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Commun. Theor. Phys. 73 (2021) 085005 (17pp) https://doi.org/10.1088/1572-9494/ac02b5

Dynamics of exact soliton solutions to the coupled nonlinear system using reliable analytical mathematical approaches

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Received 28 March 2021, revised 17 May 2021
Accepted for publication 19 May 2021
Published 2 July 2021

Abstract

Nonlinear Schrödinger-type equations are important models that have emerged from a wide variety of fields, such as fluids, nonlinear optics, the theory of deep-water waves, plasma physics, and so on. In this work, we obtain different soliton solutions to coupled nonlinear Schrödinger-type (CNLST) equations by applying three integration tools known as the $\left(\frac{G'}{G^2}\right)$-expansion function method, the modified direct algebraic method (MDAM), and the generalized Kudryashov method. The soliton and other solutions obtained by these methods can be categorized as single (dark, singular), complex, and combined soliton solutions, as well as hyperbolic, plane wave, and trigonometric solutions with arbitrary parameters. The spectrum of the solitons is enumerated along with their existence criteria. Moreover, 2D, 3D, and contour profiles of the reported results are also plotted by choosing suitable values of the parameters involved, which makes it easier for researchers to comprehend the physical phenomena of the governing equation. The solutions acquired demonstrate that the proposed techniques are efficient, valuable, and straightforward when constructing new solutions for various types of nonlinear partial differential equation that have important applications in applied sciences and engineering. All the reported solutions are verified by substitution back into the original equation through the software package Mathematica.

Keywords: soliton solutions, exact solutions, CNLST equations, $\left(\frac{G'}{G^2}\right)$-expansion function method, MDAM, generalized Kudryashov method

(Some figures may appear in colour only in the online journal)

1. Introduction

For many years, investigating the exact solutions of nonlinear partial differential equations (NLPDEs) has turned out to be a charming and challenging area of research for mathematicians and research communities, because they play a broad and significant role in the study of nonlinear physical phenomena in mathematical studies and applied physics, with essential applications in several areas of engineering and natural science including fluid mechanics, chemistry, thermodynamics, physics, electromagnetism, biomathematics, mathematical physics, and so on. Scholars have focused on exact or analytical solutions, due to their essential contribution to the analysis of the real features of nonlinear problems. Due to their wide utilization and applications in the domain of nonlinear sciences, interest in the study of NLPDEs has been increasing [1–9].

Moreover, solitons are also famous as a particular type of solitary wave, which are the solutions to various kinds of NLPDE. This incredible type of solitary waves has various and substantial use in different fields because of its specific characteristic of stability. In short, waves (mainly dispersive in nature) inelastically scatter solitary waves and lose energy due to radiation phenomena; as a result of the solitary wave's disappearance, the dispersive waves hold their shape and speed after a fully nonlinear connection. Soliton theory has made a significant contribution to the narration and expression of the physical behavior and meaning of nonlinear phenomena. Soliton theory has involved researchers in exploratory investigations due to its use in such diverse fields as media transmission, design, numerical materials science, mathematical physics, and different parts of nonlinear science [10–17]. Therefore, it has recently become more appealing for the research community to acquire exact solutions by the use of capable computational packages that alleviate the complex and tedious algebraic computations. In the literature, various powerful computational techniques have been designed to describe the natures of the diverse forms of the solutions [18–21] that have been established for nonlinear physical models.

To the best of our knowledge, it is known that in the existing literature, the solutions of the governing model have not been investigated using the $\left(\frac{G'}{G^2}\right)$-expansion function method [22], the modified direct algebraic method (MDAM) [23], and the generalized Kudryashov method [24]. Motivated by this, we employed these three mathematical techniques to obtain different forms of the solution. The proposed methodologies are powerful, reliable, capable of examining NLPDEs, consistent with computer algebra, and yield more general solutions. The discovered constructed solutions are novel and have potential applications in the nonlinear sciences.

The structure of the rest of this paper is organized as follows. In section 2, the governing equation is presented. In section 3, we discuss the application of the proposed methods. In section 4, the results and a discussion are presented. Finally, the conclusions are revealed in section 5.

2. Governing equation

This paper is concerned with the CNLST system, as presented by Ma and Geng [25],

$$\theta_{xt} = \theta_{xx} + \frac{2}{1-\vartheta^2}|\theta|^2\theta + \theta(\varphi - \psi),$$
$$\varphi_t = -\frac{(|\theta|^2)_t}{1+\vartheta} + (1+\vartheta)\varphi_x,$$
$$\psi_t = \frac{(|\theta|^2)_t}{1-\vartheta} + (1-\vartheta)\psi_x, \quad (1)$$

where $\vartheta \neq \pm 1$ is a real constant, φ and ψ represent real functions of the spatial variable x and the temporal variable t, respectively, while θ is a complex function.

In the following section, the applications of three methods are discussed.

3. Mathematical preliminaries

To solve the above equation (1), we utilize the traveling-wave transformations $\theta(x, t) = U(\eta)e^{i\Phi}$, $\varphi(x, t) = H(\eta)$, and $\psi(x, t) = Q(\eta)$, where $\Phi = \rho x - \varpi t$ and $\eta = x - c\,t + \zeta_0$. Here ρ, ϖ, c, and ζ_0 represent the wave number, frequency, velocity, and phase constant, respectively. Imposing these transformations onto equation (1) yields

$$(1+c)U'' - \rho(\rho+\varpi)U + \frac{2}{1-\vartheta^2}U^3 + (H-Q)U = 0, \quad (2)$$

$$H = -\frac{c\ U^2}{(1+\vartheta)(1+\vartheta+c)}, \quad (3)$$

$$Q = \frac{c\ U^2}{(\vartheta-1)(\vartheta-1-c)}, \quad (4)$$

$$\varpi = -\rho(2+c). \quad (5)$$

By substituting equations (3)–(5) into equation (2), we obtain the following ODE,

$$U'' - \frac{2}{\vartheta^2-(c+1)^2}U^3 + \rho^2 U = 0. \quad (6)$$

3.1. $(\frac{G'}{G^2})$-expansion method

Using the homogeneous balance rule that equation (6) yields, $n = 1$. The solution of equation (6) reduces to:

$$U(\eta) = \beta_0 + \sum_{j=1}^{n}\left(\beta_j\left(\frac{G'}{G^2}\right)^j + \delta_j\left(\frac{G'}{G^2}\right)^{-j}\right),$$

$$U(\eta) = \beta_0 + \beta_1\left(\frac{G'}{G^2}\right) + \delta_1\left(\frac{G'}{G^2}\right)^{-1}. \quad (7)$$

Substituting equation (7) into equation (6) together with $\left(\frac{G'}{G^2}\right)' = \Upsilon + \Omega\left(\frac{G'}{G^2}\right)^2$, and after comparing the coefficients, a set of strategic equations are obtained. By using a software package such as Mathematica, we get solution sets as follows:

$$Set-1: \beta_0 = 0,\ \beta_1 = \Omega\sqrt{(\vartheta-(c+1))(\vartheta+c+1)},$$
$$\delta_1 = 0,\ \rho = i\sqrt{2}\sqrt{\Upsilon}\sqrt{\Omega}.$$
$$Set-2: \beta_0 = 0,\ \beta_1 = 0,$$
$$\delta_1 = i\Upsilon\sqrt{(-\vartheta+c+1)(\vartheta+c+1)},$$
$$\rho = -i\sqrt{2}\sqrt{\Upsilon}\sqrt{\Omega}.$$
$$Set-3: \beta_0 = 0,\ \beta_1 = -i\Omega\sqrt{(-\vartheta+c+1)(\vartheta+c+1)},$$
$$\delta_1 = -i\Upsilon\sqrt{(-\vartheta+c+1)(\vartheta+c+1)},$$
$$\rho = -2\sqrt{\Upsilon}\sqrt{\Omega}.$$
$$Set-4: \beta_0 = 0,\ \beta_1 = i\Omega\sqrt{(-\vartheta+c+1)(\vartheta+c+1)},$$
$$\delta_1 = -i\Upsilon\sqrt{(-\vartheta+c+1)(\vartheta+c+1)},$$
$$\rho = -2i\sqrt{2}\sqrt{\Upsilon}\sqrt{\Omega}.$$

For **Set 1**,
Case-1 (**Trigonometric solutions**:

If $\Upsilon\Omega > 0$,

$$\theta_{1,1}(x,\,t) = \frac{\sqrt{\Upsilon}\,\sqrt{\Omega}\,\sqrt{(\vartheta - c - 1)(\vartheta + c + 1)}\,(\Xi_1 \sin(\eta\sqrt{\Upsilon\Omega}) + \Xi_2 \cos(\eta\sqrt{\Upsilon\Omega}))}{\Xi_2 \cos(\eta\sqrt{\Upsilon\Omega}) - \Xi_1 \sin(\eta\sqrt{\Upsilon\Omega})} \times \mathrm{e}^{\mathrm{i}((c+2)\rho t + \rho x)}, \tag{8}$$

$$\varphi_{1,1}(x,\,t) = -\frac{c\,|\theta_{1,1}(x,\,t)|^2}{(1+\vartheta)(1+\vartheta+c)}, \tag{9}$$

$$\psi_{1,1}(x,\,t) = \frac{c\,|\theta_{1,1}(x,\,t)|^2}{(\vartheta-1)(\vartheta-1-c)}. \tag{10}$$

Case-2 (**Hyperbolic solution**:
If $\Upsilon\Omega < 0$,

$$\theta_{1,2}(x,\,t) = -\frac{\sqrt{\vartheta^2 - c^2 - 2c - 1}\,\sqrt{|\Upsilon\Omega|}\,(\Xi_1(\sinh(2\eta\sqrt{|\Upsilon\Omega|}) + \cosh(2\eta\sqrt{|\Upsilon\Omega|})) + \Xi_2)}{\Xi_1(\sinh(2\eta\sqrt{|\Upsilon\Omega|}) + \cosh(2\eta\sqrt{|\Upsilon\Omega|})) - \Xi_2} \times \mathrm{e}^{\mathrm{i}((c+2)\rho t + \rho x)}, \tag{11}$$

$$\varphi_{1,2}(x,\,t) = -\frac{c\,|\theta_{1,2}(x,\,t)|^2}{(1+\vartheta)(1+\vartheta+c)}, \tag{12}$$

$$\psi_{1,2}(x,\,t) = \frac{c\,|\theta_{1,2}(x,\,t)|^2}{(\vartheta-1)(\vartheta-1-c)}. \tag{13}$$

For the soliton solution, taking $\Xi_1 = \Xi_2$, we get a singular wave solution as follows:

$$\theta_{1,2}(x,\,t) = -\sqrt{\vartheta^2 - (c+1)^2}\,\sqrt{|\Upsilon\Omega|}\,\coth(\sqrt{|\Upsilon\Omega|}\,\eta) \times \mathrm{e}^{\mathrm{i}((c+2)\rho t + \rho x)}, \tag{14}$$

$$\varphi_{1,2}(x,\,t) = -\frac{c\,|\theta_{1,2}(x,\,t)|^2}{(1+\vartheta)(1+\vartheta+c)}, \tag{15}$$

$$\psi_{1,2}(x,\,t) = \frac{c\,|\theta_{1,2}(x,\,t)|^2}{(\vartheta-1)(\vartheta-1-c)}. \tag{16}$$

For **Set 2**,
Case-1 (**Trigonometric solutions**:
If $\Upsilon\Omega > 0$,

$$\theta_{2,1}(x,\,t) = \frac{i\sqrt{\Upsilon}\,\sqrt{\Omega}\,\sqrt{(-\vartheta + c + 1)(\vartheta + c + 1)}\,(\Xi_2 \cos(\eta\sqrt{\Upsilon\Omega}) - \Xi_1 \sin(\eta\sqrt{\Upsilon\Omega}))}{\Xi_1 \sin(\eta\sqrt{\Upsilon\Omega}) + \Xi_2 \cos(\eta\sqrt{\Upsilon\Omega})} \times \mathrm{e}^{\mathrm{i}((c+2)\rho t + \rho x)}, \tag{17}$$

$$\varphi_{2,1}(x,\,t) = -\frac{c\,|\theta_{2,1}(x,\,t)|^2}{(1+\vartheta)(1+\vartheta+c)}, \tag{18}$$

$$\psi_{2,1}(x,\,t) = \frac{c\,|\theta_{2,1}(x,\,t)|^2}{(\vartheta-1)(\vartheta-1-c)}. \tag{19}$$

Case-2 (**Hyperbolic solution**:
If $\Upsilon\Omega < 0$,

$$\theta_{2,2}(x,\,t) = -\frac{i\Upsilon\Omega\sqrt{(-\vartheta + c + 1)(\vartheta + c + 1)}\,(\Xi_1 \sinh(2\eta\sqrt{|\Upsilon\Omega|}) + \Xi_1 \cosh(2\eta\sqrt{|\Upsilon\Omega|}) - \Xi_2)}{\sqrt{|\Upsilon\Omega|}\,(\Xi_1 \sinh(2\eta\sqrt{|\Upsilon\Omega|}) + \Xi_1 \cosh(2\eta\sqrt{|\Upsilon\Omega|}) + \Xi_2)} \times \mathrm{e}^{\mathrm{i}((c+2)\rho t + \rho x)}, \tag{20}$$

$$\varphi_{2,2}(x,\,t) = -\frac{c\,|\theta_{2,2}(x,\,t)|^2}{(1+\vartheta)(1+\vartheta+c)}, \tag{21}$$

$$\psi_{2,2}(x,\,t) = \frac{c\,|\theta_{2,2}(x,\,t)|^2}{(\vartheta-1)(\vartheta-1-c)}. \tag{22}$$

For the soliton solution, if we take $\Xi_1 = \Xi_2$, we get a dark soliton solution, as follows:

$$\theta_{2,2}(x,\, t) = -i\sqrt{|\Upsilon\Omega|}\sqrt{(-\vartheta + c + 1)(\vartheta + c + 1)} \times \tanh(\sqrt{|\Upsilon\Omega|}\eta) \times \mathrm{e}^{\mathrm{i}((c+2)\rho t + \rho x)}, \tag{23}$$

$$\varphi_{2,2}(x,\, t) = -\frac{c\,|\theta_{2,2}(x,\, t)|^2}{(1 + \vartheta)(1 + \vartheta + c)}, \tag{24}$$

$$\psi_{2,2}(x,\, t) = \frac{c\,|\theta_{2,2}(x,\, t)|^2}{(\vartheta - 1)(\vartheta - 1 - c)}. \tag{25}$$

For **Set 3**,

<u>Case-1</u> (Trigonometric solutions:

If $\Upsilon\Omega > 0$,

$$\theta_{3,1}(x,\, t) = \frac{2i\sqrt{\Upsilon}\sqrt{\Omega}\sqrt{-\vartheta^2 + c^2 + 2c + 1}\,(\Xi_1^2\sin^2(\eta\sqrt{\Upsilon\Omega}) + \Xi_2^2\cos^2(\eta\sqrt{\Upsilon\Omega}))}{\Xi_1^2\sin^2(\eta\sqrt{\Upsilon\Omega}) - \Xi_2^2\cos^2(\eta\sqrt{\Upsilon\Omega})} \times \mathrm{e}^{\mathrm{i}((c+2)\rho t + \rho x)}, \tag{26}$$

$$\varphi_{3,1}(x,\, t) = -\frac{c\,|\theta_{3,1}(x,\, t)|^2}{(1 + \vartheta)(1 + \vartheta + c)}, \tag{27}$$

$$\psi_{3,1}(x,\, t) = \frac{c\,|\theta_{3,1}(x,\, t)|^2}{(\vartheta - 1)(\vartheta - 1 - c)}. \tag{28}$$

For the soliton solution, if we take $\Xi_1 = \Xi_2$, we get a periodic wave solution, as follows:

$$\theta_{3,1}(x,\, t) = -2i\sqrt{\Upsilon}\sqrt{\Omega}\sqrt{(-\vartheta + c + 1)(\vartheta + c + 1)} \times \sec(2\sqrt{\Upsilon\Omega}\,\eta) \times \mathrm{e}^{\mathrm{i}((c+2)\rho t + \rho x)}, \tag{29}$$

$$\varphi_{3,1}(x,\, t) = -\frac{c\,|\theta_{3,1}(x,\, t)|^2}{(1 + \vartheta)(1 + \vartheta + c)}, \tag{30}$$

$$\psi_{3,1}(x,\, t) = \frac{c\,|\theta_{3,1}(x,\, t)|^2}{(\vartheta - 1)(\vartheta - 1 - c)}. \tag{31}$$

<u>Case-2</u> (Hyperbolic solution:

If $\Upsilon\Omega < 0$,

$$\theta_{3,2}(x,\, t) = \frac{i\Upsilon\Omega\sqrt{(-\vartheta + c + 1)(\vartheta + c + 1)}\,(\Xi_1\sinh(2\eta\sqrt{|\Upsilon\Omega|}) + \Xi_1\cosh(2\eta\sqrt{|\Upsilon\Omega|}) - \Xi_2)}{\sqrt{|\Upsilon\Omega|}\,(\Xi_1\sinh(2\eta\sqrt{|\Upsilon\Omega|}) + \Xi_1\cosh(2\eta\sqrt{|\Upsilon\Omega|}) + \Xi_2)} + \frac{i\sqrt{(-\vartheta + c + 1)(\vartheta + c + 1)}\sqrt{|\Upsilon\Omega|}\,(\Xi_1\sinh(2\eta\sqrt{|\Upsilon\Omega|}) + \Xi_1\cosh(2\eta\sqrt{|\Upsilon\Omega|}) + \Xi_2)}{\Xi_1\sinh(2\eta\sqrt{|\Upsilon\Omega|}) + \Xi_1\cosh(2\eta\sqrt{|\Upsilon\Omega|}) - \Xi_2} \times \mathrm{e}^{\mathrm{i}((c+2)\rho t + \rho x)}, \tag{32}$$

$$\varphi_{3,2}(x,\, t) = -\frac{c\,|\theta_{3,2}(x,\, t)|^2}{(1 + \vartheta)(1 + \vartheta + c)}, \tag{33}$$

$$\psi_{3,2}(x,\, t) = \frac{c\,|\theta_{3,2}(x,\, t)|^2}{(\vartheta - 1)(\vartheta - 1 - c)}. \tag{34}$$

For the soliton solution, if we take $\Xi_1 = \Xi_2$, we get a combined dark–singular wave solution, as follows:

$$\theta_{3,2}(x,\, t) = \frac{\mathrm{i}\sqrt{(-\vartheta + c + 1)(\vartheta + c + 1)}\tanh(\sqrt{|\Upsilon\Omega|}\,\eta)(|\Upsilon\Omega|\coth^2(\sqrt{|\Upsilon\Omega|}\,\eta) + \Upsilon\Omega)}{\sqrt{|\Upsilon\Omega|}} \times \mathrm{e}^{\mathrm{i}((c+2)\rho t + \rho x)}, \tag{35}$$

$$\varphi_{3,2}(x,\, t) = -\frac{c\,|\theta_{3,2}(x,\, t)|^2}{(1 + \vartheta)(1 + \vartheta + c)}, \tag{36}$$

$$\psi_{3,2}(x,\, t) = \frac{c\,|\theta_{3,2}(x,\, t)|^2}{(\vartheta - 1)(\vartheta - 1 - c)}. \tag{37}$$

Case-3 (**Rational solutions**:
If $\Upsilon = 0,\ \Omega \neq 0$,

$$\theta_{3,3}(x,\,t) = \frac{\mathrm{i}\Xi_1\sqrt{(-\vartheta + c + 1)(\vartheta + c + 1)}}{\Xi_1\eta + \Xi_2} \times \mathrm{e}^{\mathrm{i}((c+2)\rho t + \rho x)}, \tag{38}$$

$$\varphi_{3,3}(x,\,t) = -\frac{c\,|\theta_{3,3}(x,\,t)|^2}{(1 + \vartheta)(1 + \vartheta + c)}, \tag{39}$$

$$\psi_{3,3}(x,\,t) = \frac{c\,|\theta_{3,3}(x,\,t)|^2}{(\vartheta - 1)(\vartheta - 1 - c)}. \tag{40}$$

If we take $\Xi_1 = \Xi_2$, a plane-wave solution is obtained:

$$\theta_{3,3}(x,\,t) = \frac{\mathrm{i}\sqrt{(-\vartheta + c + 1)(\vartheta + c + 1)}}{\eta + 1} \times \mathrm{e}^{\mathrm{i}((c+2)\rho t + \rho x)}, \tag{41}$$

$$\varphi_{3,3}(x,\,t) = -\frac{c\,|\theta_{3,3}(x,\,t)|^2}{(1 + \vartheta)(1 + \vartheta + c)}, \tag{42}$$

$$\psi_{3,3}(x,\,t) = \frac{c\,|\theta_{3,3}(x,\,t)|^2}{(\vartheta - 1)(\vartheta - 1 - c)}. \tag{43}$$

For **Set 4**,
Case-1 (**Trigonometric solutions**:
If $\Upsilon\Omega > 0$,

$$\theta_{4,1}(x,\,t) = -\frac{2\mathrm{i}\sqrt{\Upsilon}\,\Xi_1\Xi_2\sqrt{\Omega}\,\sqrt{-\vartheta^2 + c^2 + 2c + 1}\,\sin(2\eta\sqrt{\Upsilon\Omega})}{\Xi_1^2\sin^2(\eta\sqrt{\Upsilon\Omega}) - \Xi_2^2\cos^2(\eta\sqrt{\Upsilon\Omega})} \times \mathrm{e}^{\mathrm{i}((c+2)\rho t + \rho x)}, \tag{44}$$

$$\varphi_{4,1}(x,\,t) = -\frac{c\,|\theta_{4,1}(x,\,t)|^2}{(1 + \vartheta)(1 + \vartheta + c)}, \tag{45}$$

$$\psi_{4,1}(x,\,t) = \frac{c\,|\theta_{4,1}(x,\,t)|^2}{(\vartheta - 1)(\vartheta - 1 - c)}. \tag{46}$$

For the soliton solution, if we take $\Xi_1 = \Xi_2$, we get a periodic wave solution, as follows:

$$\theta_{4,1}(x,\,t) = 2i\sqrt{(-\vartheta + c + 1)(\vartheta + c + 1)}\,\sqrt{\Upsilon\Omega} \times \tan(2\sqrt{\Upsilon}\,\sqrt{\Omega}\,\eta) \times \mathrm{e}^{\mathrm{i}((c+2)\rho t + \rho x)}, \tag{47}$$

$$\varphi_{4,1}(x,\,t) = -\frac{c\,|\theta_{4,1}(x,\,t)|^2}{(1 + \vartheta)(1 + \vartheta + c)}, \tag{48}$$

$$\psi_{4,1}(x,\,t) = \frac{c\,|\theta_{4,1}(x,\,t)|^2}{(\vartheta - 1)(\vartheta - 1 - c)}. \tag{49}$$

Case-2 (**Hyperbolic solution**:
If $\Upsilon\Omega < 0$,

$$\theta_{4,2}(x,\,t) = \frac{\mathrm{i}\Upsilon\Omega\sqrt{(-\vartheta + c + 1)(\vartheta + c + 1)}\,(\Xi_1\sinh(2\eta\sqrt{|\Upsilon\Omega|}) + \Xi_1\cosh(2\eta\sqrt{|\Upsilon\Omega|}) - \Xi_2)}{\sqrt{|\Upsilon\Omega|}\,(\Xi_1\sinh(2\eta\sqrt{|\Upsilon\Omega|}) + \Xi_1\cosh(2\eta\sqrt{|\Upsilon\Omega|}) + \Xi_2)} - \frac{\mathrm{i}\sqrt{(-\vartheta + c + 1)(\vartheta + c + 1)}\,\sqrt{|\Upsilon\Omega|}\,(\Xi_1\sinh(2\eta\sqrt{|\Upsilon\Omega|}) + \Xi_1\cosh(2\eta\sqrt{|\Upsilon\Omega|}) + \Xi_2)}{\Xi_1\sinh(2\eta\sqrt{|\Upsilon\Omega|}) + \Xi_1\cosh(2\eta\sqrt{|\Upsilon\Omega|}) - \Xi_2} \times \mathrm{e}^{\mathrm{i}((c+2)\rho t + \rho x)}, \tag{50}$$

$$\varphi_{4,2}(x,\,t) = -\frac{c\,|\theta_{4,2}(x,\,t)|^2}{(1 + \vartheta)(1 + \vartheta + c)}, \tag{51}$$

$$\psi_{4,2}(x,\,t) = \frac{c\,|\theta_{4,2}(x,\,t)|^2}{(\vartheta - 1)(\vartheta - 1 - c)}. \tag{52}$$

For the soliton solution, if we take $\Xi_1 = \Xi_2$, we obtain a combined dark–singular wave solution, as follows:

$$\theta_{4,2}(x,t) = -\frac{\mathrm{i}\sqrt{(-\vartheta+c+1)(\vartheta+c+1)}\tanh(\sqrt{|\Upsilon\Omega|}\,\eta)(|\Upsilon\Omega|\coth^2(\sqrt{|\Upsilon\Omega|}\,\eta)-\Upsilon\Omega)}{\sqrt{|\Upsilon\Omega|}} \times \mathrm{e}^{\mathrm{i}((c+2)\rho t+\rho x)}, \tag{53}$$

$$\varphi_{4,2}(x,t) = -\frac{c\,|\theta_{4,2}(x,t)|^2}{(1+\vartheta)(1+\vartheta+c)}, \tag{54}$$

$$\psi_{4,2}(x,t) = \frac{c\,|\theta_{4,2}(x,t)|^2}{(\vartheta-1)(\vartheta-1-c)}. \tag{55}$$

For all the above sets, $\eta = -c\,t + \zeta_0 + x$.

3.2. MDAM

By utilizing the homogeneous balance rule on equation (6), we obtain $n = 1$, which implies the solution of equation (6) is as follows:

$$U(\eta) = a_0 + a_1 Z + b_1 Z^{-1}, \tag{56}$$

where a_0, a_1, and b_1 are parameters. In order to find the parameters involved in equation (56), we substitute equation (56) along with ($Z' = \chi + Z^2$) into equation (6), and we get a cluster of equations on equating the same power coefficients of of Z. Furthermore, by using Mathematica, we obtain a cluster of solutions, as follows:

$$\begin{aligned} Set-1: & a_0 = 0,\ a_1 = \sqrt{\vartheta^2-(c+1)^2},\\ & b_1 = 0,\ \chi = -\frac{\rho^2}{2}.\\ Set-2: & a_0 = 0,\ a_1 = 0,\\ & b_1 = \frac{1}{2}\mathrm{i}\rho^2\sqrt{(-\vartheta+c+1)(\vartheta+c+1)},\\ & \chi = -\frac{\rho^2}{2}.\\ Set-3: & a_0 = 0,\ a_1 = \sqrt{\vartheta^2-(c+1)^2},\\ & b_1 = \frac{1}{8}\rho^2\sqrt{\vartheta^2-(c+1)^2},\ \chi = -\frac{\rho^2}{8}. \end{aligned}$$

For Set 1

Case-1 :

When $\chi < 0$, we get the following solutions.

Dark soliton structure:

$$\theta_1(x,t) = -\frac{\rho\sqrt{\vartheta^2-(c+1)^2}\tanh\left(\frac{\eta\rho}{\sqrt{2}}\right)}{\sqrt{2}} \times \mathrm{e}^{\mathrm{i}((c+2)\rho t+\rho x)}, \tag{57}$$

$$\varphi_1(x,t) = -\frac{c\,|\theta_1(x,t)|^2}{(1+\vartheta)(1+\vartheta+c)}, \tag{58}$$

$$\psi_1(x,t) = \frac{c\,|\theta_1(x,t)|^2}{(\vartheta-1)(\vartheta-1-c)}. \tag{59}$$

Singular wave structure:

$$\theta_2(x,t) = \frac{\rho\sqrt{\vartheta^2-(c+1)^2}\coth\left(\frac{\eta\rho}{\sqrt{2}}\right)}{\sqrt{2}} \times \mathrm{e}^{\mathrm{i}((c+2)\rho t+\rho x)}, \tag{60}$$

$$\varphi_2(x,t) = -\frac{c|\theta_2(x,t)|^2}{(1+\vartheta)(1+\vartheta+c)}, \tag{61}$$

$$\psi_2(x,t) = \frac{c|\theta_2(x,t)|^2}{(\vartheta-1)(\vartheta-1-c)}. \tag{62}$$

Case-2 :
When $\chi > 0$, then the following periodic solutions of different forms are obtained:

$$\theta_3(x, t) = -\frac{\sqrt{-\rho^2}\sqrt{\vartheta^2 - (c+1)^2}\tan\left(\frac{\eta\sqrt{-\rho^2}}{\sqrt{2}}\right)}{\sqrt{2}} \times e^{i((c+2)\rho t + \rho x)}, \tag{63}$$

$$\varphi_3(x, t) = -\frac{c|\theta_3(x, t)|^2}{(1+\vartheta)(1+\vartheta+c)}, \tag{64}$$

$$\psi_3(x, t) = \frac{c|\theta_3(x, t)|^2}{(\vartheta-1)(\vartheta-1-c)}, \tag{65}$$

and

$$\theta_4(x, t) = \frac{\sqrt{-\rho^2}\sqrt{\vartheta^2 - (c+1)^2}\cot\left(\frac{\eta\sqrt{-\rho^2}}{\sqrt{2}}\right)}{\sqrt{2}} \times e^{i((c+2)\rho t + \rho x)}, \tag{66}$$

$$\varphi_4(x, t) = -\frac{c\,|\theta_4(x, t)|^2}{(1+\vartheta)(1+\vartheta+c)}, \tag{67}$$

$$\psi_4(x, t) = \frac{c\,|\theta_4(x, t)|^2}{(\vartheta-1)(\vartheta-1-c)}. \tag{68}$$

For Set 2
Case-1 :
When $\chi < 0$, we get the singular and dark soliton structures, respectively:

$$\theta_5(x, t) = -\frac{i\rho\sqrt{(-\vartheta+c+1)(\vartheta+c+1)}\coth\left(\frac{\eta\rho}{\sqrt{2}}\right)}{\sqrt{2}} \times e^{i((c+2)\rho t + \rho x)}, \tag{69}$$

$$\varphi_5(x, t) = -\frac{c\,|\theta_5(x, t)|^2}{(1+\vartheta)(1+\vartheta+c)}, \tag{70}$$

$$\psi_5(x, t) = \frac{c\,|\theta_5(x, t)|^2}{(\vartheta-1)(\vartheta-1-c)}. \tag{71}$$

and

$$\theta_6(x, t) = -\frac{i\rho\sqrt{(-\vartheta+c+1)(\vartheta+c+1)}\tanh\left(\frac{\eta\rho}{\sqrt{2}}\right)}{\sqrt{2}} \times e^{i((c+2)\rho t + \rho x)}, \tag{72}$$

$$\varphi_6(x, t) = -\frac{c\,|\theta_6(x, t)|^2}{(1+\vartheta)(1+\vartheta+c)}, \tag{73}$$

$$\psi_6(x, t) = \frac{c\,|\theta_6(x, t)|^2}{(\vartheta-1)(\vartheta-1-c)}. \tag{74}$$

Case-2 :
When $\chi > 0$, then the periodic solutions are:

$$\theta_7(x, t) = -\frac{i\sqrt{-\rho^2}\sqrt{-\vartheta^2 + c^2 + 2c + 1}\cot\left(\frac{\eta\sqrt{-\rho^2}}{\sqrt{2}}\right)}{\sqrt{2}} \times e^{i((c+2)\rho t + \rho x)}, \tag{75}$$

$$\varphi_7(x,\,t) = -\frac{c\,|\theta_7(x,\,t)|^2}{(1+\vartheta)(1+\vartheta+c)}, \tag{76}$$

$$\psi_7(x,\,t) = \frac{c\,|\theta_7(x,\,t)|^2}{(\vartheta-1)(\vartheta-1-c)}. \tag{77}$$

and

$$\theta_8(x,\,t) = \frac{i\sqrt{-\rho^2}\sqrt{-\vartheta^2+c^2+2c+1}\,\tan\left(\frac{\eta\sqrt{-\rho^2}}{\sqrt{2}}\right)}{\sqrt{2}} \times \mathrm{e}^{\mathrm{i}((c+2)\rho t+\rho x)}, \tag{78}$$

$$\varphi_8(x,\,t) = -\frac{c\,|\theta_8(x,\,t)|^2}{(1+\vartheta)(1+\vartheta+c)}, \tag{79}$$

$$\psi_8(x,\,t) = \frac{c\,|\theta_8(x,\,t)|^2}{(\vartheta-1)(\vartheta-1-c)}. \tag{80}$$

For Set 3

Case-1 :

When $\chi < 0$, we get the following mixed hyperbolic solution:

$$\theta_9(x,\,t) = \left(-\frac{\sqrt{\rho^2}\sqrt{\vartheta^2-(c+1)^2}\,\tanh\left(\frac{\eta\sqrt{\rho^2}}{2\sqrt{2}}\right)}{2\sqrt{2}} - \frac{\sqrt{\rho^2}\sqrt{\vartheta^2-(c+1)^2}\,\coth\left(\frac{\eta\sqrt{\rho^2}}{2\sqrt{2}}\right)}{2\sqrt{2}}\right) \times \mathrm{e}^{\mathrm{i}((c+2)\rho t+\rho x)}, \tag{81}$$

$$\varphi_9(x,\,t) = -\frac{c|\theta_9(x,\,t)|^2}{(1+\vartheta)(1+\vartheta+c)}, \tag{82}$$

$$\psi_9(x,\,t) = \frac{c|\theta_9(x,\,t)|^2}{(\vartheta-1)(\vartheta-1-c)}. \tag{83}$$

Case-2 :

When $\chi > 0$, then the periodic solution is expressed as:

$$\theta_{10}(x,\,t) = \left(\frac{\sqrt{-\rho^2}\sqrt{\vartheta^2-(c+1)^2}\,\tan\left(\frac{\eta\sqrt{-\rho^2}}{2\sqrt{2}}\right)}{2\sqrt{2}} + \frac{\rho^2\sqrt{\vartheta^2-(c+1)^2}\,\cot\left(\frac{\eta\sqrt{-\rho^2}}{2\sqrt{2}}\right)}{2\sqrt{2}\sqrt{-\rho^2}}\right) \times \mathrm{e}^{\mathrm{i}((c+2)\rho t+\rho x)}, \tag{84}$$

$$\varphi_{10}(x,\,t) = -\frac{c\,|\theta_{10}(x,\,t)|^2}{(1+\vartheta)(1+\vartheta+c)}, \tag{85}$$

$$\psi_{10}(x,\,t) = \frac{c\,|\theta_{10}(x,\,t)|^2}{(\vartheta-1)(\vartheta-1-c)}. \tag{86}$$

For all the above sets, $\eta = -c\,t + \zeta_0 + x$.

3.3. Generalized Kudryashov method

In this section, we apply the above method to obtain the solutions of the governing model. To begin with, we take account of the fact that the homogeneous balance between U^3 and U'' gives the relation $T = H + 1$. In particular, for $H = 1$, we get $T = 2$.

Therefore, equation (6) along with $(G'(\eta) = G^2(\eta) - G(\eta))$ take the following form of the solution:

$$U(\eta) = \frac{a_0 + a_1 G(\eta) + a_2 G^2(\eta)}{b_0 + b_1 G(\eta)}, \tag{87}$$

where a_0, a_1, a_2, b_0 and b_1 are to be determined. Also,

$$G(\eta) = \frac{1}{1 \pm S e^{\eta}}. \tag{88}$$

Now, solving equations (6) and (87), the following system of equations is obtained:

$$\begin{cases} -\vartheta^2 a_0 b_0^2 \rho^2 + a_0 b_0^2 c^2 \rho^2 + 2a_0 b_0^2 c\rho^2 + a_0 b_0^2 \rho^2 + 2a_0^3 = 0. \\ -\vartheta^2 a_1 b_0^2 \rho^2 - 2\vartheta^2 a_0 b_0 b_1 \rho^2 - \vartheta^2 a_1 b_0^2 + \vartheta^2 a_0 b_0 b_1 + a_1 b_0^2 c^2 \rho^2 + 2a_0 b_0 b_1 c^2 \rho^2 + a_1 b_0^2 c^2 \\ -a_0 b_0 b_1 c^2 + 2a_1 b_0^2 c\rho^2 + 4a_0 b_0 b_1 c\rho^2 + 2a_1 b_0^2 c - 2a_0 b_0 b_1 c + a_1 b_0^2 \rho^2 + 2a_0 b_0 b_1 \rho^2 + a_1 b_0^2 - a_0 b_0 b_1 + 6a_0^2 a_1 = 0. \\ -\vartheta^2 a_2 b_0^2 \rho^2 - \vartheta^2 a_0 b_1^2 \rho^2 - 2\vartheta^2 a_1 b_0 b_1 \rho^2 + 3\vartheta^2 a_1 b_0^2 - 4\vartheta^2 a_2 b_0^2 - \vartheta^2 a_0 b_1^2 - 3\vartheta^2 a_0 b_0 b_1 + \vartheta^2 a_1 b_0 b_1 + a_2 b_0^2 c^2 \rho^2 \\ +a_0 b_1^2 c^2 \rho^2 + 2a_1 b_0 b_1 c^2 \rho^2 - 3a_1 b_0^2 c^2 + 4a_2 b_0^2 c^2 + a_0 b_1^2 c^2 + 3a_0 b_0 b_1 c^2 - a_1 b_0 b_1 c^2 + 2a_2 b_0^2 c\rho^2 + 2a_0 b_1^2 c\rho^2 \\ +4a_1 b_0 b_1 c\rho^2 - 6a_1 b_0^2 c + 8a_2 b_0^2 c + 2a_0 b_1^2 c + 6a_0 b_0 b_1 c - 2a_1 b_0 b_1 c + a_2 b_0^2 \rho^2 + a_0 b_1^2 \rho^2 + 2a_1 b_0 b_1 \rho^2 - 3a_1 b_0^2 \\ +4a_2 b_0^2 + a_0 b_1^2 + 3a_0 b_0 b_1 - a_1 b_0 b_1 + 36a_0 a_1^2 + 6a_0^2 a_2 = 0. \\ -\vartheta^2 a_1 b_1^2 \rho^2 - 2\vartheta^2 a_2 b_0 b_1 \rho^2 - 2\vartheta^2 a_1 b_0^2 - \vartheta^2 a_1 b_0 b_1 + 10\vartheta^2 a_2 b_0^2 + \vartheta^2 a_0 b_1^2 + 2\vartheta^2 a_0 b_0 b_1 - 3\vartheta^2 a_2 b_0 b_1 \\ +a_1 b_1^2 c^2 \rho^2 + 2a_2 b_0 b_1 c^2 \rho^2 + 2a_1 b_0^2 c^2 + a_1 b_0 b_1 c^2 - 10a_2 b_0^2 c^2 - a_0 b_1^2 c^2 - 2a_0 b_0 b_1 c^2 + 3a_2 b_0 b_1 c^2 + 2a_1 b_1^2 c\rho^2 \\ +4a_2 b_0 b_1 c\rho^2 + 4a_1 b_0^2 c + 2a_1 b_0 b_1 c - 20a_2 b_0^2 c - 2a_0 b_1^2 c - 4a_0 b_0 b_1 c + 6a_2 b_0 b_1 c + a_1 b_1^2 \rho^2 + 2a_2 b_0 b_1 \rho^2 \\ +2a_1 b_0^2 + a_1 b_0 b_1 - 10a_2 b_0^2 - a_0 b_1^2 - 2a_0 b_0 b_1 + 3a_2 b_0 b_1 + 2a_1^3 + 12a_0 a_2 a_1 = 0. \\ -\vartheta^2 a_2 b_1^2 \rho^2 - 6\vartheta^2 a_2 b_0^2 - \vartheta^2 a_2 b_1^2 + 9\vartheta^2 a_2 b_0 b_1 + a_2 b_1^2 c^2 \rho^2 + 6a_2 b_0^2 c^2 + a_2 b_1^2 c^2 - 9a_2 b_0 b_1 c^2 \\ +2a_2 b_1^2 c\rho^2 + 12a_2 b_0^2 c + 2a_2 b_1^2 c - 18a_2 b_0 b_1 c + a_2 b_1^2 \rho^2 + 6a_2 b_0^2 + a_2 b_1^2 - 9a_2 b_0 b_1 + 6a_0 a_2^2 + 6a_1^2 a_2 = 0. \\ 3\vartheta^2 a_2 b_1^2 - 6\vartheta^2 a_2 b_0 b_1 - 3a_2 b_1^2 c^2 + 6a_2 b_0 b_1 c^2 - 6a_2 b_1^2 c + 12a_2 b_0 b_1 c - 3a_2 b_1^2 + 6a_2 b_0 b_1 + 6a_1 a_2^2 = 0. \\ -2\vartheta^2 a_2 b_1^2 + 2a_2 b_1^2 c^2 + 4a_2 b_1^2 c + 2a_2 b_1^2 + 2a_2^3 = 0. \end{cases} \tag{89}$$

With the assistance of computational software such as Mathematica, we solve system 89, and a variety of solution sets is obtained, as follows:

Set 1

$$a_1 = 0,\ b_1 = -2b_0,$$
$$a_0 = -\frac{1}{2} \mathrm{i} b_0 \sqrt{(-\vartheta + c + 1)(\vartheta + c + 1)},$$
$$\rho = -\frac{1}{\sqrt{2}},\ a_2 = 0.$$

On substituting the above values of parameters into equation (87) and setting $S = 1$ in equation (88), we secure the soliton solution to equation (1) in terms of hyperbolic functions.

$$\theta_1(x, t) = -\frac{1}{2} \mathrm{i} \sqrt{(-\vartheta + c + 1)(\vartheta + c + 1)} \tanh\left(\frac{\eta}{2}\right) \times e^{\mathrm{i}((c+2)\rho t + \rho x)}, \tag{90}$$

$$\varphi_1(x, t) = -\frac{c\,|\theta_1(x, t)|^2}{(1 + \vartheta)(1 + \vartheta + c)}, \tag{91}$$

$$\psi_1(x, t) = \frac{c\,|\theta_1(x, t)|^2}{(\vartheta - 1)(\vartheta - 1 - c)}. \tag{92}$$

Set 2

$$\begin{aligned}
a_1 &= -\mathrm{i}b_0\sqrt{(-\vartheta + c + 1)(\vartheta + c + 1)},\\
b_1 &= -(1 + \sqrt{29})b_0,\\
a_0 &= -\frac{1}{2}\mathrm{i}b_0\sqrt{(-\vartheta + c + 1)(\vartheta + c + 1)},\\
\rho &= -\frac{1}{\sqrt{2}},\ a_2 = 0.
\end{aligned}$$

For $S = 1$, and solving equations (87) and (88) together, we get a soliton solution, as follows:

$$\theta_2(x, t) = \frac{i\sqrt{-\vartheta^2 + c^2 + 2c + 1}\,(\sinh(\eta) + \cosh(\eta) + 3)}{2(-\sinh(\eta) - \cosh(\eta) + \sqrt{29})} \times \mathrm{e}^{\mathrm{i}((c+2)\rho t + \rho x)}, \tag{93}$$

$$\varphi_2(x, t) = -\frac{c|\theta_2(x, t)|^2}{(1 + \vartheta)(1 + \vartheta + c)}, \tag{94}$$

$$\psi_2(x, t) = \frac{c|\theta_2(x, t)|^2}{(\vartheta - 1)(\vartheta - 1 - c)}. \tag{95}$$

Set 3

$$\begin{aligned}
a_1 &= -\frac{1}{2}\mathrm{i}b_1\sqrt{(-\vartheta + c + 1)(\vartheta + c + 1)},\\
a_0 &= 0,\ b_0 = 0,\ \rho = \frac{1}{\sqrt{2}},\\
a_2 &= \mathrm{i}b_1\sqrt{(-\vartheta + c + 1)(\vartheta + c + 1)}.
\end{aligned}$$

For $S = 1$, and solving equations (87) and (88) together, we get a singular soliton solution as follows:

$$\theta_3(x, t) = -\frac{1}{2}\mathrm{i}\sqrt{(-\vartheta + c + 1)(\vartheta + c + 1)}\coth\left(\frac{\eta}{2}\right) \times \mathrm{e}^{\mathrm{i}((c+2)\rho t + \rho x)}, \tag{96}$$

$$\varphi_3(x, t) = -\frac{c|\theta_3(x, t)|^2}{(1 + \vartheta)(1 + \vartheta + c)}, \tag{97}$$

$$\psi_3(x, t) = \frac{c|\theta_3(x, t)|^2}{(\vartheta - 1)(\vartheta - 1 - c)}. \tag{98}$$

Set 4

$$\begin{aligned}
a_1 &= 0,\ b_1 = 2b_0,\\
a_0 &= \frac{1}{2}\mathrm{i}b_0\sqrt{(-\vartheta + c + 1)(\vartheta + c + 1)},\ \rho = \frac{1}{\sqrt{2}},\\
a_2 &= -2\mathrm{i}b_0\sqrt{(-\vartheta + c + 1)(\vartheta + c + 1)}.
\end{aligned}$$

For $S = 1$, and solving equations (87) and (88) together, we get the dark soliton solution as follows:

$$\theta_4(x, t) = \frac{1}{2}i\sqrt{(-\vartheta + c + 1)(\vartheta + c + 1)}\tanh\left(\frac{\eta}{2}\right) \times \mathrm{e}^{\mathrm{i}((c+2)\rho t + \rho x)}, \tag{99}$$

$$\varphi_4(x, t) = -\frac{c\,|\theta_4(x, t)|^2}{(1 + \vartheta)(1 + \vartheta + c)}, \tag{100}$$

$$\psi_4(x,\,t)=\frac{c\,|\theta_4(x,\,t)|^2}{(\vartheta-1)(\vartheta-1-c)}. \tag{101}$$

Set 5

$$a_1=\frac{\mathrm{i}b_1\rho\sqrt{(-\vartheta+c+1)(\vartheta+c+1)}}{\sqrt{2}},$$
$$a_0=0,\ b_0=0,\ a_2=0.$$

For $S=1$, and solving equations (87) and (88) together, the plane-wave solution can be expressed as:

$$\theta_5(x,\,t)=\frac{i\rho\sqrt{(-\vartheta+c+1)(\vartheta+c+1)}}{\sqrt{2}}\times \mathrm{e}^{\mathrm{i}((c+2)\rho t+\rho x)}, \tag{102}$$

$$\varphi_5(x,\,t)=-\frac{c\,|\theta_5(x,\,t)|^2}{(1+\vartheta)(1+\vartheta+c)}, \tag{103}$$

$$\psi_5(x,\,t)=\frac{c\,|\theta_5(x,\,t)|^2}{(\vartheta-1)(\vartheta-1-c)}. \tag{104}$$

Set 6

$$a_1=2\mathrm{i}b_0\sqrt{(-\vartheta+c+1)(\vartheta+c+1)},\ b_1=-2b_0,$$
$$a_0=0,\ \rho=\mathrm{i},\ a_2=-2\mathrm{i}b_0\sqrt{(-\vartheta+c+1)(\vartheta+c+1)}.$$

For $S=1$, and solving equations (87) and (88) together, we obtain a singular wave solution as follows:

$$\theta_6(x,\,t)=-i\sqrt{(-\vartheta+c+1)(\vartheta+c+1)}\,\mathrm{csch}(\eta)\times \mathrm{e}^{\mathrm{i}((c+2)\rho t+\rho x)}, \tag{105}$$

$$\varphi_6(x,\,t)=-\frac{c\,|\theta_6(x,\,t)|^2}{(1+\vartheta)(1+\vartheta+c)}, \tag{106}$$

$$\psi_6(x,\,t)=\frac{c\,|\theta_6(x,\,t)|^2}{(\vartheta-1)(\vartheta-1-c)}. \tag{107}$$

For all the above sets, $\eta=-c\,t+\zeta_0+x$.

4. Results and discussion

In this section, a comparison is made between our outcomes and some existing results in published works. In this work, we utilized an analytical mathematical method to solve the CNLST equations. A variety of solutions were extracted, such as single (dark, singular), complex, and combined solitons as well as hyperbolic, plane wave, and trigonometric solutions. In published works, we have observed that Elboree [26] only discussed one solution for the CNLST system using He's semi-inverse variational principle, whereas Ma *et al* [25] only presented two solutions through the Darboux transformation. Yong *et al* [27] introduced eight solutions for the CNLST system using the truncated Painleve expansion and the direct quadrature method. Abdelrahman *et al* [28] discussed exact soliton solutions and employed three methods. If we compare these methods and the three proposed methods in this paper, our methods are more effective at providing many solutions than those other methods. Consequently, these methods are proficient, adequate, and robust for handling other NLPDEs in mathematical physics and applied mathematics. We observe that the results presented in this study could be helpful in explaining the physical meaning of various NLPDEs arising in the different fields of nonlinear sciences. To aid clear and good understanding, the absolute physical behaviors of some of the reported solutions are exhibited through 3D, 2D and contour graphs using suitable parameter values. To visualize the dynamics of trigonometric, dark, dark–singular, dark, and complex combined soliton and plane wave solutions which appear in equations (20), (29), (53), (57), (69), (84), (90), and (105), we present figures 1–8, respectively. From the physical description of some solutions and our discussion of the results, we conclude that our modified mathematical methods presented here are fruitful tools for investigating further results of nonlinear wave problems in applied science.

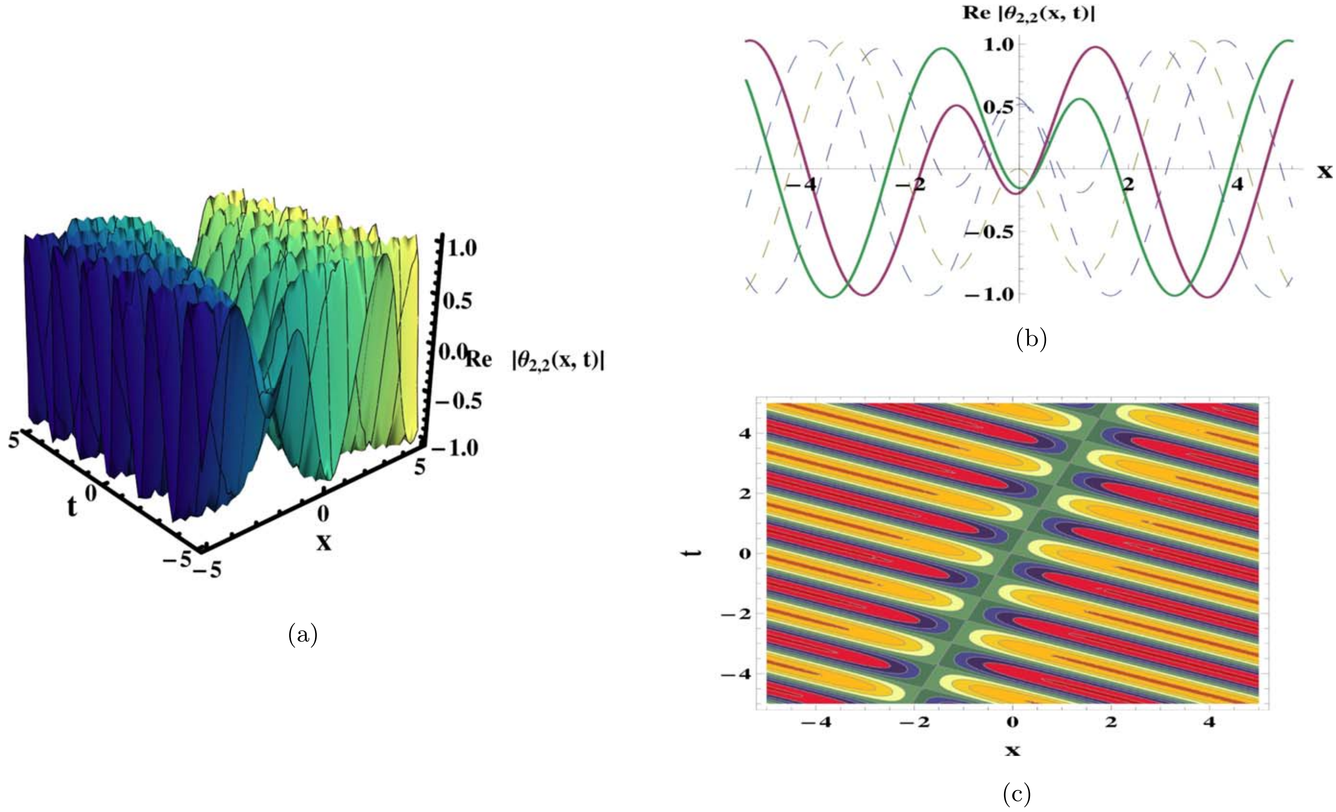

Figure 1. The plots of equation (20) are presented in 3D, 2D and contour wave profiles, respectively.

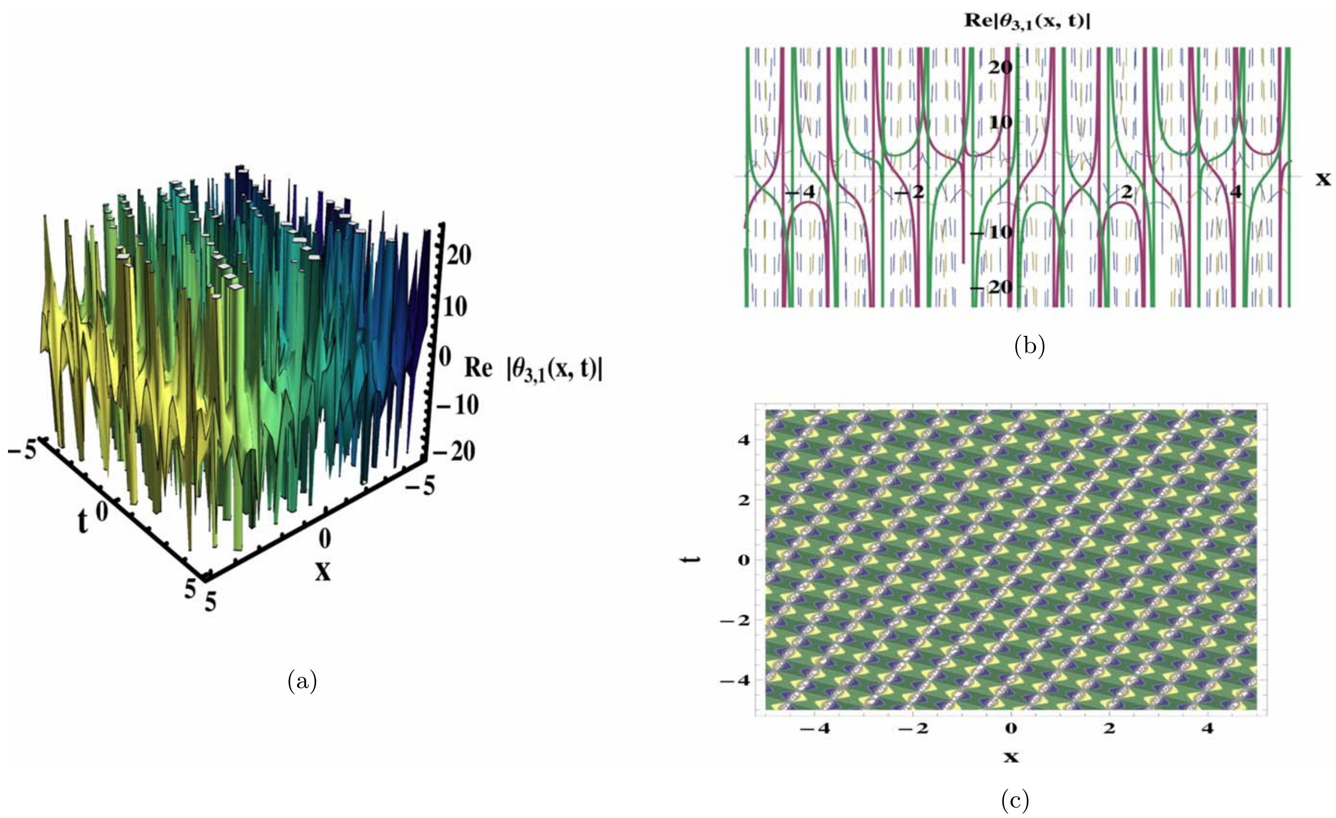

Figure 2. The plots of equation (29) are presented in 3D, 2D and contour wave profiles, respectively.

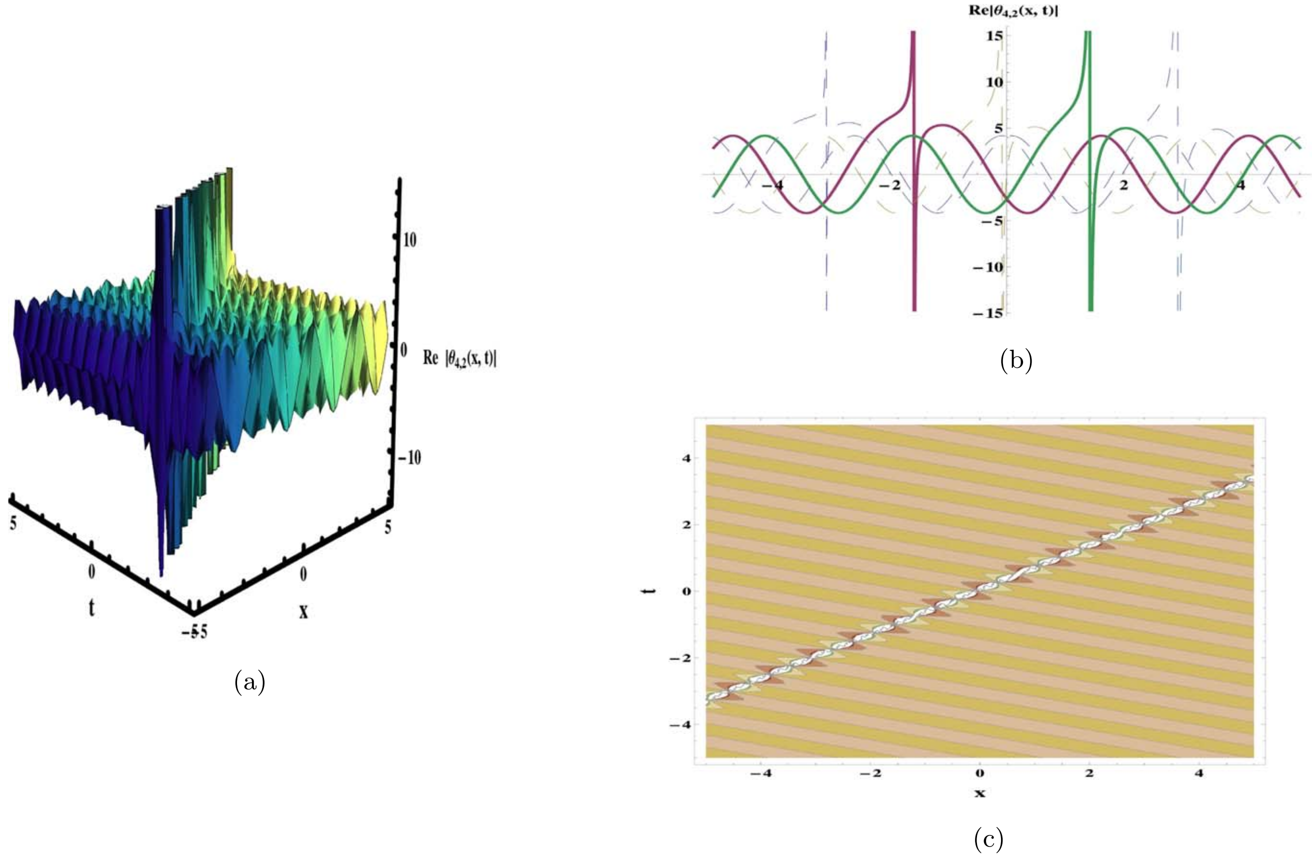

Figure 3. The plots of equation (53) are presented in 3D, 2D and contour wave profiles, respectively.

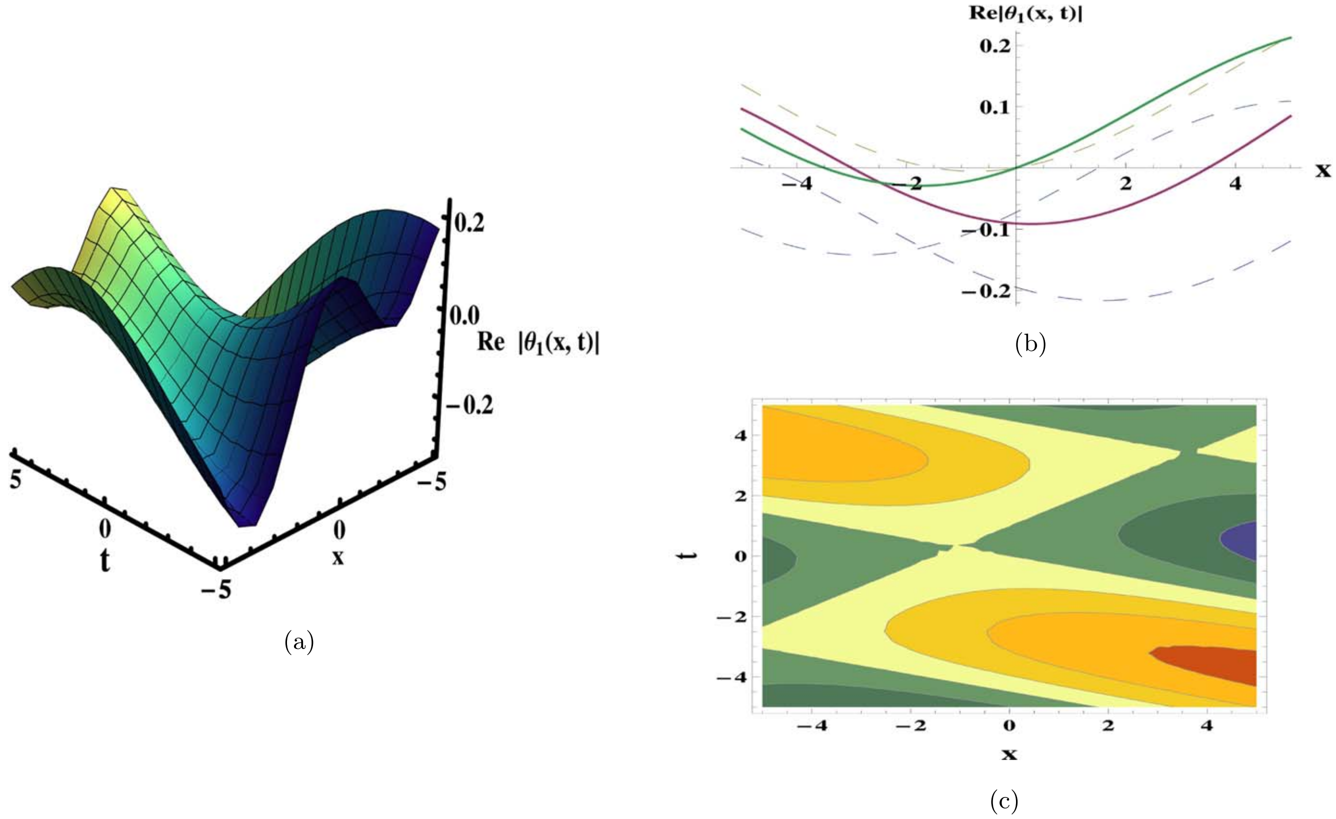

Figure 4. The plots of equation (57) are presented in 3D, 2D and contour wave profiles, respectively.

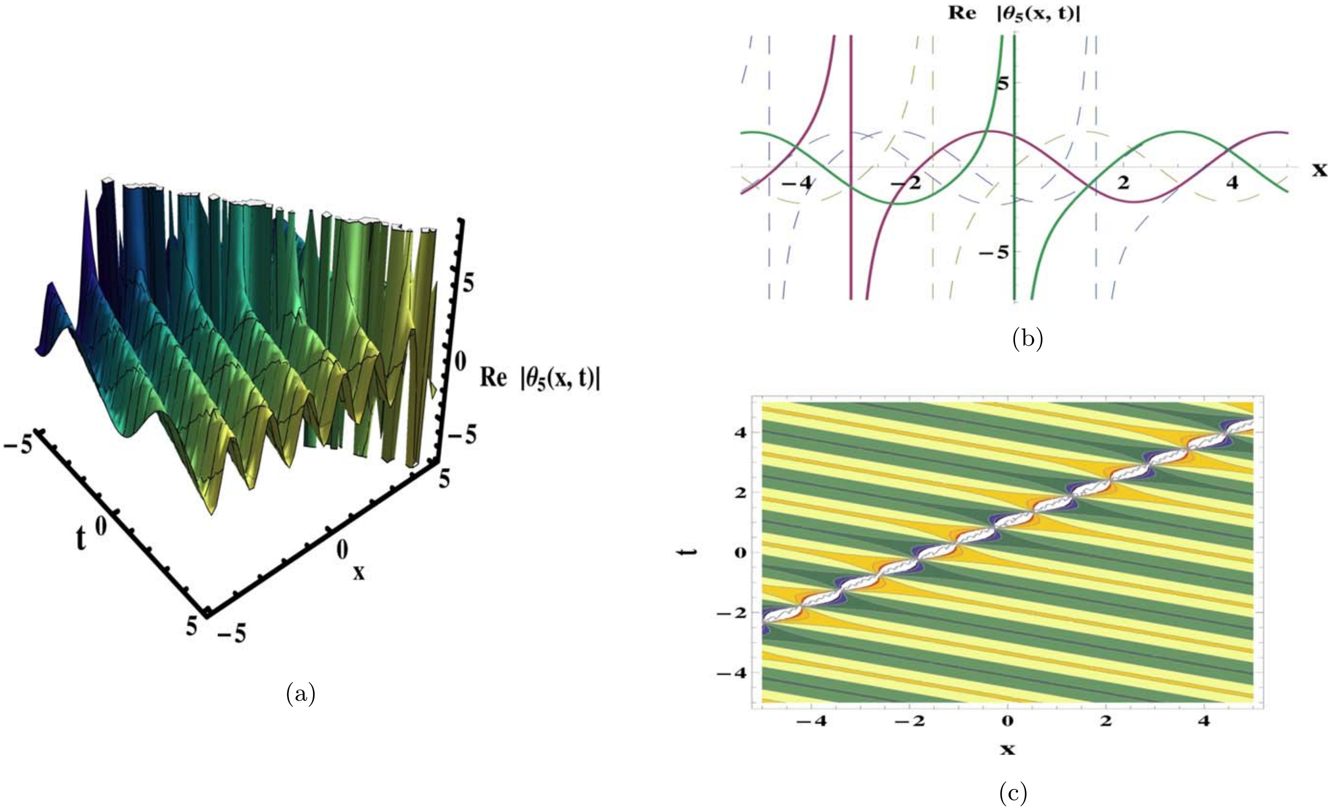

Figure 5. The plots of equation (69) are presented in 3D, 2D and contour wave profiles, respectively.

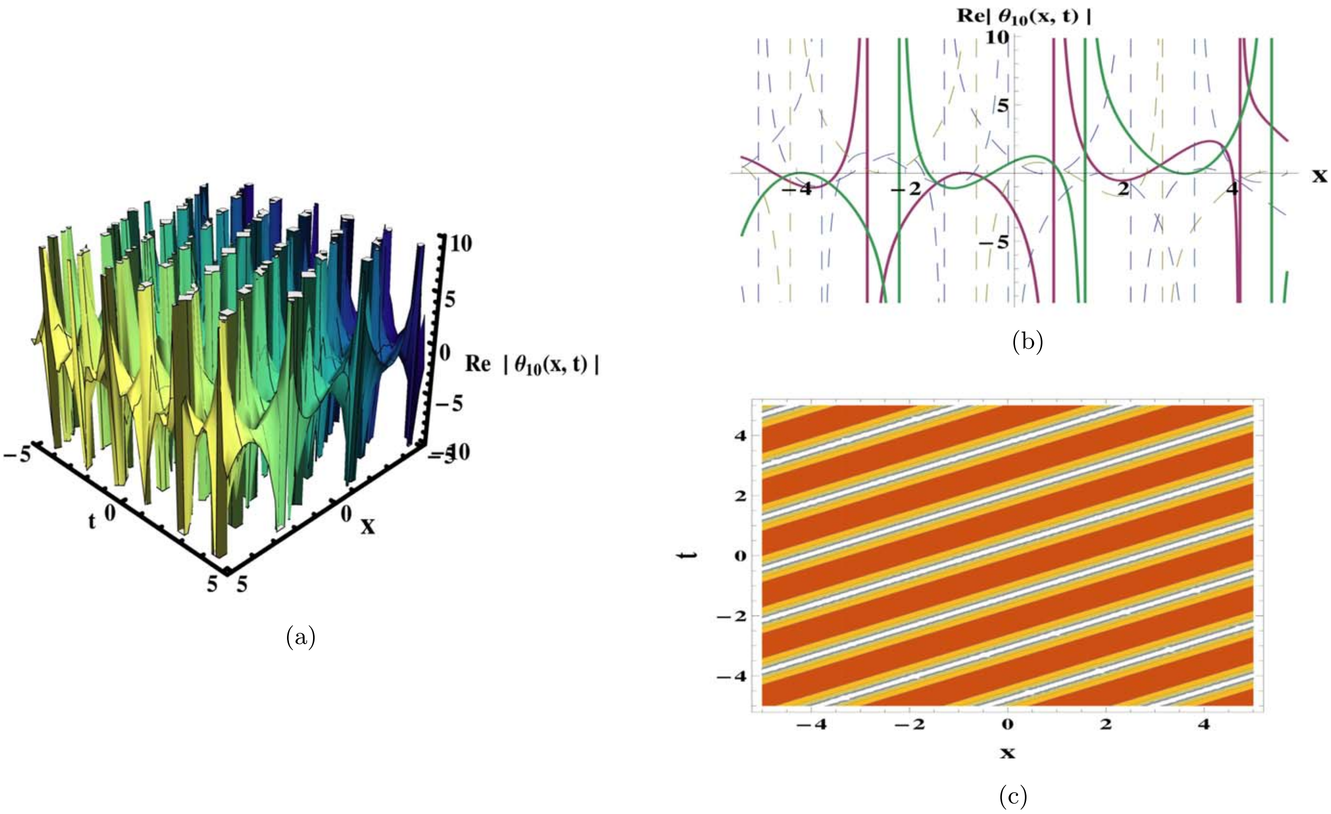

Figure 6. The plots of equation (84) are presented in 3D, 2D and contour wave profiles, respectively.

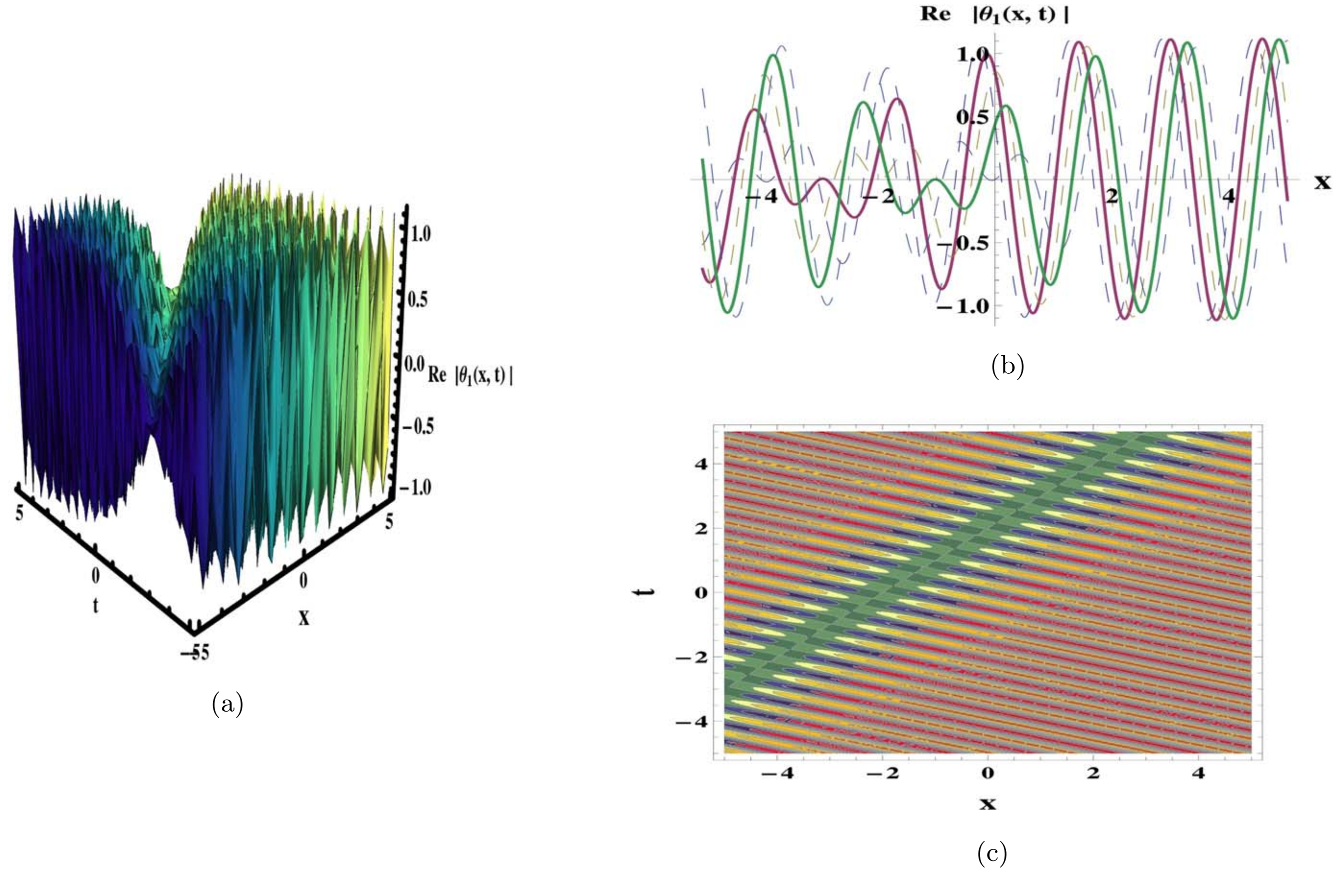

Figure 7. The plots of equation (90) are presented in 3D, 2D and contour wave profiles, respectively.

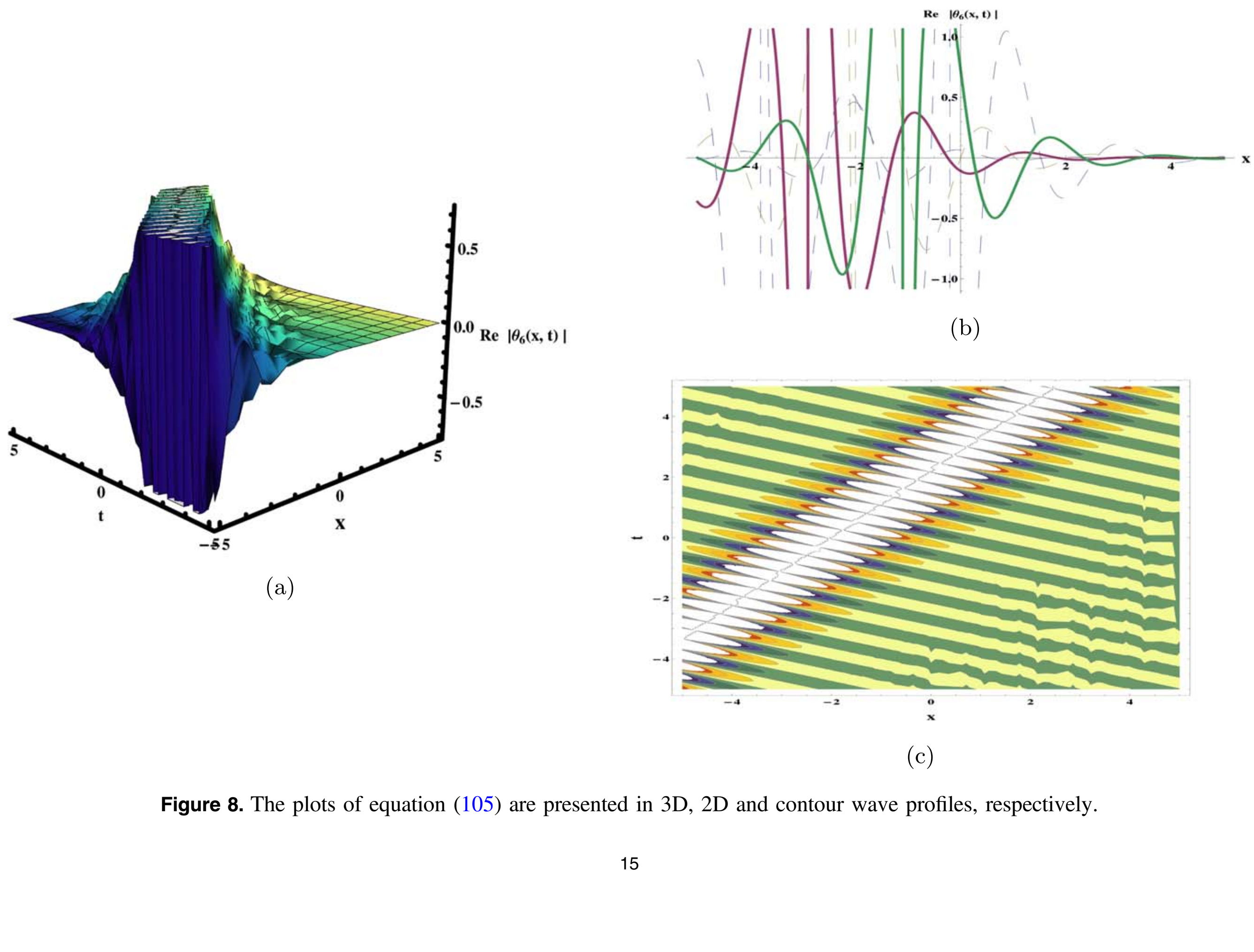

Figure 8. The plots of equation (105) are presented in 3D, 2D and contour wave profiles, respectively.

5. Conclusion

In this study, three innovative integration norms have been effectively implemented for the CNLST equations that have emerged in several fields of applied sciences. Different types of solution, such as single (dark, singular), complex, and combined solitons as well as hyperbolic, plane-wave, and trigonometric solutions were successfully obtained. All the solutions obtained were verified by substituting them back into the original equation via the software package Mathematica. The graphical representations of some solutions were also depicted using 2D, 3D and contour profiles that displayed the physical behaviors of some of the solutions obtained. These diverse solutions demonstrate the power, capability, consistency, and effectiveness of these methods and can be used to solve many other NLPDEs in mathematical physics. We observe that the results presented in this study could be helpful in explaining the physical meanings of various nonlinear evolution equations that have arisen in the different fields of nonlinear sciences. For instance, hyperbolic functions appears in different areas such as the calculation of special relativity, the Langevin function for magnetic polarization, the gravitational potential of a cylinder, and the calculation of the Roche limit in the profile of a laminar jet [29]. The wave results obtained confirm the value of this research and also have significant applications in mathematics and physics. Hence, our techniques, fortified by symbolic computation, provide an active and potent mathematical implement for solving diverse interesting nonlinear wave problems.

Acknowledgments

The authors would like to acknowledge the financial support provided for this research via the National Natural Science Foundation of China (11771407-52071298), ZhongYuan Science and Technology Innovation Leadership Program (214200510010), and the MOST Innovation Methodproject (2019IM050400). They also thank the reviewers for their valuable reviews and kind suggestions.

Conflict of interest

The authors declare that there are no conflicts of interest.

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