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Energy exchange between charged relativistic fluids in *f*(*T*) gravity

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Abstract

This study focuses on the effects of a polytropic fluid on a charged gravitational source within $f(T)$ gravity, where T is the torsion scalar. We investigate how the electromagnetic field affects the flow of energy in spherically symmetric and static celestial objects that contain relativistic fluids. By using the gravitational decoupling technique, we analyze the effects of polytropic fluid on the dynamics of the gravitational source, accompanied by the matching of the interior geometry with an exterior at the hypersurface Σ. Finally, with the help of the Tolman $\mathbb{IV}$ solution, we observe the conduct of energy conditions with the existence of charge using $f(T)$ field equations and got the intended outcomes.

Keywords: complexity, hydrodynamics, anisotropy, self-gravitating systems, energy exchange

(Some figures may appear in colour only in the online journal)

1. Introduction

Astrophysical observational data reveals several mysterious aspects of the Universe and reinforces theories regarding its expansion. The enigmatic substances, i.e. dark energy and dark matter, are regarded as the most likely ways to explain current cosmic problems. General relativity (GR) is among the foundations that offer a comprehension of astronomical phenomena and the cosmos. The existence of both dark energy and matter poses certain challenges to GR, regardless of the insightful information it has revealed regarding the Universe's evolution and its mysterious secrets. The current astronomical observations indicate that, in an accelerated way, the Universe is expanding. As a result, it indicates that GR ought to be modified so as to provide an explanation for this phenomenon. These modifications are made by adding or omitting the curvature invariants or their equivalent generic functions. These theories of gravity involve the Brans–Dicke theory [1], the $f(\mathcal{R})$ theory (where $\mathcal{R}$ is the Ricci scalar) [2], the Gauss–Bonnet (GB) theory [3], the $f(T)$ theory, etc. The $f(T)$ gravity [4] is established via modifying teleparallel gravity (TG) by assuming that Lagrangian density in relation to the torsion scalar is equivalent. Recently, it has been demonstrated that $f(T)$-gravity theories allow for the accelerated expansion, without using dark energy, of the entire cosmos [5]. The Weitzenb ö ck connection, which possesses a non-zero torsion scalar's value, is used in $f(T)$ gravity as the best substitute for illustrating inflation and the Universe's late-time rapid expansion. Yang [6] noted that the dynamical identicality of the $f(T)$ gravity is absent in TEGR Lagrangian after conformal translation. Additionally, the Di-Sitter solution is shown to arise in the context of conformal TG, as explored by Bamba *et al* [7], who additionally discussed the challenges of $f(T)$ gravity. Ruggiero and Radicella [8] demonstrated by the analysis of weak-field solutions with symmetry, i.e. spherical, in $f(T)$ gravity that perturbations occur in the Schwarzschild and Schwarzschild–de Sitter solutions, which are spherically symmetric GR solutions. They examined the effect of these perturbations in observational testing. Ilijí and Sossich [9] considered the self-

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gravitating configuration of the perfect fluid so as to investigate the characteristics of $f(T)$-gravity theory in extreme situations, such as those occurring within highly compact static spherically symmetric bodies. Bhatti *et al* [10] investigated the effects of gravitational modification, more precisely the $f(T)$ gravity, on the electromagnetic field and hyperbolic symmetry in the static fluid content. In addition, they investigated the structural scalars in $f(T)$ gravity and discovered important results in the electric charge's presence.

Different phases of the evolution of the stellar structures may be impacted by the electric charge's movement within a system [11–15]. It is generally accepted that charge acts as an extra repellent force to counteract the enormous pull of gravity required to maintain hydrostatic balance. Moffat [16] gained the field equations for new gravity by incorporating an electromagnetic field (EMF) into the Lagrangian and obtained static spherically symmetric solutions. Ivanov [17] utilized a categorized technique in the occurrence of EMF to discover the Reissner–Nordström metric solutions. Dehghani [18] provided a novel set of modified gravity solutions and a cosmic constant that is negative. These, in his opinion, were black-brane solutions. Zhang *et al* [19] investigated how charge affects the gravitational collapse in de Sitter spacetime. They investigated how two aspects affect the charge and inferred that the characteristics of gravitational collapse are not influenced by dimension. Bhatti and Yousaf [20] investigated the influence of the charge on the components of inhomogeneity for planar symmetry in $f(\mathcal{R})$ theory. For this analysis, dissipative and anisotropic fluids were employed. They showed that the electric charge has an impact on the inhomogeneity components. The impact of EMF on gravastars has recently been investigated in many modified theories, and in this scenario, the researchers discovered stable regions for gravastars [21, 22]. Yousaf and associates [23] studied the effects of electromagnetic fields on relativistic dense star formations as well as anisotropic contracting fluids in the context of string-motivated Gauss–Bonnet gravity. They created Tolman–Oppenheimer–Volkoff (TOV) equations and derived the Weyl scalar's evolution equation.

Literature has extensively scrutinized the impact of the charge on self-gravitating object's stability [24, 25]. The well-known Einstein field equations (EFE) can be evaluated to determine the composition of self-gravitating systems. As the field equations are non-linear, it is challenging to establish interior self-gravitating solutions. Several researchers in the field of gravitational physics were motivated to investigate the intrinsic features of astral objects as a result of their nature and precise composition. Anisotropy is a more plausible scenario than a completely isotropic star interior. Anisotropy refers to how a system's physical properties, such as pressure or energy density, vary depending on the direction of measurement. The inclusion of pressure anisotropy in the investigation of self-gravitating fluids (relativistic or Newtonian) is supported by the fact that it occurs in a wide range of physically significant conditions (see reference [26]). In addition, as recently demonstrated by L Herrera ([27], physical phenomena similar to those expected in star evolution would always cause pressure anisotropy, even though the system is initially considered to be isotropic. The crucial point to emphasize here is that any equilibrium structure represents the end stage of a dynamic system, and there is no reason to think that the accumulated anisotropy throughout this dynamic process will evaporate in the fin. Numerous investigations have proposed a range of mechanisms for the production of anisotropy within stellar objects, including pion condensation [28], relativistic nuclear interaction [29], phase shifts [30], superfluid core [31], extremely strong magnetic fields [32, 33], and core crystallization [34]. Raposo along with his colleagues [35] accomplished the first investigation of the dynamical behavior of self-gravitating anisotropic fluids, which is useful for testing the black hole model. They discovered that anisotropic stars may be as dense and enormous as black holes, have non-linear behavior, and often develop via gravitational wave echoes. Errehymy and collaborators [36] proposed accurate solutions for spherically symmetric anisotropic systems in GR by incorporating a static metric into a five-dimensional pseudo-Euclidean space. They determined that the class one solution embedding for anisotropic compact objects is stable and provides evidence for super-massive pulsars. Maurya and colleagues [37] investigated the vanishing complexity factor condition, which enabled the creation of solutions to EFE for celestial entities with spherically symmetric geometry. It simplified the two-metric potential system to a single-metric potential system by taking into account dark matter distortion in DM haloes. The work suggested that compact stars in galactic DM haloes can produce gravitational waves (GW) along with the impact of β on these echoes.

The gravitational decoupling (GD) by the minimal geometric deformation (MGD) method has produced significant outcomes in identifying new, substantially viable alternatives for compact spherical configuration [38]. Using the MGD technique, Morales and Tello-Ortiz [39] created an anisotropic model with a charge for compact objects. The MGD technique is an effective tool for analyzing the fundamental characteristics of stellar objects, but it has certain restrictions. For instance, it only functions when the correspondence among the matter sources is exclusively gravitational. A primary constraint is that only the radial metric potential is perturbed while keeping the time coordinate constant, which might lead to certain flaws in decoupling phenomena. Casadio *et al* [40] developed an innovative solution for spherical symmetric space-time by extending the MGD method and applying the transformation to both temporal and radial metric potentials. However, owing to the stress-energy conservation rule, this expansion is only applicable in the vacuum scenario. Ovalle [41] recently suggested an efficient extended geometric deformation (EGD) method that accomplishes each region of space-time, regardless of the presence or absence of matter, through modifying both metric components (g_{tt}, g_{rr}). He devised the Reissner–Nordström solution from Schwarzschild space-time to evaluate the consistency of his established approach. Albalahi *et al* [42] investigated the application of MGD in isolating anisotropic, self-gravitating charged entities exhibiting spherical symmetry. They employed unique methodologies, such as the zero complexity aspect and isotropization strategies, to build charged compact star simulations with the Tolman $\mathbb{IV}$ as the seed. The method was novel, assuming $Y_{TF}=0$ and using isotropization methods to simulate electrically charged anisotropic topologies.

Our article is aligned using the following approach: In the background of $f(T)$ gravity along with the existence of the existence of the EMF, we analyze the physical characteristics of static fluids using spherical symmetry. In section 2, we established the field equations with regard to EMF and hit upon the formula for the energy-momentum tensor. The $f(T)$ field equations, which also include the electromagnetic energy-momentum tensor (EMT) and an auxiliary source of gravitation not included in GR, are then obtained. By using the geometric contorsion in each radial and temporal coordinate, we introduce key concepts of the GD technique and decouple the associated field equations. In section 3, we use JCs for the external and internal metrics matching and get an expression of effective pressure at Σ. In section 4, we utilize the equation of state to find the expression for the exchange of energy in relativistic fluids. In section 5, we utilize the Tolman $\mathbb{IV}$ solution to examine the physical features of the generated results in the presence of an electromagnetic field in $f(T)$ gravity. Lastly, in section 6, we finalize our findings.

2. *f*(*T*) field equations with charge

In this portion, we shall present a conventional description of $f(T)$ gravity and derive field equations for the gravitational field. The entire summary of metric components with matter composition will be presented later. This idea has a significant impact on investigations regarding inflation and the Universe's late-time rapid expansion. In order to solve a system of differential equations that is non-linear, we apply the GD approach. The modified Einstein–Hilbert action, including charge and the unknown gravitational zone in $f(T)$ gravity, is expressed as [10, 43]

$$S_{f(T)}=\int d^4x\left(\mathcal{L}_M+\mathcal{L}_{EMF}+\mathcal{L}_\chi+\frac{f(T)}{2\kappa^2}\right)|e|, \tag{1}$$

where $\mathcal{L}_{EMF}$ is the Lagrangian of the electromagnetic EMT, $\mathcal{L}_\chi$ refers to the Lagrangian density of an entirely novel gravitational sector that is not characterized by GR, and the dynamical field of the $f(T)$ theory is the tetrad field $|e|=\det(e^\epsilon_\phi)$. In regard to the metric tensor, this orthonormal vector field set is connected by the following relation: $g_{\phi\epsilon}=\vartheta_{mn}e^m_\phi e^n_\epsilon$, where $\vartheta_{mn}=(1,-1,-1,-1)$. Now, parallel to that, the new field's action integral is provided by

$$\theta^\phi_\epsilon=\frac{2}{|e|}\frac{\delta(|e|\mathcal{L}_\chi)}{\delta e^\phi_\epsilon}=2\frac{\delta\mathcal{L}_\chi}{\delta e^\phi_\epsilon}-e^\phi_\epsilon\mathcal{L}_\chi. \tag{2}$$

The action in the tetrad field is dissimilar, which provides

$$e_i{}^\alpha S_\alpha{}^{\epsilon\phi}\partial_\epsilon Tf_{TT}+\frac{f}{4}e_i{}^\phi+\frac{f_T}{e}\partial_\epsilon(ee_i{}^\alpha S_\alpha{}^{\epsilon\phi})$$
$$+e_i{}^\rho T^\alpha_{\epsilon\rho}S_\alpha{}^{\phi\epsilon}f_T=\frac{\kappa^2}{2}e_i{}^\epsilon(T^{\phi(m)}_\epsilon+E^\phi_\epsilon), \tag{3}$$

where $f_T\equiv\frac{\partial f}{\partial T}$, $f_{TT}\equiv\frac{\partial^2 f}{\partial T^2}$, E^ϕ_ϵ represents electromagnetic EMT. Torsion scalars are generated from torsion tensors, and they are represented by T. Torsion is a measure of the variations from the Levi-Civita connection's symmetry, a mathematical technique used to characterize the curvature of space-time in GR. It is given as

$$T=S_\alpha{}^{\epsilon\phi}T^\alpha_{\epsilon\phi}, \tag{4}$$

where $T^\alpha_{\epsilon\phi}$ is a torsion tensor with the association $T^\alpha_{\epsilon\phi}=-T^\alpha_{\phi\epsilon}$ and constituted by a Weitzenböck connection that is $\Gamma^\alpha_{\phi\epsilon}=e_i{}^\alpha\partial_\phi e^i{}_\epsilon$ expressed as

$$T^\alpha_{\epsilon\phi}=\Gamma^\alpha{}_{\phi\epsilon}-\Gamma^\alpha{}_{\epsilon\phi}=e_i{}^\alpha(\partial_\phi e^i{}_\epsilon-\partial_\epsilon e^i{}_\phi), \tag{5}$$

where the expression for super-potential is given by

$$S_\alpha{}^{\epsilon\phi}=\frac{1}{2}(K^{\epsilon\phi}_{\ \ \alpha}+\delta^\epsilon_\alpha T^{\beta\phi}_{\ \ \beta}-\delta^\phi_\alpha T^{\beta\epsilon}_{\ \ \beta}). \tag{6}$$

The super-potential can help to calculate modified field equations. It presents a framework for evaluating the conservation laws related to the theory and is also crucial for comprehending the symmetries and transformations observed in the $f(T)$ theory [44]. Similarly, the contorsion tensor is given by

$$K^{\epsilon\phi}_{\ \ \alpha}=-\frac{1}{2}(T^{\epsilon\phi}_{\ \ \alpha}-T^{\phi\epsilon}_{\ \ \alpha}-T_\alpha{}^{\epsilon\phi}). \tag{7}$$

The distinction between the torsion tensor and its antisymmetric component can be determined by the contorsion tensor in $f(T)$ gravity. It serves an integral part in obtaining field equations and interpreting the dynamics of theory by providing vital information about the torsion and its connection with space-time curvature. The altered version of equation (3) is expressed as

$$G^\phi_\epsilon=\frac{\kappa^2}{f_T}T^{\phi(\mathrm{tot})}_\epsilon, \tag{8}$$

with

$$T^{\phi(\mathrm{tot})}_\epsilon=T^{\phi(\mathrm{matt})}_\epsilon+E^\phi_\epsilon+T^{\phi(T)}_\epsilon+\theta^\phi_\epsilon, \tag{9}$$

where $T^{\phi(\mathrm{matt})}_\epsilon$ represents anisotropic fluid EMT; $T^{\phi(T)}_\epsilon$ is the EMT of the field; E^ϕ_ϵ is the electromagnetic EMT; and θ^ϕ_ϵ is introduced as a new field of interaction. One can write

$$T^{\phi(T)}_\epsilon=\frac{1}{\kappa^2}\left[\frac{Tf_T-f}{2}\delta^\phi_\epsilon-f_{TT}S_\epsilon{}^{\alpha\rho}\delta^\phi_\alpha\delta^\rho_\rho\nabla_\rho T\right]. \tag{10}$$

Anisotropic matter configurations are critical for understanding a wide range of astrophysical as well as cosmological events, especially in highly compact entities such as neutron stars and strange quark stars. Despite isotropic matter, which has a uniform pressure in every direction, anisotropic matter has varying pressures in both the radial as well as tangential directions. This differentiation has a substantial impact on the composition and stability of these objects. The relevance of anisotropic matter configurations has been extensively studied in the literature [45–47]. The EMT of anisotropic fluid is given as

$$T^{\phi(\mathrm{matt})}_\epsilon=(\rho+p_t)v^\phi v_\epsilon-p_t\delta^\phi_\epsilon-(p_t-p_r)s^\phi s_\epsilon, \tag{11}$$

where p_r, ρ, and p_t are radial pressure, energy density, and tangential pressure accordingly (typically chosen as the anisotropic direction). Here v^ϕ represents four-velocity and s^ϕ is the unit vector heading in anisotropy direction. Also, the electromagnetic EMT expression is

$$E_\epsilon^\phi = \frac{1}{4\pi}\left(F^{\phi\alpha}F_{\epsilon\beta} - \frac{1}{4}F^{\alpha\beta}F_{\alpha\beta}\delta_\epsilon^\phi\right), \tag{12}$$

where $F_{\epsilon\beta}$ is the mathematical expression for the EMF tensor given by

$$F_{\epsilon\beta} = \varphi_{\beta,\epsilon} - \varphi_{\epsilon,\beta}. \tag{13}$$

In addition, four-potential is represented by $\varphi_\epsilon = \varphi(r)\delta_\epsilon^0$, and four-current density is provided by $J^\epsilon = \sigma(r)v^\epsilon$, where $\sigma(r)$ is a charge density. Now, we employ a comoving frame with a charge parameter at rest, so that the magnetic field is zero. One can write Maxwell field equations as

$$F^{\epsilon\phi}{}_{;\phi} = 4\pi J^\epsilon, \quad F_{[\epsilon\phi;\gamma]} = 0. \tag{14}$$

Now we will consider static and spherical symmetric line elements to investigate the impact of staticity and EMF in $f(T)$ theory. The static spherically symmetric metric is critical for characterizing dense stellar configurations like stars, planets, and black holes. This metric, which yields solutions such as the Schwarzschild, is fundamental to comprehending a wide range of astrophysical and cosmological events [48–50]. It is given as

$$ds^2 = e^{\omega(r)}dt^2 - e^{\lambda(r)}dr^2 - r^2d\Omega^2, \tag{15}$$

where $d\Omega^2$ is the solid angle, which is characterized as $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$. The non-vanishing components of equation (13) give

$$\phi'' - \left(\frac{\lambda'}{2} + \frac{\omega'}{2} - \frac{2}{r}\right)\phi' = 4\pi\sigma e^{\lambda+\frac{\omega'}{2}}. \tag{16}$$

By utilizing the integrating factor approach to solve equation (16), we obtain the expression as

$$q = \int 4\pi r^2\sigma e^{\frac{\lambda}{2}}dr. \tag{17}$$

The EMF tensor's components are found under

$$E_\epsilon^\phi = \text{diag}\left[\frac{q^2}{\kappa^2r^4}, \frac{q^2}{\kappa^2r^4}, -\frac{q^2}{\kappa^2r^4}, -\frac{q^2}{\kappa^2r^4}\right]. \tag{18}$$

The $f(T)$ field equations for our observed system are found as under

$$\rho + \varepsilon + \frac{q^2}{\kappa^2r^4} + \frac{1}{2\kappa^2}\times\left\{(Tf_T - f) - f_{TT}\,e^{-\lambda}\left(\frac{\omega'}{4} - \frac{2}{r}\right)T'\right\} = \frac{f_T}{\kappa^2}\left[\frac{1}{r^2} - e^{-\lambda}\left(\frac{1}{r^2} - \frac{\lambda'}{r}\right)\right], \tag{19}$$

$$p_r + \wp_r - \frac{q^2}{\kappa^2r^4} - \frac{1}{2\kappa^2}(Tf_T - f) = \frac{f_T}{\kappa^2}\left[-\frac{1}{r^2} + e^{-\lambda}\left(\frac{1}{r^2} + \frac{\omega'}{r}\right)\right], \tag{20}$$

$$-p_t - \wp_t - \frac{q^2}{\kappa^2r^4} + \frac{1}{2\kappa^2}\times\left\{(Tf_T - f) + \frac{e^{-\lambda}}{2}f_{TT}\left(\frac{3}{r} - \omega'\right)T'\right\} = -\frac{f_T}{\kappa^2}\left[\frac{e^{-\lambda}}{2}\left(\omega'' + \frac{\omega'^2}{2} - \frac{\lambda'\omega'}{2} + \frac{\omega'}{r} - \frac{\lambda'}{r}\right)\right]. \tag{21}$$

The aforementioned set of equations, which include non-zero EMF components and the stress-energy tensor, also include corrections for extra gravitational sources together with degrees of freedom provided by $f(T)$ theory. There are a total of nine unknowns (ρ, p_r, p_t, ε, $\wp_r$, $\wp_t$, λ, ω, q) in this system of non-linear differential equations, rendering it indefinite. We adopt an organized approach suggested by Ovalle [45] so as to close the system. Before employing the MGD strategy, it is vital to indicate our physical parameters, namely, $\rho^{(tot)}$, $p_r^{(tot)}$, $p_t^{(tot)}$ as

$$\rho^{(tot)} = T_0^{0(matt)} + E_0^0 + T_0^{0(T)} + \theta_0^0 = \rho + \varepsilon + \frac{q^2}{\kappa^2r^4} + \frac{1}{2\kappa^2}\times\left\{(Tf_T - f) - f_{TT}\,e^{-\lambda}\left(\frac{\omega'}{4} - \frac{2}{r}\right)T'\right\}, \tag{22}$$

$$p_r^{(tot)} = -T_1^{1(matt)} - E_1^1 - T_1^{1(T)} - \theta_1^1 = p_r + \wp_r - \frac{q^2}{\kappa^2r^4} - \frac{1}{2\kappa^2}(Tf_T - f), \tag{23}$$

$$p_t^{(tot)} = -T_2^{2(matt)} + E_2^2 - T_2^{2(T)} - \theta_2^2 = -p_t - \wp_t - \frac{q^2}{\kappa^2r^4} + \frac{1}{2\kappa^2}\times\left\{(Tf_T - f) + \frac{e^{-\lambda}}{2}f_{TT}\left(\frac{3}{r} - \omega'\right)T'\right\}. \tag{24}$$

The core of the self-gravitating structure exhibits anisotropy, which is unambiguously caused by the source $\theta_\varepsilon^\phi = 0$. As an outcome, the anisotropic component takes on the following form:

$$\Pi \equiv p_t^{(tot)} - p_r^{(tot)} = (p_r - p_t) + (\wp_r - \wp_t) + \frac{1}{2\kappa^2}\left\{\frac{e^{-\lambda}}{2}f_{TT}\left(\frac{3}{r} - \omega'\right)T'\right\} - \frac{2q^2}{\kappa^2r^4}. \tag{25}$$

2.1. The GD approach

In this section, we utilize the GD approach to find a system of non-linear differential equations (19)–(21). With this method, the system of equations will be changed such that field equations relating to the source θ_ϵ^ϕ shall present the structure of a 'quasi-gravity system'. For metric, the initial approach is to select a solution for the perfect configuration of matter for which $\theta_\epsilon^\phi = 0$. Thus, the line element is given as

$$ds^2 = e^{\zeta(r)}dt^2 - e^{\varpi(r)}dr^2 - r^2 d\Omega^2, \quad (26)$$

where

$$e^{-\varpi(r)} \equiv 1 - \frac{\kappa^2}{f_T r}\int_0^r x^2 (T_0^{0(\mathrm{matt})} + E_0^0 + T_0^{0(T)})dx = 1 - \frac{2m}{r} + \frac{q^2}{r^2}. \quad (27)$$

Equation (27) involves Misner–Sharp mass represented by $m(r)$ [51]. We notice the geometric modification in the metric element in order to observe the impact of θ_ϵ^ϕ as

$$\zeta \to \omega = \zeta + \mathrm{h}, \quad e^{-\varpi} \to e^{-\lambda} + \breve{g}, \quad (28)$$

where h and $\breve{g}$ are the distortions in the geometry of the time component together with the radial component, respectively. Observing that the deformations in equation (28) are merely radial functions ensures that the solution has spherical symmetry. In the scenario of the nominal MGD, which is equivalent to $\mathrm{h}(r) = 0$ or $\breve{g}(r) = 0$, only the radial component shall be deformed; the temporal factor remains constant. The field equation will be divided into two parts.

- **EM field equations**

The typical EM field equations for $T_\epsilon^{\phi(\mathrm{matt})} + T_\epsilon^{\phi(T)}$ are provided as

$$\rho + \frac{1}{2\kappa^2}\left\{(Tf_T - f) - f_{TT}e^{-\varpi}\left(\frac{\zeta'}{4} - \frac{2}{r}\right)T'\right\} + \frac{q^2}{\kappa^2 r^4} = \frac{f_T}{\kappa^2}\left[\frac{1}{r^2} - e^{-\varpi}\left(\frac{1}{r^2} - \frac{\varpi'}{r}\right)\right], \quad (29)$$

$$-p_r + \frac{1}{2\kappa^2}(Tf_T - f) + \frac{q^2}{\kappa^2 r^4} = \frac{f_T}{\kappa^2}\left[\frac{1}{r^2} - e^{-\varpi}\left(\frac{1}{r^2} + \frac{\zeta'}{r}\right)\right], \quad (30)$$

$$p_t - \frac{1}{2\kappa^2}\left\{(Tf_T - f) + \frac{e^{-\varpi}}{2}f_{TT}\left(\frac{3}{r} - \zeta'\right)T'\right\} - \frac{q^2}{\kappa^2 r^4} = \frac{f_T}{\kappa^2}\left[\frac{e^{-\varpi}}{4}\left(2\zeta'' + \zeta'^2 - \varpi'\zeta' + 2\frac{\zeta' - \varpi'}{r}\right)\right]. \quad (31)$$

- **Equations for θ_ϵ^ϕ**

The equations of motion for θ_ϵ^ϕ is provided as

$$\varepsilon = \frac{f_T}{\kappa^2}\left[-\frac{\breve{g}}{r^2} - \frac{\breve{g}'}{r} + \frac{f_{TT}}{2f_T}\left\{\breve{g}\left(\frac{\omega'}{4} - \frac{2}{r}\right)\right\}T' + B_0\right], \quad (32)$$

$$\wp_r = \frac{f_T}{\kappa^2}\left[\breve{g}\left(\frac{1}{r^2} + \frac{\omega'}{r}\right) + B_1\right], \quad (33)$$

$$\wp_t = \frac{f_T}{\kappa^2}\left[\frac{\breve{g}}{4}\left(2\omega'' + \omega'^2 + 2\frac{\omega'}{r}\right) + \frac{\breve{g}'}{4}\times\left(\omega' + \frac{2}{r}\right) + \frac{f_{TT}}{2f_T}\frac{\breve{g}}{2}\left(\frac{3}{r} - \omega'\right)T' + B_2\right], \quad (34)$$

where

$$B_0 = \frac{f_{TT}}{2f_T}\frac{\mathrm{h}'e^{-\varpi}}{4}T', \quad B_1 = \frac{\mathrm{h}'e^{-\varpi}}{r}, \quad (35)$$

$$B_2 = \frac{f_{TT}}{2f_T}\frac{\mathrm{h}'e^{-\varpi}}{2}T' + \frac{e^{-\varpi}}{4}\times\left(2\mathrm{h}'' + \mathrm{h}'^2 + \frac{2\mathrm{h}'}{r} + 2\zeta'\mathrm{h}' - \varpi'\mathrm{h}'\right). \quad (36)$$

We notice that for the special scenario $\mathrm{h} = 0$, equations (32)–(34) decrease to the more elementary 'quasi-gravity system' of MGD of [52] where $\breve{g}$ is solely regulated by θ_ϵ^ϕ together with unstructured metric (26), and surely the tensor θ_ϵ^ϕ disappears after putting distortions to be zero ($\mathrm{h} = \breve{g} = 0$) [53, 54]. Now we discuss the conduct of the equation of conservation in order to establish a well-off decoupling strategy. As the validity of conservation law is unaffected by the MGD technique [55, 56], for the line element (ζ, ϖ), the Bianchi identity for ideal fluid configuration including charge remains unchanged, so we get

$$\Bigg[\Bigg((T_1^{1(\mathrm{matt})} + E_1^1 + T_1^{1(T)})' - \frac{\zeta'}{2}\times((T_0^{0(\mathrm{matt})} + E_0^0 + T_0^{0(T)}) - (T_1^{1(\mathrm{matt})} + E_1^1 + T_1^{1(T)})) - \frac{2}{r}((T_2^{2(\mathrm{matt})} + E_2^2 + T_2^{2(T)}) - (T_1^{1(\mathrm{matt})} + E_1^1 + T_1^{1(T)})\Bigg)\Bigg] - \frac{\mathrm{h}'}{2}((T_0^{0(\mathrm{matt})} + E_0^0 + T_0^{0(T)}) - (T_1^{1(\mathrm{matt})} + E_1^1 + T_1^{1(T)})) + (\theta_1^1)' - \frac{\omega'}{2}(\theta_0^0 - \theta_1^1) - \frac{2}{r}(\theta_2^2 - \theta_1^1) = 0, \quad (37)$$

This provides us further

$$p'_r + \frac{2q}{\kappa^2 r^4}\left(\frac{2q}{r} - q'\right) + \frac{\zeta'}{2}(\rho + p_r) - \frac{\Phi^{(T)}}{2\kappa^2} + \frac{\mathrm{h}'}{2}(\rho + p_r) + \frac{2}{r}\Pi + \wp'_r + \frac{\omega'}{2}(\varepsilon + \wp_r) = 0. \quad (38)$$

When $f(T)$ field equations (19)–(21) are determined, the conservation equation (37) is satisfied, which may be utilized to explain the interior composition of a compact body with charged ideal fluid. If we set $q = 0$, the conservation equation for the modified $f(T)$ theory can be retrieved. In equation (38), the term $\phi^{(T)}$ indicates the presence of dark source terms whose value is presented in the appendix. It is important to remember that the MGD technique offers a decoupling methodology without the alternation of energy among the matter constituents.

3. Junction conditions

Over the hypersurface of a massive object, the appropriate conditions must be achieved, aiming to examine the physical properties of relativistic objects. These conditions suggest linear and smooth outer and inner geometrical interactions at $r = R$, where R represents the boundary of the star system. For stable stellar formations to exist, these conditions must be smoothly matched at the surface boundary. For GR junction conditions, we adopt the strategy put forward by Israel and Darmois. Equation (26) characterizes the interior metric, where the interior mass function is provided by

$$\check{m}(r) = m(r) - \frac{r}{2}\breve{g}(r), \tag{39}$$

where $\breve{g}(r)$ is the deformation factor as stated in transformation equation (28), and $m(r)$ is given in equation (27). For the exterior zone, we consider the Reissner–Nordström metric provided as

$$\mathrm{d}s_+^2 = \left(1 - \frac{2\mathbb{M}}{r} + \frac{Q^2}{r^2}\right)\mathrm{d}t^2 - \left(1 - \frac{2\mathbb{M}}{r} + \frac{Q^2}{r^2}\right)^{-1} \times \mathrm{d}r^2 - r^2\mathrm{d}\Omega^2, \tag{40}$$

where $\mathbb{M}$, Q are the total mass and charge of the exterior manifold. For the internal space-time and external metric to overlap, the first and second basic forms must remain continuous over the surface $r = R$, i.e.

$$(\mathrm{d}s_-^2)_\Sigma = (\mathrm{d}s_+^2)_\Sigma; \quad (\mathcal{K}_{ij})_- = (\mathcal{K}_{ij})_+.$$

The first fundamental form's continuation yields

$$\mathrm{e}^{\omega^-(R)} = \left(1 - \frac{2\mathbb{M}}{R} + \frac{Q^2}{R^2}\right), \quad \mathrm{e}^{-\varpi^-(R)} = \left(1 - \frac{2\mathbb{M}}{R} + \frac{Q^2}{R^2}\right). \tag{41}$$

As the fluid is charged, the electric charge both inside and out of the star ought to be equivalent, i.e. $q(R) \overset{\Sigma}{=} Q$ and also $m \overset{\Sigma}{=} \mathbb{M}$. Additionally, the extrinsic curvature must be continuous over the hypersurface, so the second condition gives

$$p_R^{(\mathrm{tot})} - \frac{f_T}{\kappa^2}\frac{Q^2}{R^4} = 0. \tag{42}$$

Now utilizing equation (23), we derive

$$p_R^{(\mathrm{tot})} - \frac{f_T}{\kappa^2}\frac{Q^2}{R^4} = \tilde{p}_R + \frac{f_T}{\kappa^2} \times \left[\breve{g}\left(\frac{1}{R^2} + \frac{\omega'}{R}\right) + \frac{\mathsf{h}'\mathrm{e}^{-\varpi}}{R}\right] - \frac{f_T}{\kappa^2}\frac{Q^2}{R^4} = 0. \tag{43}$$

This equation implies that near the surface, the effective radial pressure will diminish. Make note that, due to the field, this expression additionally incorporates a few dark source terms.

3.1. *f(T)* Junction conditions

In this section, we shall use a rigorous approach to match two distinct regions of space-time by following a technique provided by [57] in the scenario of $f(T)$ theory. For any generic $f(T)$ teleparallel theory, the junction requirements are

- The first condition entails that both induced metrics must align on the hypersurface i.e.

$$[\Upsilon_{ij}]_-^+ = 0. \tag{44}$$

- Induced tetrads are continuous across the hypersurface by the second condition i.e.

$$[\Upsilon^a_{\ i}]_-^+ = 0. \tag{45}$$

- In particular, the third junction condition is

$$[f_T \mathcal{K}^a_i]_-^+ = 0 \tag{46}$$

where, $\mathcal{K}^a_i$ is the extrinsic curvature.

Now the self-gravitating object's interior structure is considered as

$$\mathrm{d}s_-^2 = \mathrm{e}^{\omega^-(r)}\mathrm{d}t^2 - \left[1 - \frac{2\check{m}(r)}{r} + \frac{q^2}{r^2}\right]^{-1} \times \mathrm{d}r^2 - r^2\mathrm{d}\Omega^2. \tag{47}$$

For exterior zone, we consider here

$$\mathrm{d}s_+^2 = \left(1 - \frac{2\mathbb{M}}{r} + \frac{Q^2}{r^2}\right)\mathrm{d}t^2 - \left(1 - \frac{2\mathbb{M}}{r} + \frac{Q^2}{r^2}\right)^{-1}\mathrm{d}r^2 - r^2\mathrm{d}\Omega^2. \tag{48}$$

Matching of metric potential at the hypersurface yields

$$\mathrm{e}^{\omega^-(R)} = \left(1 - \frac{2\mathbb{M}}{R} + \frac{Q^2}{R^2}\right), \quad \mathrm{e}^{-\varpi^-(R)} = \left(1 - \frac{2\mathbb{M}}{R} + \frac{Q^2}{R^2}\right). \tag{49}$$

The first fundamental form, the induced metric, on the hypersurface has the following definition:

$$[\Upsilon_{ij}]_-^+ = g_{\mu\nu}\frac{\partial\chi^\mu}{\partial x^i}\frac{\partial\chi^\nu}{\partial x^j}, \tag{50}$$

where $\chi^\mu = (\chi^0, \chi^1, \chi^2, \chi^3)$ indicating the space-time coordinate, and $x^i = (x^0, x^2, x^3)$ characterizing the hypersurface's coordinates. Coinciding induce metrics at the hypersurface, in accordance with the first condition, i.e.

$$\Upsilon_{ij}\mathrm{d}x^i\mathrm{d}x^j = \left(1 - \frac{2\mathbb{M}}{r} + \frac{Q^2}{r^2}\right)\mathrm{d}t^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\phi^2, \tag{51}$$

$$\tilde{\Upsilon}_{ij}\mathrm{d}x^i\mathrm{d}x^j = \mathrm{e}^{\omega^-(r)}\mathrm{d}t^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\phi^2, \tag{52}$$

ultimately resulting in $\Upsilon_{ij} = \tilde{\Upsilon}_{ij}$, where Υ_{ij} is induced metric for interior metric and $\tilde{\Upsilon}_{ij}$ represents induced metric for exterior metric. Now considering the second criterion, namely the continuity of the induced tetrad, we can use the formula to derive the induced tetrad for our spacetimes, given as

$$\Upsilon^a_{\ i} = \mathrm{e}^a_{\ \mu}\frac{\partial\chi^\mu}{\partial x^i}, \tag{53}$$

where the set of tetrad fields for both the interior and exterior metrics is

$$\mathrm{e}^a_{\ \mu} = \mathrm{diag}(\mathrm{e}^{\frac{\omega^-}{2}}, \mathrm{e}^{\frac{\varpi^-}{2}}, r, r\sin\theta),$$

$$\tilde{\mathrm{e}}^a_{\ \mu} = \mathrm{diag}\left(\left(1-\frac{2\mathbb{M}}{r}+\frac{Q^2}{r^2}\right)^{\frac{1}{2}},\right.$$

$$\left.\left(1-\frac{2\mathbb{M}}{r}+\frac{Q^2}{r^2}\right)^{\frac{-1}{2}}, r, r\sin\theta\right), \tag{54}$$

respectively. It is thus simple to demonstrate that the induced tetrads, i.e. $\Upsilon^a_{\ \mu} = \tilde{\Upsilon}^a_{\ \mu}$, are continuous across the boundary from here. We have to find the normal vector of the hypersurface from both sides of the boundary to explore the third junction condition. For both interior and exterior metrics, we have

$$n_\mu = \mathrm{e}^{\frac{\varpi^-(r)}{2}}, \quad \tilde{n}_\mu = \left(1-\frac{2\mathbb{M}}{r}+\frac{Q^2}{r^2}\right)^{\frac{-1}{2}}. \tag{55}$$

Subsequently, the extrinsic curvature's non-zero components based on the spherically symmetric static metric and Reissner–Nordstr $\ddot{o}$m metric are attained by

$$\mathcal{K}_{ij} = n_{\mu;\nu}\frac{\partial x^\mu}{\partial x^i}\frac{\partial x^\nu}{\partial x^j}. \tag{56}$$

Implementing the third condition will finish as

$$p_R^{(\mathrm{tot})} - \frac{f_T}{\kappa^2}\frac{Q^2}{R^4} = 0. \tag{57}$$

Also, when charged fluid is distributed, the electric charge should be equivalent both within and at the star's exterior ($r = R$), i.e. $q(R) \overset{\Sigma}{=} Q$ and $m \overset{\Sigma}{=} M$. Now from equation (23), we derive

$$p_R^{(\mathrm{tot})} - \frac{f_T}{\kappa^2}\frac{Q^2}{R^4} = \tilde{p}_R + \frac{f_T}{\kappa^2}$$

$$\times\left[\breve{g}\left(\frac{1}{R^2}+\frac{\omega'}{R}\right)+\frac{\mathsf{h}'\mathrm{e}^{-\varpi}}{R}\right] - \frac{f_T}{\kappa^2}\frac{Q^2}{R^4} = 0. \tag{58}$$

Here $\tilde{p}_R = -p_R + \frac{1}{2\kappa^2}(Tf_T - f) + \frac{1}{\kappa^2}\frac{Q^2}{R^4}$. This equation has significant effects, as the compact object can only attain equilibrium in an outer charge-free space-time if the total radial pressure at the boundary has a specific value. This value contains $f(T)$ correction terms and is described in equation (58).

The JC among the two regions of space-time has been encountered by the application of GR and the known $f(T)$ theory of gravity, which is an extension of teleparallel gravity theories. In GR, standard Darmois JC was employed; conversely, in $f(T)$ gravity, the continuity of the first basic form, the induced tetrads, along with $[f_T\mathcal{K}_i^a]_-^+ = 0$, is developed. These requirements rely on the selection of the tetrads as well as the $f(T)$ model. Considering the JCs of $f(T)$ theory and GR, we can determine that the same outcomes are attained, as one can see in equations (43) and (58).

4. Equation of State

Selecting the appropriate equation of state is always important in the modeling of compact objects. Due to the high level of non-linearity inside the model, precise solutions with a polytropic equation of state for the field equations are rare. Takisa and Maharaj [58] discovered a class of precise solutions for charged anisotropic polytropic spheres. The impact of θ_ϵ^ϕ representing a polytropic fluid in this case on a different generic source $T_\epsilon^{\phi(m)} + T_\epsilon^{\phi(T)}$ can be studied through the modified field equations (29)–(31). We now examine the role of the polytropic equation of state in $f(T)$ theory. The polytropic equation of state is an important tool in the exploration of celestial structure and evolution because of its low complexity, generality, and capacity to mimic a diverse spectrum of celestial bodies. It is a power-law relationship relating pressure and density that applies to a variety of stellar regions, including convective regions with a constant adiabatic index. Various polytropic indices (n) are appropriate for representing a variety of stellar structures, such as neutron stars, white dwarfs, rocky planets, and main-sequence stars [59, 60]. The Lane–Emden equation, which defines the composition of a polytrope, offers analytical results for various polytropic indices, offering information about the features of these celestial objects. The polytropic equation of state is used in numerical models for stellar composition and development, yielding significant information about star composition as well as inner structure. For this purpose, we take

$$\wp_r = K(\varepsilon)^\Gamma \neq \wp_t, \tag{59}$$

with $\Gamma = 1 + \frac{1}{n}$, here the polytropic index is n. The thermal properties of a particular polytrope are determined by the parameter $K > 0$, which has a magnitude of length to the power of $2/n$ and implicitly includes the temperature [61]. We begin by applying equations (32) and (33) to equation (59) to produce an equation for the deformation in first-order non-linear differential as$\breve{g}$

$$\frac{\breve{g}}{r^2} + \frac{\breve{g}'}{r} = \left(\frac{\kappa^2}{f_T}\right)^{\frac{\Gamma-1}{\Gamma}}\frac{1}{K^{1/\Gamma}}$$

$$\times\left[\breve{g}\left(\frac{1}{r^2}+\frac{\omega'}{r}\right)+\frac{\mathsf{h}'\mathrm{e}^{-\varpi}}{r}\right]^{1/\Gamma}$$

$$+\frac{f_{TT}}{2f_T}\left[\breve{g}\left(\frac{\omega'}{4}-\frac{2}{r}\right)+\frac{\mathsf{h}'\mathrm{e}^{-\varpi}}{4}\right]T'. \tag{60}$$

This is a non-linear differential expression that would be used to determine the distortion factors, i.e. $\{\breve{g}, \mathsf{h}\}$. We define two auxiliary conditions to solve the system of equations (32)–(34). The constraints that one can use to solve this equation are to imply limits on pressure and density, respectively. We imply the constraint on the pressure as

$$\wp_r(r) = \beta(K, \Gamma)p_r(r), \tag{61}$$

$$\beta(K, \Gamma) = K^\Gamma, \tag{62}$$

where for each polytrope, $\beta(K, \Gamma)$ is a dimensionless characteristic function. By means of equations (61) and (62), we get

$$\wp_r(r) = K^\Gamma p_r(r). \tag{63}$$

By utilizing equations (30) and (33) in equation (63) we have

$$\mathsf{h}' = \left(\frac{1}{\mathrm{e}^{-\varpi} + \breve{g}}\right)\Bigg[(K^\Gamma \mathrm{e}^{-\varpi} - \breve{g}) \times\left(\zeta' + \frac{1}{r}\right) + \frac{rK^\Gamma}{2f_T}\left(Tf_T - f + \frac{2q^2}{r^4}\right) - \frac{K^\Gamma}{r}\Bigg], \tag{64}$$

$$\frac{\breve{g}}{r^2} + \frac{\breve{g}'}{r} = -\left(\frac{\kappa^2 K}{f_T}\right)^{\frac{\Gamma-1}{\Gamma}} \times\left[-\frac{1}{r^2} + \mathrm{e}^{-\varpi}\left(\frac{1}{r^2} + \frac{\zeta'}{r}\right) + \frac{q^2}{f_T r^4} + \frac{1}{2f_T}(Tf_T - f)\right]^{1/\Gamma} + \frac{f_{TT}}{2f_T}\Bigg[\breve{g}\left(\frac{\omega'}{4} - \frac{2}{r}\right) + \frac{\mathrm{e}^{-\varpi}}{4(\mathrm{e}^{-\varpi} + \breve{g})}\Bigg\{(K^\Gamma \mathrm{e}^{-\varpi} - \breve{g})\left(\zeta' + \frac{1}{r}\right) + \frac{rK^\Gamma}{2f_T}\left(Tf_T - f + \frac{2q^2}{r^4}\right) - \frac{K^\Gamma}{r}\Bigg\}\Bigg]T'. \tag{65}$$

By means of equations (64) and (65), we emphasize that polytrope (K, Γ) deformation effects are readily apparent for a particular charged seed solution. To assure a physically feasible solution, a condition other than equation (63), known as the 'mimic constraint for the density,' can also be imposed, which is given as

$$\varepsilon(r) = K^\Gamma \rho(r). \tag{66}$$

By making use of equations (29) and (32) in equation (66), we derive

$$\frac{\breve{g}}{r^2} + \frac{\breve{g}'}{r} = -\beta\Bigg[\frac{1}{r^2} - \mathrm{e}^{-\varpi}\left(\frac{1}{r^2} - \frac{\varpi'}{r}\right) - \frac{q^2}{f_T r^4} - \frac{1}{2f_T} \times\left\{(Tf_T - f) - f_{TT}\,\mathrm{e}^{-\varpi}\left(\frac{\zeta'}{4} - \frac{2}{r}\right)T'\right\}\Bigg] + \frac{f_{TT}}{2f_T}\left[\breve{g}\left(\frac{\omega'}{4} - \frac{2}{r}\right) + \frac{\mathsf{h}'\mathrm{e}^{-\varpi}}{4}\right]T', \tag{67}$$

$$\mathsf{h}' = \left(\frac{r}{\mathrm{e}^{-\varpi} + \breve{g}}\right)\Bigg[\Bigg\{\frac{K\kappa^2}{f_T}\Bigg[\frac{\beta f_T}{\kappa^2} \times\left\{\frac{1}{r^2} - \mathrm{e}^{-\varpi}\left(\frac{1}{r^2} - \frac{\varpi'}{r}\right) - \frac{q^2}{f_T r^4} - \frac{1}{2f_T} \times\left((Tf_T - f) - f_{TT}\,\mathrm{e}^{-\varpi}\left(\frac{\zeta'}{4} - \frac{2}{r}\right) \times T'\right)\right\}\Bigg]^\Gamma\Bigg\} - \breve{g}\left(\frac{1}{r^2} + \frac{\zeta'}{r}\right)\Bigg]. \tag{68}$$

After implementing our method, it is possible to figure out the impacts of polytropes on any general fluid expressed by $T_\epsilon^{\phi(m)} + T_\epsilon^{\phi(T)} + E_\epsilon^\phi$ and obeying modified field equations (29)–(31), irrespective of the fluid's nature. The constraint in equation (63) is more convenient than the condition in equation (66), as it reduces the coupling hindrance on the facet, which can be pivotal in a few cases. From conservation equation (37), we have

$$\nabla_\phi(T_\epsilon^{\phi(\mathrm{matt})} + E_\epsilon^\phi + T_\epsilon^{\phi(T)}) = -\nabla_\phi(\theta_\epsilon^\phi) = -\frac{\mathsf{h}'}{2}[(T_0^{0(\mathrm{matt})} + E_0^0 + T_0^{0(T)}) - (T_1^{1(\mathrm{matt})} + E_1^1 + T_1^{1(T)})]. \tag{69}$$

In the context of the gravitational source θ_ϵ^ϕ, the above equation explains the linear amalgam of charged 'quasi-relativistic' $f(T)$ equations. Additionally, it illustrates how the MGD technique makes it trivial to separate two gravitational sources. We next discuss the inherent link between GD as well as the exchange of energy for coupled and decoupled fluids. The expression describing the interchange of energy-momentum ΔE between two fluids is

$$\Delta E = \frac{\mathsf{h}' f_T}{2\kappa^2}\left[\frac{\mathrm{e}^{-\varpi}}{r}(\varpi' + \zeta')\right]. \tag{70}$$

In addition to $f(T)$ gravitational terms, this equation provides the ability to describe structural variables. This formula states that h and f_T are the primary variables that regulate the energy transfer between relativistic fluids. We can see from the formula (60) that $\mathsf{h} > 0$ results in $E > 0$. According to the conservation equation (69), this suggests that $\nabla_\phi(\theta_\epsilon^\phi) > 0$, which denotes that polytrope is transferring energy to the surroundings. The reverse behavior can be seen when $\mathsf{h} < 0$.

5. Tolman IV Solution

The Tolman IV solution [62] is a well-known conventional relativistic solution used widely in the literature. We shall modify it via the MGD technique to provide the exact analytical form of the braneworld configuration of the perfect fluid solution. The MGD methodology can be used to investigate the Braneworld interpretation of the Tolman IV solution. Among the hundreds of known accurate solutions, the Tolman IV solution can be regarded as incomparable due to its enriched physical significance. For static spherical structures, it is possible to examine the function of polytropes and $f(T)$ corrections on a charged gravitational source. Now, we take the Tolman IV solution (ζ, ϖ, $\tilde{p}$, $\tilde{\rho}$) as

$$\mathrm{e}^{\zeta(r)} = \acute{A}^2\left(1 + \frac{r^2}{\acute{B}^2}\right), \qquad \mathrm{e}^{-\varpi(r)} = \frac{(\acute{C}^2 - r^2)(\acute{B}^2 + r^2)}{\acute{C}^2(\acute{B}^2 + 2r^2)}, \tag{71}$$

where the field equations (29), (30) and (58) are utilized to determine the equations for $\tilde{\rho}(r)$, $\tilde{p}(r)$ and Q as

$$\tilde{\rho}(r) = \frac{f_T}{\kappa^2}\left(\frac{\acute{B}^2 - \acute{C}^2 + 3r^2}{\acute{C}^2(\acute{B}^2 + 2r^2)}\right), \tag{72}$$

$$\tilde{\rho}(r) = \frac{f_T}{\kappa^2}\left(\frac{3J^4 + \acute{B}^2(3\acute{C}^2 + 7r^2) + 2r^2(\acute{C}^2 + 3r^2)}{\acute{C}^2(\acute{B}^2 + 2r^2)^2}\right), \tag{73}$$

$$\frac{Q^2}{R^4} = \frac{\acute{B}^2 - \acute{C}^2 + 3R^2}{\acute{C}^2(\acute{B}^2 + 2R^2)}. \tag{74}$$

The constants $\acute{A}$, $\acute{B}$, as well as $\acute{C}$ may be computed using JCs. The constants have the following values when the interior and outer spacetimes are matched:

$$\frac{\acute{C}^2}{R^2} = \frac{R}{M}, \quad \frac{\acute{B}^2}{R^2} = \frac{R^2 + 2Q^2 - 3RM}{RM - Q^2}, \quad \acute{A}^2 = \frac{R^2 + 2Q^2 - 3RM}{R^2}, \tag{75}$$

with $1 - \frac{2\mathbb{M}}{R} + \frac{Q^2}{R^2} > 0$ and the compactness ratio $\frac{\mathbb{M}}{R} \leqslant \frac{4}{9}$ [63]. Equation (75) ensures that the Tolman $\mathbb{IV}$ solution and the outer space-time at Σ line up smoothly. Regardless, the presence of a gravitational source θ_{ϵ}^{ϕ} in the core shall affect the corresponding values. Now using the expressions of $e^{\zeta}(r)$ and $e^{-\varpi}(r)$ in equation (60), we get the value of the deformation term $\breve{g}$ as

$$\breve{g} = \frac{r^2}{3}\left[\left(-\frac{\kappa^2 K}{f_T}\right)^{\frac{\Gamma-1}{\Gamma}}\left(\frac{\acute{C}^2 - \acute{B}^2}{\acute{B}^2\acute{C}^2}\right)^{1/\Gamma} \times\left\{\frac{1}{2f_T}(Tf_T - f) + \frac{1}{f_T}\frac{q^2}{r^4}\right\}^{1/\Gamma} H(r) + G(r)\right] + \frac{1}{3r}\int r^3 G'(r)\mathrm{d}r + \frac{C_1}{r}, \tag{76}$$

$$p_r^{(\mathrm{tot})} = \frac{f_T}{\kappa^2}\left[\frac{2f_T r^4(1 - K^{\Gamma})(\acute{B}^2 + 3r^2)(R^4 + Q^2\acute{B}^2 + 2Q^2R^2) + K^{\Gamma}(r^4(Tf_T - f) + 2Q^2)}{2r^4R^4 f_T(\acute{B}^2 + 3R^2)(\acute{B}^2 + 2r^2)}\right]. \tag{83}$$

where C_1 is the constant of integration also the expressions of $H(r)$ with $G(r)$ are given in the appendix. Observe that in equations (19)–(21), the temporal component ω only occurs as functions of its derivatives, in contrast to the radial metric λ. In this regard, it is not essential to acquire the precise formation of the temporal contorsion h by equation (65) in order to identify the origin of the metric in equation (26). The elementary form's continuity yields equations (77) and (78) as

$$\acute{A}^2\left(1 + \frac{R^2}{\acute{B}^2}\right)e^{h_R} = 1 - \frac{2\mathbb{M}}{R} + \frac{Q^2}{R^2}, \tag{77}$$

$$e^{-\varpi} + \breve{g}_R = 1 - \frac{2\mathbb{M}}{R} + \frac{Q^2}{R^2}. \tag{78}$$

Now equation (76) takes its final configuration, where $C_1 = 0$ to get regularity at origin. Thus we obtain

$$\breve{g} = \frac{r^2}{3}\left[\left(-\frac{\kappa^2 K}{f_T}\right)^{\frac{\Gamma-1}{\Gamma}}\left(\frac{\acute{C}^2 - \acute{B}^2}{\acute{B}^2\acute{C}^2}\right)^{1/\Gamma} \times\left\{\frac{1}{2f_T}(Tf_T - f) + \frac{1}{f_T}\frac{q^2}{r^4}\right\}^{1/\Gamma} H(r) + G(r)\right] + \frac{1}{3r}\int r^3 G'(r)\mathrm{d}r. \tag{79}$$

However, by applying the requirement in equation (78), the Schwarzschild mass is obtained as under

$$\frac{2\mathbb{M}}{R} = \frac{2M}{R} - \breve{g}_R, \tag{80}$$

where $M = m(R)$ has been employed in equation (27). Finally by using equation (80) in equation (77), we obtain

$$\acute{A}^2\left(1 + \frac{R^2}{\acute{B}^2}\right)e^{h_R} = \frac{(R^2 - Q^2)(\acute{B}^2 + R^2)}{R^2(\acute{B}^2 + 3R^2)} + \breve{g}_R. \tag{81}$$

Now, we determine total radial pressure expression in terms of charge and $f(T)$ field equations as follows

$$p_r^{(\mathrm{tot})} = p_r^{(\mathrm{matt})} + p_r^{(T)} + E_1^1 + \wp_r, \tag{82}$$

The effective radial pressure, as represented by the aforementioned equation, depends on the polytropic index n, the thermal properties (K) of the specified polytrope, electric charge, and $f(T)$ correction terms It illustrates how temperature and charge directly influence the value of effective radial pressure. Similarly, we can evaluate other physical factors, such as energy density, etc, to obtain physically realistic outcomes.

In figures 1 and 2, the effects of the effective radial pressure and $f(T)$ corrections are depicted for the relativistic gravitational object. This explains how $p_r^{(\mathrm{tot})}$ fluctuates directly with the polytropic thermal properties K, varying in response to changes in charge values while exhibiting an inverse relationship with the polytropic parameter n. The total

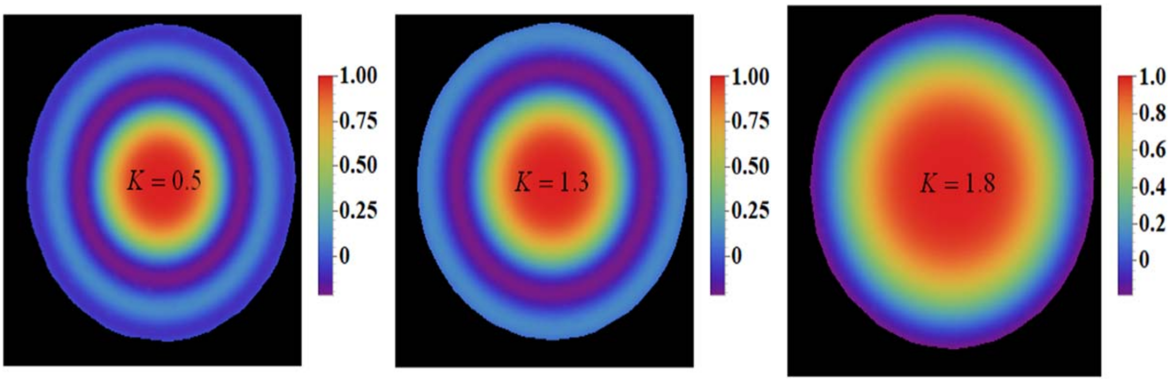

Figure 1. Radial pressure $p_r^{(\text{tot})}$ for three separate scenarios, demonstrating the impact of the polytrope. We take $n = 3$.

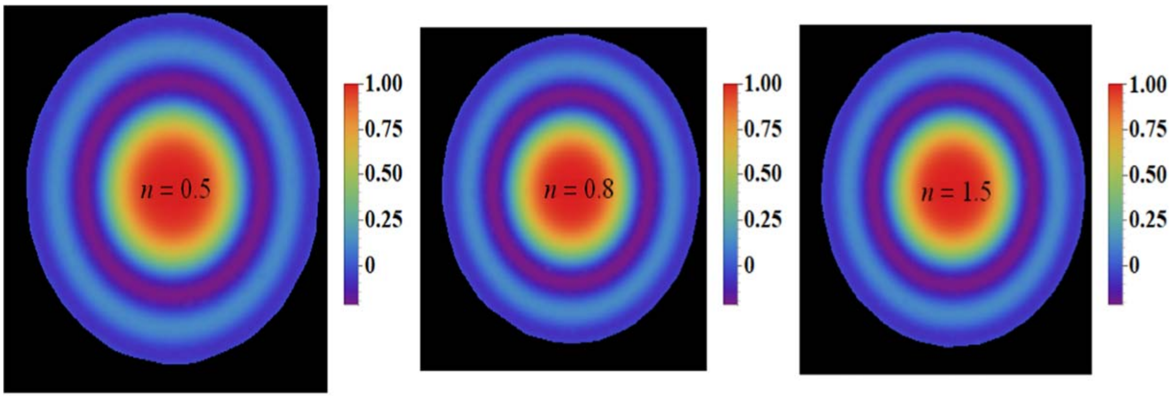

Figure 2. For K = 0.01, radial pressure $p_r^{(\text{tot})}$.

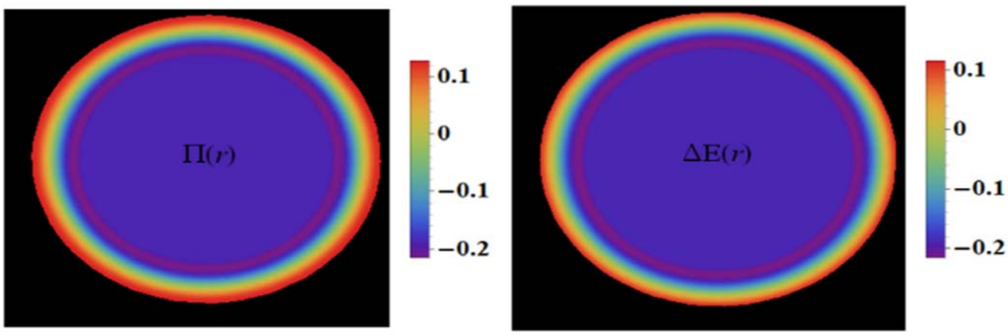

Figure 3. $\prod(r)$ and ΔE graphs for $K = 0.02$ and $K = 0.05$ while keeping the value of n constant i.e $n = 2$.

radial pressure at multiple points within a stellar object is illustrated in figures 1 and 2 at various levels. The figures demonstrate that the effective radial pressure $p_r^{(\text{tot})}$ within stellar objects is not uniform and decreases as we approach the stellar surface. Anisotropy is generated within the system due to the varying levels of effective pressure, as is evident from figure 3.

Also, for various values of K, the gradient of energy is shown in figure 3. The graph illustrates that, while keeping the value of n constant, the rate at which energy is exchanged between fluids inversely correlates with temperature. As a result, it is clear that the stellar object's surface is where energy exchange is most active.

6. Conclusion

It is worthy of mention that many scholars are interested in seeking solutions to the anisotropic self-gravitating systems. Finding definite solutions for the inner composition of star bodies has recently become significantly accessible due to the minimal GD approach. In the framework of $f(T)$ theory, we have investigated precise solutions from well-known isotropic models for the charged anisotropic field equations. In section 2, we investigated the characteristics of matter, which are static and spherically symmetric, in the existence of EMF together with $f(T)$ gravity. To accomplish this, an additional source θ_ϵ^ϕ is included in the charged isotropic EMT such that

$(T_{\epsilon}^{\phi(m)} + T_{\epsilon}^{\phi(T)} + E_{\epsilon}^{\phi}) \longrightarrow (T_{\epsilon}^{\phi(m)} + T_{\epsilon}^{\phi(T)} + E_{\epsilon}^{\phi} + \theta_{\epsilon}^{\phi})$, which results in the requisite field equations with an anisotropic distribution of matter. Then, for radial and temporal metric functions of metric, we established a geometric deformation $\{\breve{g}, \mathsf{h}\}$, respectively. The field equations in two sets result from this deformation: One is in line with the new source θ_{ϵ}^{ϕ} and the deformed metric coefficient, while the other is equivalent to the typical Einstein equations for isotropically charged sources that are $(T_{\epsilon}^{\phi(m)} + T_{\epsilon}^{\phi(T)} + E_{\epsilon}^{\phi})$. We derive the conclusion that the GD can successfully decouple the two sources $(T_{\epsilon}^{\phi(m)} + T_{\epsilon}^{\phi(T)} + E_{\epsilon}^{\phi})$ and θ_{ϵ}^{ϕ}. In section 3, with the use of Darmois constraints calculated in [64], we combined exterior geometry regarded as Reissner–Nordstr öm with interior geometry accounted for static spherically symmetric and evaluated their physical characteristics. As a consequence, we get an equation for radial pressure, which involves charge and a few field-induced dark source components.

In section 4, in a summary of $f(T)$ theory, the equation of state was utilized to figure out the energy exchange between fluids. For the deformation $\breve{g}$, we obtain a first-order nonlinear differential equation with two unknowns. Therefore, we used some limitations on the radial pressure and density, respectively, to solve this equation. We found that the energy density mimic limitation is more appropriate than the radial pressure counterpart since it results in an easy-to-solve linear differential equation. The coupling hindrance on the facet, which is feasibly challenging in a few conditions, is significantly minimized when the condition (53) is used, which is very advantageous. We investigated the transfer of energy among the fluids and found that the energy exchange is minimal in the center and rises as it departs beyond the surface. Additionally, we concluded that the energy transfer is towards the perfect fluid from polytrope. To be more specific, the polytrope's energy gradients are favorable for the perfect fluid, indicating that the polytrope must sacrifice energy to couple with the perfect fluid and be suitable for the exterior solution.

In section 5, in the existence of an electromagnetic field, we utilized a renowned Tolman $\mathbb{IV}$ solution and regarded it as a seed. We noticed the contact of polytropes with perfect fluids. We computed the equation for deformation $\breve{g}$ that took into account charge and a few dark source factors due to field, i.e. $f(T)$. We may assess the equation for the deformation h in a similar manner. Following that, we used the junction condition to obtain the exterior mass expression. The expression of radial pressure in terms of charge and a few dark source terms was finally computed. The equation for tangential pressure may also be found to determine anisotropy; however, it is too massive to present. As we can see, the cooperation between the two fluids is enhanced significantly close to the stellar boundary, and an energy gradient in the direction of the radius is present. This may be reckoned as the role required by polytrope to maintain the ideal fluid inside the stellar dimension. In order to draw a conclusion, we must indicate that every region inside the star distribution satisfies the strong energy criteria. In order to visualize our findings, we also employed graphics. In figures 1 and 2, the physical meaning of effective pressure is shown in relation to various charge ranges. We note that while the rise in charge causes a drop in the value of effective pressure, the pressure is at its highest value near the core of the star ($r = 0$) and progressively falls on the way to the surface. The rate of fluid energy exchange inside a compact object is seen in figure 3. The gravitational effects between two relativistic fluids become more significant with a drop in charge as we circle the spherical structure's center more and farther. The increase in the energy transfer between the fluid's positive values is the reason behind this. This served as more proof that even in the presence of charge and under $f(T)$ gravity, the polytrope strives to maintain the ideal fluid in the framework of a spherical figure. Therefore, the criteria for equilibrium and constancy of compact objects are improved by the existence of anisotropy and static electric field [65]. certainly, as each of these components opposes the effects of gravity. This feature prevents a spherical and symmetric distribution of matter from collapsing into a point singularity in the course of a gravitational crash. If we take $Q = 0$ all outcomes will reduce to [66]. All of our results reduce to GR [67] under certain limits.

Gravitational decoupling is a straightforward but effective methodology for producing anisotropic solutions via simple ideal fluid seed metrics. We discovered that the torsion scalar's extra contributions modify the effective pressure, distribution of energy, and thermal parameters of the star object. Unlike in GR, wherein the gravitational field is simply impacted by space-time curvature, $f(T)$ gravity incorporates fluctuations owing to torsion, possibly resulting in anisotropic pressures regardless of whether the initial matter composition is isotropic. These modifications generate multiple density profiles, energy exchange mechanisms, and stability criteria, influencing the star's formation and behavior. The value of the deformation factor in our work indicates a complicated interaction of enhanced and diminished gravitational effects across areas, resulting in unique observable phenomena not expected by GR. The gravitational decoupling approach presents a viable direction for further investigation in the framework of $f(T)$ theory. By utilizing various seed solutions or deformation functions, the method can be used to produce new solutions in $f(T)$ theory, such as anisotropic solutions or solutions with particular energy-momentum tensors. Additionally, it is worthwhile to investigate the consequences of gravitational decoupling in $f(T)$ theory for cosmological scenarios, including the early universe or epochs dominated by dark energy. This could entail researching how decoupling affects dark energy behavior and the Universe's evolution. The behavior of gravitational decoupling in $f(T)$ theory can also be explored by computational simulations, especially in cases where analytical solutions are hard to come by.

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Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Appendix

The following are the $f(T)$ terms that result from equations (41) and (69).

$$G(r)=\frac{f_{TT}}{2f_T}\left[\breve{g}\left(\frac{\omega'}{4}-\frac{2}{r}\right)+\frac{e^{\varpi}}{4e^{-\varpi}+\breve{g}}\times\left\{(K^{\Gamma}e^{-\varpi}-\breve{g})\left(\zeta'+\frac{1}{r}\right)+\frac{rK^{\Gamma}}{2f_T}\left(Tf_T-f+\frac{2q^2}{r^4}\right)-\frac{K^{\Gamma}}{r}\right\}T'\right],$$

$$H(r)=\left[1-\frac{(\acute{B}^2+2\acute{C}^2)r^2}{(\acute{C}^2-\acute{B}^2)(\acute{B}^2+2r^2)}\right]^{1/\Gamma}\times\left(\frac{2f_T r^4}{r^4(Tf_T-f)+2q^2}+\frac{(\acute{B}^2+2r^2)\acute{C}^2}{\acute{C}^2-\acute{B}^2-3r^2}\right)^{1/\Gamma}-\frac{1}{r^3}\left(\frac{2f_T r^4}{r^4(Tf_T-f)+2q^2}\right)^{1/\Gamma}\times\int r^3\frac{d}{dr}\left\{\left(1-\frac{(\acute{B}^2+2\acute{C}^2)r^2}{(\acute{C}^2-\acute{B}^2)(\acute{B}^2+2r^2)}\right)\times\left(\frac{(\acute{B}^2+2r^2)\acute{C}^2}{\acute{C}^2-\acute{B}^2-3r^2}\times\frac{r^4(Tf_T-f)+2q^2}{2r^4f_T}\right)^{1/\Gamma}\right\},$$

$$\Phi^{(T)}=(Tf_T-f)'+\frac{\omega' f_{TT}}{2}e^{-\lambda}\left(\frac{\omega'}{4}-\frac{2}{r}\right)T'.$$

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