© 2024 Institute of Theoretical Physics CAS, Chinese Physical Society and IOP Publishing Printed in China Communications in Theoretical Physics

Commun. Theor. Phys. **76** (2024) 105501 (6pp) https://doi.org/10.1088/1572-9494/ad5527

The combined effects of quantum and strong coupling on the nonlinear collective excitation in quantum dusty plasmas

Chengkai Li, Youyou Kang and Yunliang Wang

Department of Physics, School of Mathematics and Physics, University of Science and Technology Beijing, Beijing 100083, China

E-mail: ylwang@ustb.edu.cn

Received 12 April 2024, revised 4 June 2024
Accepted for publication 7 June 2024
Published 31 July 2024

Abstract

The quantum hydrodynamic model for electrons and ions and the generalized hydrodynamic model for the strongly coupled dust particles are proposed in the strongly coupled quantum dusty plasma, where the combined quantum effects of quantum diffraction, quantum statistic pressure, as well as electron exchange and correlation effects are all considered in the quantum hydrodynamic model. The shear and bulk viscosity effects are included in the viscoelastic relaxation, which leads to the decay of the dust-ion-acoustic waves. The approximate time-dependent solitary solution is obtained by the momentum conservation law in the presence of viscosity.

Keywords: quantum plasma, strongly coupled plasma, dusty plasma

1. Introduction

Quantum effects in plasmas should be taken into account when the Fermi energy of the charged particles in the plasma exceeds its thermal energy [1–3]. Accordingly, quantum plasmas have been widely investigated in the laser solid interaction [4], and in astrophysical objects like neutron stars and white dwarfs star, in quantum x-ray free-electron lasers [5], in nuclear fusion reaction [6, 7], in warm dense matter [8, 9], and in streaming and wake effects [10].

In order to measure the interparticle interactions between electrons or ions, the quantum coupling parameter is preferred in the quantum plasma as $\Gamma_{e,i} = 4\pi e^2 n_{e,i}^{1/3} / E_{\mathrm{Fe,Fi}}$ with $E_{\mathrm{Fe,Fi}} = \hbar^2 (3\pi^2 n_{e,i})^{2/3} / 2m_{e,i}$ being the Fermi energy, where e is the electron charge, $n_{e,i}$ is the number density of electrons or ions, $m_{e,i}$ is the rest mass of electron or ion, and $\hbar$ is the reduced Plancks constant. The qantum coupling parameters of electrons and ions implies that a quantum plasma becomes even more ideal at higher densities [11–13]. The quantum diffraction effect stands for the fluctuation of microscopic particles, which was experimentally verified by x-ray Thomson scattering [14]. When the plasma density is high enough that the distance between charged particles is comparable or smaller than the De Broglie wavelength of the charged particles, the relativistic degeneracy pressure, which arises from the Pauli exclusion mechanism, must be taken into acount. The presence of upper limit of the mass of a white dwarf is one of the first important evidence which is supported by the pressure of degenerate plasma [15, 16]. The spin effects is also important in quantum plasmas, especially in astrophysical invironments such as magnetars and pulsars [17], which was studied by spin magnetohydrodynamic approach by using a non-relativistic Pauli equation for spin-$1/2$ particles [18]. The investigation of the magnetorotational instability should be also taken into acount with the effect of spin magnetization in a differentially rotating degenerate quantum plasma [19]. The combined effects of spin and polarization on the laser C plasma interactions is important for the quantum electrodynamics plasmas [20], where spin-polarized quantum radiation reaction strongly affects the dynamics of the plasma.

Dusty particles with nano-sized or micron-sized solid particles suspended in a plasma environment will deeply modify the collective excitation in the dusty plasma. Due to the large mass of the dusty particles, it should be taken that the Coulomb coupling strength for dusty plasma can be defined as $\Gamma_d = (Z_d^2 e^2 / k_{\mathrm{B}} T_d L) \exp(-L/\lambda_d)$ with the dusty Debye length $\lambda_d = (k_{\mathrm{B}} T_d / 4\pi n_d Z_d^2 e^2)^{1/2}$ and the mean

0253-6102/24/105501+6$33.00

interparticle distance L. The coupling strength will increase with the increase of the dusty plasma density. The dusty plasma can be described by the fluid model [21, 22] or the visco-elastic fluid model [23, 24] according to the magnitude of the Coulomb coupling strength. The strongly coupling effects can cause ring structural transition [25] and the formation of a void [26]. A pair of counter-rotating symmetric vortices was observed in the wake in a flowing dusty plasma, where the strongly coupled dusty plasmas flow with controllable velocity [27]. For dusty plasma in a perpendicular magnetic field, the shear viscosity can be modified substantially, which shows that the viscosity of the dusty plasma increases at low temperatures if the magnetic field is increased, while at high temperatures the viscosity of the dusty plasma decreases [28]. The strong correlation effects contribute to stabilizing the growth rate of linear Rayleigh–Taylor instability [29] and Jeans instability [30] in the strongly coupled dusty plasma.

Accordingly, for the ultradense dusty plasma, the quantum coupling effects of electrons and ions and the Coulomb coupling effects will dominate the dynamics of the strongly coupled dusty plasma [31, 32]. We will consider the combined quantum effects of quantum diffraction, quantum statistic pressure, as well as electron exchange and correlation effects due to the spin of electrons and ions on the nonlinear collective excitation of dust-ion-acoustic waves (DIAWs) in the strongly coupled dusty plasma. As will be shown, the strongly coupling effects will lead to the attenuation of DIAWs. The quantum effects will also affect the dispersion and nonlinearity of the DIAWs.

2. Governing equations

We consider the nonlinear DIAW propagation in a strongly coupled dust quantum plasma with arbitrary charged cold dust fluid, and electron and ion fluids. The electrons and ions are modeled by quantum hydrodynamic equation, where the quantum diffraction, quantum statistic pressure, as well as electron exchange and correlation effects due to electron spin are all considered in our model. The quantum hydrodynamic equations of the electrons and ions are given as [33–35]

$$\frac{\partial n_j}{\partial t}+\frac{\partial(n_j u_j)}{\partial x}=0, \tag{1}$$

$$\frac{\partial u_e}{\partial t}+u_e\frac{\partial u_e}{\partial x}-\frac{\partial\phi}{\partial x}+\Gamma_{\mathrm{xce}}\frac{\partial V_{\mathrm{xce}}}{\partial x}+\frac{V_{\mathrm{Fe}}^2}{C_{\mathrm{se}}^2}n_e^{-\frac{1}{3}}\frac{\partial n_e}{\partial x} -2H_e^2\frac{\partial}{\partial x}\left(\frac{1}{\sqrt{n_e}}\frac{\partial^2\sqrt{n_e}}{\partial x^2}\right)=0, \tag{2}$$

$$\frac{\partial u_i}{\partial t}+u_i\frac{\partial u_i}{\partial x}+M\frac{\partial\phi}{\partial x}+\Gamma_{\mathrm{xci}}\frac{\partial V_{\mathrm{xci}}}{\partial x}+\frac{V_{\mathrm{Fi}}^2}{C_{\mathrm{si}}^2}n_i^{-\frac{1}{3}}\frac{\partial n_i}{\partial x} -2H_i^2\frac{\partial}{\partial x}\left(\frac{1}{\sqrt{n_i}}\frac{\partial^2\sqrt{n_i}}{\partial x^2}\right)=0, \tag{3}$$

where $n_j(j=e,\ i,\ d)$ (normalized by n_{j0}) is the electron, ion, or dust number density, respectively. The n_{j0}) is the equilibrium number density. $u_j(j=e,\ i,\ d)$ (normalized by Fermi speed V_{Fe}) is the electron, ion, or dust fluid velocity, respectively. The time and space variables are normalized by $\lambda_{\mathrm{DFe}}=(k_{\mathrm{B}}T_{\mathrm{Fe}}\varepsilon_0/e^2n_{e0})^{1/2}$ and inverse electron plasma frequency $\omega_{\mathrm{pe}}^{-1}=(e^2n_{e0}/m_e\varepsilon_0)^{-1/2}$. Here, T_{Fe} is the electron Fermi-temperature, k_{B} is the Boltzmann constant, $m_{e,i}$ is electron (ion) rest mass. In equations (2) and (3) the third term stands for the electrostatic field force from the charge separation between electrons, ions, and dust particles. The fourth term in equations (2) and (3) is related to the electron and ion exchange-correlation force due to electron spin, of which the coefficients are $\Gamma_{\mathrm{xce}}=0.985e^2/\epsilon r_{e0}k_{\mathrm{B}}T_{\mathrm{Fe}}$, and $\Gamma_{\mathrm{xci}}=0.985e^2M/\epsilon r_{i0}k_{\mathrm{B}}T_{\mathrm{Fe}}$, where the $r_{e0,i0}=n_{e0,i0}^{-1/3}$ are the Wigner–Seitz radius. The non-dimensional exchange-correlation potential $V_{xce,i}$ are given as [36]

$$V_{\mathrm{xce},i}=n_{e,i}^{\frac{1}{3}}+\frac{0.034}{a_{\mathrm{Be},i}}\ln(1+18.37a_{\mathrm{Be},i}n_{e,i}^{\frac{1}{3}}), \tag{4}$$

where $M=m_e/m_i$ and the Bohr radius is $a_{\mathrm{Be},i}=\epsilon\hbar^2/m_{e,i}e^2r_{0e,i}$, which is normalized by the Wigner–Seitz radius. ϵ is the dielectric constant in vacuum. The fifth and sixth term of equations (2) and (3) is the degenerate pressure and quantum reoil force, respectively. Here we consider non-relativistic degenerate plasma and then the pressure can be given as $P_{e,i}=K_{e,i}n_{e,i}^{5/3}$ and $K_{e,i}=(3/5)(\pi/3)^{1/3}\pi\hbar^2/m_{e,i}$ [37]. $\hbar$ is the Planck constant. The parameters in the coefficient $V_{\mathrm{Fe},i}^2/C_{\mathrm{se},i}^2$ are the Fermi speed $V_{\mathrm{Fe},i}=(1/3)^{1/6}(\hbar/m_{e,i}^*)(\pi^2n_{e0,i0})^{1/3}$ and quantum acoustic velocity $C_{\mathrm{se},i}=(k_{\mathrm{B}}T_{\mathrm{Fe}}/m_{e,i}^*)^{1/2}$. The quantum recoil force is related to the Bohm potential, which stands for the electrons or ion tunneling effects. The quantum diffraction parameters are given as $H_e=(\hbar\omega_{\mathrm{pe}}/2k_{\mathrm{B}}T_{\mathrm{Fe}})$ and $H_i=(\hbar\omega_{\mathrm{pe}}/2k_{\mathrm{B}}T_{\mathrm{Fe}})M$ [38], which stands for the fluctuation of microscopic particles in the quantum mechanical effects.

For strongly coupled dust particles, a generalized hydrodynamic model is considered as

$$\left(1+\overline{\tau}_m\frac{\mathrm{d}}{\mathrm{d}t}\right)\left[n_d\frac{\mathrm{d}u_d}{\mathrm{d}t}+A_\gamma\frac{\partial n_d}{\partial x}-A_\beta n_d\frac{\partial\phi}{\partial x}\right]=\eta^*\frac{\partial^2u_d}{\partial x^2}, \tag{5}$$

where the coefficients are $A_\gamma=\gamma_d m_e T_d/m_d T_{\mathrm{Fe}}$ and $A_\beta=m_e Z_d/m_d$. Here m_d is the mass of the dust particles, and Z_d is the charged number of dust particles, and γ_d is the adiabatic index of the dust fluid, and T_d is the temperature of the dusty plasma, and $\eta^*=m_e\omega_{\mathrm{pe}}\Big/(n_{d0}m_dk_{\mathrm{B}}T_{\mathrm{Fe}})\left(\xi+\frac{4}{3}\eta\right)$ is the viscosity effects. ξ and η stand for shear and bulk viscosity effects, respectively. The non-dimensional $\overline{\tau}_m=\eta^*(T_e/T_d)\left[1-\mu_d+\frac{5}{14}u(\Gamma)\right]^{-1}$ refers to the visco-elastic relaxation time [39], in which $\mu_d=1+\frac{1}{3}u(\Gamma)+\frac{\Gamma}{9}\frac{\partial u(\Gamma)}{\partial\Gamma}$ is the compressibility and $u(\Gamma)$ is a measure of the excess internal energy of the system. The system is enclosed by the following Poisson's equations:

$$\frac{\partial^2\phi}{\partial x^2}=n_e-\alpha n_i-(1-\alpha)n_d. \tag{6}$$

Here, $\alpha = n_{i0}/n_{e0}$ and ϕ is the electrostatic potential normalized by $k_B T_{\rm Fe}/e$. The equilibrium quasi-neutrality condition $n_{e0} + Z_d n_{d0} = n_{i0}$ is considered.

3. The solution of the modified KdV of the DIAWs

In order to study the nonlinear set of equations (1)–(6) we introduce the stretched variables ξ and τ with $\xi = \varepsilon^{\frac{1}{2}}(x - \lambda t)$, $\tau = \varepsilon^{\frac{3}{2}}t$, where ε is small parameter standing for the weak nonlinearity and λ is the phase velocity of the DIAWs [40]. We then expand the physical variables n_j, u_j, and ϕ in terms of the expansion parameter ε as

$$f = f^{(0)} + \sum_{n=1}^{\infty} \varepsilon^n f^{(n)}, \tag{7}$$

where $f = n_j$, u_j, ϕ, $f^{(0)} = 1$ for n_j, and $f^{(0)} = 0$ for $f = u_j$, ϕ. The expansion (7) shows that the physical quantities have a small deviation from the equilibrium state f_0, whereas ε characterizes the strength of the nonlinearity. The equilibrium state of the physical quantities can be written as unperturbed quantities. Accordingly, the unperturbed number density is normalized by their own equilibrium quantities as $n_e^{(0)} = n_i^{(0)} = n_d^{(0)} = 1$. While the unperturbed velocity of the quantum fluids and the electrostatic potential are $u_e^{(0)} = u_i^{(0)} = u_d^{(0)} = \phi^{(0)} = 0$. Then, we will use equation (7) in equations (1)–(6) to expand the physical parameters in various orders in ε. We will take the ordering of $\overline{\tau}_m$ and η^* on the expansion. We consider the limits as $\overline{\tau}_m = \omega_{\rm pe}\tau_m \gg 1$, $\eta^* \sim O(1)$. In this limit, we use stretched variables and equation (7) in equations (1)–(6) and obtain the following equations in the first two lowest order in ε from each of the equations (1)–(6) as

$$\frac{\partial u_j^{(1)}}{\partial \xi} - \lambda \frac{\partial n_j^{(1)}}{\partial \xi} = 0, \tag{8}$$

$$\frac{\partial u_j^{(2)}}{\partial \xi} - \lambda \frac{\partial n_j^{(2)}}{\partial \xi} + \frac{\partial n_j^{(1)}}{\partial \tau} + \frac{\partial n_j^{(1)} u_j^{(1)}}{\partial \xi} = 0, \tag{9}$$

$$-\lambda \frac{\partial u_e^{(1)}}{\partial \xi} - \frac{\partial \phi^{(1)}}{\partial \xi} + \left(\frac{V_{\rm Fe}^2}{C_{\rm se}^2} + \frac{C_{\rm xce}^2}{C_{\rm se}^2}\right)\frac{\partial n_e^{(1)}}{\partial \xi} = 0, \tag{10}$$

$$\begin{aligned} &-\lambda \frac{\partial u_e^{(2)}}{\partial \xi} - \frac{\partial \phi^{(2)}}{\partial \xi} + \left(\frac{V_{\rm Fe}^2}{C_{\rm se}^2} + \frac{C_{\rm xce}^2}{C_{\rm se}^2}\right)\frac{\partial n_e^{(2)}}{\partial \xi} + \frac{\partial u_e^{(1)}}{\partial \tau} \\ &+\frac{1}{2}\frac{\partial u_e^{(1)} u_e^{(1)}}{\partial \xi} - \frac{V_{\rm Fe}^2}{6C_{\rm se}^2}\frac{\partial n_e^{(1)} n_e^{(1)}}{\partial \xi} - H_e^2 \frac{\partial^3 n_e^{(1)}}{\partial \xi^3} = 0, \end{aligned} \tag{11}$$

$$-\lambda \frac{\partial u_i^{(1)}}{\partial \xi} + M\frac{\partial \phi^{(1)}}{\partial \xi} + \left(\frac{V_{\rm Fi}^2}{C_{\rm si}^2} + \frac{C_{\rm xci}^2}{C_{\rm si}^2}\right)\frac{\partial n_i^{(1)}}{\partial \xi} = 0, \tag{12}$$

$$\begin{aligned} &-\lambda \frac{\partial u_i^{(2)}}{\partial \xi} + M\frac{\partial \phi^{(2)}}{\partial \xi} + \left(\frac{V_{\rm Fi}^2}{C_{\rm si}^2} + \frac{C_{\rm xci}^2}{C_{\rm si}^2}\right)\frac{\partial n_i^{(2)}}{\partial \xi} + \frac{\partial u_i^{(1)}}{\partial \tau} \\ &+\frac{1}{2}\frac{\partial u_i^{(1)} u_i^{(1)}}{\partial \xi} - \frac{V_{\rm Fi}^2}{6C_{\rm si}^2}\frac{\partial n_i^{(1)} n_i^{(1)}}{\partial \xi} - H_i^2 \frac{\partial^3 n_i^{(1)}}{\partial \xi^3} = 0, \end{aligned} \tag{13}$$

$$(\eta^* - \lambda^2 \overline{\tau}_m)\frac{\partial^2 u_d^{(1)}}{\partial \xi^2} = \lambda \overline{\tau}_m \left[A_\beta \frac{\partial^2 \phi^{(1)}}{\partial \xi^2} - A_\gamma \frac{\partial^2 n_d^{(1)}}{\partial \xi^2}\right], \tag{14}$$

$$\begin{aligned} &(\eta^* - \lambda^2 \overline{\tau}_m)\frac{\partial^2 u_d^{(2)}}{\partial \xi^2} = \lambda \overline{\tau}_m \left[A_\beta \frac{\partial^2 \phi^{(2)}}{\partial \xi^2} - A_\gamma \frac{\partial^2 n_d^{(2)}}{\partial \xi^2}\right] \\ &-\lambda \overline{\tau}_m \frac{\partial}{\partial \xi}\left(u_d^{(1)} \frac{\partial u_d^{(1)}}{\partial \xi}\right) + \lambda^2 \overline{\tau}_m \frac{\partial}{\partial \xi}\left(n_d^{(1)} \frac{\partial u_d^{(1)}}{\partial \xi}\right) \\ &+\lambda \overline{\tau}_m A_\beta \frac{\partial}{\partial \xi}\left(n_d^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi}\right) + \overline{\tau}_m A_\gamma u_d^{(1)} \frac{\partial^2 n_d^{(1)}}{\partial \xi^2} \\ &-\overline{\tau}_m A_\beta u_d^{(1)} \frac{\partial^2 \phi^{(1)}}{\partial \xi^2} - 2\lambda \overline{\tau}_m \frac{\partial}{\partial \tau}\left(\frac{\partial u_d^{(1)}}{\partial \xi}\right) \\ &+\overline{\tau}_m A_\gamma \frac{\partial}{\partial \tau}\left(\frac{\partial n_d^{(1)}}{\partial \xi}\right) - \overline{\tau}_m A_\beta \frac{\partial}{\partial \tau}\left(\frac{\partial \phi^{(1)}}{\partial \xi}\right), \end{aligned} \tag{15}$$

$$n_e^{(1)} - \alpha n_i^{(1)} - (1 - \alpha) n_d^{(1)} = 0, \tag{16}$$

$$n_e^{(2)} - \alpha n_i^{(2)} - (1 - \alpha) n_d^{(2)} = \frac{\partial^2 \phi}{\partial \xi^2}. \tag{17}$$

From the lowest order expansions, we obtain the relationship between the first-order quantities as

$$\begin{aligned} n_e^{(1)} &= -\frac{1}{D_e}\phi_1, \\ u_e^{(1)} &= -\frac{\lambda}{D_e}\phi_1, \\ n_i^{(1)} &= -\frac{M}{D_i}\phi_1, \\ u_i^{(1)} &= -\frac{\lambda M}{D_i}\phi_1, \\ n_d^{(1)} &= \frac{A_\beta \overline{\tau}_m}{D_d}\phi_1, \\ u_d^{(1)} &= \frac{\lambda A_\beta \overline{\tau}_m}{D_d}\phi_1, \end{aligned} \tag{18}$$

where the parameters standing for quantum oscillation dispersion are determined as $D_e = \lambda^2 - (V_{\rm Fe}^2/C_{\rm se}^2 + C_{\rm xce}^2/C_{\rm se}^2)$, $D_i = \lambda^2 - (V_{\rm Fi}^2/C_{\rm si}^2 + C_{\rm xci}^2/C_{\rm si}^2)$, and $D_d = -\lambda^2 \overline{\tau}_m + A_\gamma \overline{\tau}_m + \eta^*$. From the first-order quantities, we can obtain the dispersion relation as

$$\frac{1}{D_e} + \frac{\alpha M}{D_i} + \frac{(1 - \alpha) A_\beta \overline{\tau}_m}{D_d} = 0. \tag{19}$$

Eliminating $n_j^{(2)}$, $u_j^{(2)}$, and $\phi^{(2)}$ from equations (9), (11), (13), (15), (17) and making use of equations (8), (10), (12), (14), (16), the following modified KdV (mKdV) equation is obtained:

$$\frac{\partial}{\partial \xi}\left[A\frac{\partial \phi^{(1)}}{\partial \tau} + B\phi^{(1)}\frac{\partial \phi^{(1)}}{\partial \xi} + C\frac{\partial^3 \phi^{(1)}}{\partial \xi^3}\right] = D\left(\frac{\partial \phi^{(1)}}{\partial \xi}\right)^2, \tag{20}$$

where coefficients A, B, C, and D of equation (18) can be written as follows:

$$\begin{aligned}
A &= Y_e - \alpha Y_i - (1-\alpha)Y_{d1},\\
B &= X_e - \alpha X_i - (1-\alpha)X_{d1},\\
C &= Z_e - \alpha Z_i - 1,\\
D &= (1-\alpha)Z_{d1},
\end{aligned} \tag{21}$$

where the other coefficients are given in the appendix.

As an exact analytic solution of equation (20) is absent, we are devoted to seek for an approximate time-dependent soliton solution of equation (20), where we consider the effect of D is small limit, i.e., $A \sim B \gg D$. Here the coefficient D stands for the importance of strong coupling. As will be shown in the approximate solution, the coefficient D will cause the dissipation of the solitary waves. The coefficients A, and B represent nonlinear and dispersion effects, respectively. From the analytical expression of A, and B, the nonlinear and dispersion effects come from the quantum effects of electrons and ions. Accordingly one can expect that the approximate solution will be reasonable when quantum effects are much stronger than strong coupling effects. Then we will consider the limit $X_e - \alpha X_i \gg (1-\alpha)(X_{d1} + Z_{d1})$ to obtain the approximate solution, where the quantities X_e and X_i stand for the quantum statistic and electron exchange and correlation effects and the quantities X_{d1}, and Z_{d1} represent the strong coupling effects. In this limit, we can rewrite equation (20) after integrating with respect to ξ as

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A\phi^{(1)}\frac{\partial \phi^{(1)}}{\partial \xi} + B\frac{\partial^3 \phi^{(1)}}{\partial \xi^3} = D\int_{-\infty}^{\infty}\left(\frac{\partial \phi^{(1)}}{\partial \xi'}\right)^2 \mathrm{d}\xi', \tag{22}$$

where we have used the boundary conditions: $\phi^{(1)} \to 0$, $\partial\phi^{(1)}/\partial\xi \to 0$ as $\xi \to \pm\infty$. Now, in absence of D, a travelling wave (stationary soliton) solution of equation (20) can be obtained as

$$\Phi \equiv \phi^{(1)} = \Psi\,\mathrm{sech}^2\left(\frac{\xi - U\tau}{W}\right), \tag{23}$$

where $\Psi = 3U/A$ is the amplitude, $W = (12B/\Psi A)^{1/2} \equiv \sqrt{4B/U}$ is the width and $U = \Psi A/3$ is the constant phase speed of the DIAWs.

Next, to find the solitary wave solution of equation (22) with the effect of a small amount of D, we first integrate equation (22) with respect to ξ to obtain

$$\frac{\partial}{\partial \tau}\int_{-\infty}^{+\infty}\Phi\,\mathrm{d}\xi = 0. \tag{24}$$

This shows that equation (22) conserves the total number of particles. Furthermore, multiplying equation (22) by Φ and integrating over ξ yields

$$\frac{\partial}{\partial \tau}\int_{-\infty}^{\infty}\Phi^2(\xi, \tau)\mathrm{d}\xi \leqslant 0, \tag{25}$$

in which the equality sign holds only when $\Phi = 0$ for all ξ. Equation (25) states that an initial perturbation of the form (23) for which

$$\int_{-\infty}^{+\infty}\Phi^2\,\mathrm{d}\xi < \infty, \tag{26}$$

will decay to zero. Thus, the wave amplitude Ψ is not a constant, but decreases slowly with time. Here, we perform a perturbation analysis of equation (22) assuming that D is a small parameter with $1 \sim A \sim B \gg D$ (in magnitudes). So, we introduce a new space coordinate χ in a frame moving with the solitary wave and normalized to its width as $\chi = \xi - U(\tau)\tau$ The amplitude Ψ is assumed to vary slowly with time and $\Psi = \Psi(D, \tau)$. We also assume that $\Phi \equiv \Phi(\chi, \tau)$. To determine the time-dependent speed $U(\tau)$, we use momentum conservation law in the presence of viscosity as [41]

$$\frac{\mathrm{d}I}{\mathrm{d}\tau} = D\int_{-\infty}^{+\infty}\phi\left[\int_{-\infty}^{\chi}\left(\frac{\partial \phi}{\partial \chi'}\right)^2 \mathrm{d}\chi'\right]\mathrm{d}\chi, \tag{27}$$

where the momentum of KdV system $I = (1/2)\int_{-\infty}^{+\infty}\phi^2\mathrm{d}X$. Substituting equation (18) into equation (19) we obtain the analytical expression for time-dependent speed as

$$U(\tau) = U_0\left(1 - \frac{\tau}{\tau_0}\right)^{-\frac{2}{3}}, \tag{28}$$

where the parameters $\tau_0 = 5A\sqrt{B}\big/U_0^{3/2}D$ and U_0 is the velocity of the DIAWs when $\tau = 0$. From the expression of the speed $U(\tau)$ one can see that the amplitude of DIA solitary waves will decrease with time going on.

4. Conclusion

In conclusion, the DIAWs solitary in the kinetic regime are investigated in strongly coupled quantum dusty plasmas. We used the quantum hydrodynamic model for electrons and ions and the generalized hydrodynamic model for the strongly coupled dust particles. According to the quantum coupling parameter, the electrons and ions can be considered by quantum plasma, where the combined quantum effects of quantum diffraction, quantum exchange-correlation were concluded in our model. For dusty particles with nano-sized or micron-sized solid particles, we introduce the Coulomb coupling parameters to determine the generalized hydrodynamic model. In order to consider the spin effects in the ultradense quantum plasma, we introduce the electron and ion exchange-correlation force in the quantum hydrodynamic model. By using the reductive perturbation method, we derive a mKdV equation for the DIAWs in the presence of strong coupling effects. We firstly obtain the travelling wave (stationary soliton) solution of the mKdV equation. Consequently, we perform a perturbation analysis of the mKdV equation and assume that the shear and bulk viscosity effects are very weak. In the weak visco-elastic effect limit, we obtained an analytically approximate time-dependent solution of the mKdV equation for the dust-ion-acoustic solitary waves, which shows that the amplitude will increase as time goes on.

Acknowledgments

This research is supported by the National Natural Science Foundation of China (NSFC) (Grant No. 11974043) and the

Fundamental Research Funds for the Central Universities (Grant No. FRF-BR-19-002B).

Appendix. The coefficients in equation (21)

$$X_e = \frac{3\lambda^2 - \frac{V_{\rm Fe}^2}{C_{\rm se}^2}}{3\left[\lambda^2 - \left(\frac{V_{\rm Fe}^2}{C_{\rm se}^2} + \frac{C_{\rm xce}^2}{C_{\rm se}^2}\right)\right]^3}, \tag{A1}$$

$$Y_e = -\frac{2\lambda}{\left[\lambda^2 - \left(\frac{V_{\rm Fe}^2}{C_{\rm se}^2} + \frac{C_{\rm xce}^2}{C_{\rm se}^2}\right)\right]^2}, \tag{A2}$$

$$Z_e = \frac{H_e^2}{\left[\lambda^2 - \left(\frac{V_{\rm Fe}^2}{C_{\rm se}^2} + \frac{C_{\rm xce}^2}{C_{\rm se}^2}\right)\right]^2}, \tag{A3}$$

$$X_i = M^2\frac{3\lambda^2 - \frac{V_{\rm Fi}^2}{C_{\rm si}^2}}{3\left[\lambda^2 - \left(\frac{V_{\rm Fi}^2}{C_{\rm si}^2} + \frac{C_{\rm xci}^2}{C_{\rm si}^2}\right)\right]^3}, \tag{A4}$$

$$Y_i = \frac{2\lambda M}{\left[\lambda^2 - \left(\frac{V_{\rm Fi}^2}{C_{\rm si}^2} + \frac{C_{\rm xci}^2}{C_{\rm si}^2}\right)\right]^2}, \tag{A5}$$

$$Z_i = -\frac{H_i^2 M}{\left[\lambda^2 - \left(\frac{V_{\rm Fi}^2}{C_{\rm si}^2} + \frac{C_{\rm xci}^2}{C_{\rm si}^2}\right)\right]^2}, \tag{A6}$$

$$X_{d1} = \frac{A_\beta^2 A_\gamma \bar{\tau}_m^3 + 2A_\beta^2 \bar{\tau}_m^2(\eta^* - \lambda^2\bar{\tau}_m)}{(\eta^* - \lambda^2\bar{\tau}_m + A_\gamma\bar{\tau}_m)^3}, \tag{A7}$$

$$Y_{d1} = \frac{A_\beta A_\gamma \bar{\tau}_m^2 - 2\lambda A_\beta \bar{\tau}_m^2 + A_\beta\bar{\tau}_m(\eta^* - \lambda^2\bar{\tau}_m)}{\lambda(\eta^* - \lambda^2\bar{\tau}_m + A_\gamma\bar{\tau}_m)^2} - \frac{A_\beta\bar{\tau}_m}{\lambda(\eta^* - \lambda^2\bar{\tau}_m + A_\gamma\bar{\tau}_m)}, \tag{A8}$$

$$Z_{d1} = \frac{A_\beta^2\bar{\tau}_m^2}{(\eta^* - \lambda^2\bar{\tau}_m + A_\gamma\bar{\tau}_m)^2} - \frac{A_\beta^2 A_\gamma\bar{\tau}_m^3}{(\eta^* - \lambda^2\bar{\tau}_m + A_\gamma\bar{\tau}_m)^3}. \tag{A9}$$

References

[1] Bethkenhagen M, Witte B B L, Schörner M, Röpke G, Döppner T, Kraus D, Glenzer S H, Sterne P A and Redmer R 2020 Carbon ionization at gigabar pressures: an ab initio perspective on astrophysical high-density plasmas *Phys. Rev. Research* **2** 023260

[2] Akbari-Moghanjoughia M 2024 Phase-space evolution of quasiparticle excitations in electron gas *Phys. Plasmas* **31** 032109

[3] El-Labany S K, El-Taibany W F, Behery E E and Abd-Elbaki R 2020 Oblique collision of ion acoustic solitons in a relativistic degenerate plasma *Sci. Rep.* **10** 16152

[4] Vinko S M *et al* 2012 Creation and diagnosis of a solid-density plasma with an x-ray free-electron laser *Nature* **482** 59

[5] Eliasson B and Shukla P K 2012 Relativistic x-ray free-electron lasers in the quantum regime *Phys. Rev.* E **85** 065401

[6] Hora H, Sadighi-Bonabi R and Yazdani E 2012 Effect of quantum correction on the acceleration and delayed heating of plasma blocks *Phys. Rev.* E **85** 036404

[7] Lee M-J and Jung Y-D 2017 Quantum shielding effects on the Gamow penetration factor for nuclear fusion reaction in quantum plasmas *Phys. Plasmas* **24** 014502

[8] Fletcher L B *et al* 2015 Ultrabright x-ray laser scattering for dynamic warm dense matter physics *Nat. Photonics* **9** 274

[9] Dornheim T, Groth S, Sjostrom T, Malone F D, Foulkes W M C and Bonitz M 2016 Ab initio quantum monte carlo simulation of the warm dense electron gas in the thermodynamic limit *Phys. Rev. Lett.* **117** 156403

[10] Moldabekov Z, Ludwig P, Bonitz M and Ramazanov T 2015 Ion potential in warm dense matter: wake effects due to streaming degenerate electrons *Phys. Rev.* E **91** 023102

[11] Walia K 2023 Propagation characteristics of a high-power beam in weakly relativistic-ponderomotive thermal quantum plasma *Commun. Theor. Phys.* **75** 95501

[12] Akbar M A, Kayum M A and Osman M S 2021 Bright, periodic, compacton and bell-shape soliton solutions of the extended QZK and (3 + 1)-dimensional ZK equations *Commun. Theor. Phys.* **73** 105003

[13] Shahmansouri M and Misra A P 2019 Polarized debye sheath in degenerate plasmas *Commun. Theor. Phys.* **71** 1341

[14] Glenzer S H and Redmer R 2009 X-ray Thomson scattering in high energy density plasmas *Rev. Mod. Phys.* **81** 1625

[15] Chandrasekhar S and Tooper R F 1964 The dynamical instability of the white-dwarf configurations approaching the limiting mass *Astrophys. J.* **139** 1396

[16] Mamun A and Shukla P K 2010 Solitary waves in an ultrarelativistic degenerate dense plasma *Phys. Plasmas* **17** 104504

[17] Kouveliotou C *et al* 1998 An x-ray pulsar with a superstrong magnetic field in the soft x-ray repeater SGR1806-20 *Nature (London)* **393** 235

[18] Brodin G and Marklund M 2007 Spin magnetohydrodynamics *New J. Phys.* **9** 277

[19] Usman S, Mushtaq A and Jan Q 2018 Magnetorotational instability in spin quantum plasmas *Phys. Rev.* E **98** 033202

[20] Seipt D and Thomas A G R 2023 Kinetic theory for spin-polarized relativistic plasmas *Phys. Plasmas* **30** 093102

[21] Rao N N, Shukla P K and Yu M Y 1990 Dust-acoustic waves in dusty plasmas *Planet. Space Sci.* **38** 543

[22] Ma J X, Liu J and Yu M Y 1997 Fluid theory of the boundary of a dusty plasma *Phys. Rev.* E **55** 4627

[23] Diaw A and Murillo M S 2015 Generalized hydrodynamics model for strongly coupled plasmas *Phys. Rev.* E **92** 013107

[24] Dharodi V S 2020 Rotating vortices in two-dimensional inhomogeneous strongly coupled dusty plasmas: shear and spiral density waves *Phys. Rev.* E **102** 043216

[25] Dharodi V and Kostadinova E 2023 Ring structural transitions in strongly coupled dusty plasmas *Phys. Rev.* E **107** 055208

[26] Bailung Y, Deka T, Boruah A, Sharma S K, Pal A R, Chutia J and Bailung H 2018 Characteristics of dust voids in a strongly coupled laboratory dusty plasma *Phys. Plasmas* **25** 053705

[27] Bailung Y, Chutia B, Deka T, Boruah A, Sharma S K, Kumar S, Chutia J, Nakamura Y and Bailung H 2020 Vortex formation in a strongly coupled dusty plasma flow past an obstacle *Phys. Plasmas* **27** 123702

[28] Feng Y, Lin W and Murillo M S 2017 Viscosity of two-dimensional strongly coupled dusty plasma modified by a perpendicular magnetic field *Phys. Rev.* E **96** 053208

[29] Dolaia B and Prajapatib R P 2018 The rotating Rayleigh-Taylor instability in a strongly coupled dusty plasma *Phys. Plasmas* **25** 083708

[30] Dolai B and Prajapati R P 2020 Effects of dust-charge gradient and polarization forces on the waves and Jeans instability in strongly coupled dusty plasma *Phys. Lett.* A **384** 126462

[31] Bezbaruah P, Das P, Borah P and Das N 2018 Jeans instability of self gravitating dust cloud in presence of effective electrostatic pressure *Commun. Theor. Phys.* **70** 209

[32] Shahmansouri M, Farokhi B and Khodabakhshi N 2017 Modified potential around a moving test charge in strongly coupled dusty plasma *Commun. Theor. Phys.* **68** 111

[33] Zeba I, Yahia M E, Shukla P K and Moslem W M 2012 ElectronChole two-stream instability in a quantum semiconductor plasma with exchange-correlation effects *Phys. Lett.* A **376** 2309

[34] Moslem W M, Zeba I and Shukla P K 2012 Solitary acoustic pulses in quantum semiconductor plasmas *Appl. Phys. Lett.* **101** 032106

[35] Wang Y and Lü X 2014 Modulational instability of electrostatic acoustic waves in an electron-hole semiconductor quantum plasma *Phys. Plasmas* **21** 022107

[36] Brey L, Dempsey J, Johnson N F and Halperin B I 1990 Infrared optical absorption in imperfect parabolic quantum wells *Phys. Rev.* B **42** 1240

[37] Shukla P K, Mamun A A and Mendis D A 2011 Nonlinear ion modes in a dense plasma with strongly coupled ions and degenerate electron fluids *Phys. Rev.* E **84** 026405

[38] Shukla P K and Eliasson B 2011 Colloquium: Nonlinear collective interactions in quantum plasmas with degenerate electron fluids *Rev. Mod. Phys.* **83** 885

[39] Kaw P K and Sen A 1998 Low frequency modes in strongly coupled dusty plasmas *Phys. Plasmas* **5** 3552

[40] Washimi H and Taniuti T 1966 Propagation of ion-acoustic solitary waves of small amplitude *Phys. Rev. Lett.* **17** 996

[41] Janaki M S, Dasgupta B, Gupta M R and Som B K 1992 Solitary magnetosonic waves with Landau damping *Phys. Scripta* **45** 368