Lett.* B **660** 118
[9] Wu J P, Ma D Z and Ling Y 2008 *Phys. Lett.* B **663** 152
[10] Zhang J, Zhang X and Liu H 2008 *Eur. Phys. J.* C **54** 303
[11] Cui J *et al* 2010 *Chin. Phys.* B **19** 019802
[12] Sheykhi A 2009 *Phys. Lett.* B **680** 113
[13] Sheykhi A *et al* 2010 *J. Cosmol. Astropart. Phys.* JCAP09(2010)017
[14] Sun C Y and Yue R H 2011 *Phys. Rev.* D **83** 107302
[15] Tsallis C and Cirto L J L 2013 *Eur. Phys. J.* C **73** 2487
[16] Tavayef M, Sheykhi A, Bamba K and Moradpour H 2018 *Phys. Lett.* B **781** 195
[17] Abdollahi Zadeh M, Sheykhi A and Moradpour H 2019 *Mod. Phys. Lett.* A **34** 1950086
[18] Zimdahl W, Pavon D and Chimento L P 2001 *Phys. Lett.* B **521** 133
[19] Guo Z K and Zhang Y Z 2005 *Phys. Rev.* D **71** 023501
[20] Wei H and Cai R G 2009 *Eur. Phys. J.* C **59** 99
[21] Cai R G and Wang A 2005 *J. Cosmol. Astropart. Phys.* JCAP03(2005)002
[22] Chen X, Gong Y and Saridakis E N 2009 *J. Cosmol. Astropart. Phys.* JCAP04(2009)001
[23] He J H, Wang B and Zhang P 2009 *Phys. Rev.* D **80** 063530
[24] Chimento L P 2010 *Phys. Rev.* D **81** 043525
[25] Setare M R 2006 *Phys. Lett.* B **642** 1
[26] Jamil M, Saridakis E N and Setare M R 2010 *Phys. Rev.* D **81** 023007
[27] Cai R G and Su Q 2010 *Phys. Rev.* D **81** 103514
[28] Copeland E J, Sami M and Tsujikawa S 2006 *Int. J. Mod. Phys.* D **15** 1753
[29] Frieman J, Turner M and Huterer D 2008 *Annu. Rev. Astron. Astrophys.* **46** 385
[30] Wei H 2011 *Nucl. Phys.* B **845** 381
[31] Li Y H and Zhang X 2011 *Eur. Phys. J.* C **71** 1700
[32] Guo J J, Zhang J F, Li Y H, He D Z and Zhang X 2018 *Sci. China Phys. Mech. Astron.* **61** 030011
[33] Sahni V, Saini T D, Starobinsky A A and Alam U 2003 *JETP Lett.* **77** 201
[34] Caldwell R R and Linder E V 2005 *Phys. Rev. Lett.* **95** 141301
[35] Wei H 2011 *Commun. Theor. Phys.* **56** 972
[36] Zhang X 2005 *Phys. Lett.* B **611** 1
[37] Zhang X 2005 *Int. J. Mod. Phys.* D **14** 1597
[38] Zhang X 2005 *Commun. Theor. Phys.* **44** 573
[39] Setare M R, Zhang J and Zhang X 2007 *J. Cosmol. Astropart. Phys.* JCAP03(2007)007
[40] Zhang J, Liu H and Zhang X 2008 *Phys. Lett.* B **659** 26
[41] Zhang L, Cui J, Zhang J and Zhang X 2010 *Int. J. Mod. Phys.* D **19** 21
[42] Chang B R, Liu H Y, Xu L X, Zhang C W and Ping Y L 2007 *J. Cosmol. Astropart. Phys.* JCAP01(2007)016
[43] Wei H and Cai R G 2007 *Phys. Lett.* B **655** 1
[44] Sharma U K and Pradhan A 2019 *Mod. Phys. Lett.* A **34** 1950101
[45] Varshney G, Sharma U K and Pradhan A 2019 *New Astron.* **70** 36
[46] Zhang N *et al* 2019 (arXiv:1905.04299)
[47] Abdollahi Zadeh M, Sheykhi A, Moradpour H and Bamba K 2019 (arXiv:1901.05298)
[48] Arabsalmani M and Sahni V 2011 *Phys. Rev.* D **83** 043501
[49] Li J, Yang R and Chen B 2014 *J. Cosmol. Astropart. Phys.* JCAP12(2014)043
[50] Zhang J F, Cui J L and Zhang X 2014 *Eur. Phys. J.* C **74** 3100
[51] Cui J L, Yin L, Wang L F, Li Y H and Zhanga X 2015 *J. Cosmol. Astropart. Phys.* JCAP09(2015)024
[52] Yu F, Cui J L, Zhang J F and Zhang X 2015 *Eur. Phys. J.* C **75** 274