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Multiple soliton solutions and symmetry analysis of a nonlocal coupled KP system

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Abstract

A nonlocal coupled Kadomtsev–Petviashivili (ncKP) system with shifted parity ($\hat{P}_s^x$) and delayed time reversal ($\hat{T}_d$) symmetries is generated from the local coupled Kadomtsev–Petviashivili (cKP) system. By introducing new dependent variables which have determined parities under the action of $\hat{P}_s^x \hat{T}_d^d$, the ncKP is transformed to a local system. Through this way, multiple even number of soliton solutions of the ncKPI system are generated from N-soliton solutions of the cKP system, which become breathers by choosing appropriate parameters. The standard Lie symmetry method is also applied on the ncKPII system to get its symmetry reduction solutions.

Keywords: nonlocal coupled Kadomtsev–Petviashivili system, N-soliton solutions, symmetry reduction solutions

(Some figures may appear in colour only in the online journal)

1. Introduction

In 2013 Ablowitz and Musslimani introduced a new nonlocal nonlinear Schrödinger (nNLS) equation [1]

$$\mathrm{i}q_t(x,t) = q_{xx}(x,t) \pm q(x,t)q^*(-x,t)q(x,t), \tag{1}$$

with $q^*(-x,t)$ being complex conjugate of $q(-x,t)$, which is proved to be integrable under the meaning that it has a Lax pair and an infinite number of conservation laws. Contrary to local equations where dependent variables have the same independent variables, dependent variables of a nonlocal equation have two or more independent variables which are usually linked by space and/or time reversion, such as the variables of $(-x,t)$ and (x,t) in equation (1). Since the work of [1], nonlocal versions of many famous nonlinear systems, such as the Korteweg de-Vries (KdV) and modified KdV equation, the sine-Gordon equation, the Kadomtsev–Petviashivili (KP) equation, Sasa-Satsuma equation, etc are introduced and studied by applying various methods including inverse scattering transform [2, 3], Riemann–Hilbert method [4, 5], the Hirota's bilinear method [6–9], the Darboux transformations [10–12], Wronskian technique [13], symmetry analysis [14], deep learning neural network framework [15] and so on.

In recent years, Lou and Huang proposed the concept of the Alice–Bob (AB) system to describe two correlated events which can be assumed to be related by an operator $\hat{f}$, e.g. $A = \hat{f}B$, where $\hat{f}$ can be taken as shifted parity and delayed time reversal and so forth [16]. In other words, there exist at least two spacetime coordinates in one AB system. In this context, many AB-type nonlocal systems are constructed including the AB-KdV equation [17], AB-mKdV equation [18], AB-AKNS system [17], etc. In [19], a consistent correlated bang (CCB) method is proposed from which one can generate nonlocal systems from known local ones [20, 21].

The coupled KP (cKP) system [22–25] takes the form

$$\left(u_t + \frac{1}{4}u_{xxx} - \frac{3}{2}uu_x - \frac{3}{4}(vw)_x\right)_x + \sigma^2 u_{yy} = 0, \tag{2a}$$

$$\left(v_t + \frac{1}{4}v_{xxx} - \frac{3}{2}(uv)_x\right)_x + \sigma^2 v_{yy} = 0, \tag{2b}$$

$$\left(w_t + \frac{1}{4}w_{xxx} - \frac{3}{2}(uw)_x\right)_x + \sigma^2 w_{yy} = 0, \tag{2c}$$

which was first appeared in a paper of Jimbo and Miwa in Hirota bilinear form [26], it has N-soliton solutions expressed

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in terms of Pfaffians [27]. The cKP system (2) can be categorized as cKPI by taking $\sigma^2 = -1$ and cKPII by taking $\sigma^2 = 1$. In [23, 28], a host of solitonic interactions of the cKP are obtained, among which peculiar spider-web solutions are obtained and analyzed. In this paper, inspired by the CCB method, we introduce a nonlocal coupled KP (ncKP) system as

$$\begin{aligned}&A_{xt} - \frac{9}{8}A_x^2 - \frac{3}{4}A_xB_x + \frac{3}{8}B_x^2 - \frac{3}{8}(3F_x + G_x)C_x\\&+\frac{3}{8}(G_x - F_x)E_x - \frac{3}{8}(3A + B)A_{xx} + \sigma^2A_{yy}\\&+\frac{3}{8}(B - A)B_{xx} - \frac{3}{16}(3F + G)C_{xx} + \frac{3}{16}(G - F)E_{xx}\\&-\frac{3}{16}(E + 3C)F_{xx} - \frac{3}{16}(C - E)G_{xx} + \frac{1}{4}A_{xxxx} = 0,\end{aligned}\tag{3}$$

$$\begin{aligned}&B_{xt} - \frac{9}{8}B_x^2 - \frac{3}{4}A_xB_x + \frac{3}{8}A_x^2 - \frac{3}{8}(3G_x + F_x)E_x\\&+\frac{3}{8}(F_x - G_x)C_x - \frac{3}{8}(3B + A)B_{xx} + \sigma^2B_{yy}\\&+\frac{3}{8}(A - B)A_{xx} - \frac{3}{16}(3G + F)E_{xx} + \frac{3}{16}(F - G)C_{xx}\\&-\frac{3}{16}(C + 3E)G_{xx} - \frac{3}{16}(E - C)F_{xx} + \frac{1}{4}B_{xxxx} = 0,\end{aligned}\tag{4}$$

$$\begin{aligned}&C_{xt} - \frac{3}{4}(C_x - E_x)A_x - \frac{3}{4}(E_x + 3C_x)B_x - \frac{3}{8}(3C + E)A_{xx}\\&+\frac{3}{8}(E - C)B_{xx} - \frac{3}{8}(3A + B)C_{xx} + \sigma^2C_{yy}\\&+\frac{3}{8}(B - A)E_{xx} + \frac{1}{4}C_{xxxx} = 0,\end{aligned}\tag{5}$$

$$\begin{aligned}&E_{xt} - \frac{3}{4}(E_x - C_x)B_x - \frac{3}{4}(C_x + 3E_x)A_x - \frac{3}{8}(3E + C)B_{xx}\\&+\frac{3}{8}(C - E)A_{xx} - \frac{3}{8}(3B + A)E_{xx} + \sigma^2E_{yy}\\&+\frac{3}{8}(A - B)C_{xx} + \frac{1}{4}E_{xxxx} = 0,\end{aligned}\tag{6}$$

$$\begin{aligned}&F_{xt} - \frac{3}{4}(F_x - G_x)A_x - \frac{3}{4}(F_x + 3G_x)B_x - \frac{3}{8}(3F + G)A_{xx}\\&+\frac{3}{8}(G - F)B_{xx} - \frac{3}{8}(3A + B)F_{xx} + \sigma^2F_{yy}\\&+\frac{3}{8}(B - A)G_{xx} + \frac{1}{4}F_{xxxx} = 0,\end{aligned}\tag{7}$$

$$\begin{aligned}&G_{xt} - \frac{3}{4}(G_x - F_x)B_x - \frac{3}{4}(G_x + 3F_x)A_x - \frac{3}{8}(3G + F)B_{xx}\\&+\frac{3}{8}(F - G)A_{xx} - \frac{3}{8}(3B + A)G_{xx} + \sigma^2G_{yy}\\&+\frac{3}{8}(A - B)F_{xx} + \frac{1}{4}G_{xxxx} = 0,\end{aligned}\tag{8}$$

with

$$\begin{aligned}B &= \hat{P}_s^x\hat{T}_dA(x, y, t) = A(-x + x_0, y, -t + t_0),\\E &= \hat{P}_s^x\hat{T}_dC(x, y, t) = C(-x + x_0, y, -t + t_0),\\G &= \hat{P}_s^x\hat{T}_dF(x, y, t) = F(-x + x_0, y, -t + t_0),\end{aligned}\tag{9}$$

and probe its exact solutions and symmetry properties.

The paper is organized as follows. In section 2, we convert the ncKPI ($\sigma^2 = -1$) system into a local system by introducing some new variables with definite parity properties to replace the variables of the ncKPI system. Then we use N-soliton solutions of the cKP system to generate an even number of N-soliton solutions of the ncKPI system. In section 3, we apply the standard Lie symmetry method on the ncKPII ($\sigma^2 = 1$) system to give its Lie symmetry group and similarity reduction solutions. The last section is devoted to a summary.

2. Multiple soliton solutions of the ncKPI system

To convert the ncKPI system into a local system, considering the relation (9), we take

$$\begin{aligned}&A = u + u_1,\ B = u - u_1,\ C = v + v_1,\\&E = v - v_1,\ F = w + w_1,\ G = w - w_1,\end{aligned}\tag{10}$$

with

$$\begin{aligned}&\hat{P}_s^x\hat{T}_du = u,\ \hat{P}_s^x\hat{T}_du_1 = -u_1,\\&\hat{P}_s^x\hat{T}_dv = v,\ \hat{P}_s^x\hat{T}_dv_1 = -v_1\\&\hat{P}_s^x\hat{T}_dw = w,\\&\hat{P}_s^x\hat{T}_dw_1 = -w_1.\end{aligned}\tag{11}$$

Substituting equation (10) into the ncKP system (3)–(9) we split it into the following equations

$$\left(u_t + \frac{1}{4}u_{xxx} - \frac{3}{2}uu_x - \frac{3}{4}(vw)_x\right)_x - u_{yy} = 0,\tag{12}$$

$$\left(v_t + \frac{1}{4}v_{xxx} - \frac{3}{2}(uv)_x\right)_x - v_{yy} = 0,\tag{13}$$

$$\left(w_t + \frac{1}{4}w_{xxx} - \frac{3}{2}(uw)_x\right)_x - w_{yy} = 0,\tag{14}$$

$$\begin{aligned}&-3u_{1,x}u_x - \frac{3}{2}u_{xx}u_1 - \frac{3}{2}uu_{1,xx} - \frac{3}{4}v_1w_{xx}\\&-\frac{3}{4}vw_{1,xx} - \frac{3}{2}w_xv_{1,x} - \frac{3}{4}v_{xx}w_1\\&-\frac{3}{4}wv_{1,xx} - \frac{3}{2}v_xw_{1,x} - u_{1,yy} + u_{1,xt} + \frac{1}{4}u_{1,xxxx} = 0,\end{aligned}\tag{15}$$

$$\begin{aligned}&-v_{1,yy} - 3u_xv_{1,x} + 3v_xu_{1,x} - \frac{3}{2}vu_{1,xx} - \frac{3}{2}u_{xx}v_1 + v_{1,xt}\\&-\frac{3}{2}uv_{1,xx} - \frac{3}{2}v_{xx}u_1\\&+\frac{1}{4}v_{1,xxxx} = 0,\end{aligned}\tag{16}$$

$$
\begin{aligned}
&-w_{1,yy}-3u_x w_{1,x}+3w_x u_{1,x}-\frac{3}{2}wu_{1,xx}\\
&-\frac{3}{2}u_{xx}w_1+w_{1,xt}-\frac{3}{2}uw_{1,xx}-\frac{3}{2}w_{xx}u_1\\
&+\frac{1}{4}w_{1,xxxx}=0. \qquad (17)
\end{aligned}
$$

It can be seen that equations (12)–(14) are just the cKPI system (2) with $\sigma^2=-1$ while equations (15)–(17) are linearized equations of the cKPI system.

The cKP system (2) has the following N-soliton solution [22]

$$
\begin{aligned}
&u=-(\ln F)_{xx},\ v=\alpha(\ln F)_{xx},\ w=\frac{1}{\alpha}(\ln F)_{xx},\\
&F=\sum_{\nu}\exp\Big(\sum_{j=1}^{N}\nu_j\xi_j+\sum_{1\leqslant i'<j}^{N}\nu_{i'}\nu_j\theta_{i'j}\Big), \qquad (18)
\end{aligned}
$$

with α being a nonzero real number, which is an extension of N-soliton solutions of the KP equation, the summations should be done for all permutations of $\nu_{i'}=0,\ 1(i'=1,\ 2,\ 3,\cdots N)$, and

$$
\begin{aligned}
&\xi_j=k_jx+r_jy-\frac{k_j^4+4\sigma^2r_j^2}{4k_j}t+\xi_{j0},\ \exp(\theta_{i'j})\\
&=\frac{3k_{i'}^2k_j^2(k_{i'}-k_j)^2-4\sigma^2(k_{i'}r_j-k_jr_{i'})^2}{3k_{i'}^2k_j^2(k_{i'}+k_j)^2-4\sigma^2(k_{i'}r_j-k_jr_{i'})^2},
\end{aligned}
$$

with arbitrary constants k_j, r_j, ξ_{j0}, $(j=1,\ 2,\ 3,\ \cdots,N)$.

Considering that u , v and w in equation (18) are invariant under the following transformation

$$F\to\beta\exp(Kx+\Omega t+X_0)F,$$

we rewrite

$$\xi_j=\eta_j-\frac{1}{2}\sum_{i'=1}^{j-1}\theta_{i'j}-\frac{1}{2}\sum_{i'=j+1}^{N}\theta_{ji'},$$

where

$$
\begin{aligned}
\eta_j&=k_j\Big(x-\frac{x_0}{2}\Big)+r_j\Big(y-\frac{y_{0j}}{2}\Big)\\
&-\frac{k_j^4+4r_j^2}{4k_j}\Big(t-\frac{t_0}{2}\Big)+\eta_{0j},
\end{aligned}
$$

with arbitrary constants y_{0j} and η_{0j}. So the N-soliton solutions of the cKPI system can be rewritten as [17]

$$
\begin{aligned}
u&=-\frac{1}{\alpha}v=-\alpha w=-\Bigg[\ln\sum_{\nu}K_\nu\\
&\times\cosh\Bigg(\frac{1}{2}\sum_{j=1}^{N}\nu_j\eta_j\Bigg)\Bigg]_{xx}\equiv(\ln F_N)_{xx}, \qquad (19)
\end{aligned}
$$

where the summation of ν being done for all non-dual permutations of $\nu_{i'}=1,\ -1,\ (i'=1,\ 2,\ 3,\cdots N)$ and

$$K_\nu=\prod_{i'>j}\sqrt{3k_{i'}^2k_j^2(k_{i'}-\nu_{i'}\nu_jk_j)^2+4(k_{i'}r_j-k_jr_{i'})^2}.$$

In order to give $\hat{P}_s^x\hat{T}_d$ invariant part of the N-soliton solutions in equation (18), additional restrictions should be given as

$$N=2n,\ k_{n+i'}=-k_{i'},\ r_{n+i'}=r_{i'},\ y_{0(n+i')}=y_{0i'}. \qquad (20)$$

It is clear that solutions of u_1, v_1, and w_1 in equations (15)–(17) are symmetries of the cKP system, which can be taken as

$$
\begin{aligned}
u_1&=\Big(\frac{1}{3}\dot{f}_1x+\frac{1}{6}\ddot{f}_1y^2+\frac{1}{2}\dot{f}_2y+f_3\Big)u_x+\Big(\frac{2}{3}\dot{f}_1y+f_2\Big)u_y\\
&+f_1u_t+\frac{2}{3}\dot{f}_1u+\frac{2}{9}\ddot{f}_1x+\frac{1}{9}\dddot{f}_1y^2+\frac{1}{3}\ddot{f}_2y+\frac{2}{3}\dot{f}_3,\\
v_1&=\Big(\frac{1}{3}\dot{f}_1x+\frac{1}{6}\ddot{f}_1y^2+\frac{1}{2}\dot{f}_2y+f_3\Big)v_x\\
&+\Big(\frac{2}{3}\dot{f}_1y+f_2\Big)v_y+f_1v_t+\frac{2}{3}\dot{f}_1v,\\
w_1&=\Big(\frac{1}{3}\dot{f}_1x+\frac{1}{6}\ddot{f}_1y^2+\frac{1}{2}\dot{f}_2y+f_3\Big)w_x\\
&+\Big(\frac{2}{3}\dot{f}_1y+f_2\Big)w_y+f_1w_t+\frac{2}{3}\dot{f}_1w,
\end{aligned}
\qquad (21)
$$

where f_1, f_2 and f_3 are arbitrary functions of t satisfying

$$\hat{T}_d\{f_1,f_2,f_3\}=\{f_1,-f_2,f_3\}. \qquad (22)$$

It can be verified that equation (21) with equation (22) satisfies the condition of equation (11). So, N-soliton solutions of the ncKPI system (12)–(14) can be expressed by equation (10) with equations (19) and (21).

By the condition of equation (20), odd number solitons of the ncKPI system are prohibited. As for $N=2,\ 4$, the explicit expressions of F_N in equation (19) are

$$
\begin{aligned}
F_2&=\sqrt{3k_1^4+4r_1^2}\cosh\Big[r_1\Big(y-\frac{y_{01}}{2}\Big)\Big]\\
&+2\cosh\Big[k_1\Big(x-\frac{x_0}{2}\Big)-\frac{(k_1^4-4r_1^2)}{4k_1}\Big(t-\frac{t_0}{2}\Big)\Big], \qquad (23)
\end{aligned}
$$

and

$$
\begin{aligned}
F_4&=\delta_{14,-}\delta_{24,-}\delta_{34,-}\delta_{13,-}\delta_{23,-}\delta_{12,-}\\
&\times\cosh\Big[r_1\Big(y-\frac{y_{01}}{2}\Big)+r_2\Big(y-\frac{y_{02}}{2}\Big)\Big]\\
&+\delta_{14,+}\delta_{24,-}\delta_{34,-}\delta_{13,+}\delta_{23,-}\delta_{12,+}\\
&\times\cosh\Big[k_1\Big(x-\frac{x_0}{2}\Big)-\frac{k_1^4-4r_1^2}{4k_1}\Big(t-\frac{t_0}{2}\Big)-r_2\Big(y-\frac{y_{02}}{2}\Big)\Big]\\
&+\delta_{14,-}\delta_{24,+}\delta_{34,-}\delta_{13,-}\delta_{23,+}\delta_{12,+}\\
&\times\cosh\Big[r_1\Big(y-\frac{y_{01}}{2}\Big)-k_2\Big(x-\frac{x_0}{2}\Big)+\frac{(k_2^4-4r_2^2)}{4k_2}\Big(t-\frac{t_0}{2}\Big)\Big]
\end{aligned}
$$

$$
\begin{aligned}
&+\delta_{14,-}\delta_{24,-}\delta_{34,+}\delta_{13,+}\delta_{23,+}\delta_{12,-}\\
&\times\cosh\left[k_1\left(x-\frac{x_0}{2}\right)-\frac{(k_1^4-4r_1^2)}{4k_1}\left(t-\frac{t_0}{2}\right)+r_2\left(y-\frac{y_{02}}{2}\right)\right]\\
&+\delta_{14,+}\delta_{24,+}\delta_{34,+}\delta_{13,-}\delta_{23,-}\delta_{12,-}\\
&\times\cosh\left[r_1\left(y-\frac{y_{01}}{2}\right)+k_2\left(x-\frac{x_0}{2}\right)-\frac{k_2^4-4r_2^2}{4k_2}\left(t-\frac{t_0}{2}\right)\right]\\
&+\delta_{14,+}\delta_{24,+}\delta_{34,-}\delta_{13,+}\delta_{23,+}\delta_{12,-}\\
&\times\cosh\left[k_1\left(x-\frac{x_0}{2}\right)-\frac{(k_1^4-4r_1^2)}{4k_1}\left(t-\frac{t_0}{2}\right)+k_2\left(x-\frac{x_0}{2}\right)\right.\\
&\left.-\frac{(k_2^4-4r_2^2)}{4k_2}\left(t-\frac{t_0}{2}\right)\right]\\
&+\delta_{14,+}\delta_{24,-}\delta_{34,+}\delta_{13,-}\delta_{23,+}\delta_{12,+}\\
&\times\cosh\left[r_1\left(y-\frac{y_{01}}{2}\right)-r_2\left(y-\frac{y_{02}}{2}\right)\right]\\
&+\delta_{14,-}\delta_{24,+}\delta_{34,+}\delta_{13,+}\delta_{23,-}\delta_{12,+}\\
&\times\cosh\left[k_1\left(x-\frac{x_0}{2}\right)-\frac{(k_1^4-4r_1^2)}{4k_1}\left(t-\frac{t_0}{2}\right)-k_2\left(x-\frac{x_0}{2}\right)\right.\\
&\left.+\frac{(k_2^4-4r_2^2)}{4k_2}\left(t-\frac{t_0}{2}\right)\right],
\end{aligned}
\tag{24}
$$

where

$$\delta_{i'j,\pm}=\sqrt{3k_{i'}^2k_j^2(k_{i'}\pm k_j)^2+4(k_{i'}r_j-k_jr_{i'})^2},$$

and

$$
\begin{aligned}
&k_4=-k_2,\ k_3=-k_1,\ r_4=r_2,\\
&r_3=r_1,\ y_{04}=y_{02},\ y_{03}=y_{01}.
\end{aligned}
$$

At time $t=0$, for $N=2$ case, figures 1(a) and 2(a) give density plot and three-dimensional plot of A of the ncKP system expressed by equations (10) with equations (19) and (21) where the parameters are fixed by

$$
\begin{aligned}
&f_1=f_2=x_0=t_0=\eta_{01}=\eta_{02}=y_{01}=y_{02}=0,\\
&f_3=2,\ \alpha=2,\ k_1=-k_2=1,\ r_1=r_2=1;
\end{aligned}
\tag{25}
$$

as for $N=4$ case, figures 1(b) and 2(b) give density plot and three-dimensional plot of A of the ncKPI system where the parameters are fixed by

$$
\begin{aligned}
&f_1=f_2=x_0=t_0=\eta_{0j}=y_{0j}=0,\ (j=1,2,\cdots,4),\\
&f_3=2,\ \alpha=1,\ k_1=-k_3=1,\\
&k_2=-k_4=2,\ r_1=r_2=r_3=r_4=1.
\end{aligned}
\tag{26}
$$

Because F_2 (or F_4) in equations (23) (or (24)) depends similarly on the coordinates of x, y and t, the multiple soliton interaction behaviors of A depending on other variable pairs (x, t) and (y, t) are similar to those in figures 1 and 2.

When $r_j (j=1, 2, 3, \cdots, N)$ in equation (19) are taken to be a pure imaginary number, these N-soliton solutions become breather solutions. To illustrate this point, for the $N=2$ case, when we take the parameters as

$$
\begin{aligned}
&f_1=f_2=x_0=y_{01}=y_{02}=t_0=\eta_{01}=\eta_{02}=0,\\
&f_3=2,\ k_1=-k_2=1,\\
&r_1=r_2=\sqrt{-1},\ \alpha=2,
\end{aligned}
\tag{27}
$$

we get breathers at time $t=0$ in figure 3.

It is well known that the KP equation has lump solutions, we can verify that the cKPI system has the following solution

$$
\begin{aligned}
&u=-\frac{1}{\alpha}v=-\alpha w=\frac{L_1}{L_2},\\
&L_1=32d^2x^2+32[d^2(dt_0-x_0)-2d^3t]x-32d^3y^2\\
&+64d^3yy_0+32d^4t^2-32d^3(dt_0-x_0)t\\
&+8d(d^3t_0^2-2d^2t_0x_0-4d^2y_0^2+dx_0^2-3),\\
&L_2=[4dx^2+4d[(dt_0-x_0)-2dt]x+4d^2y^2-8d^2yy_0\\
&+4d^3t^2-4d^2(dt_0-x_0)t+d(dt_0-x_0)^2+4d^2y_0^2+3]^2,
\end{aligned}
\tag{28}
$$

with arbitrary constants d, x_0, y_0, t_0, which leads to lump-type solutions of the ncKP system by substituting equation (28) into equations (10) with equation (21). Figure 4 demonstrates a lump-type solution of the ncKPI system for the variable A, where the parameters are fixed by

$$f_1=f_2=f_3=x_0=y_0=t_0=0,\ d=1. \tag{29}$$

3. Symmetry reduction solutions of the the ncKPII system

Symmetry analysis plays an important role in solving nonlinear systems [29, 30], in this section we apply the standard Lie symmetry method on the ncKPII system. To this end, we first give the Lie point symmetry of this system in the form

$$
\begin{aligned}
V=&X\frac{\partial}{\partial x}+Y\frac{\partial}{\partial y}+T\frac{\partial}{\partial t}+\Gamma_1\frac{\partial}{\partial A}+\Lambda_1\frac{\partial}{\partial B}\\
&+\Gamma_2\frac{\partial}{\partial C}+\Lambda_2\frac{\partial}{\partial E}+\Gamma_3\frac{\partial}{\partial F}+\Lambda_2\frac{\partial}{\partial G},
\end{aligned}
\tag{30}
$$

where X, Y, T, Γ_1, Γ_2, Γ_3, Λ_1, Λ_2, Λ_3 are functions of x, y, t, A, B, C, E, F, G that needs to be determined. In other words, the ncKPII system is invariant under the following transformation

$$
\begin{aligned}
&\{x, y, t, A, B, C, E, F, G\}\\
&\quad\to\{x+\epsilon X, y+\epsilon Y, t+\epsilon T, A+\epsilon\Gamma_1, B+\epsilon\Lambda_1,\\
&\quad C+\epsilon\Gamma_2, E+\epsilon\Lambda_2,\\
&\quad F+\epsilon\Gamma_3, G+\epsilon\Lambda_3\},
\end{aligned}
$$

with infinitesimal parameter ϵ. The symmetry of equation (30) can be written in function form as

$$\sigma_A=XA_x+YA_y+TA_t-\Gamma_1, \tag{31a}$$

$$\sigma_B=XB_x+YB_y+TB_t-\Lambda_1, \tag{31b}$$

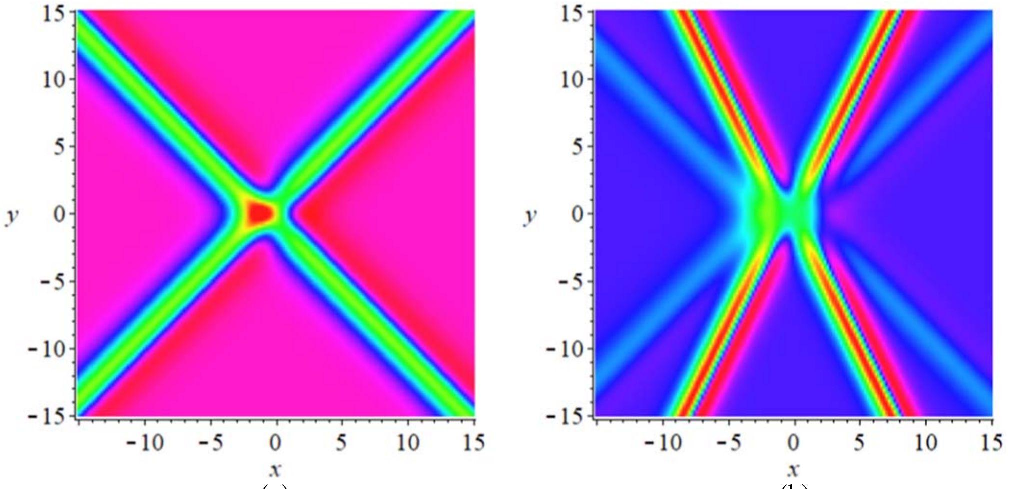
$$\sigma_C=XC_x+YC_y+TC_t-\Gamma_2, \tag{31c}$$

$$\sigma_E=XB_x+YB_y+TB_t-\Lambda_2, \tag{31d}$$

$$\sigma_F=XF_x+YF_y+TF_t-\Gamma_3, \tag{31e}$$

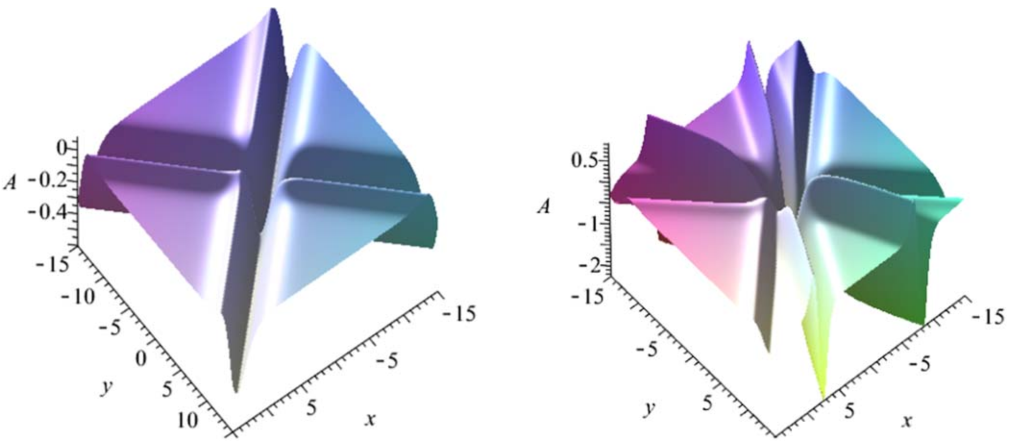
$$\sigma_G=XG_x+YG_y+TG_t-\Lambda_3, \tag{31f}$$

which satisfy the linearized equations of the ncKPII system



(a) (b)

Figure 1. (a) The density plot of the solution A of the ncKPI system at time $t=0$ for $N=2$ case with parameters being fixed by equation (25); (b) the density plot of the solution A of the ncKPI system at time $t=0$ for $N=4$ case with parameters being fixed by equation (26).



(a) (b)

Figure 2. (a) The three-dimensional plot of the solution A of the ncKPI system at time $t=0$ for $N=2$ case with parameters being fixed by equation (25); (b) the three-dimensional plot of the solution A of the ncKPI system at time $t=0$ for $N=4$ case with parameters being fixed by equation (26).

$$
\begin{aligned}
&\frac{3}{4}(\sigma_{E,x}-\sigma_{C,x})A_x-\frac{3}{4}(\sigma_{E,x}+3\sigma_{C,x})B_x\\
&\quad-\frac{3}{4}(3\sigma_{B,x}+\sigma_{A,x})C_x+\frac{3}{4}(\sigma_{A,x}-\sigma_{B,x})E_x\\
&-\frac{3}{8}(3\sigma_C+\sigma_E)A_{xx}+\frac{3}{8}(\sigma_E-\sigma_C)B_{xx}\\
&\quad-\frac{3}{8}(3\sigma_A+\sigma_B)C_{xx}+\frac{3}{8}(\sigma_B-\sigma_A)E_{xx}\\
&\quad+\frac{1}{4}\sigma_{C,xxxx}-\frac{3}{8}(3C+E)\sigma_{A,xx}\\
&\quad+\frac{3}{8}(E-C)\sigma_{B,xx}+\sigma_{C,xt}-\frac{3}{8}(3A+B)\sigma_{C,xx}\\
&\quad+\sigma_{C,yy}+\frac{3}{8}(B-A)\sigma_{E,xx}=0,
\end{aligned}
\qquad (32a)
$$

$$
\begin{aligned}
&\frac{3}{4}(\sigma_{C,x}-\sigma_{E,x})B_x-\frac{3}{4}(\sigma_{C,x}+3\sigma_{E,x})A_x\\
&\quad-\frac{3}{4}(3\sigma_{A,x}+\sigma_{B,x})E_x+\frac{3}{4}(\sigma_{B,x}-\sigma_{A,x})C_x\\
&\quad-\frac{3}{8}(3\sigma_E+\sigma_C)B_{xx}+\frac{3}{8}(\sigma_C-\sigma_E)A_{xx}\\
&\quad-\frac{3}{8}(3\sigma_B+\sigma_A)E_{xx}+\frac{3}{8}(\sigma_A-\sigma_B)C_{xx}\\
&\quad+\frac{1}{4}\sigma_{E,xxxx}-\frac{3}{8}(3E+C)\sigma_{B,xx}\\
&\quad+\frac{3}{8}(C-E)\sigma_{A,xx}+\sigma_{E,xt}-\frac{3}{8}(3B+A)\sigma_{E,xx}\\
&\quad+\sigma_{E,yy}+\frac{3}{8}(A-B)\sigma_{C,xx}=0,
\end{aligned}
\qquad (32b)
$$

$$\begin{aligned}
&\frac{3}{4}(3\sigma_{F,x}+\sigma_{G,x})A_x+\frac{3}{4}(\sigma_{F,x}-\sigma_{G,x})B_x\\
&\quad+\frac{3}{4}(\sigma_{B,x}+3\sigma_{A,x})F_x\\
&\quad-\frac{3}{4}(\sigma_{B,x}-\sigma_{A,x})G_x+\frac{3}{8}(3\sigma_F+\sigma_G)A_{xx}\\
&+\frac{3}{8}(\sigma_F-\sigma_G)B_{xx}+\frac{3}{8}(3\sigma_A+\sigma_B)F_{xx}+\frac{3}{8}(\sigma_A-\sigma_B)G_{xx}\\
&\quad-\frac{1}{4}\sigma_{F,xxxx}+\frac{3}{8}(3F+G)\sigma_{A,xx}+\frac{3}{8}(F-G)\sigma_{B,xx}\\
&-\sigma_{F,xt}+\frac{3}{8}(3A+B)\sigma_{F,xx}-\sigma_{F,yy}+\frac{3}{8}(A-B)\sigma_{G,xx}=0,
\end{aligned}\tag{32c}$$

$$\begin{aligned}
&\frac{3}{4}(3\sigma_{G,x}+\sigma_{F,x})B_x+\frac{3}{4}(\sigma_{G,x}-\sigma_{F,x})A_x\\
&\quad+\frac{3}{4}(\sigma_{A,x}+3\sigma_{B,x})G_x\\
&\quad-\frac{3}{4}(\sigma_{A,x}-\sigma_{B,x})F_x+\frac{3}{8}(3\sigma_G+\sigma_F)B_{xx}\\
&+\frac{3}{8}(\sigma_G-\sigma_F)A_{xx}+\frac{3}{8}(3\sigma_B+\sigma_A)G_{xx}\\
&\quad+\frac{3}{8}(\sigma_B-\sigma_A)F_{xx}-\frac{1}{4}\sigma_{G,xxxx}\\
&\quad+\frac{3}{8}(3G+F)\sigma_{B,xx}+\frac{3}{8}(G-F)\sigma_{A,xx}\\
&-\sigma_{G,xt}+\frac{3}{8}(3B+A)\sigma_{G,xx}-\sigma_{G,yy}+\frac{3}{8}(B-A)\sigma_{F,xx}=0,
\end{aligned}\tag{32d}$$

$$\begin{aligned}
&\sigma_{A,xt}-\frac{3}{4}(\sigma_{B,x}+3\sigma_{A,x})A_x\\
&\quad+\frac{3}{4}(\sigma_{B,x}-\sigma_{A,x})B_x\\
&\quad-\frac{3}{8}(3\sigma_{F,x}+\sigma_{G,x})C_x+\frac{3}{8}(\sigma_{G,x}-\sigma_{F,x})E_x\\
&\quad-\frac{3}{8}(\sigma_{E,x}+3\sigma_{C,x})F_x+\frac{3}{8}(\sigma_{E,x}-\sigma_{C,x})G_x-\frac{3}{8}(3\sigma_A\\
&\quad+\sigma_B)A_{xx}+\frac{3}{8}(\sigma_B-\sigma_A)B_{xx}-\frac{3}{16}(3\sigma_F+\sigma_G)C_{xx}\\
&\quad+\frac{3}{16}(\sigma_G-\sigma_F)E_{xx}-\frac{3}{16}(\sigma_E+3\sigma_C)F_{xx}\\
&\quad-\frac{3}{16}(\sigma_C-\sigma_E)G_{xx}+\frac{1}{4}\sigma_{A,xxxx}-\frac{3}{8}(3A+B)\sigma_{A,xx}+\sigma_{A,yy}\\
&\quad+\frac{3}{8}(B-A)\sigma_{B,xx}-\frac{3}{16}(3F+G)\sigma_{C,xx}\\
&\quad+\frac{3}{16}(G-F)\sigma_{E,xx}\\
&\quad-\frac{3}{16}(E+3C)\sigma_{F,xx}-\frac{3}{16}(C-E)\sigma_{G,xx}=0,
\end{aligned}\tag{32e}$$

$$\begin{aligned}
&\sigma_{B,xt}-\frac{3}{4}(\sigma_{A,x}+3\sigma_{B,x})B_x+\frac{3}{4}(\sigma_{A,x}-\sigma_{B,x})A_x\\
&\quad-\frac{3}{8}(3\sigma_{G,x}+\sigma_{F,x})E_x+\frac{3}{8}(\sigma_{F,x}-\sigma_{G,x})C_x\\
&-\frac{3}{8}(\sigma_{C,x}+3\sigma_{E,x})G_x+\frac{3}{8}(\sigma_{C,x}-\sigma_{E,x})F_x-\frac{3}{8}(3\sigma_B\\
&\quad+\sigma_A)B_{xx}+\frac{3}{8}(\sigma_A-\sigma_B)A_{xx}-\frac{3}{16}(3\sigma_G+\sigma_F)E_{xx}\\
&\quad+\frac{3}{16}(\sigma_F-\sigma_G)C_{xx}-\frac{3}{16}(\sigma_C+3\sigma_E)G_{xx}\\
&\quad-\frac{3}{16}(\sigma_E-\sigma_C)F_{xx}+\frac{1}{4}\sigma_{B,xxxx}-\frac{3}{8}(3B+A)\sigma_{B,xx}+\sigma_{B,yy}\\
&\quad+\frac{3}{8}(A-B)\sigma_{A,xx}-\frac{3}{16}(3G+F)\sigma_{E,xx}\\
&\quad+\frac{3}{16}(F-G)\sigma_{C,xx}\\
&\quad-\frac{3}{16}(C+3E)\sigma_{G,xx}-\frac{3}{16}(E-C)\sigma_{F,xx}=0,
\end{aligned}\tag{32f}$$

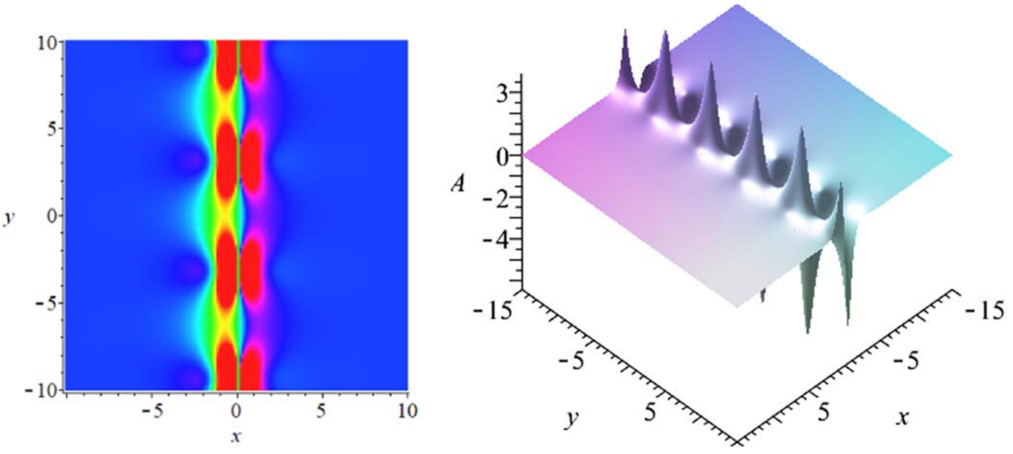
and also the nonlocal condition

$$\begin{aligned}
&\sigma_A=\sigma_B(-x+x_0,\,y,\,-t+t_0),\\
&\sigma_C=\sigma_E(-x+x_0,\,y,\,-t+t_0),\\
&\sigma_F=\sigma_G(-x+x_0,\,y,\,-t+t_0).
\end{aligned}\tag{32g}$$

By substituting equations (31) into equation (32) and eliminating A_{xt}, B_{xt}, C_{xt}, E_{xt}, F_{xt}, G_{xt} by the ncKPII system, we obtain a system of the functions X, Y, T, Γ_1, Γ_2, Γ_3, Λ_1, Λ_2, Λ_3. By vanishing all independent partial derivatives of variables A, B, C, E, F, G we obtain a system of over determined linear equations, which can be solved by software like *maple*. After considering the nonlocal relation of equation (32*g*), we have

$$\begin{aligned}
&X=\frac{1}{3}\dot{f}_1x-\frac{1}{6}\ddot{f}_1y^2-\frac{1}{2}\dot{f}_2y+f_3,\\
&Y=\frac{2}{3}\dot{f}_1y+f_2,\,T=f_1,\\
&\Gamma_1=\frac{1}{9}\dddot{f}_1y^2-\frac{2}{3}\dot{f}_1A-\frac{2}{9}\ddot{f}_1x+\frac{1}{3}\ddot{f}_2y-\frac{2}{3}\dot{f}_3,\\
&\Lambda_1=\frac{1}{9}\dddot{f}_1y^2-\frac{2}{3}\dot{f}_1B-\frac{2}{9}\ddot{f}_1x\\
&\qquad+\frac{1}{3}\ddot{f}_2y-\frac{2}{3}\dot{f}_3,\\
&\Gamma_2=-\frac{2}{3}C\dot{f}_1-e_1E-e_1C,\,\Lambda_2=-\frac{2}{3}\dot{f}_1E+e_1C+e_1E,\\
&\Gamma_3=-\frac{2}{3}\dot{f}_1F+e_1G+e_1F,\\
&\Lambda_3=-\frac{2}{3}\dot{f}_1G-e_1G-e_1F,
\end{aligned}$$

where f_1, f_2, f_3 are arbitrary functions of t satisfying the condition of (22). So the explicit expressions of equation (31) are



(a) (b)

Figure 3. Breather solutions of the ncKPI system at time $t=0$ for: (a) the density plots of the variable A; (b) the three-dimensional plots of the variable A. The parameters are fixed by equation (27).

$$\begin{aligned}\sigma_A = {} & f_1 A_t + \left(\frac{1}{3}\dot{f}_1 x - \frac{1}{6}\ddot{f}_1 y^2 - \frac{1}{2}\dot{f}_2 y + f_3\right)A_x \\ & + \left(\frac{2}{3}\dot{f}_1 y + f_2\right)A_y - \frac{1}{9}\dddot{f}_1 y^2 + \frac{2}{3}\dot{f}_1 A \\ & + \frac{2}{9}\ddot{f}_1 x - \frac{1}{3}\ddot{f}_2 y + \frac{2}{3}\dot{f}_3,\end{aligned} \tag{33a}$$

$$\begin{aligned}\sigma_B = {} & f_1 B_t + \left(\frac{1}{3}\dot{f}_1 x - \frac{1}{6}\ddot{f}_1 y^2 - \frac{1}{2}\dot{f}_2 y + f_3\right)B_x \\ & + \left(\frac{2}{3}\dot{f}_1 y + f_2\right)B_y - \frac{1}{9}\dddot{f}_1 y^2 + \frac{2}{3}\dot{f}_1 B \\ & + \frac{2}{9}\ddot{f}_1 x - \frac{1}{3}\ddot{f}_2 y + \frac{2}{3}\dot{f}_3,\end{aligned} \tag{33b}$$

$$\begin{aligned}\sigma_C = {} & f_1 C_t + \left(\frac{1}{3}\dot{f}_1 x - \frac{1}{6}\ddot{f}_1 y^2 - \frac{1}{2}\dot{f}_2 y + f_3\right)C_x \\ & + \left(\frac{2}{3}\dot{f}_1 y + f_2\right)C_y + \frac{2}{3}C\dot{f}_1 + e_1(C+E),\end{aligned} \tag{33c}$$

$$\begin{aligned}\sigma_E = {} & f_1 E_t + \left(\frac{1}{3}\dot{f}_1 x - \frac{1}{6}\ddot{f}_1 y^2 - \frac{1}{2}\dot{f}_2 y + f_3\right)E_x \\ & + \left(\frac{2}{3}\dot{f}_1 y + f_2\right)E_y + \frac{2}{3}E\dot{f}_1 \\ & - e_1(C+E),\end{aligned} \tag{33d}$$

$$\begin{aligned}\sigma_F = {} & f_1 F_t + \left(\frac{1}{3}\dot{f}_1 x - \frac{1}{6}\ddot{f}_1 y^2 - \frac{1}{2}\dot{f}_2 y + f_3\right)F_x \\ & + \left(\frac{2}{3}\dot{f}_1 y + f_2\right)F_y + \frac{2}{3}F\dot{f}_1 - e_1(F+G),\end{aligned} \tag{33e}$$

$$\begin{aligned}\sigma_G = {} & f_1 G_t + \left(\frac{1}{3}\dot{f}_1 x - \frac{1}{6}\ddot{f}_1 y^2 - \frac{1}{2}\dot{f}_2 y + f_3\right)G_x \\ & + \left(\frac{2}{3}\dot{f}_1 y + f_2\right)G_y + \frac{2}{3}G\dot{f}_1 \\ & + e_1(F+G).\end{aligned} \tag{33f}$$

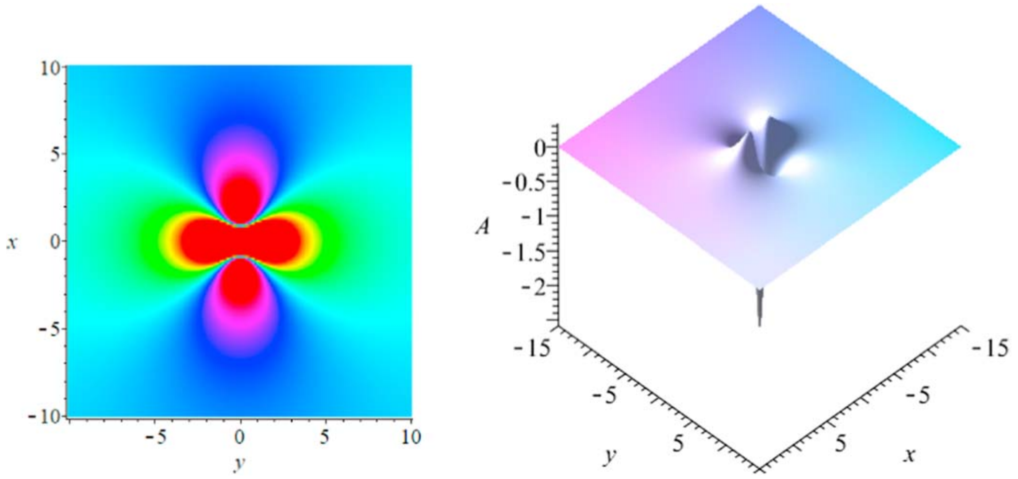
Group invariant solutions of the ncKPII system can be obtained by solving equation (33) under the condition $\sigma_A=\sigma_B=\sigma_C=\sigma_E=\sigma_F=\sigma_G=0$, which is equivalent to solving the characteristic equation

$$\begin{aligned}& \frac{dx}{\frac{1}{3}\dot{f}_1 x - \frac{1}{6}\ddot{f}_1 y^2 - \frac{1}{2}\dot{f}_2 y + f_3} = \frac{dt}{f_1} \\ & = \frac{dA}{\frac{1}{9}\dddot{f}_1 y^2 - \frac{2}{3}\dot{f}_1 A - \frac{2}{9}\ddot{f}_1 x + \frac{1}{3}\ddot{f}_2 y - \frac{2}{3}\dot{f}_3} \\ & = \frac{dB}{\frac{1}{9}\dddot{f}_1 y^2 - \frac{2}{3}\dot{f}_1 B - \frac{2}{9}\ddot{f}_1 x + \frac{1}{3}\ddot{f}_2 y - \frac{2}{3}\dot{f}_3} \\ & = \frac{dC}{-\frac{2}{3}C\dot{f}_1 - e_1 E - e_1 C} \\ & = \frac{dE}{-\frac{2}{3}\dot{f}_1 E + e_1 C + e_1 E} \\ & = \frac{dF}{-\frac{2}{3}\dot{f}_1 F + e_1 G + e_1 F} \\ & = \frac{dG}{-\frac{2}{3}\dot{f}_1 G - e_1 G - e_1 F}.\end{aligned} \tag{34}$$

After solving equation (34) we get symmetry reduction solutions of the ncKPII system

$$\begin{aligned}A = {} & -\frac{(m_1+\eta)^2\dddot{m}_3}{9\dot{m}_3^{\frac{7}{3}}} + \frac{(m_1+\eta)\ddot{m}_1}{3\dot{m}_3^{\frac{4}{3}}} \\ & + \left[-\frac{4(m_1+\eta)\dot{m}_1}{9\dot{m}_3^{\frac{7}{3}}} + \frac{m_2+2\xi}{9\dot{m}_3^{\frac{4}{3}}}\right]\ddot{m}_3 \\ & + \frac{5(m_1+\eta)^2\ddot{m}_3^{\,2}}{27\dot{m}_3^{\frac{10}{3}}} + \frac{\dot{m}_1^{\,2}}{6\dot{m}_3^{\frac{4}{3}}} - \frac{\dot{m}_2}{3\dot{m}_3^{\frac{1}{3}}} + \dot{m}_3^{\frac{2}{3}}A_1,\end{aligned} \tag{35}$$

$$\begin{aligned}B = {} & -\frac{(m_1+\eta)^2\dddot{m}_3}{9\dot{m}_3^{\frac{7}{3}}} + \frac{(m_1+\eta)\ddot{m}_1}{3\dot{m}_3^{\frac{4}{3}}} \\ & + \left[-\frac{4(m_1+\eta)\dot{m}_1}{9\dot{m}_3^{\frac{7}{3}}} + \frac{m_2+2\xi}{9\dot{m}_3^{\frac{4}{3}}}\right]\ddot{m}_3 \\ & + \frac{5(m_1+\eta)^2\ddot{m}_3^{\,2}}{27\dot{m}_3^{\frac{10}{3}}} + \frac{\dot{m}_1^{\,2}}{6\dot{m}_3^{\frac{4}{3}}} \\ & - \frac{\dot{m}_2}{3\dot{m}_3^{\frac{1}{3}}} + \dot{m}_3^{\frac{2}{3}}B_1,\end{aligned} \tag{36}$$



(a) (b)

Figure 4. Plots of lump solution of A of the ncKPI system at time $t = 0$, while the parameters being fixed by equation (29): (a) density plot; (b) three-dimensional plot.

$$C = -\dot{m}_3^{\frac{2}{3}}[(e_1 m_3 - 1)C_1 + E_1], \tag{37}$$

$$E = \dot{m}_3^{\frac{2}{3}}(e_1 m_3 C_1 + E_1), \tag{38}$$

$$F = \dot{m}_3^{\frac{2}{3}}(e_1 m_3 G_1 + F_1), \tag{39}$$

$$G = -\dot{m}_3^{\frac{2}{3}}[(e_1 m_3 - 1)G_1 + F_1], \tag{40}$$

where A_1, B_1, C_1, E_1, F_1, G_1 are invariant functions of two new invariant variables

$$\begin{aligned}\xi = {} & \frac{\dot{m}_1}{2\dot{m}_3^{\frac{1}{3}}}y + \dot{m}_3^{\frac{1}{3}}x - \frac{\ddot{m}_3}{6(\dot{m}_3)^{\frac{2}{3}}}y^2 \\ & -\frac{1}{2}m_2,\ \eta = -m_1 + \dot{m}_3^{-\frac{2}{3}}y.\end{aligned} \tag{41}$$

In equation (41), m_1, m_2, m_3 are arbitrary functions of t which related to f_1, f_2, f_3 by

$$f_1 = \frac{1}{\dot{m}_3}, f_2 = \frac{\dot{m}_1}{\dot{m}_3^{\frac{5}{3}}}, f_3 = -\frac{\dot{m}_1^{\,2}}{2\dot{m}_3^{\frac{7}{3}}} + \frac{\dot{m}_2}{2\dot{m}_3^{\frac{4}{3}}}, \tag{42}$$

and satisfy

$$\hat{T}_d\,\{m_1, m_2, m_3\} = \{m_1, -m_2, -m_3\}. \tag{43}$$

By substituting equations (35)–(40) into the ncKPII system, we get corresponding symmetry reduction equations

$$\begin{aligned}&-18A_{1,\xi}^2 - 12A_{1,\xi}B_{1,\xi} + 6B_{1,\xi}^2 - 6(G_{1,\xi} + 2F_{1,\xi})C_{1,\xi} \\ &\quad +12G_{1,\xi}E_{1,\xi} + 16A_{1,\eta\eta} - 6(B_1 + 3A_1)A_{1,\xi\xi} \\ &\quad +6(B_1 - A_1)B_{1,\xi\xi} - 3(G_1 + 2F_1)C_{1,\xi\xi} + 6E_{1,\xi\xi}G_1 \\ &\quad -6F_{1,\xi\xi}C_1 + 3(2E_1 - C_1)G_{1,\xi\xi} + 4A_{1,\xi\xi\xi\xi} = 0,\end{aligned} \tag{44}$$

$$\begin{aligned}&-18B_{1,\xi}^2 - 12A_{1,\xi}B_{1,\xi} + 6A_{1,\xi}^2 - 6(G_{1,\xi} - 2F_{1,\xi})C_{1,\xi} \\ &\quad -12G_{1,\xi}E_{1,\xi} + 16B_{1,\eta\eta} - 6(B_1 - A_1)A_{1,\xi\xi} \\ &\quad -6(3B_1 + A_1)B_{1,\xi\xi} - 3(G_1 - 2F_1)C_{1,\xi\xi} - 6E_{1,\xi\xi}G_1 \\ &\quad +6F_{1,\xi\xi}C_1 - 3(2E_1 + C_1)G_{1,\xi\xi} + 4B_{1,\xi\xi\xi\xi} = 0,\end{aligned} \tag{45}$$

$$\begin{aligned}&[6(2e_1 m_3 - 1)C_{1,\xi} + 12E_{1,\xi}]A_{1,\xi} + [6(2m_3 e_1 - 3)C_{1,\xi} \\ &\quad +12E_{1,\xi}]B_{1,\xi} - 8C_{1,\xi}e_1 \\ &\quad +3(2C_1 e_1 m_3 + 2E_1 - 3C_1)A_{1,\xi\xi} + 3(2C_1 e_1 m_3 - C_1 \\ &\quad +2E_1)B_{1,\xi\xi} - 8(m_3 e_1 - 1)C_{1,\eta\eta} \\ &\quad +3(2B_1 m_3 e_1 + 2A_1 m_3 e_1 - B_1 - 3A_1)C_{1,\xi\xi} - 8E_{1,\eta\eta} \\ &\quad +6(B_1 + A_1)E_{1,\xi\xi} - 2(m_3 e_1 - 1)C_{1,\xi\xi\xi\xi} \\ &\quad -2E_{1,\xi\xi\xi\xi} = 0,\end{aligned} \tag{46}$$

$$\begin{aligned}&[6(2e_1 m_3 + 1)C_{1,\xi} + 12E_{1,\xi}]A_{1,\xi} \\ &\quad +[6(2m_3 e_1 - 1)C_{1,\xi} + 12E_{1,\xi}]B_{1,\xi} - 8C_{1,\xi}e_1 \\ &\quad +3(2C_1 e_1 m_3 + 2E_1 - C_1)A_{1,\xi\xi} + 3(2C_1 e_1 m_3 \\ &\quad +C_1 + 2E_1)B_{1,\xi\xi} - 8m_3 e_1 C_{1,\eta\eta} \\ &\quad +3(2B_1 m_3 e_1 + 2A_1 m_3 e_1 + B_1 - A_1)C_{1,\xi\xi} - 8E_{1,\eta\eta} \\ &\quad +6(B_1 + A_1)E_{1,\xi\xi} - 2m_3 e_1 C_{1,\xi\xi\xi\xi} - 2E_{1,\xi\xi\xi\xi} = 0,\end{aligned} \tag{47}$$

$$\begin{aligned}&6[2F_{1,\xi} + (2m_3 e_1 + 1)G_{1,\xi}]A_{1,\xi} \\ &\quad +6[2F_{1,\xi} + (2m_3 e_1 - 1)G_{1,\xi}]B_{1,\xi} - 8G_{1,\xi}e_1 \\ &\quad +3(2G_1 e_1 m_3 + G_1 + 2F_1)A_{1,\xi\xi} \\ &\quad +3(2G_1 e_1 m_3 - G_1 + 2F_1)B_{1,\xi\xi} - 8F_{1,\eta\eta} \\ &\quad +6(B_1 + A_1)F_{1,\xi\xi} - 8G_{1,\eta\eta}m_3 e_1 \\ &\quad +3(2A_1 m_3 e_1 + 2B_1 m_3 e_1 + A_1 - B_1)G_{1,\xi\xi} \\ &\quad -2G_{1,\xi\xi\xi\xi}e_1 m_3 - 2F_{1,\xi\xi\xi\xi} = 0,\end{aligned} \tag{48}$$

$$\begin{aligned}&6[2F_{1,\xi} + (2m_3 e_1 - 1)G_{1,\xi}]A_{1,\xi} + 6[2F_{1,\xi} \\ &\quad +(2m_3 e_1 - 3)G_{1,\xi}]B_{1,\xi} - 8G_{1,\xi}e_1 \\ &+3(2G_1 e_1 m_3 - G_1 + 2F_1)A_{1,\xi\xi} \\ &\quad +3(2G_1 e_1 m_3 - 3G_1 + 2F_1)B_{1,\xi\xi} - 8F_{1,\eta\eta} \\ &+6(B_1 + A_1)F_{1,\xi\xi} - 8(m_3 e_1 - 1)G_{1,\eta\eta} \\ &\quad +3(2A_1 m_3 e_1 + 2B_1 m_3 e_1 - A_1 - 3B_1)G_{1,\xi\xi} \\ &-2(e_1 m_3 - 1)G_{1,\xi\xi\xi\xi} - 2F_{1,\xi\xi\xi\xi} = 0,\end{aligned} \tag{49}$$

along with

$$
\begin{aligned}
&A_1(-\xi,\eta)=B_1(\xi,\eta),\ C_1(-\xi,\\
&\eta)=C_1(\xi,\eta),\ E_1(-\xi,\eta)=C_1(\xi,\eta)-E_1(\xi,\eta),\\
&F_1(-\xi,\eta)=G_1(\xi,\eta)-F_1(\xi,\eta),\\
&G_1(\xi,\eta)=G_1(-\xi,\eta).
\end{aligned}
$$

4. Summary

In summary, a nonlocal coupled KP system is introduced and studied by converting it into a localized system. Via this method, new solutions of the ncKP system are generated from known ones of the cKP system. An even number of singular soliton solutions are obtained in a general form, among which $N=2$ and $N=4$ soliton solutions are plotted and analyzed. By fixing appropriate parameters, soliton solutions of the ncKPI system become breathers and we also attained lump-type solutions. The standard Lie symmetry method is carried on the ncKPII system to obtain symmetry reduction solutions.

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Compliance with ethical standards

Conflict of interest statement

The authors declare that they have no conflicts of interest to this work. There is no professional or other personal interest of any nature or kind in any product that could be construed as influencing the position presented in the manuscript entitled.

References

[1] Ablowitz M J and Musslimani Z H 2013 Integrable nonlocal nonlinear Schrödinger equation *Phys. Rev. Lett.* **110** 064105

[2] Gerdjikov V S and Saxena A 2017 Complete integrability of nonlocal nonlinear Schrödinger equation *J. Math. Phys.* **58** 013502

[3] Ablowitz M J and Musslimani Z H 2016 Inverse scattering transform for the integrable nonlocal nonlinear Schrödinger equation *Nonlinearity* **29** 915

[4] Yang B and Chen Y 2018 Dynamics of high-order solitons in the nonlocal nonlinear Schrödinger equations *Nonlinear Dyn.* **94** 489

[5] Wang M and Chen Y 2021 Dynamic behaviors of general N-solitons for the nonlocal generalized nonlinear Schrödinger equation *Nonlinear Dyn.* **104** 2621

[6] Feng B F, Luo X D, Ablowitz M J and Musslimani Z H 2018 General soliton solution to a nonlocal nonlinear Schrödinger equation with zero and nonzero boundary conditions *Nonlinearity* **31** 5385

[7] Ma L Y and Zhu Z N 2016 N-soliton solution for an integrable nonlocal discrete focusing nonlinear Schrödinger equation *Appl. Math. Lett.* **59** 115

[8] Yan Z 2015 Integrable PT-symmetric local and nonlocal vector nonlinear Schrödinger equations: a unified two-parameter model *Appl. Math. Lett.* **47** 61

[9] Xu Z X and Chow K W 2016 Breathers and rogue waves for a third order nonlocal partial differential equation by a bilinear transformation *Appl. Math. Lett.* **56** 72

[10] Huang X and Ling L 2016 Soliton solutions for the nonlocal nonlinear Schrödinger equation *Eur. Phys. J. Plus* **131** 148

[11] Li M and Xu T 2015 Dark and antidark soliton interactions in the nonlocal nonlinear Schrödinger equation with the self-induced parity-time-symmetric potential *Phys. Rev.* E **91** 033202

[12] Song C Q, Xiao D M and Zhu Z N 2017 Reverse space-time nonlocal Sasa–Satsuma equation and its solutions *J. Phys. Soc. Jpn.* **86** 054001

[13] Chen K and Zhang D 2018 Solutions of the nonlocal nonlinear Schrödinger hierarchy via reduction *Appl. Math. Lett.* **75** 82

[14] Liu X Z, Yu J, Lou Z M and Qian X M 2019 A nonlocal Burgers equation in atmospheric dynamical system and its exact solutions *Chin. Phys.* B **28** 010201

[15] Li J and Li B 2021 Solving forward and inverse problems of the nonlinear Schrödinger equation with the generalized *PT*-symmetric Scarf-II potential via PINN deep learning *Commun. Theor. Phys.* **73** 125001

[16] Lou S Y and Huang F 2017 Alice–Bob physics: coherent solutions of nonlocal KdV systems *Sci. Rep.* **7** 869

[17] Lou S Y 2018 Alice-Bob systems, P^−T^−Ĉ symmetry invariant and symmetry breaking soliton solutions *J. Math. Phys.* **59** 083507

[18] Li C C, Lou S Y and Jia M 2018 Coherent structure of Alice–Bob modified Korteweg de-Vries equation *Nonlinear Dyn.* **93** 1799–808

[19] Lou S Y 2017 From nothing to something II: nonlinear systems via consistent correlated bang *Chin. Phys. Lett.* **34** 060201

[20] Lou S Y 2020 Multi-place physics and multi-place nonlocal systems *Commun. Theor. Phys.* **72** 057001

[21] Wu W B and Lou S Y 2019 Exact Solutions of an Alice–Bob KP Equation *Commun. Theor. Phys.* **71** 629

[22] Wazwaz A M 2010 Completely integrable coupled KdV and coupled KP systems *Commun. Nonlinear Sci. Numer. Simul.* **15** 2828

[23] Isojima S, Willox R and Satsuma J 2002 On various solutions of the coupled KP equation *J. Phys. A: Math. Gen.* **35** 6893

[24] Hietarinta J 1990 *Partially Integrable Evolution Equations in Physics* ed R Conte and N Boccara (Dordrecht: Kluwer) pp 459–78

[25] Yang G Y and Liu Q P 2008 A Darboux transformation for the coupled Kadomtsev–Petviashvili equation *Chin. Phys. Lett.* **25** 1

[26] Jimbo M and Miwa T 1983 Solitons and infinite dimensional lie algebras *Publ. Res. Inst. Math. Sci. Kyoto Univ.* **19** 943

[27] Hirota R and Ohta Y 1991 Hierarchies of coupled soliton equations *J. Phys. Soc. Jpn.* **60** 798

[28] Isojima S, Willox R and Satsuma J 2003 Spider-web solutions of the coupled KP equation *J. Phys. A: Math. Gen.* **36** 9533

[29] Olver P J 1993 *Applications of Lie Groups to Differential Equations* (NewYork: Springer)

[30] Bluman G W and Kumei S 1989 *Symmetries and Differential Equation* (Berlin: Springer)