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Commun. Theor. Phys. **75** (2023) 035001 (11pp) https://doi.org/10.1088/1572-9494/acb205

Dynamical rational solutions and their interaction phenomena for an extended nonlinear equation

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Received 29 October 2022, revised 3 January 2023
Accepted for publication 11 January 2023
Published 17 March 2023

Abstract

In this paper, we analyze the extended Bogoyavlenskii-Kadomtsev-Petviashvili (eBKP) equation utilizing the condensed Hirota's approach. In accordance with a logarithmic derivative transform, we produce solutions for single, double, and triple M-lump waves. Additionally, we investigate the interaction solutions of a single M-lump with a single soliton, a single M-lump with a double soliton, and a double M-lump with a single soliton. Furthermore, we create sophisticated single, double, and triple complex soliton wave solutions. The extended Bogoyavlenskii-Kadomtsev-Petviashvili equation describes nonlinear wave phenomena in fluid mechanics, plasma, and shallow water theory. By selecting appropriate values for the related free parameters we also create three-dimensional surfaces and associated counter plots to simulate the dynamical characteristics of the solutions offered.

Keywords: simplified Hirota's method, lump solution, mixed solution, complex multiple soliton, eBKP equation

(Some figures may appear in colour only in the online journal)

1. Introduction

Nonlinear partial differential equations (NPDEs) are used to explain diverse significant nonlinear phenomena in nature that demonstrate significant properties, such as the existence of numerous conservation laws, soliton solutions, bi-Hamiltonian structures, and various symmetries [1]. The quest for analytical solutions to nonlinear partial differential equations is essential in scientific and engineering applications since it provides a wealth of information on the mechanisms of complicated physical phenomena. Numerous effective methods have been devised to seek exact solutions for NPDEs in mathematical physics, such as the Burgan *et al* method [2], the similarity transformations [3], the parabolic equation method [4], the new modified unified auxiliary equation method [5], the $(1/G')-$ expansion method [6, 7], Jacobi elliptic function expansion (JEFE) method [8], the simplified Hirota's method [9], the Kudrayshov approach and its modified version [10, 11], the modified auxiliary expansion method [12], and the generalized exponential rational function method and its modified version [13, 14], as well as some numerical methods [15–18]. Each of these methods has its characteristics, and the simplified Hirota method is commonly used owing to its efficiency and directness. In [19–28], the

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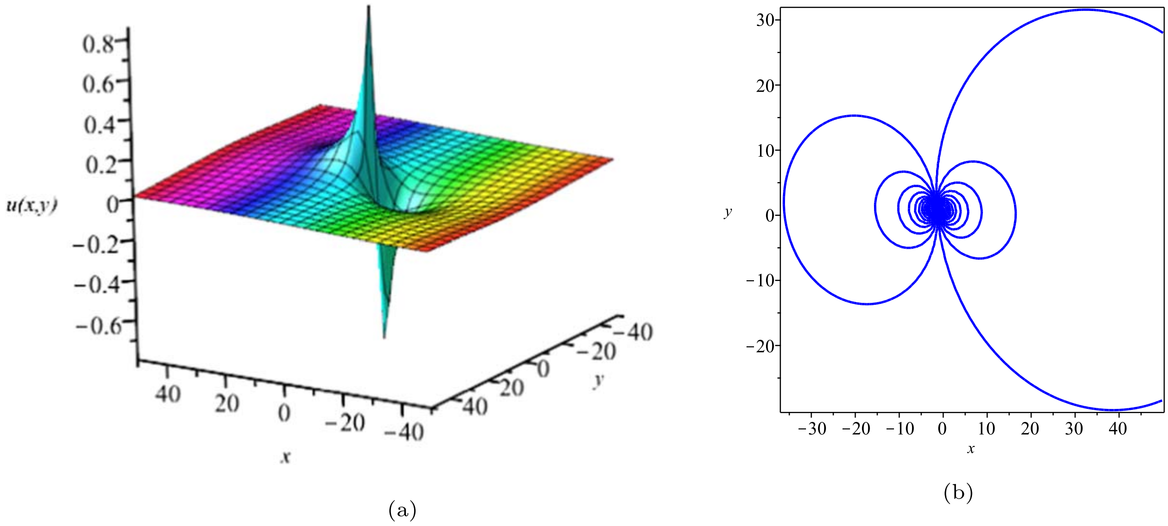


Figure 1. 3D profile and counter plot of equation (15) using $t=1$, $b=1.2$ $a=0.1$, $\beta=2$, $\alpha=2$ and $\gamma=2$.

authors have constructed multiple solitons, complexiton solutions, fusions, breather solutions, lump solutions, and mixed kink-lump and periodic lump solutions of some NPDEs by using the simplified Hirota's method. The Bogoyavlenskii-Kadomtsev-Petviashvili (BKP) equation is given by [29]

$$u_{xxt} + u_{xxxy} + 12u_{xx}u_{xy} + 8u_x u_{xxy} + 4u_{xxx}u_y = \alpha u_{yyy}, \tag{1}$$

Equation (1) represents nonlinear wave phenomena in fluid mechanics, plasma physics, and shallow water theory. The solution function $u(x, y, t)$ stands for the wave amplitude. When $\alpha=0$, equation (1) reduces to the Calogero-Bogoyavlenskii-Schiff (CBS) equation. When $\alpha\neq 0$, equation (1) represents a modification of the CBS equation, often known as a modification to the Kadomtsev-Petviashvili (KP) equation [30]. Equation (1) was actually extracted in [31–33] by reducing the renowned (3 + 1)-dimensional KP equation. Equation (1) represents the spread of non-linear waves in several scientific fields, like fluid mechanics, plasma, and shallow waves.

Wazwaz developed an extended form of the BKP equation (1). The Painlevé analysis has been used to explore the integrability of the eBKP equation. The eBKP has the following form [34]:

$$u_{xxt} + u_{xxxxy} + 12u_{xx}u_{xy} + 8u_x u_{xxy} + 4u_{xxx}u_y = \alpha u_{yyy} + \beta u_{xxx} + \gamma u_{xxy}. \tag{2}$$

Equivalently, it reads

$$(u_{xt} + u_{xxxy} + 8u_x u_{xy} + 4u_{xx}u_y)_x = \alpha u_{yyy} + (\beta u_x + \gamma u_y)_{xx}. \tag{3}$$

It is clear that the extended form equation (3) acknowledges two extra weak terms of dispersion, namely u_{xxx} and u_{xxy}. When $\alpha=\beta=\gamma=0$, equation (3) is simplified to the CBS equation [34]

$$u_{xt} + u_{xxxy} + 8u_x u_{xy} + 4u_{xx}u_y = 0. \tag{4}$$

Nevertheless, for $\beta=\gamma=0$, equation (3) is reduced to the BKP equation (1). More specifically, for $\alpha=0$ the eBKP equation (3) can be simplified to [34]:

$$u_{xt} + u_{xxxy} + 8u_x u_{xy} + 4u_{xx}u_y = (\beta u_x + \gamma u_y)_x. \tag{5}$$

In [34], multiple soliton solutions of equation (2) have been extracted via the Hirota approach. In this paper, we will study multiple M-lump waves and their interactions with single, and double soliton solutions. Also, we will derive complex single, double, and triple soliton solutions. Lump waves are rational solutions that locate in all directions of space [35–39]. For the first time, Hirota used a direct method to obtain a soliton solution [40, 41]. Manakov *et al* were the first to discover the lump wave solutions [42]. Then, Satsuma and Ablowitz *et al* developed the long-wave limit method to construct the multiple lump (M-lump) waves solutions [43]. Zhang *et al*, developed the extended long-wave limit method to study the high-order M-lump solutions [44]. Subsequently, the interaction of lump waves and solitons has developed and so have many interactive solutions, including lump-kink and lump-strip solutions [45, 46]. Ma used the positive quadratic function to construct the single lump wave of the KP equation [47]. Thereupon, several functions are used to construct different types of rational solutions [48, 49].

2. Rational solutions to the eBKP equation

In this part, we propose to provide single, double, and triple M-lump solutions and also generate single, double, and triple complex soliton solutions to the eBKP equation. We also investigate the mixed single M-lump wave with single and double soliton solutions from the perspective of the suggested equation.

Consider the following transformation:

$$u(x, y, t) = (\ln f(x, y, t))_x. \tag{6}$$

$$u(x,\, y,\, t) = \frac{2x + r_1 y + r_2 y + t(r_1^3\alpha + \beta + r_1\gamma) + t(r_2^3\alpha + \beta + r_2\gamma)}{-\frac{4}{(r_1 - r_2)^2\alpha} + (x + r_1 y + t(r_1^3\alpha + \beta + r_1\gamma))(x + r_2 y + t(r_2^3\alpha + \beta + r_2\gamma))}, \tag{15}$$

Plugging equation (6) into equation (2), we obtain

$$\begin{aligned}
&-6\alpha f_y^3 f_x + 6\alpha ff_y{}_y^2 f_{xy} + 6f_x^2(-2f_{xy}f_{xx} + f_x(f_t - \beta f_x + 2f_{xxy}))\\
&+6f(-f_t f_x f_{xx} + f_{xy}f_{xx}^2 + f_x^2(-f_{xt} + \gamma f_{xy} + 2\beta f_{xx} - 2f_{xxxy}))\\
&+f^2(-\alpha f_{yyy}f_x - 3\alpha f_{yy}f_{xy} + 3f_{xt}f_{xx} - 3\gamma f_{xy}f_{xx} - 3\beta f_{xx}^2\\
&+3f_x f_{xxt} - 3\gamma f_x f_{xxy} + f_t f_{xxx} - 4\beta f_x f_{xxx} - 2f_{xxy}f_{xxx}\\
&+2f_{xx}f_{xxxy} + f_{xy}f_{xxxx} + 5f_x f_{xxxxy})\\
&+f_y(-6f_x(\gamma f_x^2 + f_2^2 - 2f_x f_{xxx}) + 6ff_x(\alpha f_{yy} + \gamma f_{xx} - f_{xxxx})\\
&+f^2(-3\alpha f_{xyy} - \gamma f_{xxx} + f_{xxxxx}))\\
&+f^3(\alpha f_{xyyy} - f_{xxxt} + \gamma f_{xxxy} + \beta f_{xxxx} - f_{xxxxy}) = 0.
\end{aligned} \tag{7}$$

Motivated by Hirota's bilinear method, the N-soliton solution in general form is given by [50]

$$f = f_N = \sum_{\mu=0,1} \exp\left(\sum_{1\leqslant i<j}^{N} \mu_i\mu_j A_{ij} + \sum_{i=1}^{N}\mu_i v_i\right) \tag{8}$$

where

$$v_i = q_i(x + r_i y + s_i t) + \alpha_i, \tag{9}$$

$$\mathrm{e}^{A_{ij}} = 1 - \frac{4q_i q_j}{(q_i + q_j)^2 + (r_i - r_j)^2\alpha}. \tag{10}$$

The $\sum_{\mu=0,1}$ notation indicates summing over all possible combinations of $\mu_1 = 0,\ 1,\ \mu_2 = 0,\ 1,\ldots,\mu_N = 0,\ 1$; the $\sum_{i<j}^{(N)}$ summation is over all possible combinations of the N elements with the specific condition $i < j$.

The first three terms of equation (8) have the following forms:

$$f_1 = 1 + \mathrm{e}^{v_1}, \tag{11}$$

$$f_2 = 1 + \mathrm{e}^{v_1} + \mathrm{e}^{v_2} + \mathrm{e}^{v_1+v_2+A_{12}}, \tag{12}$$

$$\begin{aligned}f_3 = {}&1 + \mathrm{e}^{v_1} + \mathrm{e}^{v_2} + \mathrm{e}^{v_3} + \mathrm{e}^{v_1+v_2+A_{12}} + \mathrm{e}^{v_1+v_3+A_{13}}\\ &+ \mathrm{e}^{v_2+v_3+A_{23}} + c\,\mathrm{e}^{v_1+v_2+v_3+A_{123}}.\end{aligned} \tag{13}$$

2.1. M-lump solutions

We explore a single, double, and triple M-lump solution of equation (2) in this portion. To obtain one M-lump solution, the long-wave limit method will be used, and taking $q_i \to 0$, $\frac{q_1}{q_2} = O(1)$, and $\mathrm{e}^{\alpha_i} = -1$, $(i = 1, 2)$, so equation (11) reduces to

$$f_2 = \phi_1\phi_2 + B_{12}, \tag{14}$$

where $\phi_1 = x + r_1 y + s_1 t, \phi_2 = x + r_2 y + s_2 t$, $B_{12} = -\frac{4}{(r_1 - r_2)^2\alpha}$, $s_1 = r_1^3\alpha + \beta + r_1\gamma$, and $s_2 = r_2^3\alpha + \beta + r_2\gamma$,

Plugging equation (14) in equation (6), one M-lump solution will be obtained,

where $r_1 = a + ib$, and $r_2 = a - ib$. equation (15) represents a one M-lump solution of equation (2) as shown in figure 1.

To obtain a 2-M-lump solution of equation (2), we take $q_m \to 0$, $\alpha_i = -1(i = 1, 2, 3, 4)$ in equation (7), and we get

$$\begin{aligned}f_4 = {}&\phi_1\phi_2\phi_3\phi_4 + B_{12}\phi_3\phi_4 + B_{13}\phi_2\phi_4 + B_{14}\phi_2\phi_3\\ &+ B_{23}\phi_1\phi_4 + B_{24}\phi_1\phi_3 + B_{34}\phi_1\phi_2 + B_{12}\phi_{34}\\ &+ B_{13}B_{24} + B_{14}B_{23},\end{aligned} \tag{16}$$

$$\phi_i = x + r_i y + s_i t, \tag{17}$$

$$B_{ij} = -\frac{4}{(r_i - r_j)^2\alpha}, \tag{18}$$

and

$$s_i = r_i^3\alpha + \beta + r_i\gamma. \tag{19}$$

Using equation (16) into equation (6), we have a two M-lump solution of equation (2) as given in figure 2.

To generate a 3-M-lump solution of equation (2), we take $q_m \to 0$, $\alpha_i = -1(i = 1, 2, 3, 4, 5, 6)$ in equation (7), which give

$$\begin{aligned}f_6 = {}&\phi_1\phi_2\phi_3\phi_4\phi_5\phi_6 + B_{12}B_{34}B_{56} + B_{12}B_{35}B_{46}\\
&+ B_{12}B_{45}B_{36} + B_{13}B_{24}B_{56} + B_{13}B_{25}B_{46} + B_{13}B_{45}B_{26}\\
&+ B_{23}B_{14}B_{56} + B_{14}B_{25}B_{36} + B_{14}B_{35}B_{26} + B_{24}B_{15}B_{36}\\
&+ B_{34}B_{15}B_{26} + B_{23}B_{15}B_{46} + B_{23}B_{45}B_{16} + B_{24}B_{35}B_{16}\\
&+ B_{34}B_{25}B_{16} + \phi_2\phi_3\phi_4\phi_5 B_{16} + \phi_2\phi_3\phi_5\phi_6 B_{14} + \phi_2\phi_3\phi_4\phi_6 B_{15}\\
&+ \phi_3\phi_4\phi_5\phi_6 B_{12} + \phi_2\phi_4\phi_5\phi_6 B_{13} + \phi_1\phi_2\phi_4\phi_6 B_{35}\\
&+ \phi_1\phi_2\phi_4\phi_5 B_{36} + \phi_1\phi_4\phi_5\phi_6 B_{23} + \phi_1\phi_3\phi_5\phi_6 B_{24} + \phi_1\phi_3\phi_4\phi_6 B_{25}\\
&+ \phi_1\phi_3\phi_4\phi_5 B_{26} + \phi_1\phi_2\phi_3\phi_4 B_{56} + \phi_1\phi_2\phi_3\phi_6 B_{45}\\
&+ \phi_1\phi_2\phi_3\phi_5 B_{46} + \phi_1\phi_2\phi_5\phi_6 B_{34} + \phi_1\phi_2 B_{34}B_{56} + \phi_1\phi_2 B_{35}B_{46}\\
&+ \phi_1\phi_2 B_{45}B_{36} + \phi_1 B_{23}\phi_4 B_{56} + \phi_1 B_{23}B_{45}\phi_6\\
&+ \phi_1 B_{23}\phi_5 B_{46} + \phi_1\phi_3 B_{24}B_{56} + \phi_1\phi_6 B_{24}B_{35} + \phi_1\phi_5 B_{24}B_{36}\\
&+ \phi_1\phi_3 B_{25}B_{46} + \phi_1\phi_6 B_{34}B_{25} + \phi_1\phi_4 B_{25}B_{36}\\
&+ \phi_1\phi_3 B_{45}B_{26} + \phi_1\phi_5 B_{34}B_{26} + \phi_1\phi_4 B_{35}B_{26} + \phi_4\phi_5 B_{12}B_{36}\\
&+ \phi_3\phi_4 B_{12}B_{56} + \phi_3\phi_6 B_{12}B_{45} + \phi_3\phi_5 B_{12}B_{46}\\
&+ \phi_5\phi_6 B_{12}B_{34} + \phi_4\phi_6 B_{12}B_{35} + \phi_5\phi_6 B_{13}B_{24} + \phi_4\phi_6 B_{13}B_{25}\\
&+ \phi_4\phi_5 B_{13}B_{26} + \phi_2\phi_4 B_{13}B_{56} + \phi_2\phi_6 B_{13}B_{45}\\
&+ \phi_2\phi_5 B_{13}B_{46} + \phi_2\phi_3 B_{14}B_{56} + \phi_2\phi_6 B_{14}B_{35} + \phi_2\phi_5 B_{14}B_{36}\\
&+ \phi_5\phi_6 B_{23}B_{14} + \phi_3\phi_6 B_{14}B_{25} + \phi_3\phi_5 B_{14}B_{26}\\
&+ \phi_4\phi_6 B_{23}B_{15} + \phi_3\phi_6 B_{24}B_{15} + \phi_3\phi_4 B_{15}B_{26} + \phi_2\phi_3 B_{15}B_{46}\\
&+ \phi_2\phi_6 B_{34}B_{15} + \phi_2\phi_4 B_{15}B_{36} + \phi_2\phi_4 B_{35}B_{16}\\
&+ \phi_4\phi_5 B_{23}B_{16} + \phi_3\phi_5 B_{24}B_{16} + \phi_3\phi_4 B_{25}B_{16}\\
&+ \phi_2\phi_3 B_{45}B_{16} + \phi_2\phi_5 B_{34}B_{16},\end{aligned} \tag{20}$$

where B_{ij}, ϕ_i, and s_i are given in the earlier steps.

Plugging equation (20) into equation (6), we have a three M-lump solution of equation (2) as seen in figure 3.

2.2. Mixed between one M-lump solution and one soliton solution

We generate the mixed single M-lump solution with a single soliton solution in this part. To do that, we will take

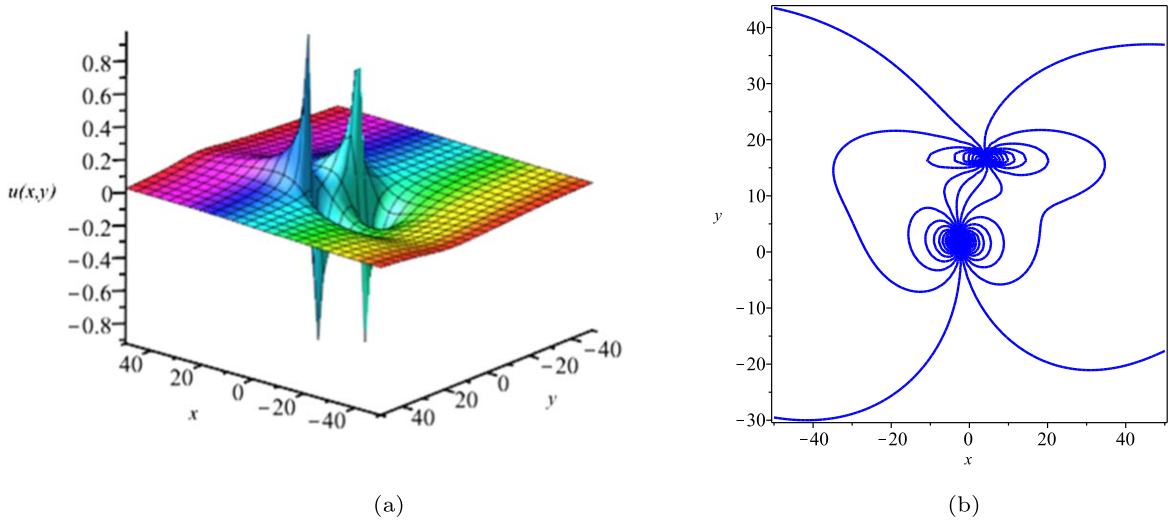


Figure 2. 3D profile and counter plot of equation (2) using $t = 2$, $a_1 = 0.1$, $b_1 = 1.2$, $\beta = 1.5$, $a_2 = 0.2$, $b_2 = 3$, $\gamma = 0.5$ and $\alpha = 1$.

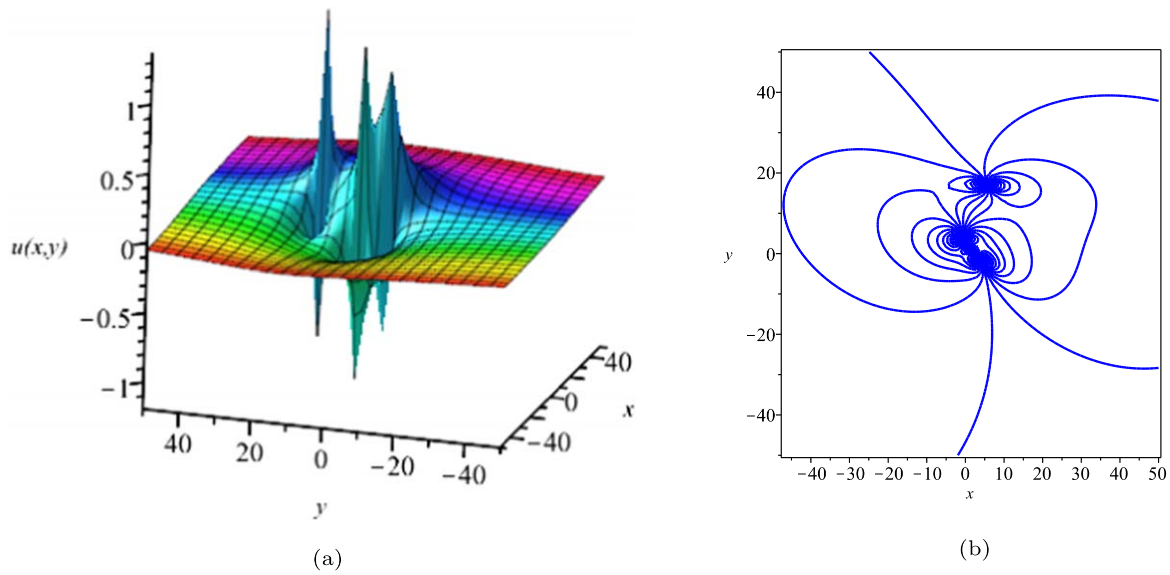


Figure 3. 3D profiles and counter plot of equation (2), using $t = 2$, $a_1 = 0.1$, $b_1 = 1.52$, $a_2 = 0.2$, $b_2 = 31.9$, $\gamma = 2$, $\beta = 1.5$, $\alpha = 3$, $a_3 = 0.5$ and $b_3 = 1$.

equation (12) into account and use the limit $q_\delta \to 0$, $(\delta = 1, 2)$ and $\frac{q_1}{q_2} = O(1)$. Then f_3 can be rewritten as follows:

$$f_3 = \phi_1\phi_2 + B_{12} + (\phi_1\phi_2 + B_{12} + C_{23}\phi_1 + C_{13}\phi_2 + C_{13})e^{v_3}, \tag{21}$$

where B_{12} is given in the previous subsection, and v_3 is given in equation (8). The constants C_{13} and C_{23} are given as follows

$$C_{13} = -\frac{4q_3}{q_3^2 + r_1^2\alpha - 2r_1r_3\alpha + r_3^2\alpha},$$
$$C_{23} = -\frac{4q_3}{q_3^2 + r_2^2\alpha - 2r_2r_3\alpha + r_3^2\alpha}. \tag{22}$$

Using equation (21) into equation (6), we get mixed of single M-lump solution with a single soliton solution as seen in figure 4.

2.3. Mixed solutions between single M-lump solution and double soliton solution

We provide the mixed single M-lump solution and double soliton solution in this part. For this purpose, we take equation (7) into account and take $q_n \to 0$, $(n = 1, 2$, and $\frac{q_1}{q_2} = O(1))$. Then f_4 may be rewritten as follows

$$f_4 = \phi_1\phi_2 + B_{12} + \Omega_1 e^{v_3} + \Omega_2 e^{v_4} + A_{34}e^{v_3+v_4}(\Omega_1 + \Omega_2 - \phi_1\phi_2 + B_{12} + C_{13}C_{24} + C_{14}C_{23}), \tag{23}$$

where $\Omega_2 = (\phi_1\phi_2 + B_{12} + C_{24}\phi_1 + C_{14}\phi_2 + C_{14}C_{24})$, ϕ_1, ϕ_2, and B_{12} are given in the previous section, v_3, v_4 are given

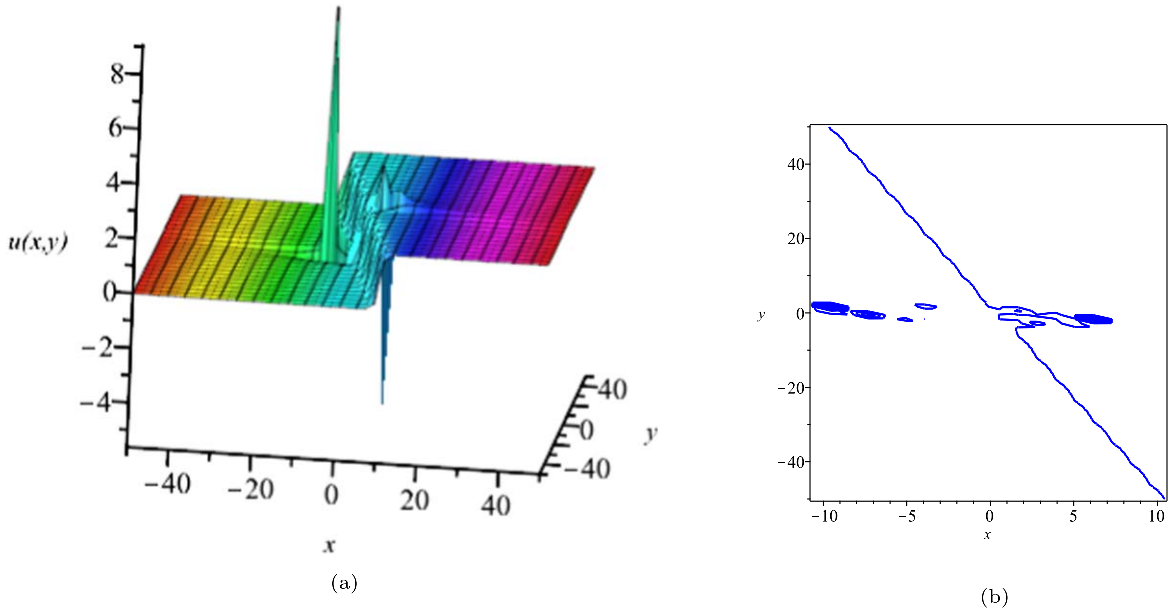


Figure 4. 3D profile of mixed of single M-lump solution with single soliton solution of equation (2) for the values of $t=2$, $a=\frac{1}{2}$, $b=2$, $q_3=2$, $r_3=2$, $\gamma=3$, $\beta=1$ and $\alpha=-\frac{1}{2}$.

in equation (8), and C_{j4}, $(j=1,2)$ is given as follows

$$C_{j4}=-\frac{4q_4}{q_4^2+r_j^2\alpha-2r_jr_4\alpha+r_4^2\alpha}. \tag{24}$$

The interaction between a single M-lump solution and a double soliton solution is presented in figure 5.

2.4. Mixed between a two M-lump solution and a one soliton solution

Here, we extract the mixed two M-lump solution and one soliton solution motivated by [23], we use

$$\begin{aligned} f_5=&\phi_1\phi_2\phi_3\phi_4+B_{34}\phi_1\phi_2+B_{24}\phi_1\phi_3\\ &+B_{23}\phi_1\phi_4+B_{14}\phi_2\phi_3+B_{13}\phi_2\phi_4\\ &+B_{12}\phi_3\phi_4+Q\mathrm{e}^{q_5(x+r_5y+s_5t)+\alpha_5}+B_{14}B_{23}\\ &+B_{13}B_{24}+B_{12}B_{34}, \end{aligned} \tag{25}$$

where $r_3=e+if$, $r_4=e-if$, and $Q=\phi_1\phi_2\phi_3\phi_4+C_{45}\phi_1\phi_2\phi_3+C_{15}\phi_2\phi_3\phi_4+C_{25}\phi_1\phi_3\phi_4+C_{35}\phi_1\phi_2\phi_4+(B_{34}+C_{35}C_{45})\phi_1\phi_2+(B_{24}+C_{25}C_{45})\phi_1\phi_3+(B_{14}+C_{15}C_{45})\phi_2\phi_3+(B_{23}+C_{25}C_{35})\phi_1\phi_4+(B_{13}+C_{15}C_{35})\phi_2\phi_4+(B_{12}+C_{15}C_{25})\phi_3\phi_4+(B_{34}C_{25}+B_{24}C_{35}+B_{23}C_{45}+C_{25}C_{35}C_{45})\phi_1+(B_{34}C_{15}+B_{14}C_{35}+B_{13}C_{45}+C_{15}C_{35}C_{45})\phi_2+(B_{24}C_{15}+B_{14}C_{25}+B_{12}C_{45}+C_{15}C_{25}C_{45})\phi_3+(B_{23}C_{15}+B_{13}C_{25}+B_{12}C_{35}+C_{15}C_{25}C_{35})\phi_4+B_{14}B_{23}+B_{13}B_{24}+B_{12}B_{34}+B_{34}C_{15}C_{25}+B_{24}C_{15}C_{35}+B_{14}C_{25}C_{35}+B_{23}C_{15}C_{45}+B_{13}C_{25}C_{45}+B_{12}C_{35}C_{45}+C_{15}C_{25}C_{35}C_{45}$.

Plugging equation (25) into equation (6), we get the mix between the two M-lump solution and the one soliton wave solution as presented in figure 6.

2.5. Complex soliton solutions

In this subsection, we generate complex multiple soliton solutions of equation (2). Rotschild *et al* illustrated experimentally that the lone range of nonlocality enables the formation of several scalar solitons possessing complex features from dipole tripoles to quadrupoles, to necklaces [51]. Wazwaz introduced a complex algorithm of Hirota's simple method in order to determine multiple complex solutions [52].

To derive mentioned solutions, one may use

$$s_n=-q_n^2r_n+r_n^3\alpha+\beta+r_n\gamma,\quad n=1,2,3. \tag{26}$$

To identify complex single, double, and triple soliton solutions, one uses

$$f_1=\mathrm{i}+\mathrm{e}^{q_1(x+r_1y+s_1t+\alpha_1)}, \tag{27}$$

$$f_2=\mathrm{i}+\mathrm{e}^{\nu_1}+\mathrm{e}^{\nu_2}+\mathrm{e}^{\nu_1+\nu_2+A_{12}}, \tag{28}$$

$$\begin{aligned} f_3=&\mathrm{i}+\mathrm{e}^{\nu_1}+\mathrm{e}^{\nu_2}+\mathrm{e}^{\nu_3}+\mathrm{e}^{\nu_1+\nu_2+A_{12}}+\mathrm{e}^{\nu_1+\nu_3+A_{13}}\\ &+\mathrm{e}^{\nu_2+\nu_3+A_{23}}+\mathrm{i}\mathrm{e}^{\nu_1+\nu_2+\nu_3+A_{123}}, \end{aligned} \tag{29}$$

where

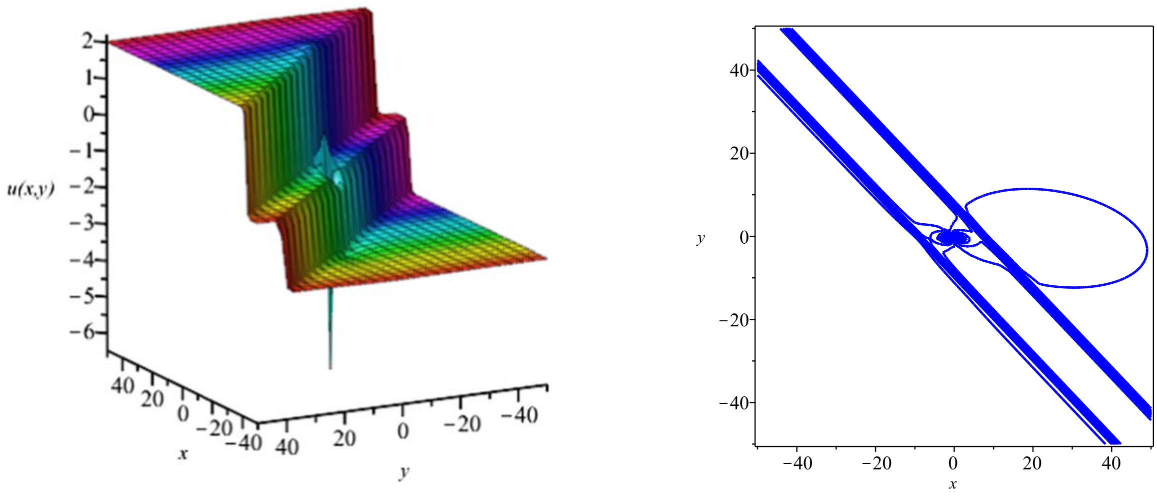
$$\mathrm{e}^{A_{ij}}=\frac{4\mathrm{i}q_iq_j}{(q_i+q_j)^2+(r_i-r_j)^2\alpha}-\mathrm{i},\quad \text{and } \mathrm{i}=\sqrt{-1}. \tag{30}$$

Inserting equation (27) in equation (6), we have a complex 1-soliton solution as shown in figure 7.

$$u(x,y,t)=\frac{\mathrm{e}^{q_1(x+r_1y+t(-q_1^2r_1+r_1^3\alpha+\beta+r_1\gamma))}q_1}{\mathrm{i}+\mathrm{e}^{q_1(x+r_1y+t(-q_1^2r_1+r_1^3\alpha+\beta+r_1\gamma))}}. \tag{31}$$

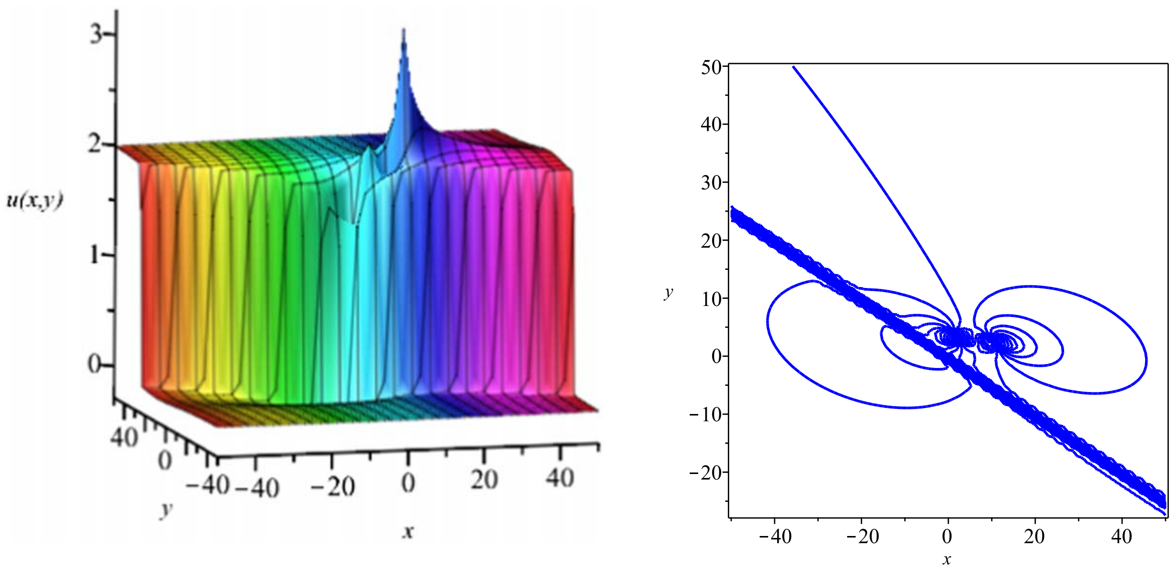
Inserting equation (28) into equation (6), we have a complex two-soliton solution, as seen in figure 8.

Using equation (29) in equation (6), we have a complex three-soliton solution, as given in figure 9.



(a) (b)

Figure 5. 3D profile and their counter plot of the mixed of single M-lump wave and double soliton solution, for the values of $t = 1$, $a = \frac{1}{2}$, $b = 3$, $r_3 = 1$, $q_3 = 4$, $r_4 = 1$, $q_4 = 3$, $\gamma = 1$, $\beta = 1$ and $\alpha = \frac{1}{2}$.



(a) (b)

Figure 6. 3D profile and counter plot of the mixed double M-lump solution and single soliton solution, when $a_1 = \frac{1}{2}$, $b_1 = 2$, $q_5 = 2$, $r_5 = 2$, $a_2 = \frac{1}{4}$, $b_2 = 2$, $\gamma = 1$, $\beta = \frac{1}{2}$, $t = 1$ and $\alpha = 1$.

3. Conclusion

In this study, we analyze the extended Bogoyavlenskii-Kadomtsev-Petviashvili equation using the streamlined Hirota's approach. First, we turn the suggested equation into a quadratic form using a logarithmic derivative transformation. Then, we produce M-lump solutions with one, two, and three lumps. The interactions between a single M-lump wave and a single soliton solution, a single M-lump wave, and a double soliton solution, and a double M-lump wave and a single soliton solution are also investigated. Furthermore, we have developed sophisticated single, double, and triple solitons. The created 3D profiles and associated contour plots help to better comprehend the dynamical properties of the built solutions. All built-in solutions, to the best of our knowledge, satisfy equation (2). In future work, some new features and physical patterns for the governing equation will be considered from a different point of view such as fractional calculus and some extended classical derivatives which include M-truncated, conformable, and beta derivatives among others.

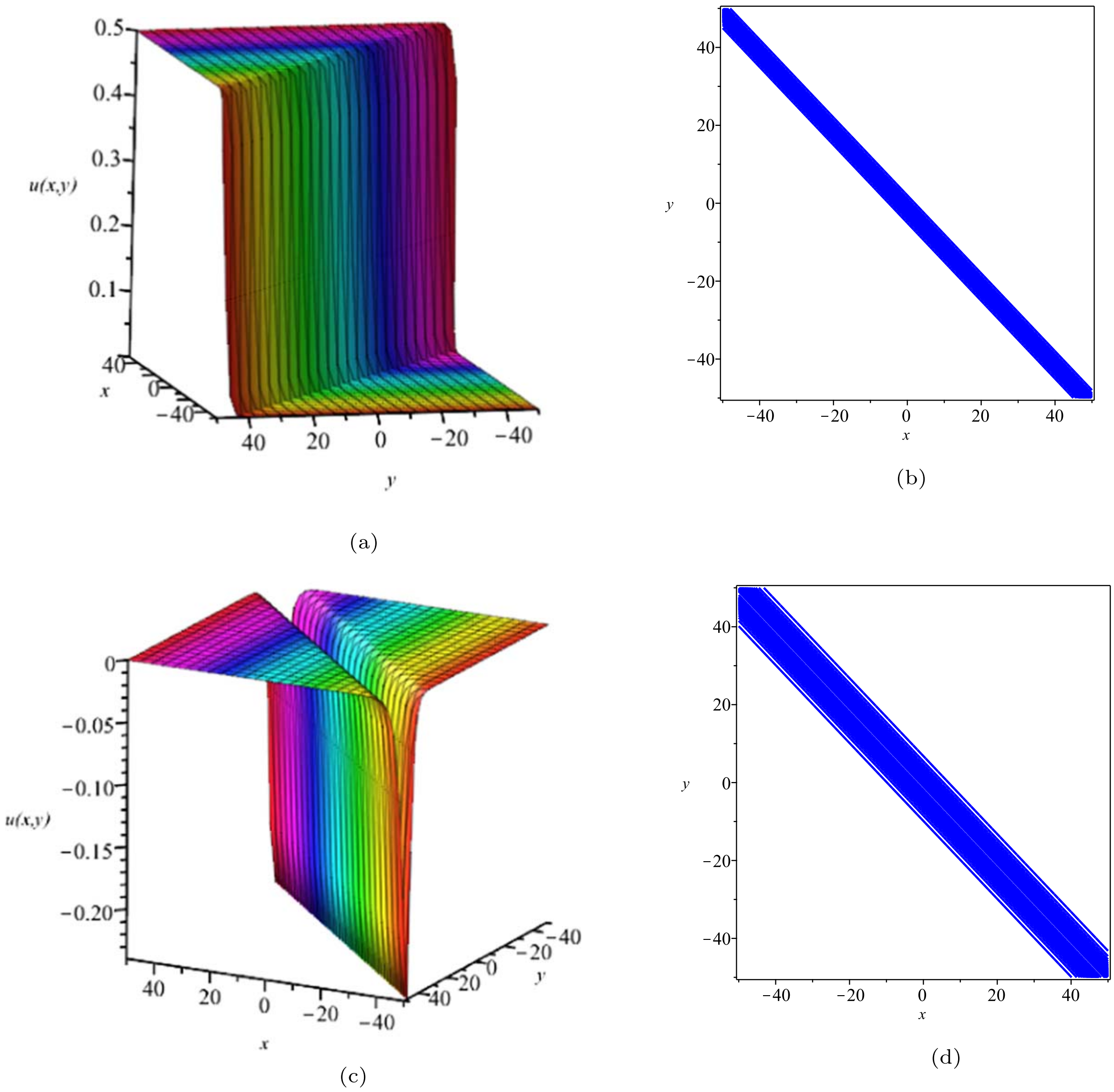


Figure 7. 3D profile and counter plot of complex one soliton solution of equation (31) when $t = 2$, $\gamma = -2$, $\beta = 2$, $r_1 = 1$, $q_1 = 0.5$, and $\alpha = 1$.

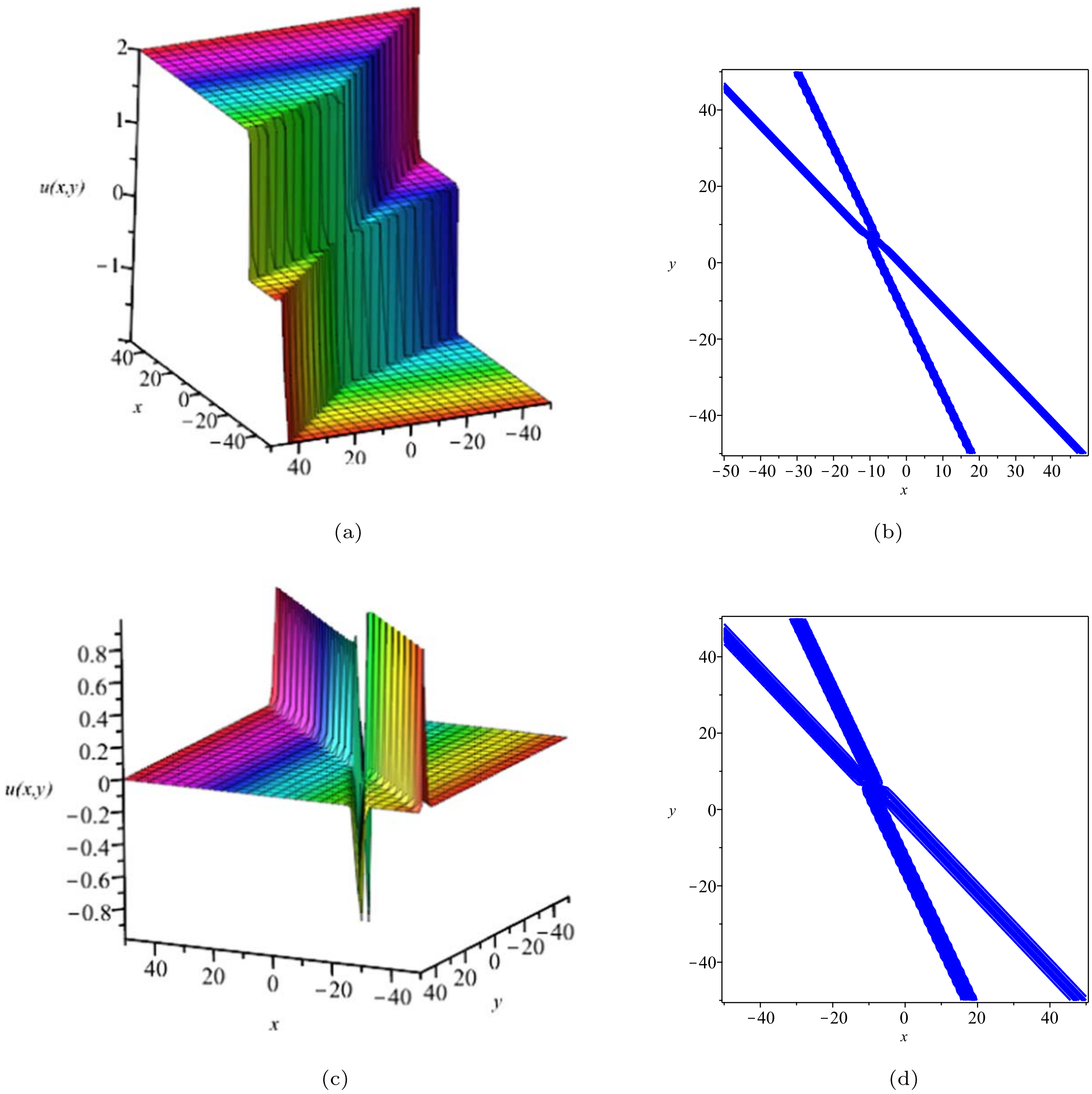


Figure 8. 3D profile and their counter plot of complex two soliton solution of equation (2) of the values of $t = 2$, $q_1 = -2$, $q_2 = 2$, $r_1 = 1$, $r_2 = 0.5$, $\alpha_1 = -1.5$, $\alpha_2 = 2$, $\epsilon = 0.5$, $\gamma = 2$, $\beta = 3$ and $\alpha = 0.5$.

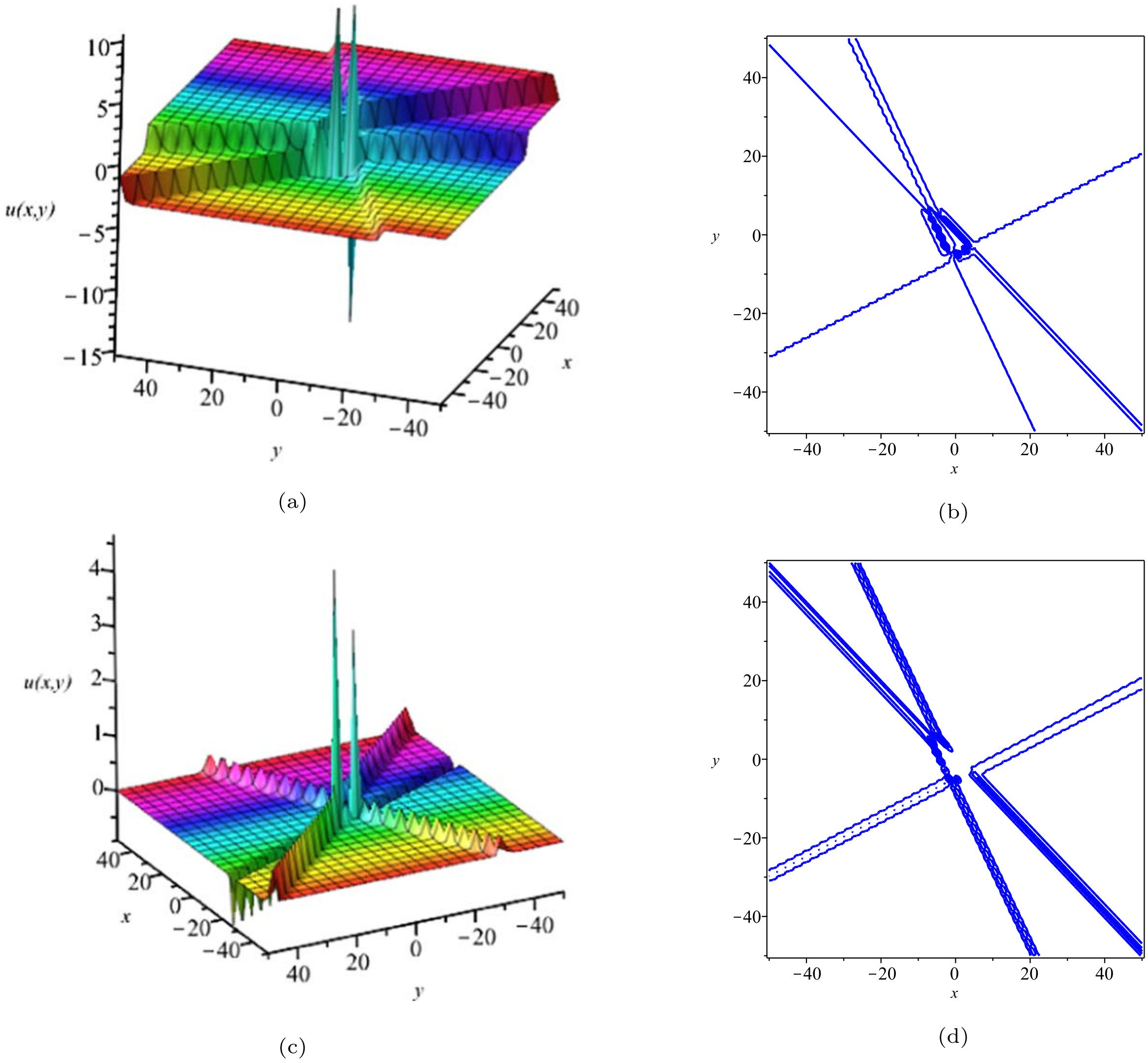


Figure 9. 3D profile of complex three soliton solution of equation (2) for $t = 2$, $q_1 = -2$, $q_2 = 2$, $q_3 = -1$, $r_1 = 1$, $r_2 = 0.5$, $r_3 = -2$, $\alpha_1 = -1.5$, $\alpha_2 = 1$, $\alpha_3 = 2$, $\epsilon = 0.5$, $\gamma = 2$, $\beta = 1.5$ and $\alpha = 0.5$.

Ethical approval

Not applicable.

Competing interests

It is affirmed that there are no financial or other conflicts of interest.

Authors' contributions

Karmina K. Ali: Conceptualization, Formal analysis, Writing—original draft; Abdullahi Yusuf: Formal analysis, Writing—original draft, Writing—review and editing; Wen-Xiu Ma: Conceptualization, Writing—original draft, Writing—review and editing.

Funding

Not applicable.

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