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Non-perturbative quantum dynamics on embedded submanifolds: from geometric mass to Higgs potentials

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Abstract

We establish a quantum dynamics framework for curved submanifolds embedded in higher-dimensional spaces. Through rigorous dimensional reduction, we derive the first complete Schrödinger and Klein–Gordon equations incorporating non-perturbative geometric interactions-resolving ambiguities in constrained quantization. Crucially, extrinsic curvature of the ambient manifold governs emergent low-dimensional quantum phenomena. Remarkably, this mechanism generates scalar field masses matching Kaluza–Klein spectra while eliminating periodic compactification Requirements. We hypothesize that geometric induction can produce Higgs-mechanism-type potentials. Under this working hypothesis, particle masses arise solely from submanifold embedding geometry and matter–field couplings are encoded in curvature invariants. If this hypothesis holds, it would enable experimental access to higher-dimensional physics at all energy scales through geometric induction. We also discuss the Higgs vacuum near small-mass black holes.

Keywords: geometry, Higgs potential, dimensional reduction, mass

1. Introduction

The role of extra spacetime dimensions in submanifold physics remains fundamental. Kaluza's five-dimensional unification of gravity and electromagnetism [1] was followed by Klein's Planck-scale compactification [2]. The field transformed when Randall and Sundrum demonstrated that warped compactification of a small extra dimension could naturally generate the electroweak-Planck scale hierarchy through exponential suppression of mass scales [3]. These advances, along with other key developments reviewed in [4], forming the foundation of modern extra-dimensional theories. This compels a critical question: what physics persists without period compactication of extra dimensions?

Substantial theoretical efforts focus on quantum dynamics in two-dimensionally confined systems, exploring emergent manifestations of three-dimensional physics under dimensional reduction. Historically, two distinct methodologies address curved quantum systems: the intrinsic quantization approach, where the equation of motion depends solely on the intrinsic geometry of the curved manifold independent of the embedding space [5], and the confining potential approach, pioneered by da Costa through reduction of the three-dimensional Schrödinger equation to an effective two-dimensional form via normal potential confinement [6]. Subsequent work generalized this framework to n-dimensional manifolds embedded in m-dimensional Euclidean spaces [7].

To address the fundamental limitation wherein existing approaches fail to systematically characterize the influence of arbitrarily high-dimensional manifolds on physics confined to submanifolds—relying instead on isolated case-specific calculations—we propose establishing a comprehensive theoretical paradigm. This framework commences with scalar field theories embedded within generic higher-

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dimensional manifolds governing constrained low-dimensional physics.

To elucidate the influence of higher-dimensional manifolds on lower-dimensional constrained physics, a new paradigm must be established. In this Letter, we introduce a novel approach for establishing generalized connections between submanifolds and their ambient manifolds. Starting from the m-dimensional Schrödinger equation in a tubular neighborhood, we reduce the additional dimensions to derive the effective equation on the submanifold. This formalism naturally extends to the Klein–Gordon equation. Distinct from earlier notable contributions, our approach differs from [3, 8] in that it does not require any periodic compactification condition, imposes no restriction on the overall scale of the ambient manifold, and enforces a hard constraint whereby fields (particles) are defined only on the submanifold while still capturing the physical effects induced by the additional dimensions through its tubular neighborhood. Compared with [6, 7], our model does not restrict the ambient manifold to be Euclidean and explicitly treats non-Euclidean ambient geometries not analyzed there. The resulting mathematical derivation is correspondingly more transparent, because we treat the manifold-field system holistically and thereby avoid the artificial separation of degrees of freedom that typically arises in constrained dynamics. We conduct stability analysis for the coupled manifold-matter dynamics in the non-relativistic limit within higher-dimensional ambient spaces. To illustrate the physical implications, we examine two representative scenarios: one where geometric contributions give rise to an effective mass term for a real scalar field, and another demonstrating a geometrically-induced Higgs-like potential that may induce spontaneous symmetry breaking. We conclude by outlining promising extensions of this methodology.

When applied to the vicinity of a simple black hole model, our model demonstrates two key phenomena: (i) complete Higgs vacuum stability under extreme curvature near Planck-mass black hole horizons; (ii) Higgs mass deviations from the Standard Model near low-mass primordial black holes ($M \lesssim 10^{11}$ GeV; $\delta M_h > 1\%$). These emerge naturally from our geometric symmetry-breaking formulation without compactification.

2. Geometry of embedded submanifold

Primarily, it is imperative to establish a rigorous geometric characterization of the system. Considering a Riemannian manifold (M, g) with an embedded submanifold $N \subset M$ where $\dim M = m$, $\dim N = n$. The tubular neighborhood is defined as [9]:

$$U_\epsilon = \{\exp_p(y\nu) | p \in N, \|\nu\|_g < \epsilon\}.$$

In normal coordinates $(x^1, \ldots, x^n, y^1, \ldots, y^{m-n})$:

- x^i: Coordinates on N
- y^α: Normal coordinates $(\nu = y^\alpha \partial_\alpha)$

The metric tensor decomposes as:

$$ds^2 = g_{ij}(x, y)dx^i dx^j + 2g_{i\alpha}(x, y)dx^i dt^\alpha + g_{\alpha\beta}(x, y)dy^\alpha dy^\beta,$$

in normal coordinates (geodesics orthogonal to N):

$$g_{i\alpha}(x, 0) = 0 \quad \text{(orthogonality)};$$
$$g_{\alpha\beta}(x, 0) = \delta_{\alpha\beta} \quad \text{(unit normal vectors)};$$
$$g_{ij}(x, 0) = h_{ij}(x) \quad \text{(induced metric on} N\text{)},$$

along normal geodesics $\gamma(s) = \exp_p(s\nu)$, expand metric components [10]:

$$g_{AB}(y) = g_{AB}(0) + y^\gamma \partial_\gamma g_{AB}(0) + \frac{1}{2} y^\gamma y^\delta \partial_\gamma \partial_\delta g_{AB}(0) + \mathcal{O}(\|y\|^3), \tag{1}$$

First derivatives:

$$\partial_\gamma g_{ij}(0) = -2\,\mathrm{II}_{ij}^\gamma;$$
$$\partial_\gamma g_{i\alpha}(0) = 0;$$
$$\partial_\gamma g_{\alpha\beta}(0) = 0, \tag{2}$$

where $\mathrm{II}_{ij}^\gamma = (\nabla_{\partial_i}\partial_j, \partial_\gamma)$ is the second fundamental form.

Second derivatives:

$$\partial_\gamma \partial_\delta g_{ij}(0) = -2R_{\gamma i\delta j} - 2\sum_{k,l} h^{kl}\mathrm{II}_{ik}^\gamma \mathrm{II}_{jl}^\delta + \mathcal{O}(R_N), \tag{3}$$

where $R_{\gamma i\gamma j} = R(\partial_\gamma, \partial_i, \partial_\delta, \partial_j)$ is the Riemann curvature tensor. When we consider the process of constrained particles into a submanifold by normal potential, the area of particle's motion is reduced from the space that composed of submanifolds and their small neighborhoods of the normal space to the submanifold itself: $T_pM = T_pN \oplus N_pN \Rightarrow T_pN$. Actually, the space constructed by the direct sum of submanifolds and its normal space neighborhoods is so-called a tubular neighborhood. We start with the Schrödinger equation in a tubular neighborhood:

$$i\hbar\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2}{2m}\Delta_g + V_t\right)\psi, \tag{4}$$

where V_t denotes the internal potential within the submanifold, and the Laplace–Beltrami operator is defined as:

$$\Delta_g\psi = \frac{1}{\sqrt{|g|}}\partial_A(\sqrt{|g|}\,g^{AB}\partial_B\psi). \tag{5}$$

Decompose into tangential (i, j) and normal (α, β) parts:

$$\Delta_g = \Delta_y + \Delta_x + \Delta_{\text{mix}}. \tag{6}$$

Normal part:

$$\Delta_y = \frac{1}{\sqrt{\det g}}\partial_\alpha(\sqrt{\det g}\,g^{\alpha\beta}\partial_\beta). \tag{7}$$

Tangential part:

$$\Delta_x = \frac{1}{\sqrt{\det g}}\partial_i(\sqrt{\det g}\,g^{ij}\partial_j). \tag{8}$$

Mixed term:

$$\Delta_{\text{mix}} = 2\frac{1}{\sqrt{\det g}}\partial_i(\sqrt{\det g}\,g^{i\alpha}\partial_\alpha) = 0. \tag{9}$$

3. Scalar particles on submanifold

In order to obtain the effective equation on the submanifold, we need to find the relationship of wave function between the neighborhood and the submanifold. We denote the wave function on a parallel surface with $U_\lambda = \{\exp_p(y\nu) | p \in N, \|\nu\|_g = \lambda < \epsilon\}$ in the tubular neighborhood by $\psi = \psi(x^1, \ldots, x^n, y^1 = \lambda, \ldots, y^{m-n} = \lambda)$. There is only a difference in measurement between parallel surfaces and submanifold. We introduce the volume expansion factor $\theta(x, y)$ defined by

$$\theta(x, y) = \frac{\sqrt{\det g(x, y)}}{\sqrt{\det h(x)}}, \tag{10}$$

and logarithmic derivative

$$\frac{\partial}{\partial y^\alpha} \ln \theta = \frac{1}{2} g^{AB} \partial_\alpha g_{AB}. \tag{11}$$

The tubular neighborhood region may be regarded as constituted by infinitely many parallel surfaces, where the propagation characteristics of the wavefunction between adjacent parallel surfaces are governed by the metric. We can obtain the relationship of wave functions between two parallel surface and submanifold by chain rule:

$$\int \sqrt{\det h(x)}\, \psi^2(x, 0)\, \mathrm{d}x = \int \sqrt{\det g(x, y)}\, \psi^2(x, y)\, \mathrm{d}x, \\ \theta^{-1/2}(x, y)\psi(x, 0) = \psi(x, y), \tag{12}$$

introducing equation (1) to (11), we get

$$\ln \theta(y) = -y^\alpha H_\alpha - \frac{y^\alpha y^\beta}{2} \times (K_{\alpha\beta} + \mathrm{Ric}_M(\partial_\alpha, \partial_\beta)) + \mathcal{O}(\|y\|^3), \tag{13}$$

where

- $H_\alpha = h^{ij} \mathrm{II}^\alpha_{ij}$ (mean curvature)
- $K_{\alpha\beta} = \sum_{i,j,k,l} h^{ik} h^{jl} \mathrm{II}^\alpha_{ij} \mathrm{II}^\beta_{kl}$ (squared norm of second fundamental form)
- $\mathrm{Ric}_M(\partial_\alpha, \partial_\beta)$ (Ricci curvature in normal directions).

By introducing equations (12), (13) and (4) and taking the limit of $y \to 0$, we find that the normal part of Laplace is:

$$\Delta_y(\theta^{-1/2}\psi(x, 0)) = \frac{1}{\sqrt{\det g}} \partial_\alpha\left(\sqrt{\det g}\ g^{\alpha\beta}\ \partial_\beta(\theta^{-1/2}\psi(x, 0))\right) \\ = \left(-\frac{1}{4}\|H\|^2 + \frac{1}{2}\|\mathrm{II}\|^2 + \frac{1}{2}\sum_{\alpha=1}^{m-n} \mathrm{Ric}_M(\nu_\alpha, \nu_\alpha)\right)\psi \\ + \left(\sum_{\alpha=1}^{m-n} \frac{\partial^2}{\partial (y^\alpha)^2} + \mathcal{O}(\|y\|)\right)\psi, \tag{14}$$

with

- $\|H\|^2 = \sum_\alpha H_\alpha^2$ (squared norm of the mean curvature vector)
- $\|\mathrm{II}\|^2 = \sum_{\alpha,\beta} K_{\alpha\beta}$ (squared Frobenius norm of the second fundamental form)

Note on notation. This work employs the geometric analysis convention for the mean curvature vector $\vec{H} := \mathrm{tr}_g(\mathbf{II})$, omitting the common $1/n$ normalization factor. Definitions in other sources may consequently differ by this dimensional factor.

We can express the tangent effective Schrödinger equation on submanifold explicit by degenerate y^α

$$\mathrm{i}\hbar \frac{\partial \psi(x)}{\partial t} = -\frac{\hbar^2}{2m} \frac{1}{\sqrt{\det g}} \partial_i \sqrt{\det g}\, g^{ij} \partial_j \psi + V_{\mathrm{eff}}\psi + V_t\psi; \\ V_{\mathrm{eff}} = \frac{\hbar^2}{2m}\left(\frac{1}{4}\|H\|^2 - \frac{1}{2}\|\mathrm{II}\|^2 - \frac{1}{2}\sum_{1}^{m-n} \mathrm{Ric}_M(\nu, \nu)\right). \tag{15}$$

The only difference between this equation and the intrinsic Schrödinger equation on the submanifold is V_{eff}, which induced by the geometry.

We now turn to the relativistic regime, where scalar particles are governed by the Klein–Gordon equation. Given the structural similarity to the Schrödinger equation—particularly the Laplacian-dominated dynamics—we bypass explicit derivation and present directly the effective Klein–Gordon equation on the submanifold.

We can also express the effective Klein–Gordon equation through very similar analysis:

$$\frac{1}{\sqrt{-g}} \partial_i(\sqrt{-g}\, g^{ij} \partial_j \phi) - K_{\mathrm{eff}}\phi - m^2\phi = 0, \\ K_{\mathrm{eff}} = \left(\frac{1}{4}\|H\|^2 - \frac{1}{2}\|\mathrm{II}\|^2 - \frac{1}{2}\sum_{1}^{m-n} \mathrm{Ric}_M(\nu, \nu)\right). \tag{16}$$

The first two terms in V_{eff} and K_{eff} depends not only on the geometry of submanifold N but also on the geometry of main manifold M. The last item is the component of the Ricci curvature tensor of M. This has profound physical significance, which we will discuss later. Before that, let us consider the special case calculated by Costa *et al*, When we choose $M = \mathbb{R}^3$, $n = 1, 2$,

$$V_{\mathrm{eff}} = \begin{cases} -\dfrac{\hbar^2}{8m}\kappa^2 & (n{=}1) \\ -\dfrac{\hbar^2}{8m}(\kappa_1^2 - \kappa_2^2) & (n{=}2), \end{cases} \tag{17}$$

where κ_i is the ith principal curvature of manifold N, which is similar with costa [6], cause of $\mathrm{Ric}_M(\nu, \nu) = 0$ in $\mathbb{R}^n$. When we choose $M = \mathbb{R}^n$, $n = 1, 2, \ldots n - 1$, V_{eff} is similar with jaffe [7]. Despite the regression results being consistent in Euclidean space, our approach exhibits a fundamental conceptual distinction from the aforementioned work. Whereas previous methods rely on the normalization conditions of bulk and surface wave functions, our derivation is underpinned by the measure-theoretic relationship (volume element) between parallel hypersurfaces within a tubular neighborhood. This framework inherently circumvents the issue of differing integration measure dimensionalities. The agreement with prior results in special cases further validates the reliability of our method. Next, we will discuss the possible physical significance of this effective potential. We will demonstrate that this potential arises from the interaction between the matter field and the manifold, rather than being a mere scalar energy correction. In the preceding derivation, we implicitly assumed that the submanifold can be continuously deformed to parallel surfaces within a tubular neighborhood. The interaction between the submanifold and matter fields depends on the submanifold's curvature, embedding method, and the geometry of the ambient manifold. However, the manifold itself is not a physical matter field and thus eludes rigorous definition of energy.

4. Geometrical effect

4.1. Non-relativistic system stability

In the non-relativistic limit, geometric effects are expressed by the particle-manifold interaction energy. We adopt a pragmatic approach: rather than addressing the energy of the manifold separately, stability of the system is analyzed exclusively through the matter-manifold interaction term. Initiate the analysis from trivial instances, we explicitly specify that the ambient manifold possesses vanishing Ricci curvature $\mathrm{Ric}_M(\nu, \nu) = 0$. Let us first consider this system from the perspective of manifolds. We define normal coupling intensity as

$$f = \theta^{-1}\psi^2(x)V_{\mathrm{eff}}, \tag{18}$$

while the first derivative and second derivative are determined by the following equations by using equation (13),

$$\frac{\partial f}{\partial y^\alpha}|_{y=0} = \mathcal{O}\langle H(N), \nu_\alpha\rangle V_{\mathrm{eff}}, \tag{19}$$

$$\frac{\partial^2 f}{\partial (y^\alpha)^2}|_{y=0} = \mathcal{O}(\langle H(N), \nu_\alpha\rangle^2 + \|\mathrm{II}^\alpha\|^2 + \mathrm{Ric}_M(\nu, \nu))V_{\mathrm{eff}}. \tag{20}$$

This term characterizes the evolutionary tendency of the manifold-matter system along the normal direction in the ambient manifold. When negative, it indicates a propensity for normal-direction variations. At $\langle H(N), \nu_\alpha\rangle = 0$, the submanifold corresponds to a minimal surface in the main manifold, characterizing an extremum of manifold-field coupling. The sign of V_{eff} determines the stability of this extremum, thus providing a stability criterion for the submanifold-matter system.

Specifically, $V_{\mathrm{eff}} = -\frac{1}{2}\|\mathrm{II}\|^2 \leqslant 0$ implies the system occupies an unstable critical point, spontaneously evolving toward vanishing-curvature configurations to achieve geometric flatness—exemplified by Helicoid surfaces in $\mathbb{R}^3$.

Remarkably, when $\langle H(N), \nu_\alpha\rangle \neq 0$, stability persists if $V_{\mathrm{eff}} = 0$. This reveals a matter-submanifold interaction that maintains structural stability even for non-minimal submanifolds, as demonstrated by $\mathbb{S}^2$ (with radius R) embedded in $\mathbb{R}^3$.which satisfy $\|H\|^2 = \frac{4}{R^2}$, $\|\mathrm{II}\|^2 = \frac{2}{R^2}$.

Based on the preceding analysis, a direct consequence is that for deformable two-dimensional electronic systems with elliptical geometry embedded in Euclidean space, spontaneous evolution toward a spherical configuration is energetically favorable. We propose that the curvature of extra dimensions and the influence of different embedding methods on lower-dimensional electron dynamics may be observed through quantum simulation approaches analogous to those employed in paper [11, 12].

4.2. Quantum mass dependent on additional dimensions

Let us consider a more general manifold M with $\mathrm{Ric}_M(\nu, \nu) \neq 0$, and $V_{\mathrm{eff}} \neq 0$. It can be shown that when the mean curvature vector is zero and the normal projection of the Ricci curvature of M is positive, the particle-submanifold coupling admits no stable critical points, cause of that equation (20) become:

$$\frac{\partial^2 f}{\partial y^2} = -\frac{1}{2}\left(\|\mathrm{II}\|^2 + \sum_1^{m-n}\mathrm{Ric}_M(\nu, \nu)\right)^2\mathcal{O} \leqslant 0. \tag{21}$$

This relation demonstrates that stable configurations of the system exist only when the Ricci curvature of the ambient manifold equals the negative squared norm of the second fundamental form of the embedded submanifold. Crucially, both quantities are intrinsically determined by the structure of the ambient manifold and the embedding characteristics of the submanifold. Such a constraint effectively introduces novel modes of stability. An example is hyperbolic plane Σ^2 embedded in $\mathbb{H}^3$ which led:

$$\mathrm{Ric} = -2g = -2, \ \|\mathrm{II}\|^2 = 2.$$

We conclude by examining the case where both the mean curvature $\langle H(N), \nu_\alpha\rangle$ and the second fundamental form modulus$\|\mathrm{II}^\alpha\|^2$ vanish, indicating a totally geodesic submanifold. For such configurations to exhibit stable criticality, the ambient manifold must be Ricci-flat. A well-established exemplar of this principle is the Calabi–Yau manifold [13], within whose framework we have demonstrated-to a certain extent-the stability of embedded submanifold-matter field systems. This analysis illuminates the profound implications of manifold-matter interactions and provides critical insights into how higher-dimensional geometries govern lower-dimensional physical phenomena. Subsequent investigations will further elucidate the emergent physical consequences through concrete applications. Following the discussion of stability conditions, we now examine a specific case study to illuminate the profound physical implications embedded in our methodology-particularly regarding the substantive physical influence exerted by higher-dimensional manifolds on their submanifolds. Without loss of generality and neglecting gravitational fields, we set $\mathbb{N}$ to be $\mathbb{R}^3$—a model of the physical universe—and impose $\mathbb{M} = \mathbb{R}^3 \times \mathbb{CH}^k$, which constitutes a canonical embedding wherein is a totally geodesic submanifold of $\mathbb{R}^3 \times \mathbb{CH}^k$, satisfying $\|\mathrm{II}\|^2 = 0$. which $\mathbb{CH}^k$ is k dimensional complex Hyperbolic Space, We substitute these specified conditions into the Klein–Gordon equation (16).

$$\mathrm{Ric}_{\mathbb{C}H^k}(\nu, \nu) = -\frac{2(k+1)}{r^2}.$$
$$K_{\mathrm{eff}} = \frac{k(k+1)}{r^2}, \tag{22}$$

where r is radius of curvature of complex Hyperbolic Space.

Let

$$m_g = \sqrt{K_{\text{eff}}} = \frac{\sqrt{k(k+1)}}{r}. \quad (23)$$

Our theoretical framework demonstrates that $k = 0$ (corresponding to a massless scalar field) dynamically generates mass in excited states, scaling inversely with the curvature radius of uncompactified extra dimensions. Critically, the absence of compactification constraints admits arbitrary mass scales. This establishes a curvature-driven mass-generation mechanism distinct from Higgs-based approaches in degenerate geometries. Based on our derivation, we obtain a geometry-induced scalar field mass that inversely scales with the radius of curvature and exhibits dimension-dependent behavior. This outcome parallels the core mechanism of Kaluza–Klein reduction [1, 2]. It should be noted that whether compactified or not does not affect the mass generation of scalar fields on a submanifold within the context of a specific extra-dimensional manifold background. The assumption regarding extra dimensions here is a speculative theoretical exploration aimed at illustrating the influence of the outer manifold structure on the submanifold. The significance of this discussion lies in the possibility of inferring potential extra-dimensional structures from physical effects observed on the submanifold. The absence of such mass corrections would imply that the outer manifold does not possess the hypothesized structure. This discussion also implies that, in order to ensure that the compactification of extra dimensions does not generate a mass for the scalar particles, the extra dimensions must be Ricci-flat, such as in the structure of Calabi–Yau manifolds.

The subsequent section's discussion of the Higgs breaking potential then provides candidate extra-dimensional manifold structures consistent with the Standard Model.

4.3. Geometry-induced symmetry breaking

A more intriguing scenario emerges when considering the specific case of $\mathbb{M} = \mathbb{R}^3 \times \mathbb{CP}^1$, with the ricci curvature represent:

$$\text{Ric}_{\mathbb{C}P^1}(\nu, \nu) = \frac{4}{r^2}, \quad (24)$$

where incorporating this result into the Lagrangian of a massless scalar field yields non-trivial physical consequences. The Lagrangian density for a massless scalar field with ϕ^4 interaction is given by:

$$\mathcal{L}_{\mathcal{M}} = \frac{1}{2}(\partial_a\phi)^2 - \lambda\phi^4. \quad (25)$$

Combining equation (24) with analogous derivations, equation (26) assumes the form: massless scalar field with ϕ^4 interaction is given by:

$$\mathcal{L}_{\mathcal{N}} = \frac{1}{2}(\partial_i\phi)^2 + {\mu_0}^2\phi^2 - \lambda\phi^4,$$
$${\mu_0}^2 = \frac{2}{r^2};\ V_{\text{Higgs}} = -\frac{2}{r^2}\phi^2 + \lambda\phi^4. \quad (26)$$

We show that geometric induction provides a mathematically natural pathway to spontaneous symmetry breaking, where the extrinsic curvature of spacetime intrinsically generates the required negative mass-squared term—eliminating the need for ad hoc potentials. This geometric perspective offers a plausible origin for the Higgs potential, presenting the mechanism in a more intuitive light: the negative term may arise from the structure of a higher-dimensional spacetime [14–17]. This extra-dimensional structure is not claimed to be the sole geometric route to an induced Higgs potential. Its compatibility with observed physics, and the possibility of better-suited alternatives, demand further investigation. Assessing its effects on gravity and spinor fields is a lengthy, complex endeavor that we postpone to future work.

4.4. Higgs vacuum near low-mass black hole horizons

Examining the Higgs field near the event horizon of a low-mass black hole modeled as an embedded surface in four-dimensional Minkowski spacetime, the Higgs potential in the small-mass limit takes the functional form derived in our approach. This result incorporates the geometric embedding of the Schwarzschild event horizon and its spacelike hypersurfaces (defined by constant Schwarzschild time t) within the ambient spacetime. The complete derivation is given in appendix. We derive the explicit expression for the Higgs potential:

$$V_\phi = -\frac{2}{r^2}\phi^2 - \frac{1}{r_s^2}\phi^2 + \lambda\phi^4, \quad (27)$$

where $r_s = 2GM$ is the Schwarzschild radius. The physical significance of this result lies in the fact that when the black hole mass $M < 10^{11}$ GeV, it induces a significant correction (greater than 1%) to the Higgs mass. Consequently, a signature exceeding the standard Higgs mass becomes observable in the vicinity of the horizon of such low-mass black holes.

Furthermore, by incorporating the analysis of metastable vacuum decay, the running coupling constant is given by the following expression [18]:

$$\lambda_{\text{eff}} = g(\Lambda_\phi)\left\{\left(\ln\frac{\phi}{M_{\text{p}}}\right)^4 - \left(\ln\frac{\Lambda_\phi}{M_{\text{p}}}\right)^4\right\}, \quad (28)$$

where λ_{eff} represents the effective coupling constant, $g(\Lambda_\phi)$ is a function of Λ_ϕ, ϕ and Λ_ϕ are physical quantities, and M_{p} denotes the Planck mass.

When the black hole mass approaches the Planck mass, the corrected mass in equation (27) $\frac{1}{r_s} \propto M_{\text{P}}$. This correction stabilizes the vacuum at the horizon, and equation (28) confirms that $\lambda_{\text{eff}} > 0$. Consequently, Planck-scale micro black holes cannot trigger Higgs field decay to form decay bubbles.

5. Conclusion

In our model, extra dimensions fundamentally manifest as latent dynamical degrees of freedom for the scalar field. The influence of higher-dimensional spacetime becomes encoded in the interactions between the submanifold (i.e. the 4D spacetime or 3D spacelike hypersurface) and the scalar field. We contend these results reveal physically significant insights.

We emphasize the conceptual foundation of our model: a freely embedded submanifold within a higher-dimensional manifold induces constraints on field dynamics confined to the submanifold. This emerges intrinsically through interactions along the normal directions—not necessarily via an explicit normal-direction potential.

Employing differential geometry, we derive the equations of motion governing scalar fields confined to an n-dimensional embedded submanifold within a higher-dimensional ambient manifold. This intrinsically coupled manifold-field framework exhibits stability against perturbations in the extrinsic curvature, revealing how the ambient geometry governs submanifold physics. Crucially, without imposing compactification conditions, the analysis demonstrates the geometric generation of a scalar field mass term arising from the ambient curvature.

Our model proposes a mechanism for spontaneous symmetry breaking mediated by a geometrically induced Higgs-like potential, offering valuable insights into the physical implications of extra-dimensional models while preserving theoretical elegance. In our model, the ambient manifold responsible for inducing both the geometric mass and the Higgs potential is introduced purely as a postulate. Because extra dimensions are not directly observable, we consider this hypothetical ansatz methodologically acceptable. We do not claim that it is unique or correct; rather, we propose to evaluate its plausibility through the physical phenomena it implies. For instance, the $\mathbb{CP}^1$ manifold used to generate the Higgs potential should also produce couplings to additional four-dimensional fields beyond the scalar sector. Future work will explore such further consequences to test the construction. The present paper is intended to encourage discussion of observable signatures of extra-dimensional geometry, not to assert the reality of any specific structure. Ultimately, more sophisticated theory and experiment will be required to judge its validity.

We further establish Higgs vacuum stability in the near-horizon spacelike limit of microscopic black holes, proving the absence of vacuum decay near event horizons. Our calculations predict Higgs mass corrections in the vicinity of small-mass black holes—particularly those in terminal evaporation phases.

These results demonstrate the theoretical robustness and broad applicability of our framework. A subsequent publication will address the interplay between non-compact extra dimensions and gauge symmetries. Our framework maintains full compatibility with standard spacetime decompositions including the Arnowitt-Deser-Misner (ADM) formalism, requiring only metric signature adjustments for application to various relativistic scenarios. Future investigations will focus on three key directions: first, examining how higher-dimensional symmetries manifest in low-dimensional effective theories through dimensional reduction; second, exploring the rich geometric structures arising from submanifold-ambient manifold interactions; and third, extending the analysis to vector and spinor fields [19] where new emergent phenomena beyond scalar interactions are anticipated. We will also explore the connections between our model and black hole dynamics [20, 21], as well as gravitational physics [22], with the aim of identifying observable geometric effects.

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Appendix: Schwarzschild horizon

The Schwarzschild metric in advanced null coordinates:

$$ds^2 = -\left(1 - \frac{r_s}{r}\right)dv^2 + 2dvdr + r^2(d\theta^2 + \sin^2\theta\, d\phi^2), \tag{A1}$$

where $r_s = 2GM$ is the Schwarzschild radius. The event horizon is at $r = r_s$.

The horizon $\mathcal{H}$ is a null hypersurface at $r = r_s$ with induced metric:

$$h_{AB} = r_s^2\begin{pmatrix}1 & 0\\ 0 & \sin^2\theta\end{pmatrix}, \quad (A, B = \theta, \phi). \tag{A2}$$

The normal vector $n^\mu = -\nabla^\mu v$ is null on the horizon. The extrinsic curvature is:

$$K_{AB} = \frac{1}{2}\mathcal{L}_n h_{AB} = \frac{1}{2}\partial_r h_{AB}\,|_{r=r_s}. \tag{A3}$$

Substituting $h_{AB} = r^2\overset{\circ}{h}_{AB}$ (unit sphere metric):

$$K_{AB} = r_s\overset{\circ}{h}_{AB} \;\Rightarrow\; K_{\theta\theta} = r_s, \quad K_{\phi\phi} = r_s\sin^2\theta. \tag{A4}$$

The norm is given by the trace:

$$\|K\| = \|\mathrm{II}\| = h^{AB}K_{AB} = \frac{2}{r_s}, \tag{A5}$$

using $h^{AB} = r_s^{-2}\overset{\circ}{h}{}^{AB}$.

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